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A Practical Method for Determining the Pseudo-Rigid-Body Parameters of Spatial Compliant Mechanisms via CAE Tools

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Abstract

Compliant Mechanisms (CMs) are employed in several applications requiring high precision and reduced number of parts. For a given topology, CM analysis and synthesis may be developed resorting to the Pseudo-Rigid Body (PRB) approximation, where flexible members are modelled via a series of spring-loaded revolute joints, thus reducing computational costs during CM simulation. Owing to these considerations, this paper reports about a practical method to determine accurate PRB models of CMs comprising out-of-plane displacements and distributed compliance. The method leverages on the optimization capabilities of modern CAE tools, which provide built-in functions for modelling the motion of flexible members. After the validation of the method on an elementary case study, an industrial CM consisting of a crank mechanism connected to a fully-compliant four-bar linkage is considered. The resulting PRB model, which comprises four spherical joints with generalized springs mounted in parallel, shows performance comparable with the deformable system.

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Keywords: Compliant Mechanisms, Pseudo-Rigid Body Models, Carachteristic Parameters, Flexible Multi-Body Dynamics, CAD/CAE Tools

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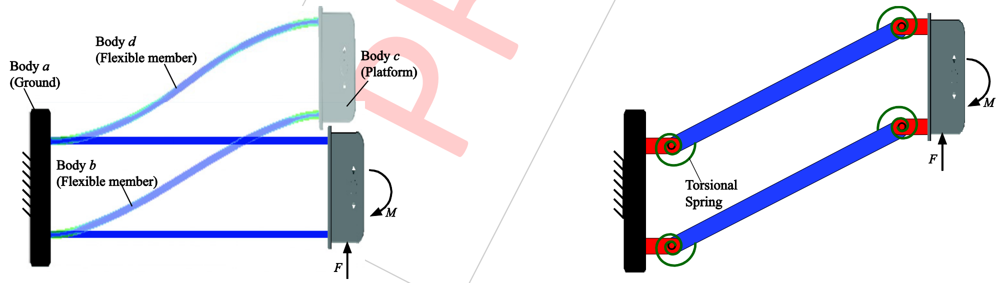
1. Introduction

Differently from rigid-body mechanisms, which transmit motion and force employing traditional kinematic pairs based on conjugate surfaces, Compliant Mechanisms (CMs) gain at least some of their mobility from the deflection of flexible members [1-3]. Potential CM advantages when compared to their rigid-body counterpart can be outlined into two categories, namely *cost reduction* and *increased performance*. For what concerns economic perspectives, CMs require fewer components to achieve the desired mobility (possibly leading to one-piece manufactured solutions) with consequent reduction of assembly time/cost. For what concerns performance improvement, the absence of contacts between rigid surfaces reduces wear, need of lubrication and possible backlash, which might be beneficiary in terms of mechanism precision. Moreover, the development of new materials, production technologies (e.g. additive manufacturing [4]) and new fields of application (e.g. miniaturized assembly [5,6]) largely justifies the increased studies in this area during the last twenty years of research. Nonetheless, aside this numerous advantages, a series of challenging issues must be considered, namely: a) the analysis and design of CMs is more complex when compared to traditional mechanisms; b) continuous rotational motions cannot be achieved by means of flexible members; c) in many applications, CM resistance to fatigue must be carefully addressed.

From a terminology standpoint, also adopted in [7], generic CM may be classified into three categories: *flexure-based* CMs, *flexible beam-based* CMs and *fully compliant elastic continua*. As for flexure-based CMs, they are characterized by compliant structures whose elastic deformation is localized in "small regions", the term *flexural joint* or *flexure* being used to indicate the location at which the deflection is concentrated. Flexural joints are usually obtained by machining one or two cutouts in a blank material with constant width, making it possible to obtain (if desired) monolithic solutions. In parallel, the term flexible beam-based CMs is used whenever the compliance is distributed along slender beam-like segments designed to undergo rather large deformations. These latter solutions may comprise both flexible and rigid links connected with traditional kinematic pairs. At last, a further generalization of the distributed CM concept paves the way to the so-called fully compliant elastic continua, which generically indicate structures of complex shape, that are designed to undergo a desired deformation upon the application of known external loads.

As it may be self-evident, CM design is primarily made difficult by the presence of finite nonlinear deflections of the flexible members. Traditional techniques such as Finite Element Method (FEM) and, where available, analytical solutions, might be very accurate but are far too complicated to be used in either the conceptual design stage or in the industrial scenario, where tools providing ease-of-use and low computational costs are largely needed. In this context, a powerful method for CM analysis/design is the Pseudo-Rigid-Body (PRB) approach. Generically speaking, a PRB model describes a complaint mechanism by a series of rigid links connected through Revolute (R) joints (called *characteristic pivots*) each having a torsional spring with defined stiffness mounted in parallel. For example, Fig. 1 depicts a fully compliant four-bar-linkage along with its corresponding PRB model, which comprises four spring-loaded revolute joints. The PRB parameters are evaluated by means of optimization routines, whose aim is to assess the numerical values of both spring stiffness and pivot locations. In the particular case depicted in Fig.1, for instance, these parameters have been determined so as to minimize the difference in the trajectory of body *c* (platform) in the two cases (i.e. CM and PRB model). As clearly highlighted in [8], PRB techniques have been successfully used for identifying bi-stability [9], evaluating CM workspaces [10], characterizing their dynamic behaviors [11], and comparing compliant joints morphologies [12].



(a) Compliant Four-Bar Linkage (b) Pseudo-Rigid Body Model

Fig. 1. (a) fully compliant four-bar-linkage c; (b) related 4R PRB.

For what concerns possible optimization routines, several techniques have been presented and compared in the literature. As an instance, considering a slender cantilever beams with generic end loads, an interesting PRB consisting of four rigid links joined by three torsional springs (i.e. a 3R PRB) has been proposed in [8,13]. In this case, joint positions have been found by means of a discrete three-dimensional search routine and then used to compute the optimal values of the stiffness coefficients. Owing to the large amount of past works, at present, either analytical or empirical expressions for evaluating the PRB parameters are available for what concerns simple flexible geometries, such as the abovementioned straight beams [2] or flexures characterized by circular, elliptical, parabolic, or hyperbolic profiles [3].

In any case, a careful assessment of the previous literature mainly highlights the following needs:

- most of the past works dealing with PRB techniques take into account CMs subjected to planar motions only. On the other hand, despite their practical relevance, investigations on PRB for spatial compliant mechanisms are instead quite limited;
- the determination of the optimal PRB parameters starts from the knowledge of the load-deflection characteristics of the CM under investigation, which is usually derived resorting to analytical slender-beam models. This approach is accurate but fails to provide useful information for generic flexure geometries;
- in terms of computer-aided design of mechanisms, there has been a number of numerical solvers developed throughout the years (see [14] for a review), comprising nonlinear FEM and Multi-Body Dynamics (MBD) packages. Nonetheless, practical methods, also employable in industry, which take advantage of the built-in capabilities of general-purpose CAE tools when specifically applied for CM analysis and design have been scarcely described and should be furtherly investigated.

In the attempt to overcome the abovementioned limitations, this paper proposes a method for defining the PRB parameters resorting to a commercial CAE package. In particular, all simulations are performed by means of the software *Recurdyn* [15], which allows to analyze flexible members during their motion and provides integrated optimization routines. Initially, a simple case study, based on fixed slender beam with end moment and vertical force loading, is used for comparing this CAE-based method with analytical results taken from [2]. Then, a more complex case is considered, for testing the tool accuracy in case of deformable parts with complex geometry comprising out of plane motions. The considered system consists of a crank mechanism connected to a compliant four bar linkages subjected to both in-plane and out-of-plane loads. This particular case study actually replicates a design solution found in an automatic machine for packaging. The resulting PRB model comprises four spherical joints with three-dimensional rotational springs. A properly defined objective function, based on a kinematical error, along with the *Recurdyn* built-in optimization routine yield the PRB parameters that allow to fully reproduce the trajectory of a point of interest when compared to the actual compliant model.

The rest of this paper is organized as follows: Section 2 briefly presents background theory regarding a specific PRB model; the optimization routine is outlined in Section 3; Section 4 describes the method validation on a theoretical case taken from the literature; Section 5 reports about the method implementation on the abovementioned spatial mechanism, whereas Section 6 provides the concluding remarks.

2. Basic Background on PRB Models for Fixed-Guided Beams

As previously said, several PRB models of increasing complexity (i.e. number of degrees of freedom) have been reported (see [16], for a review). For validation purposes, in this section, a specific case is recalled, namely the modelling of a *fixed-guided flexible* segments, whose schematic layout and set of boundary conditions are depicted in Fig. 2. In particular, one end of the beam is clamped to the ground, whereas the other end is “guided” so as to maintain absence of rotation at all times. In order to obtain this configuration, a resultant moment, M, must be applied at the beam end point in addition to a vertical force, F, the magnitude of the moment being obtained by considering the expressions of the beam end rotation when either a force or a moment load are respectively applied. In particular, the following relations hold:

$$\boldsymbol{\vartheta_P} = \frac{\boldsymbol{Fa^2}}{\boldsymbol{2EJ}} = \frac{\boldsymbol{Ma}}{\boldsymbol{EJ}} = \boldsymbol{\vartheta_M} \implies \boldsymbol{M} = \frac{\boldsymbol{Fa}}{\boldsymbol{2}} \tag{1}$$

where $\boldsymbol{\vartheta_P}$ and $\boldsymbol{\vartheta_M}$ are, respectively, the beam end rotation in case of force or moment loads, $\boldsymbol{E}$ is the Young modulus of the beam material, $\boldsymbol{J}$ is the moment of inertia of the beam cross section, and $\boldsymbol{a}$ is the horizontal distance of the beam free end to the fixed end (see Fig. 2). As said, the application of a vertical force requires the simultaneous application of a clockwise moment whose magnitude is given by Eq. (1). The resulting deflected shape is antisymmetric at its centerline, as shown in Fig. 2a. In this particular situation, a 2R-PRB model, consisting of three rigid links connected with two revolute pairs, is shown in Fig.2b. Due to symmetry, the lateral links (i.e. grounded link and link connected to the end-point) are characterized by the same length. Furthermore, two torsional springs with same stiffness are located over the revolute joints in order to approximate the beam compliance. Therefore, this 2R-PRB model requires two characteristic parameters to describe the kinematic and the force-deflection behavior of the related CM. By employing the same notations suggested in [2], the PRB parameters are indeed the *characteristic radius factor* ($\boldsymbol{\gamma}$) and the *stiffness coefficient* ($\boldsymbol{K_\Theta}$). Within the PRB approximation, the length of the links (i.e. $\boldsymbol{l_1} = \boldsymbol{l_3}$ and $\boldsymbol{l_2}$, see Fig. 2b) and, consequently, the horizontal, $\boldsymbol{a}$, and vertical, $\boldsymbol{b}$, positions of the beam end point (see Fig. 2a) can be defined as function of $\boldsymbol{\gamma}$ as follows:

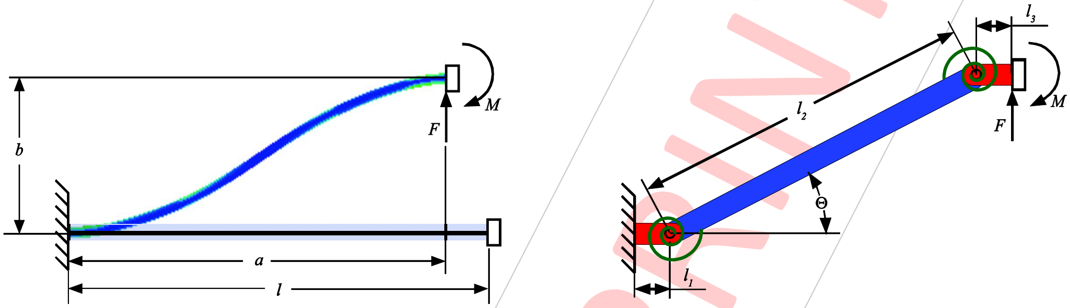
$$\boldsymbol{l_1 = l_3 = 0.5(1-\gamma)l;\ \ l_2 = \gamma l;} \tag{2}$$

$$\boldsymbol{a = l\big(1-\gamma(1-\cos(\Theta))\big);\ \ \ b = \gamma l \sin(\Theta);} \tag{3}$$

In parallel, the stiffness of the torsional springs can be expressed as function of the dimensionless stiffness coefficient as follows:

$$\boldsymbol{K = 2\gamma K_\Theta \frac{EJ}{l}} \tag{4}$$

Both $\boldsymbol{\gamma}$ and $\boldsymbol{K_\Theta}$ can be assessed via empirical equations reported in [2], which have been obtained by means of optimization techniques aiming at providing PRB models which can optimally replicate the beam-end trajectory during deformation.



(a) fixed-guided flexible beam (b) 2R PRB model

Fig. 2. (a) fixed-guided flexible beam (undeflected + deflected configurations); (b) related 2R PRB.

3. Estimation of PRB Parameters through CAE Built-in Optimizer

As previously recalled, the determination of the PRB parameters can be achieved by means of a variety of optimization techniques (see e.g. [8,16]). Generically speaking, a design optimization routine requires the definition of an objective function (performance index), lower and upper bounds for the design variables, and a set of constraints. For what concerns the analysis of CM by means of the PRB method, once the PRB topology is defined, a set of external loads or displacements are applied to both CM (comprising flexible elements), and PRB system (comprising only rigid links and lumped springs). On one hand, the CM can be analysed by means of nonlinear FEM or, if possible, via theoretical methods (such as the Euler-Bernoulli beam theory). In parallel, the PRB system can be analysed resorting to the free-body diagram approach, to the principle of virtual works [2], or by means of MBD tools. Subsequently, in order to set-up an optimization problem, upper and lower limits of both link dimensions and spring stiffness should be decided upon in order to reduce the extent of the design space. The trajectory of one point of interest is then measured on CM and, subsequently, on the PRB system, in order to define a performance index. As a particular case, the optimization problem considered in this paper may be formulated as follows:

$$\text{Minimize}\quad \boldsymbol{e_{tra}(l_i, K_j) = \frac{1}{Q}\sum_{\kappa=1}^{Q}\left\{\sum_{m=1}^{3}\frac{|x_m^C - x_m^P|}{l} + \sum_{n=1}^{3}|\theta_n^C - \theta_n^P|\right\}} \tag{5}$$

$$\text{Subject to} \begin{cases} l_{i,min} \leq l_i \leq l_{i,max} \\ K_{i,min} \leq K_i \leq K_{i,max} \end{cases}$$

where l_i and K_i are the *i*-th link length and spring stiffness, $l_{i,min}$, $l_{i,max}$, $K_{i,min}$, $K_{i,max}$ are the related lower and upper bounds for the design variables, x_m^C, x_m^P, θ_n^C, θ_n^P are, respectively translations and rotations (defined, for instance, via the Euler angles convention) of a properly chosen reference frame related to either the CM (superscript C) or the PRB (superscript P), whereas e_{tra} is a measure of the mean trajectory error (namely, the optimization performance index). Note that the parametric MBD model is built so as to satisfy the condition $\sum_i l_i = l$ at all times, whereas forces/moments and/or displacements are applied in a series of Q simulation steps. The performance index, therefore, represents a mean value of the trajectory error computed on a number Q of static configurations of the mechanism, so that the optimization results depend on the maximum deflections imposed to the CM.

This generic minimization problem can be effectively tackled by means of CAE tools capable of computing numerical results coming from nonlinear FEM and MBD within a single simulation environment. A possible solution, widely employed in modern design optimization software, is based on the use of *meta-models* [17, 18]. With reference to Fig. 3, once external loads and displacements are defined, the CM behaviour is simulated in order to compute the values of x_m^C, θ_n^C. These value are then fed to the PRB model optimizer along with a design space (i.e. variable bounds). A finite set of sample points is generated (*DOE phase*), the PRB system responses and, consequently, the objective function being computed on these sample points by means of the MBD solver. Note that, in all these iterations, the same external loads/displacements previously imposed on the CM are applied to the PRB models. Subsequently, an approximate response surface is built on the basis of the computed results (*meta-modelling phase* [19]) and a gradient-based numerical optimizer is used to find the minimum of such meta-model (*optimum search*), thus providing an approximate optimum (denoted as l_i^*, K_j^*). A new MBD analysis is then performed with the parameters corresponding to the approximate optimum, and the obtained response is checked against a convergence criterion. If the convergence criterion on the objective function or on the constraints is not satisfied, a new response surface meta model is built, by including the approximate optimum, along with it computed response, $e_{tra}(l_i^*, K_j^*)$, to the finite set of points generated during the iteration previous steps. In the present study, the abovementioned optimization procedure is performed by the built-in Recurdyn Optimizer (*AutoDesign*).

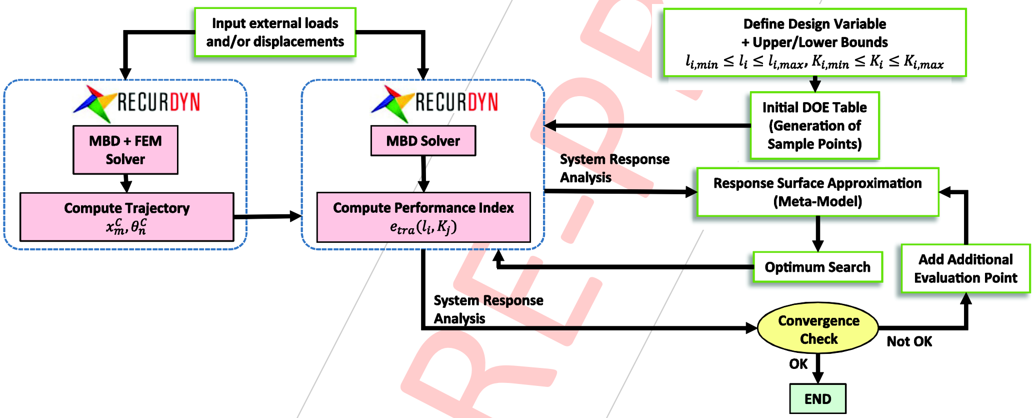
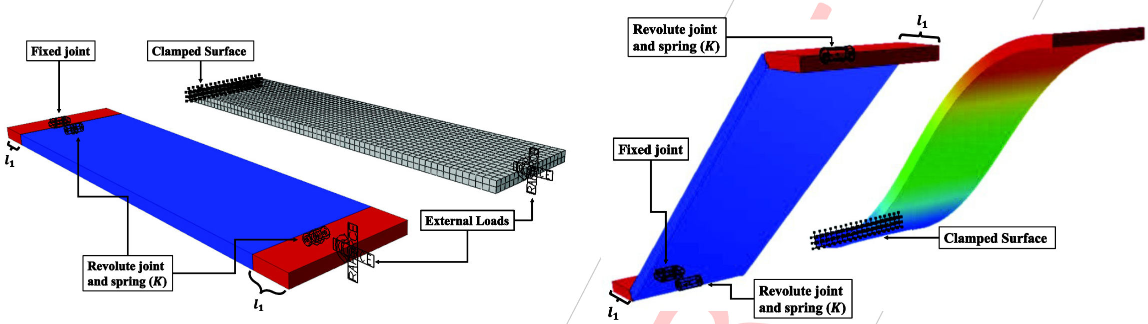


Fig. 3. Schematic of the optimization loop

4. Validation of the Method on a Theoretical Case

Before analyzing the spatial CM described in the next section, it is necessary to validate the optimization procedure against consolidated results obtained on a simple theoretical case study (shown in Fig. 4). Therefore, a cantilever beam with rectangular section, as the one depicted in Fig. 3, is taken into account. The beam length is $l = 20mm$, whereas cross section's width and height are, respectively, $w = 5\ mm$ and $h = 0.5\ mm$. As for the employed material, the Young's modulus and Poisson's ratio are, respectively, $E = 200\ Gpa$ and $\nu = 0.33$. Regarding the *Recurdyn* FEM model, a mapped mesh with brick elements has been defined ($0.3\ mm$ as max element size). After a mesh convergence analysis, the employed mesh consists of 3673 elements and 2279 nodes, the Linear Elastic Model algorithm being chosen in order to ensure a linear relationship between stresses and deformations. For what concerns the loads acting on both CM and related PRB, with reference to Fig. 2b, a vertical upward force, $\boldsymbol{F = 280\ N}$, and a clockwise moment, $\boldsymbol{M = 2211\ Nmm}$, are applied in a series to in a series of $\boldsymbol{Q}$ simulation steps to the beam end, ensuring a maximum deformation characterized by $\boldsymbol{\Theta_{max} = 42.5°}$, along with vertical and horizontal deflections respectively equaling $\boldsymbol{a/l = 0.79}$ and $\boldsymbol{b/l = 0.56}$. For each load step, the position error is measured and, at the end of the simulation, the objective function is computed with reference to the complete trajectory. As for the convergence check, the objective function change rate in consecutive iterations is set to $\boldsymbol{5 \cdot 10^{-2}}$. In particular, referring to this specific case study, the values for the *characteristic radius factor* and for the *stiffness coefficient* suggested in the literature are, respectively, $\boldsymbol{K_\Theta = 2.68}$ and $\boldsymbol{\gamma = 0.8517}$ [2], the corresponding PRB links length and torsional stiffness being readily obtained by means of Eqs. (2) and (4). These PRB parameters are computed considering the Euler-Bernoulli beam theory, whose limits are discussed in e.g. [20]. In parallel, the results achieved by means of the AutoDesign tool (considering the above-mentioned 3D FEM model) are summarized in Table 1. The same table provides the trajectory error, as computed from Eq. (5), in the two cases, highlighting that the proposed CAE-based optimizer can provide reliable results, whenever the corresponding trajectory error is acceptable for the considered application. Note that: *i)* if the approximations related to the Euler-Bernoulli beam theory are accepted, then a FEM model comprising 1D beam element can be employed, thus furtherly reducing the computational times; *ii)* lower trajectory errors can be obtained by limiting the maximum flexure deformation.



(a) Undeflected configurations (b) Deflected configurations

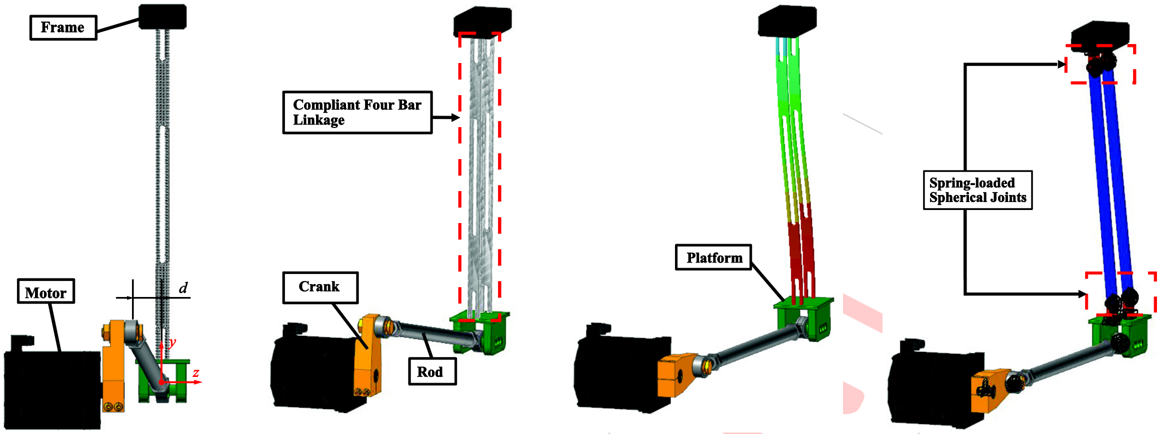
Fig. 4. Fixed-guided beam: MBD and nonlinear FEM

Table 1. Optimization results as compared to theoretical case [2].

Pseudo-Rigid-Body Parameters	*Analytical*	*Numerical*
$l_1 = 0.5(1-\gamma)l$	$1.48\ mm$	$1.84\ mm$
$K = 2\gamma K_\Theta EJ/l$	$2378\ Nmm/rad$	$2256\ Nmm/rad$
Trajectory error (e_{tra})	0.795	0.585

5. Determination of PRB Parameters for a Spatial Compliant Mechanism

As a case study comprising a spatial CM and flexures of complex geometry, a crank mechanism connected to a fully-compliant four bar linkage is considered. The actual CM CAD model, along with its PRB counterpart, is depicted in Fig. 5, whereas the compliant four-bar linkage (after model meshing) is depicted in Fig. 6. The CM model is composed of three moving rigid bodies (crank, rod and platform), two rigid bodies fixed to the ground (motor and frame), and two flexible members (similarly to Fig. 1a). For what concerns the flexible elements, they are shaped as slender beams with length, $l = 400\ mm$, width, $b = 50\ mm$, and height, $h = 2\ mm$. Nonetheless, it is necessary to underline that, as clearly highlighted in Fig. 6, the beam cross section is not constant throughout the beam length (due to the presence of hollow parts). The length of crank and connecting rod are, respectively, $70\ mm$ and $356\ mm$. In addition, a misalignment of $d = 36.5\ mm$ in the z-direction between the connecting rod and the platform is present (see Fig. 5a), causing out plane loads (and, therefore, motions) acting on the platform. Concerning the kinematic pairs, the crank is connected to the motor via a revolute joint, whereas the rod is connected to crank and platform via spherical joints. Deformable members are made of Spring Steel with Young's Module $E = 207\ GPa$ and a Poisson's ratio $\nu = 0.30$. After a mesh convergence analysis, the employed mesh consists of 32338 brick elements and 48900 nodes for each member. An example of the displacement magnitude contour plot is shown in Fig. 5c. As for the PRB system (depicted in Fig. 5d and 7), in order to capture the spatial motion of the platform, four equally-spaced spherical joints are introduced, each having a generalized rotational spring mounted in parallel. These four springs are characterized by the same rotational stiffness constants, denoted as $K_{\vartheta x}$,$K_{\vartheta y}$,$K_{\vartheta z}$. Subsequently, with the purpose of measuring the trajectory error, e_{tra}, between CM and PRB system, a reference frame is placed on a point of interest located on the platform (as depicted in Fig. 6), while a rotation with relatively low constant velocity is enforced along the rotation axis of the crank (*0.25 rev/s*). Owing to these assumptions, the PRB model requires the determination of four design parameters, namely l_1, $K_{\vartheta x}$,$K_{\vartheta y}$,$K_{\vartheta z}$, where l_1 is the distance along the y-direction of a spherical joint to the frame (see Fig. 7).



(a) CM side view (b) CM 3D view (c) CM deflected (d) PRB deflected

Fig. 5. Crank Mechanism connected to a fully compliant four-bar linkage: 3D model view and PRB model.

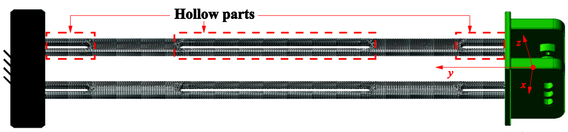


Fig. 6. Compliant four bar linkage: geometry and mesh

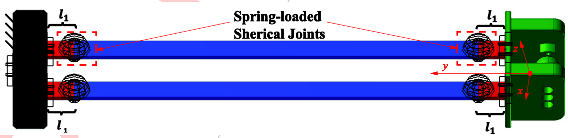
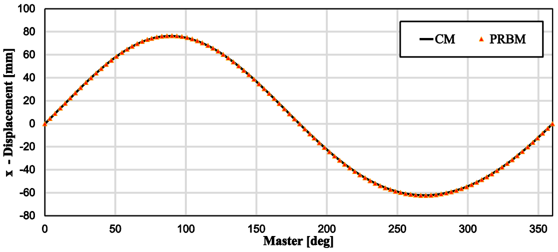


Fig. 7. Compliant four bar linkage: spatial PRB model

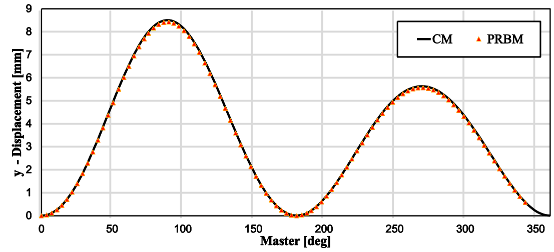
The results obtained by means of the AutoDesign tool are summarized in Table 2, whereas Fig. 8a-8c respectively report the behavior of both CM and optimal PRB for what concerns spatial translations of the platform (along *x*, *y*, and *z* directions, shown in Figs. 5a, 6 and 7). Similarly to the previous case, for what concerns the convergence check, the objective function change rate in consecutive iterations is set to $5 \cdot 10^{-2}$. For all variables, the PRB model captures the CM behavior with good accuracy, highlighting that the proposed CAE-based optimizer can provide reliable results whenever the corresponding trajectory errors are acceptable for the considered application.

Table 2. Optimization results for the Crank Mechanism connected to a compliant four-bar linkage

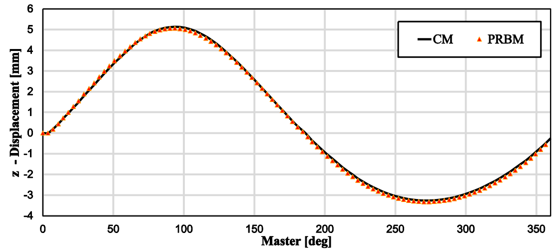
Pseudo-Rigid-Body Parameters	l_1	$K_{\vartheta x}$	$K_{\vartheta y}$	$K_{\vartheta z}$
	$23.51\ mm$	$1165\ Nmm/rad$	$1295\ Nmm/rad$	$241\ Nmm/rad$
Trajectory error (e_{tra})	0.152			



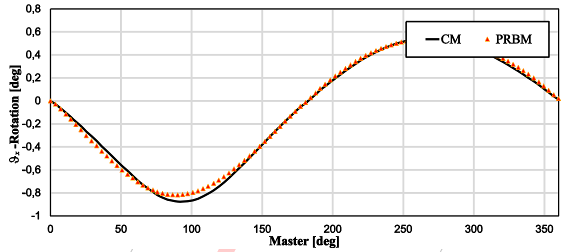
(a) Displacement along the *x*-direction.



(b) Displacement along the *y*-direction.



(c) Displacement along the *z*-direction.



(d) Rotation along the *z*-axis.

Fig. 8. Optimization results: comparison of CM and PRB model.

For what concerns computational times, all the simulations have been performed on a Notebook PC with an Intel I7-4719HQ CPU @ 2.50GHz and 16GB RAM. The CM model is solved in 1843.52s, whereas the optimized PRB model is simulated in 0.37s, further highlighting the usefulness of the PRB approximation whenever computational efficiency is sought after (e.g. applications requiring model-based control of systems comprising compliant members).

6. Conclusion

In this paper, a practical method for determining the PRB parameters of planar and spatial compliant mechanisms has been reported. The method leverages on the capabilities of modern CAE tools, which allow to simulate the motion of both flexible and rigid bodies by means of nonlinear FEM and MBD solvers. In addition, meta-model based optimization techniques allow to optimize a set of design parameters for a user-defined performance index. In the specific case of PRB, the design parameters are represented by the location and stiffness of a series of spring-loaded kinematic pairs (either revolute or spherical joints). The procedure has been tested on both planar and spatial compliant mechanisms with distributed compliance. Numerical results confirm the usability of the method also for systems comprising a set of rigid and flexible bodies, concurrently simulated within the abovementioned CAE tool. Since the proposed approach is largely based on the use of a general purpose MBD/FEM software, it shows good potentials for the quick and efficient generation of reliable PRB models in a large variety of applications.

References

[1] Kota, S., Ananthasuresh, G. K.: "Designing compliant mechanisms", *Mechanical Engineering-CIME,* 117(11), pp. 93-97, 1995.
[2] Howell, L.L.: *Compliant mechanisms*. John Wiley & Sons, 2001
[3] Lobontiu, N.: *Compliant Mechanisms: Design of Flexure Hinges*. CRC Press, 2002.
[4] Gibson, I., Rosen, D., Stucker, B.: *Additive Manufacturing Technologies: Rapid Prototyping to Direct Digital Manufacturing*. Boston, MA, USA, Springer, 2010.
[5] Bruzzone, L., Molfino, R.M.: "A novel parallel robot for current microassembly applications", *Assembly Automation,* 26(4), pp. 299-306, 2006.
[6] Bruzzone, L., Bozzini, G., "A flexible joints microassembly robot with metamorphic gripper", *Assembly Automation*, 30(3), pp. 240-247, 2010.
[7] Yin, L., Ananthasuresh, G. K.: "Design of Distributed Compliant Mechanisms", *Mechanics Based Design of Structures and Machines*, 31(2), pp. 151-179, 2003

[8] Chen, G., Botao X., Xinbo H.: "Finding the optimal characteristic parameters for 3R pseudo-rigid-body model using an improved particle swarm optimizer", *Precision Engineering,* 35(3), pp. 505-511, 2011.
[9] Jensen B.D., Howell L.L.: "Identification of compliant pseudo-rigid-body mechanism configurations resulting in bistable behaviour", *ASME Journal of Mechanical Design*, 125(4), pp. 701-708, 2003.
[10] Midha A, Howell L.L, Norton T.W.: "Limit positions of compliant mechanisms using the pseudo-rigid-body model concept", *Mechanisms and Machine Theory*, 35(1), pp. 99-115, 2000.
[11] Yu Y.Q., Howell L.L., Lusk C., Yue Y., He M.G.: "Dynamic modeling of compliant mechanisms based on the pseudo-rigid-body model. *ASME Journal of Mechanical Design*, 27(7), pp. 760–765, 2005.
[12] Berselli, G., Piccinini, M., Vassura, G.: "Comparative evaluation of the selective compliance in elastic joints for robotic structures", *IEEE International Conference on Robotics and Automation*, pp. 4626-4631, 2011.
[13] Su, H.: "A pseudorigid-body 3R model for determining large deflection of cantilever beams subject to tip loads", *Journal of Mechanisms and Robotics,* 1(2), pp. 021008, 2009.
[14] Turkkan, O.A., Su, H.: "DAS-2D: a concept design tool for compliant mechanisms", *Mechanical Sciences,* 7(2), pp. 135-148, 2016.
[15] www.functionbay.org, accessed: 03/03/2017.
[16] Kalpathy, V.V., and Su, H.: "A parameter optimization framework for determining the pseudo-rigid-body model of cantilever-beams", *Precision Engineering,* 40, pp. 46-54, 2015.
[17] Kim, M.S., Kang, D.O., Heo, S.J., "Innovative design optimization strategy for the automotive industry*", International Journal of Automotive Technology*, 15(2), pp. 291-301, 2014.
[18] Simpson, T. W., Peplinski, J. D., Koch, P. N. and Allen, J. K.: "Metamodels for computer-based engineering design: survey and recommendations", *Engineering with Computers*, 17, pp. 129-150, 2001.
[19] Wang, J. G. and Liu, G. R. "A point interpolation meshless method based on radial basis functions", *International. Journal of Numerical Methods in Engineering*, 54, pp. 1623-1648, 2002.
[20] Berselli, G., Meng, Q., Vertechy, R., Parenti Castelli, V.: "An improved design method for the dimensional synthesis of flexure-based compliant mechanisms: optimization procedure and experimental validation". *Springer Meccanica*, 51(5), pp. 1-17, 2016.