Probabilistic Load Flow for Uncertainty Based Grid Operation

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***Abstract*-- Traditional algorithms used in grid operation and planning only evaluate one deterministic state. Uncertainties introduced by the increasing utilization of renewable energy sources have to be dealt with when determining the operational state of a grid. From this perspective the probability of certain operational states and of possible bottlenecks is important information to support the grid operator or planner in their daily work. From this special need the field of application for Probabilistic Load Flow methods evolved. Uncertain influences like power plant outages, deviations from the forecasted injected wind power and load have to be considered by their corresponding probability. With the help of probability density functions an integrated consideration of the partly stochastic behaviour of power plants und loads is possible.**

***Index Terms*-- probabilistic load flow**

I. Nomenclature

x	real valued scalar
$\overline{x}$	complex valued scalar
$\underline{X}, \underline{x}$	Matrix, vector
$\underline{X}^{\dagger}$	Pseudoinverse of matrix $\underline{X}$
δ_i	nodal voltage angle
$\delta(t)$	dirac-impulse
$p_x(x)$	probability density function of real scalar x
$p_{\overline{x}}(\overline{x})$	two-dimensional probability function of complex-valued scalar $\overline{x}$

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II. Introduction

IN this paper, after a brief discussion of the Probabilistic Load Flow (PLF) problem and introduction of a newly developed algorithm for solving the PLF problem, the possible applications of PLF techniques will be discussed.

In contrast to the regular Load Flow (LF) calculation the PLF calculation does not only determine the operational state of a grid for a single, sharply determined state. The PLF uses information about the probability of certain nodal power injections or consumptions rather than a single value. The objective of the PLF is to determine all possible operational states of the modelled network and their corresponding probability. To achieve this, the nodal behaviour is given as a Probability Density Function (PDF). The task of the PLF is to determine the PDF of network parameters from this input data. Therefore all possible combinations of nodal power injections and consumptions have to be considered in the extreme case.

Starting from PDF for every node sampled with 100 values for the actual power balance of that node, this leads to 100^d combinations on a network with *d* nodes that all have to be evaluated with PF techniques. With traditional approaches this is unsolvable for networks of a reasonable size. A first relieve is to not evaluate all possible combinations but to apply a Monte-Carlo approach by randomly choosing only some possible combinations for a subsequent evaluation. This leads to an unpredictable imprecision of the results and is also applicable for rather small networks only.

In this paper the mathematical basis of a newly developed approach to solve the PLF problem will be presented briefly. This approach allows for the calculation of a line's current PDF independently from all other lines of the network. It does not require a regular LF, but makes use of convolution techniques leading to a hugely reduced computation effort.

Furthermore the approach allows for a qualified selection of nodes to be taken into account, reducing the computation effort even further. Only those nodes having a significant impact on the loading of a certain line have to be included into the calculation. The deviation introduced by limiting the number of nodes considered can be estimated from the underlying network model.

III. Presented Algorithm

While most other approaches to solve the PLF problem with means of standard convolution techniques base on Jacobian matrices as a network model [1]-[3], [6], the presented approach bases on a linear map between node and line currents, denoted $\underline{Y}_{nl}$ in the following. This map is generated by combining the inverse nodal admittance matrix $\underline{Y}_n$ (1) and the line admittance matrix $\underline{Y}_l$ (2) stated for the exemplary network depicted in Fig. 1.

$$\underline{Y}_n = \begin{pmatrix} \overline{Y}_{1,0} + \sum_{i=1}^{n} \overline{Y}_{1i} & -\overline{Y}_{12} & \cdots & -\overline{Y}_{1n} \\ -\overline{Y}_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ -\overline{Y}_{n1} & \cdots & \cdots & \overline{Y}_{n,0} + \sum_{i=1}^{n} \overline{Y}_{ni} \end{pmatrix} \quad (1)$$

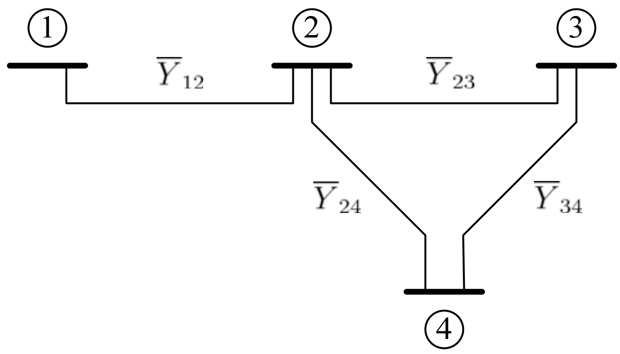

Fig. 1: Exemplary four node topology

$$\underline{Y}_l = \begin{pmatrix} \overline{Y}_{12} & -\overline{Y}_{12} & & \\ & \overline{Y}_{23} & -\overline{Y}_{23} & \\ & & \overline{Y}_{34} & -\overline{Y}_{34} \\ & \overline{Y}_{24} & & -\overline{Y}_{24} \end{pmatrix} \quad (2)$$

Depending on whether or not line shunt elements are considered $\underline{Y}_{nl}$ can be generated using either (3) (with consideration of shunt elements) or (4) (without consideration of shunt elements).

$$\underline{Y}_{nl} = \underline{Y}_l \cdot \underline{Y}_n^{-1} \quad (3)$$

$$\underline{Y}_{nl} = \underline{Y}_l \cdot \underline{Y}_n^{\dagger} \quad (4)$$

For both cases the relationship between nodal currents and line currents is determined by (5).

$$\underline{I}_l = \underline{Y}_{nl} \cdot \underline{I}_n \quad (5)$$

One of the most important issues is that there is usually no information available on the behavior of the reference node. It has to be determined form the available information for the remaining nodes. The current of the reference node – indexed with r in the following – can be calculated as the negative-signed sum of all remaining nodal currents plus the negative signed sum of all currents through the shunt elements (6).

$$\overline{I}_{n,r} = -\sum_{\substack{i=1 \\ i \neq r}}^{n} \overline{I}_{n,i} - \sum_{i=1}^{n} \overline{Y}_{i,0} \cdot \overline{V}_i \quad (6)$$

With the reference node current calculated using (6) the mapping between nodal currents and line currents can be stated like (7).

$$\underline{I}_{l,j} = \sum_{i=1}^{n} \overline{y}_{nl,ji} \cdot \overline{I}_{n,i} \quad (7)$$

Combining (6) and (7) results in (8).

$$\underline{I}_{l,j} = \sum_{\substack{i=1 \\ i \neq r}}^{n} \left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr} \right) \cdot \overline{I}_{n,i} - \overline{y}_{nl,jr} \cdot \sum_{i=1}^{n} \overline{Y}_{i,0} \cdot \overline{V}_i \quad (8)$$

In order to be able to state a regular convolution scheme it is necessary to calculate the normalized nodal current $\overline{X}_{ji}$, as being the influence of nodal current $\overline{I}_{ji}$ weighted by the line dependent factors of (8). This leads to the expression (9).

$$\overline{X}_{ji} = \left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr}\right) \cdot \overline{I}_{n,i} \tag{9}$$

$$\overline{I}_{l,j0} = -\overline{y}_{nl,jr} \cdot \sum_{i=1}^{n} \overline{Y}_{i,0} \cdot \overline{V}_{i} \tag{10}$$

Using (9) and (10), (8) can be restated as (11).

$$\overline{I}_{l,j} = \sum_{\substack{i=1\\ i\neq r}}^{n} \overline{X}_{ji} + \overline{I}_{l,j0} \tag{11}$$

Based on (11) it is then possible to formulate the convolution scheme stated in (12) in order to calculate the PDF $p_{\overline{I}_{l,j}}$ of the complex valued line current $\overline{I}_{l,j}$.

$$p_{\overline{I}_{l,j}} = p_{\overline{X}_{j1}} * \cdots * p_{\overline{X}_{jn}} * \delta\left(\overline{I}_{l,0}\right) \tag{12}$$

As (12) is stated with the PDF $p_{\overline{X}_{ji}}$ of the normalized line current $\overline{X}_{ji}$, $p_{\overline{X}_{ji}}$ has to be determined from the PDF $p_{\overline{I}_{n,i}}$ of nodal current $\overline{I}_{n,i}$ in a previous step using (13) and (14).

$$\overline{I}_{n,i} = \frac{\overline{X}_{ji}}{\left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr}\right)} \tag{13}$$

$$p_{\overline{X}_{ji}}\left(\overline{X}_{ji}\right) = \left|\left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr}\right)\right| \cdot p_{\overline{I}_{n,i}}\left(\frac{\overline{X}_{ji}}{\left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr}\right)}\right) \tag{14}$$

The factor $\left|\left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr}\right)\right|$ in (14) assures that the following basic probabilistic condition is still met:

$$\iint p_{\overline{X}_{ji}}\left(\overline{X}_{ji}\right) \cdot d\overline{X}_{ji} = 1 \tag{15}$$

With the methodology described before it is possible to calculate the PDF of line currents from the PDF of nodal currents. But as the input data to a PLF calculation is usually the PDF of nodal power, the PDF of nodal currents have to be determined from them.

As the link between nodal power and nodal current is nodal voltage (16), the following part of this paper will focus on the determination of nodal current PDF from the nodal power PDF with the help of a nodal voltage profile.

$$\overline{S}_{i} = \overline{V}_{i} \cdot \overline{I}_{n,i}^{*} \tag{16}$$

As this paper mainly addresses the possible applications of PLF computation, only an approach using a static voltage profile will be presented. This leads to some deviations from the accurate PLF solution, but increases the readability of this paper.

The calculation described before could also be performed with all nodal voltages equal to nominal voltage. In this case the PDF of nodal current could be calculated using (17).

$$p_{\overline{I}_{n,i}}\left(\overline{I}_{n,i}\right) = p_{\overline{S}_{i}}\left(1\,pu \cdot \overline{I}_{n,i}^{*}\right) \tag{17}$$

An improved accuracy can be achieved by estimating the voltage profile resulting from the injected and absorbed power at all nodes. In order to maintain a regular convolution scheme the voltage profile corresponding to the expected values of nodal power will be used and assumed constant. The voltage profile will be derived from the expected values for active and reactive power of each node by a DC-like approach, based on a simplification of the Fast Decoupled Load Flow (FDLF).

In [7] a detailed derivation of the used matrices is given. The following equations describe the resulting relationship between active power and voltage angle on the one hand, and reactive power and absolute voltage on the other hand.

$$\underline{P} = \underline{B}' \cdot \Delta\underline{\delta} \tag{18}$$

$$\underline{Q} = \underline{B}'' \cdot \Delta\underline{V} \tag{19}$$

This can be used to estimate the complex voltage argument and the absolute voltage from the vectors of active and reactive power. As matrix $\underline{B}'$ and $\underline{B}''$ are both singular there does not exist an inverse matrix, but only the pseudoinverse matrix $\underline{B}'^{\dagger}$ (20) and $\underline{B}''^{\dagger}$ (21) respectively.

$$\Delta\underline{\delta} = \underline{B}'^{\dagger} \cdot \underline{P} \tag{20}$$

$$\Delta\underline{V} = \underline{B}''^{\dagger} \cdot \underline{Q} \tag{21}$$

Using the notation of (22) and (23), (20) and (21) can be restated to (24) and (25) in order to emphazise their origin in the expected values.

$$E(\underline{P}) = \begin{bmatrix} E(P_1) \\ \vdots \\ E(P_n) \end{bmatrix} \quad E(\underline{Q}) = \begin{bmatrix} E(Q_1) \\ \vdots \\ E(Q_n) \end{bmatrix} \tag{22}$$

$$\underline{E}(\underline{\Delta\delta}) \begin{bmatrix} E(\Delta\delta_1) \\ \vdots \\ E(\Delta\delta_n) \end{bmatrix} \quad \underline{E}(\underline{\Delta V}) \begin{bmatrix} E(\Delta V_1) \\ \vdots \\ E(\Delta V_n) \end{bmatrix} \tag{23}$$

$$\underline{\mathrm{E}}(\underline{\Delta\delta}) = \underline{B}'^{\dagger} \cdot \underline{E}(\underline{P}) \tag{24}$$

$$\underline{\mathrm{E}}(\underline{\Delta V}) = \underline{B}''^{\dagger} \cdot \underline{E}\left(\underline{Q}\right) \tag{25}$$

Due to the used matrices being singular, the resulting vectors are only one of an infinite number of possible solutions for the given vectors of expected values. It is necessary to define the values for one node as a reference for all other elements in order to find the solution actually searched for. Usually the absolute value of the voltage at the reference node is set to 1 pu and its complex argument to 0. To determine the corresponding solution it is necessary to apply a common shift to the vectors calculated using (24) and (25), so that the absolute value at the reference node equals 1 pu and the complex value equals 0. With the help of (26) and (27) this can be achieved.

$$E(\delta_i) = E(\Delta\delta_i) - E(\Delta\delta_{ref}) \tag{26}$$

$$E(V_i) = E(\Delta V_i) + \left(1 - E(\Delta V_r)\right) \tag{27}$$

With the voltage profile estimated using (26) and (27) the shunt element related current can be estimated with (28).

$$\overline{I}_{l,j0} = -\overline{y}_{nl,jr} \cdot \sum_{i=1}^{n} \overline{Y}_{i,0} \cdot E(V_i) \cdot e^{j \cdot E(\delta_i)} \tag{28}$$

As the input parameters to the PLF-calculation are the PDF of nodal power, the PDF of nodal currents have to be derived from this input parameters. With the estimated voltage profile (30) and the basic nodal power equation (29) it is possible to estimate the complex-valued nodal power using (31).

$$\overline{S}_i = \overline{V}_i \cdot \overline{I}_{n,i}^{*} \tag{29}$$

$$E(\overline{V}_i) = E(V_i) \cdot e^{j \cdot E(\delta_i)} \tag{30}$$

$$\begin{aligned} \overline{S}_{ji} &= E(\overline{V}_i) \cdot \overline{I}_{n,i}^{*} \\ &= E(\overline{V}_i) \cdot \frac{\overline{X}_{ji}^{*}}{\left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr}\right)^{*}} \end{aligned} \tag{31}$$

The nodal power $\overline{S}_{ji}$ is double indexed here because this nodal power corresponds to the normalized nodal current calculated for line j.

Based on the considerations and findings described before, it is possible to determine the PDF of the normalized nodal current $\overline{X}_{ji}$ as stated in (32).

$$\begin{aligned} p_{\overline{X}_{ji}}(\overline{X}_{ji}) = &\left| \frac{\left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr}\right)}{E(\overline{V}_i)} \right| \\ &\cdot p_{\overline{S}_{n,i}}\left(E(\overline{V}_i) \cdot \frac{\overline{X}_{ji}^{*}}{\left(\overline{y}_{nl,ji} - \overline{y}_{nl,jr}\right)^{*}} \right) \end{aligned} \tag{32}$$

The leading factor is again needed to fulfill the probabilistic condition (15).

With the previously calculated values and PDF it is possible to state the calculation of the PDF of complex-valued line current $\overline{I}_{l,j}$ using standard convolution technique as stated in (33).

$$p_{\overline{I}_{l,j}} = p_{\overline{X}_{j1}} * \cdots * p_{\overline{X}_{jn}} * \delta(\overline{I}_{l,j0}) \tag{33}$$

As the PDF of interest is the PDF of absolute line current (34) has to be applied to the result of (33) in order to calculate the PDF of $|\bar{I}_{l,j}|$

$$p_{|\bar{I}_{l,j}|}\left(|\bar{I}_{l,j}|\right) = \int_{0}^{2\pi} \frac{p_{\bar{I}_{l,j}}\left(|\bar{I}_{l,j}| \cdot e^{j \cdot \delta}\right)}{2\pi \cdot |\bar{I}_{l,j}|} \cdot d\delta \qquad (34)$$

Summary

The presented algorithm possesses the following valuable features:

1. It allows for the computation of $p_{|\bar{I}_{l,j}|}$ for a given line j independently from all other lines.
2. It allows for the selection of the most important nodes by analysis of the model matrix $\underline{Y}_{nl}$ prior to the main computation. This allows reducing significantly the computational complexity.
3. For the nodes excluded from the computation an estimation of the introduced deviation can be given by analysing $\underline{Y}_{nl}$ together with the extent of the PDF of the excluded nodes.
4. The probability of each value of a complex-valued line current $\bar{I}_{l,j}$ can be calculated separately, allowing for the parallelization of the computation.

IV. Exemplary Results

In order to proof the validity of the presented approach examples computed for two test cases will be discussed in the following. The results computed using the presented approach will be verified against the reference results of a Monte-Carlo (MC) based algorithm. The first test case involved a radial network with 5 nodes, like sketched in Fig. 2.

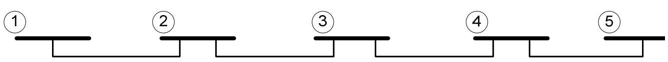

Fig. 2: Test network No. 1

The results show a good correlation to the reference MC results. This is true for the line next to the reference node (node number one) (see Fig. 3), as well as for one of the intermediate lines (see Fig. 4).

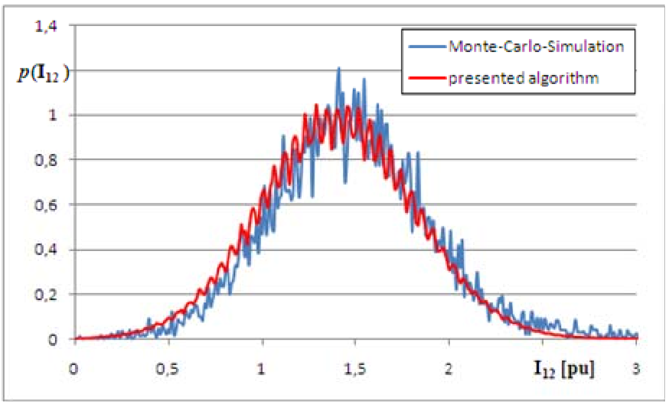

Fig. 3: Precision comparison test network 1, line 1-2

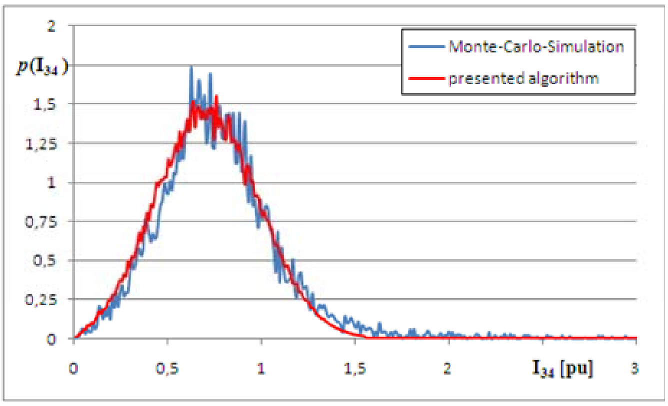

Fig. 4: Precision comparison test grid 1, line 3-4

Also the results for the second test case, based on the meshed network sketched in Fig. 5, show good correlation with the reference result (see Fig. 6).

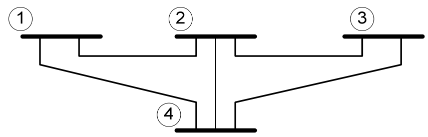

Fig. 5: Test network No. 2

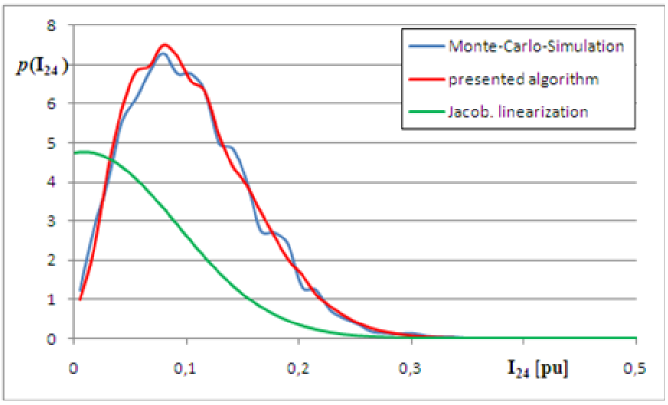

Fig. 6: Precision comparison meshed grid, line 2-4

To proof advancement to the state of the art in PLF computation, introduced by the presented approach, Fig. 6 also displays the result for $p_{\bar{I}_{24}}$ calculated with an algorithm based on the linearization of the Jacobian matrix as a network model. The severe deviation from the reference result is clearly visible. Not only the fact that there is a mayor deviation, but also that these deviations are not predictable is one of the main reasons for the limited practical usability of this class of algorithms.

V. Possible Applications

In network operation and network extension planning Probabilistic Load Flow computation provides much more complex information about the possible operational state of a network than conventional deterministic Load Flow computation. Instead of focusing on deterministic states caused by a selected number of scenarios, it provides an operator or planner with information about all possible states and their corresponding probability. The results not only show whether or not a certain extreme loading is possible, but also its probability and thus a mean to estimate the duration of appearance. It is then the operators or planners decision if any countermeasures are necessary or not. Keeping in mind that it is actually not an over-current that harms a line, but the produced and accumulated heat, it becomes clear that the information about the probability – and thus expected duration of appearance – is extremely valuable information.

Fig. 7 sketches the way the results of PLF computation have to be interpreted. As the result is a Probability Density Function, actual probability can only be determined for given intervals on the absolute line current axis. In the given example the probability of an absolute line current above 0.8 pu equals the area between the PDF and the line current's axis, starting from 0.8 pu. In this qualitative example the probability of a absolute line current of 0.8 pu and more equals to 20%, while the probability of an absolute line current of 1.1 pu and above is only 1%. It is now the operators or planners decision to intervene or to back up to short term mitigations in case an inacceptable line loading really occurs. Information about weather conditions might also be taken into account, as low temperature and high wind speed conditions improve the cooling of overhead lines.

Network expansion planning

In case of network expansion planning, the results can be used to assess the probability – and thus the average time of occurrence – of network contingencies. This information is of great importance for the investment decision as contingency costs can be estimated on the basis of the PLF results [4], [5]. Furthermore the most important nodes, strongly involved in the appearance of the contingency – can be identified by analysing the before mentioned matrix $\overline{Y}_{nl}$.

The so called N-1 criterion, used in conventional network extension planning can be included by the line current PDF of lines at risk for multiple, relevant network topologies. In this case it is necessary to take into account the probability of the selected topologies as well. By weighting the respectively resulting PDF with the probability of occurrence of the underlying topology, it is possible to estimate the average duration of overload even with consideration of line outages.

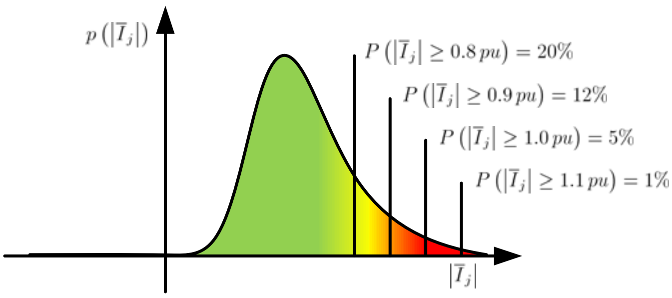

Fig. 7: Interpretation of the results

On-line operation

Another possible field of application is the calculation of the probability of line loading under the strong influence of stochastic power injection (e.g. feed-in from wind turbines). Wind forecasts can then be directly turned into line loading forecasts, helping the network operator to assess the risk of line overloads. It is then his decision whether or not he wants to limit the feed-in. For this decision the results of the PLF computation is the only qualified basis.

Apart from the influence of power injections with a stochastic character, PLF allows for the risk assessment of generator outages. As the probability of an outage can be easily modelled with the PDF of injected power, the consideration of generator outages is a natural feature of PLF computation. For networks without frequency control, the algorithm presented in this paper provides a fairly good approximation (see section IV). If a frequency control has to be modelled as well, it becomes a necessary extension to the algorithm presented in this paper to include the dependencies between generator feed-ins introduced by the frequency control mechanism.

VI. Conclusions

As in many other academic and technical fields, also in power systems the use of probabilistic mathematic is pushing forward. Two different main causes can be identified. On the one hand there is a huge pressure towards a more efficient usage of existing or future networks, while on the other hand the increasing usage of uncontrolled generation unit with a stochastic nature makes the power flow patterns more and more diverse and complex. The coincidence of both prepares the ground for PFL techniques in power systems.

The Probabilistic Load Flow problem was addressed over decades now, but previously available approaches either suffered from severe, hardly predictable deviations from the actual results, or their application was limited to extremely small networks. Main source of the deviations occurring in those algorithms applicable also to networks of a reasonable size are mainly introduced by network model simplifications.

In contrast to the previously presented approaches, the approach presented in this paper incorporates an exact linear model of the underlying network. The issue of non-linearity of the power flow equations is addressed by determining network states in terms of voltage and nodal current profiles, for which the corresponding complex-valued nodal power profile is determined in the very last step. This circumvents the lack of a function inverse to the power flow equations. While in this paper an approach using static voltage estimation was presented as an introduction into the field of PLF computation, an approach including a dynamic voltage profile computation is presently under verification and will be published soon.

VII. References

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