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A MULTISCALE STRUCTURAL MODEL FOR COHESIVE DELAMINATION OF MULTILAYERED BEAMS

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Abstract. *A homogenized approach is formulated for studying delamination fracture problems in laminated beams and plates. The idea is tested on an edge-cracked homogeneous and orthotropic beam under mode II dominant conditions. In order to define the field variables, the domain is discretized only in the in-plane direction and a multiscale structural theory with three unknown variables is used to homogenize and analyze the cracked and intact portions of the laminate. An explicit expression for the energy release rate in terms of force and moment resultants and rotations is derived through an application of the J-integral using the local fields; the expression coincides with that obtained through discrete approaches. A comparison with accurate two-dimensional elasticity results highlights the predictive capabilities of the model and its limitations.*

1 INTRODUCTION

Multilayered structures are often prone to delaminate due to their inhomogeneous structure and through-thickness stresses caused by in-service loads and impacts or to the presence of flaws caused by manufacturing errors. Onset and propagation of delaminations may be accurately studied introducing cohesive interfaces, which are governed by cohesive traction laws [1, 2]. The laws relate the interfacial tractions to the relative displacements between the adjacent layers and can be used to model all linear and nonlinear mechanics taking place at the interfaces. Introducing cohesive interfaces implies a discrete description and a through-thickness discretization of the problem, which complicate the solution of the problem especially when multiple delaminations are present in thin layered structures.

In this paper, a homogenized approach is proposed for studying cohesive delamination fracture of layered systems, which removes the need for the through-thickness discretization. The idea is explored using the multiscale structural model formulated in [3, 4] for beams and plates with cohesive interfaces. The model is based on the original zigzag theory in [5], and assumes a multiscale displacement field described by global variables (first-order equivalent single layer theory) and local perturbations to account for the layered structure and the displacement jumps at the cohesive interfaces. A homogenization technique, which imposes continuity of the tractions at the layer interfaces and the cohesive traction laws, is then applied to define the local variables as functions of the global ones. The number of unknown functions in the model is independent of the number of layers and delaminations, and is equal to that of the global model. The accuracy of the model in predicting the local fields in layered plates with continuous, imperfect and fully debonded interfaces and subjected to stationary thermo-mechanical loads was verified in [3, 4].

In this work, the capability of the model [3] to predict the fracture parameters in plates with finite length delaminations is investigated through the analysis of the brittle fracture of an edge-cracked homogeneous and orthotropic layer subjected to end forces and under mode II dominant conditions. The main assumption of the work is explained in Fig. 1, which shows the actual cracked element (Fig. 1a) and the homogenized element used in the multiscale model (Fig. 1b). The energy release rate associated to the collinear propagation of the crack in Fig. 1a is derived through an application of the J-integral in the homogenized representation of the problem and using the local fields derived a posteriori from the global variables of the multiscale model. The resulting expression coincides with that obtained through the solution of the discrete problem [6].

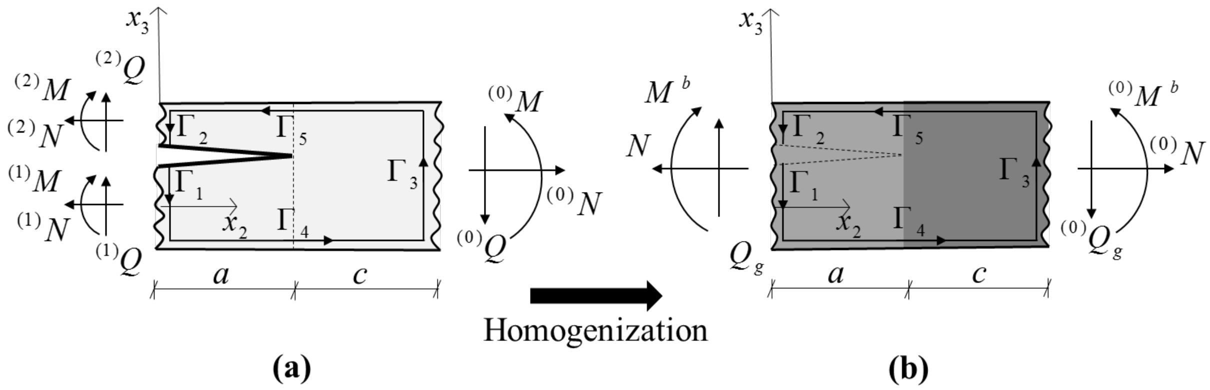

Figure 1: (a) a cracked element, and (b) its homogenized representation.

2 FRACTURE PARAMETERS IN AN EDGE-CRACKED LAYER

Consider the traction-free edge-cracked layer shown in Fig. 1a. The layer is subjected to end forces applied per unit width and has global thickness $h = h_1 + h_2$, with h_1 and h_2 the thicknesses of the lower and upper crack arms. The forces act at distances *a* and *c* from the crack tip which are assumed to be sufficiently long to ensure that the stress field at the crack tip is unaffected by their actual distributions and depend only on the force and moment resultants. A system of Cartesian coordinates $x_1 - x_2 - x_3$ is introduced with origin at the mid-plane of the lower crack arm. The layer is linearly elastic, homogenous and orthotropic with principal material axes parallel to the geometrical axes, e.g. a unidirectionally reinforced laminate, and under plane-strain conditions parallel to the plane $x_2 - x_3$. The problem in Fig. 1a is studied using the multiscale structural theory formulated in [3]. A cohesive interface is first introduced along the crack line which divides the geometry into two sub-layers. The following constitutive and compatibility equations are assumed:

$$\begin{array}{l} {}^{(k)}\sigma_{22} = \bar{C}_{22}\,{}^{(k)}\varepsilon_{22}; \quad {}^{(k)}\sigma_{23} = C_{44}\,2\,{}^{(k)}\varepsilon_{23} \\ {}^{(k)}\varepsilon_{22} = {}^{(k)}v_{2,2}; \quad 2\,{}^{(k)}\varepsilon_{23} = {}^{(k)}v_{2,3} + {}^{(k)}v_{3,2} \end{array} \tag{1}$$

with $\bar{C}_{22} = \left(C_{22} - C_{23}C_{32}/C_{33}\right)$, C_{ij} the stiffness coefficients, ${}^{(k)}\sigma_{ij}$, ${}^{(k)}\varepsilon_{ij}$ and ${}^{(k)}v_i$ the stress, strain and displacement components, *i*, *j* = 2, 3, 4 (the transverse normal stresses have been assumed to be negligible).

In this paper the interface is assumed to be rigid against relative opening displacements and the following piecewise linear interfacial traction law is used:

$$\hat{\sigma}_S(x_2) = \begin{cases} K_S \hat{v}_2(x_2) & \text{for } \hat{v}_2(x_2) \le \hat{v}_{2c} \\ 0 & \text{for } \hat{v}_2(x_2) \ge \hat{v}_{2c} \end{cases} \tag{2}$$

with $\hat{v}_{2c}$ the critical sliding displacement. The law relates the tangential traction, $\hat{\sigma}_S$, to the sliding displacement, $\hat{v}_2(x_2, x_3 = h_1/2) = {}^{(2)}v_2(x_2, x_3 = h_1/2) - {}^{(1)}v_2(x_2, x_3 = h_1/2)$, with K_S the interfacial tangential stiffness (see Fig. 2b). The law is composed of an initial stiff branch which is used to model the perfect bonding of the two sub-layers in the intact region, and a branch with zero interfacial stiffness which is used to model the traction-free delamination. The mode II fracture energy, $\mathcal{G}_{IIc}$, is defined by the area under the curve. Neglecting the relative opening displacements allows to use the simplified version of the multiscale theory in [3] and to model mode II dominant problems. This assumption, which is used here to preliminarily assess the accuracy of the homogenized description of the fracture problem, will be removed later to study general fracture problems.

For the solution of the problem [3], the small-scale displacement field in the layer in Fig. 1b is defined as:

$$
\begin{aligned}
&{}^{(k)}v_2(x_2,x_3) = v_{02}(x_2) + x_3\varphi_2(x_2) + \sum_{i=1}^{k-1}\hat{v}_2(x_2) \\
&{}^{(k)}v_3(x_2,x_3) = w_0(x_2)
\end{aligned}
\tag{3}
$$

where $v_{02}(x_2)$, $\varphi_2(x_2)$ and $w_0(x_2)$, are the global variables (first-order shear deformation theory) and $\sum_{i=1}^{k-1}\hat{v}_2(x_2)$ is the local perturbation which accounts for the jump at the layer interface. The jump is derived in terms of the global variables by using the interfacial traction law, Eq. (2), and the compatibility and constitutive equations (1), and by imposing continuity of the transverse shear tractions of the layers at their surfaces. The macro-scale displacement field in the sub-layer k follows as:

$$
\begin{aligned}
&{}^{(k)}v_2(x_2,x_3) = v_{02}(x_2) + x_3\varphi_2(x_2) + \left[w_0,_2(x_2) + \varphi_2(x_2)\right]R_S^k \\
&{}^{(k)}v_3(x_2,x_3) = w_0(x_2)
\end{aligned}
\tag{4}
$$

where $R_S^2 = C_{44}/K_S$ and $R_S^1 = 0$. The equilibrium equations are derived using the Principle of Virtual Work. They are presented here, in terms of global displacements:

$$
\begin{aligned}
&hv_{02},_{22} + h_2\left[\left(0.5h + R_S^2\right)\varphi_2,_{22} + R_S^2 w_0,_{222}\right] = 0 \\
&h_2(0.5h + R_S^2)v_{02},_{22} + \frac{\left(h_1\right)^3 + \left(h + h_2\right)^3}{24}\varphi_2,_{22} + h_2R_S^2\left[\left(h + R_S^2\right)\varphi_2,_{22} + (0.5h + R_S^2)w_0,_{222}\right] - \\
&-\frac{C_{44}}{\bar{C}_{22}}\left[k_{44}h + \frac{C_{44}}{K_S}\right](w_0,_2 + \varphi_2) = 0 \\
&R_S^2 h_2 v_{02},_{222} + h_2R_S^2\left[(0.5h + R_S^2)\varphi_2,_{222} + R_S^2 w_0,_{2222}\right] - \frac{C_{44}}{\bar{C}_{22}}\left[k_{44}h + \frac{C_{44}}{K_S}\right](w_0,_{22} + \varphi_2,_2) = 0
\end{aligned}
\tag{5}
$$

A shear correction factor, k_{44}, has been introduced to improve the approximate description of the shear strain of the global first order model. The equilibrium equations (5) are solved in the different regions of the layer and boundary and continuity conditions are imposed to calculate the unknown constants of the solutions.

In the cohesive-crack modeling, the energy release rate is typically calculated directly from the cohesive traction law, as $\mathcal{G}_{II} = \int_0^{\hat{v}_2}\hat{\sigma}_S d\hat{v}_2$. In this paper, the energy release rate is derived through an application of the J-integral. In the cracked layer of Fig. 1a, the energy release rate is then calculated as, $\mathcal{G}_{II} = J = \int_\Gamma (W dx_3 - \sigma_{ij}n_j \frac{\partial v_i}{\partial x_2} d\Gamma)$, with Γ a path surrounding the crack tip, W the strain energy density, n_j the components of the unit outward vector normal to the path, $\sigma_{ij}n_j$ the tractions along the contour, and $J_4 = J_5 = 0$. In the homogenized problem of Fig. 1b, the energy release rate is defined calculating J along a similar path.

Upon substitutions of the macro-scale displacement and strain components of the intact and cracked regions into the given expression for the J-integral, and some algebraic manipulations, the energy release rate, $\mathscr{G}_{II} = J_1 + J_2 + J_3$, is expressed in terms of the variables of the homogenized model as:

$$J_1 = \frac{1}{2}\int_{-h_1/2}^{h_1/2}\left[{}^{(1)}\sigma_{22}\left(v_{02},_2 + x_3\varphi_2,_2\right) + {}^{(1)}\sigma_{23}^{post}\left(\varphi_2 + w_0,_2\right)\right]dx_3 - \varphi_2\int_{-h_1/2}^{h_1/2}{}^{(1)}\sigma_{23}^{post}dx_3 \tag{6}$$

$$\begin{aligned} J_2 = &\frac{1}{2}\int_{h_1/2}^{h_2+h_1/2}\left[{}^{(2)}\sigma_{22}\left(v_{02},_2 + x_3\varphi_2,_2 + \left[w_0,_{22} + \varphi_2,_2\right]R_S^2\right) + {}^{(2)}\sigma_{23}^{post}\left(\varphi_2 + w_0,_2\right)\right]dx_3 - \\ &-\varphi_2\int_{h_1/2}^{h_2+h_1/2}{}^{(2)}\sigma_{23}^{post}dx_3 \end{aligned} \tag{7}$$

$$J_3 = -\frac{1}{2}\int_{-h_1/2}^{h_2+h_1/2}\left[\sigma_{22}\left(v_{02},_2 + x_3\varphi_2,_2\right) + \sigma_{23}^{post}\left(\varphi_2 + w_0,_2\right)\right]dx_3 + \varphi_2\int_{-h_1/2}^{h_2+h_1/2}\sigma_{23}^{post}dx_3 \tag{8}$$

where ${}^{(k)}\sigma_{22}$ is defined through the constitutive equation (1) and ${}^{(k)}\sigma_{23}^{post}$ through local equilibrium ${}^{(k)}\sigma_{22},_2 + {}^{(k)}\sigma_{23,3}^{post} = 0$.

In order to prove that the energy release rate of the actual problem in Fig. 1a can be accurately predicted from its homogenized representation Fig. 1b through Eqs. (6)-(8), the equations are expressed in terms of force and moment resultants acting on the three paths, Γ_1, Γ_2 and Γ_3. The procedure is explained here only for the path Γ_2, Eq. (7); a similar procedure should be followed for the other paths. Equation (7) is rewritten by introducing a local coordinate at its mid-plane, $Z = x_3 - \left(h_1 + h_2\right)/2$, and adding and subtracting $\varphi_2,_2\left(h_1 + h_2\right)/2$:

$$J_2 = \frac{1}{2}\int_{-h_2/2}^{h_2/2}\left({}^{(2)}\sigma_{22}\left[{}^{(2)}v_2\left(Z=0\right),_2 + Z\varphi_2,_2\right] + {}^{(2)}\sigma_{23}^{post}\left(\varphi_2 + w_0,_2\right)\right)dZ - \varphi_2\int_{-h_2/2}^{h_2/2}{}^{(2)}\sigma_{23}^{post}dZ \tag{9}$$

where ${}^{(2)}v_2\left(Z=0\right)$ is the longitudinal displacement at the mid-plane of the upper crack arm (see Eq. (4)). The terms in Eq. (9) which depend on the global displacements, are independent of Z and related to the force and moment resultants acting in the upper sub-layer, ${}^{(2)}v_2\left(Z=0\right),_2 = \frac{{}^{(2)}N}{\bar{C}_{22}h_2}$, $\varphi_2,_2 = \frac{{}^{(2)}M^b}{\bar{C}_{22}h_2^3/12}$ and $\left(\varphi_2 + w_0,_2\right) = \frac{{}^{(2)}Q_g}{k_{44}C_{44}h_2}$ where $\left({}^{(2)}N, {}^{(2)}M^b\right) = \int_{-h_2/2}^{h_2/2}{}^{(2)}\sigma_{22}\left(1, Z\right)dZ$, and ${}^{(2)}Q_g = \int_{-h_2/2}^{h_2/2}{}^{(2)}\sigma_{23}^{post}dZ$. Therefore, Eqs. (7) and (9) can be written as:

$$J_2 = \frac{1}{2}\left[\frac{\left({}^{(2)}N\right)^2}{\bar{C}_{22}h_2} + \frac{12\left({}^{(2)}M^b\right)^2}{\bar{C}_{22}h_2^3} + \frac{\left({}^{(2)}Q_g\right)^2}{k_{44}C_{44}h_2} - 2\left({}^{(2)}Q_g\right)\varphi_2\right] \tag{10}$$

where φ_2 is the rotation of the upper sub-layer at the left edge. Operating on the other paths and renaming the rotations of the end cross-sections using ${}^{(i)}\beta$, the following expression of the energy release rate is derived:

$$\mathcal{G}_{II}=\frac{1}{2}\left[\sum_{i=1}^{2}\left(\frac{\left({}^{(i)}N\right)^2}{\bar{C}_{22}h_i}+\frac{12\left({}^{(i)}M^b\right)^2}{\bar{C}_{22}h_i^3}+\frac{\left({}^{(i)}Q_g\right)^2}{k_{44}C_{44}h_i}-2\,{}^{(i)}Q_g\,{}^{(i)}\beta\right)-\right.$$
$$\left.-\left(\frac{\left({}^{(0)}N\right)^2}{\bar{C}_{22}h}+\frac{12\left({}^{(0)}M^b\right)^2}{\bar{C}_{22}h^3}+\frac{\left({}^{(0)}Q_g\right)^2}{k_{44}C_{44}h}-2\,{}^{(0)}Q_g\,{}^{(0)}\beta\right)\right] \tag{11}$$

where $\left({}^{(0)}N,{}^{(0)}M^b\right)=\int_{-h_1/2}^{h_2+h_1/2}\sigma_{22}\left(1,x_3-h_2/2\right)dx_3$ and ${}^{(0)}Q_g=\int_{-h_1/2}^{h_2+h_1/2}\sigma_{23}^{post}dx_3$. The expression coincides with that derived in [6, 7] for the cracked element in Fig. 1a if the rotations of the three end sections coincide. This occurs in all cases when the relative rotations of the beam arms at the crack tip in Fig. 1a are negligible so that the arms can be assumed as clamped at the crack tip cross section.

3 APPLICATION TO AN ENF SPECIMEN

The model presented in Sect. 2 is applied to the End Notched Flexural specimen shown in Fig. 2a. The energy release rate is calculated using Eq. (11) and for $h_1=h_2=h$ turns out to be $\mathcal{G}_{II}\bar{C}_{22}h/P^2=9/16(a/h)^2$. The dimensionless energy release rate is presented and compared with accurate 2D solutions [7] on varying the dimensionless crack length in Fig. 3a; relative percent errors are presented in Fig. 3b.

The energy release rate coincides with the classical solution from the literature which assumes the two delaminated arms of the beam as clamped at the crack tip cross-section. This limitation of the solution is a consequence of the assumptions of the multiscale model, which imposes continuity of the rotations of the sub-layers at the crack tip cross-section. The model then neglects the relative rotations of the arms at the crack tip, or root-rotations, which play an important role in the presence of short cracks and/or thick beams, as shown in Fig. 3b. The limitation may be overcome by improving the solution a posteriori using the root rotations defined in [7]. It is expected that this limitation of the multiscale treatment will be partially overcome in layered systems where zigzag perturbations of the displacements are included in the description of the problem which leads to relative rotations of the sub-layers.

The load-deflection response of a specimen with geometrical and material properties defined in the caption is shown in Fig. 3c. The homogenized model is able to capture the snap-back instability which appears in the post-peak response.

As explained above, the energy release rate in the cohesive interface modeling of fracture is typically derived using measures of the interfacial tractions and relative crack displacements. In order to preliminary verify the capability of the proposed approach to analyze cohesive delamination fracture, the energy release rate has also been calculated using the J-integral and a path around the crack surfaces. This calculation is conveniently performed using Bueckner's superposition principle, which requires the calculation of the transverse shear tractions in a specimen with no crack and the relative sliding displacements in the cracked specimen. The resulting energy release rate coincides with that given by Eq. (11).

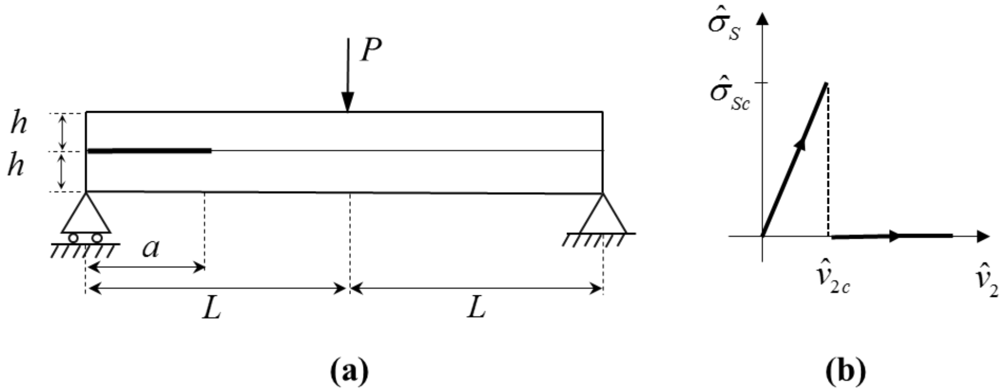

Figure 2: (a) ENF specimen, and (b) interfacial traction law used for modelling perfectly brittle mode II fracture.

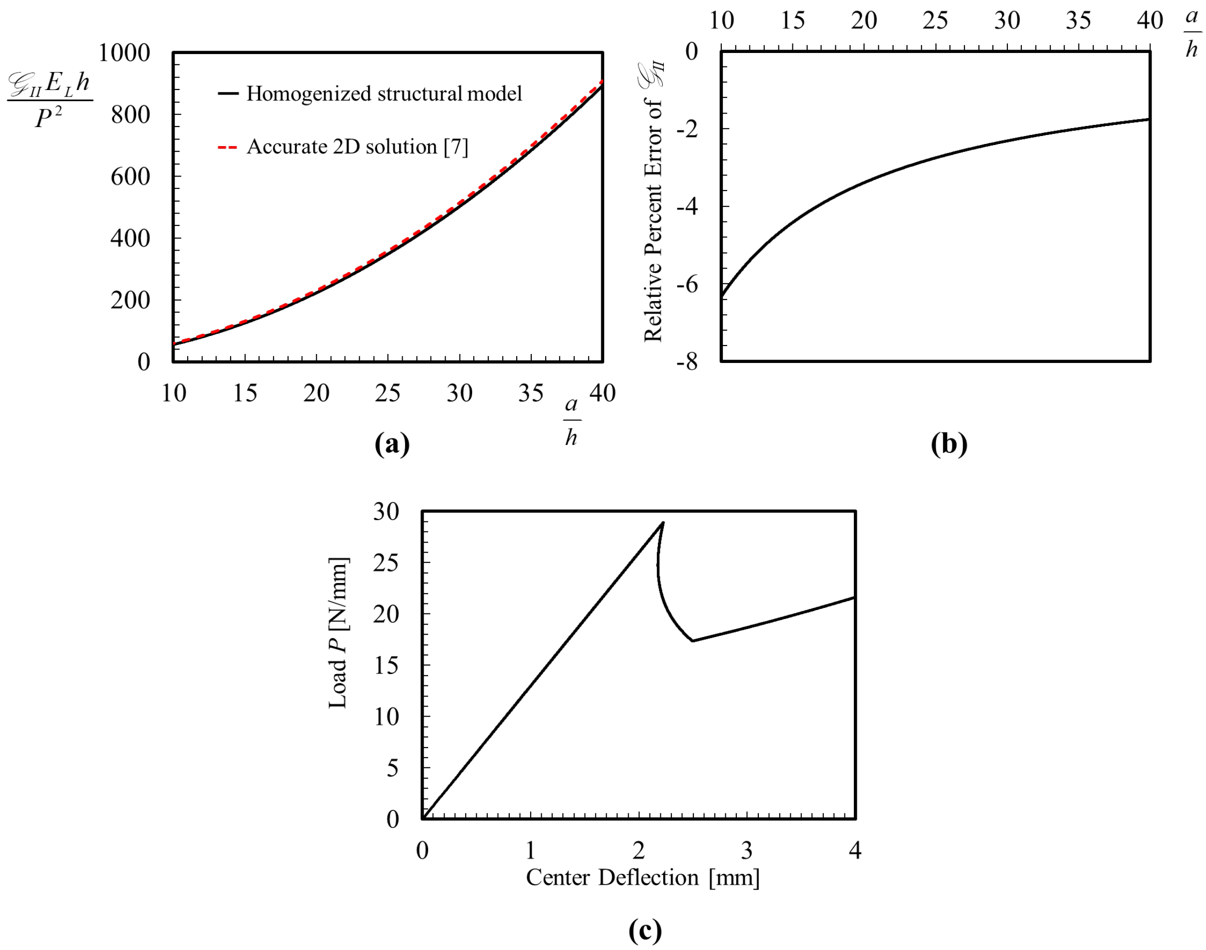

Figure 3: (a) dimensionless energy release rates, and (b) relative percent error with respect to accurate 2D solutions [7] in an unidirectionally reinforced ENF specimen with elastic constants $E_T/E_L = 0.071$, $G_{LT}/E_L = 0.033$, $\nu_{LT} = 0.32$, and $\nu_{TT} = 0.45$ (subscripts *L* and *T* indicate in-plane principal material directions). (c) load-deflection response of a specimen made of IM7/8552 graphite/epoxy with $E_L = 161$ GPa, $\mathscr{G}_{IIc} = 0.774$ N/mm, *h* = 1.5 mm, *L* = 50 mm, initial crack length = 30 mm, shear correction factor, $k_{44} = 5/6$.

4 CONCLUSIONS

A novel homogenized approach has been proposed to study delamination fracture problems. The idea has been put into practice through a multiscale structural model with a reduced number of variables which removes the need for the through-thickness discretization typically used in the cohesive-interface approaches. For a preliminary validation of the idea an edge-cracked homogeneous and orthotropic layer under mode II dominant conditions has been considered and the energy release rate has been derived through an application of the J-integral using the local fields calculated through the multiscale model. We showed that the derived expression of the energy release rate coincides with the classical solution of the problem. The expression neglects the contributions of the root rotations, which can be calculated a posteriori using the equations derived in [7]. The model is expected to partially account for the root rotations in multilayered structures where zigzag functions are introduced to enrich the global displacement fields. Numerical results have been presented for an ENF specimen and compared with accurate 2D solutions. The model captures the load-displacement response of the specimen including the snap-back instability.

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