Linear Programming and Practicable Farm Plans—A Case Study in the Goondiwindi District, Queensland

by

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LINEAR PROGRAMMING AND PRACTICABLE FARM PLANS

INTRODUCTION

In the decade following the introduction of linear programming to problems in agriculture, an extensive body of literature has been built up. Candler & Musgrave (1960) and Musgrave (1963) cite references to material of this kind. Articles dealing with the problems of developing profit-maximizing plans for farms are numerous—for example those by Peterson (1955), Heady *et al.* (1956), Puterbaugh *et al.* (1957), McFarquhar & Evans (1957), Waring *et al.* (1963), and Camm & Rothlisberger (1965). Most of this literature, however, is concerned not so much with generation of practicable plans for individual farms, but rather with presenting solutions for average types of farms. Swanson (1961) summarizes this situation as follows:

> In spite of the voluminous list of agricultural "applications" of linear programming, one finds virtually no documentation of commercial applications . . . the solutions apply to typical (in most cases, hypothetical) farms and the principal purpose of the work has been to analyze relationships within the firm.

The lack of commercial applications has a number of possible explanations. Initially, many authors were merely attempting to fit the particular problems of the farm firm into the theoretical framework of the method. Heady (1954) is an early example. As well, extension workers who were the people closely concerned with everyday farm planning frequently did not have the background necessary to capitalize on the method, nor ready access to computers. A further complication was the evidence that formulation of individual farm plans was relatively costly in terms of the time and effort required to estimate input-output coefficients and for use of the computer itself.

In an attempt to minimize some of these difficulties, the standard or benchmark plan was put forward. A benchmark plan is an optimum plan for a hypothetical or average farm which is subsequently used as a basis for advice in particular farm situations. However, this approach only gave acceptable results when the farms involved were homogeneous with respect to resource supplies and technology and where the production functions for individual enterprises were approximately identical among farms. In practice such conditions do not normally prevail.

As a consequence of the lack of published studies in which practicable farm plans were presented, some people began to doubt the usefulness of the technique. Thus Clarke & Simpson (1959) put forward a "simpler" alternative, while Defries (1959) stated bluntly: "I am doubtful of the use of (such) elaborate mathematical tools in production economics".

An alternative and less pessimistic view is presented here: namely, that a major shortcoming of farm plans by linear programming has been incomplete specification of the problem. In turn this has led to recommendations which cannot be applied in practice. For example, the literature appears to include only two studies (Woodworth (1957) and Coutou & Bishop (1957)) in which treatment of the problem of soil heterogeneity is explicit and none in which planning is on a paddock by paddock basis. To suggest that activities can be allocated among paddocks or soil types after the programme has been run nullifies the whole purpose of the exercise. Admittedly such restrictions make more complex an already complex situation, but, as computer capacity and efficiency is constantly increasing, so can the programming of the farm situation be made more realistic.

If programming is to be of maximum practical use to extension and advisory workers, farm complexity and variability will need to be more completely specified than hitherto. The extension worker should also consider the initially computed farm plan as a draft subject to modification by discussion with the farmer and probably recomputation at least once before the final working plan is decided on.

This paper is concerned with some aspects of these problems. Specifically the paper includes soil types and paddock areas as restraints and indicates the necessity for discussion of successively more sensible draft plans with the farmer. This more realistic type of approach is one which extension workers will need to adopt if programming is to become a practical aid in advisory work.

Of course the cost and time required for this approach may well preclude its use by publicly financed extension services in isolated studies of farm planning. As Musgrave (1963) points out, the most expensive aspect of programming studies in an area is the accumulation of information and experience from which the basic programming matrix is constructed. It would be reasonable to expect that the cost of further studies in an area would rapidly diminish. Hence it is the authors' opinion that, if plans for a number of farms in a district could be generated by use of edited forms of the basic matrix, then this would be an economically feasible method of district farm planning.

The current study, in which an individual property is considered, was commenced early in 1964 and entails the use of a static linear programming model to generate a practicable farm plan for the ensuing twelve months. The analysis proceeds as follows: firstly the case study property is described and the problem delimited; next, activities and restraints are specified and a draft plan computed. Initially the restraints include soil type but not paddock acreages. The impracticability of such a plan is demonstrated even though the inclusion of soil type makes it more realistic than most plans found in the literature. Paddocks are then introduced as a restraint and the ensuing plan presented to the farmer for assessment before finally arriving at a practicable plan.

THE CASE STUDY PROPERTY

The property chosen, "Trevanna Downs",[1] is located in part of the "brigalow belt" approximately 30 miles north of Goondiwindi. Like most properties in this district, "Trevanna Downs" is characterized by the heterogeneous nature of its soils and the wide range of enterprises which are successful on these soils. This leads to a multiplicity of production opportunities. Hence the farm manager is faced with an extremely complex decision problem in determining an optimum allocation of scarce farm resources among alternative production processes.

At the commencement of this study, most of the property had been cleared of native scrub and 2,200 acres were available for immediate cultivation. An accompanying map, Figure 1, shows the range of soil types, their distribution and paddock layout in 1964. This map was constructed after identifying soil-production types and cultivation land on a scaled aerial photograph. Shadelines, wasteground, and subdivision fences were also located. A planimeter was then used to measure the acreage of soil types and paddocks.

Having established the land resources of the case property, the operator was then interviewed in order to qualify the level of production restraints. In addition, estimates of production coefficients for enterprises which had actually been carried out on any one of the six soil production types were determined. These data were supplemented with information provided by members of a district survey,[2] local technical experts, and also from market reports. In this way a relatively accurate description of all the production opportunities available to management was built up.

CONSTRUCTION OF MATRIX A

The case study property demonstrates some of the problems of realistic matrix construction. Activities and restraints are therefore discussed in some detail. As far as soil type is concerned, each type has a unique set of input-output relationships for alternate crops which are available to the operator. Thus in constructing the initial programming matrix (Matrix A) the acreage of each soil type acts as a resource restraint.

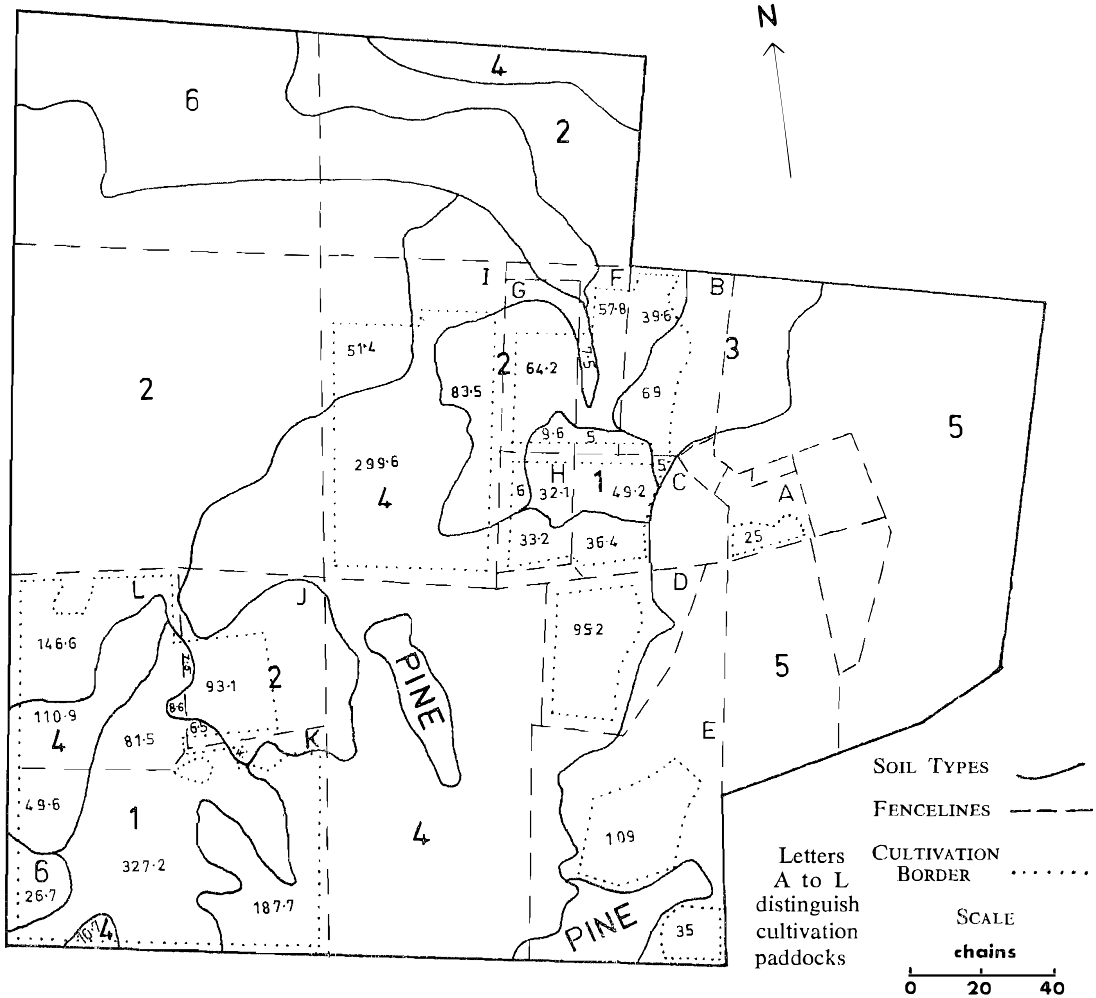
The programmed restraints

Five discrete soil production types were under cultivation on the case study property in 1964. (The cultivation portion of soil production type 6 has been amalgamated with that of soil production type 2 since these two soils have identical crop productivity once the gilgais have been removed from the former soil type.) The acreages of these soil types comprised the restraints R_1 to R_5.

In addition to cultivation land it was estimated that 5,494 acres of the property were under cleared native pasture in 1964. This land was subdivided into three

[1]The plans in this study relate to the 1964 planning period, and hence the original grazing homestead-block of 8,333 acres is considered. More recently the size of the property has been reduced by compulsory resumption of 1,000 acres.

[2]A survey of production enterprises on eight neighbouring properties was conducted in order to provide a more comprehensive pool of information from which the appropriate coefficients could be selected.



No.	Mapping Unit	Cultivated Acreage	Major Characteristics of Dominant Soil
1	Production type 1	519	Heavy dark grey self-mulching clays.
2	Production type 2	555	Weakly gilgaied brown clays with strongly acid subsoils.
3	Production type 3	74	Deep heavy grey clays with little profile differentiation.
4	Production type 4	730	Weakly solonized red brown earths.
5	Production type 5	169	Weakly solonized soils related to solodized-solonetz, red brown earths, and grey and brown soils of heavy texture.
6	Production type 6	27	Very strongly gilgaied grey clays with strongly acid subsoils.

Fig. 1.—Soil production types, paddock code, fencelines, and cultivation acreages—Trevanna Downs—May, 1964.

types on the basis of pasture productivity, and the acreage of each type constituted the restraints R_6 to R_8.

R_9. The maximum stud ewe restraint

The number of stud ewes was restricted to 500 as this was the maximum number that the operator could manage in the 1964 period.

R_{10}. The lucerne restraint

This restraint ensures that there will be adequate supply of lucerne feed available for those stock requiring it for strategic grazing purposes.

R_{11}. The maximum late grazing sorghum restraint

The operator has expressed a preference for a restricted acreage of this crop and has indicated that 160 acres would be the maximum he would be prepared to plant in any one year.

R_{12}. The maximum wheat restraint

Wheat is harvested with an auto-header capable of handling 600 bags daily. The safe harvesting period is considered to be twelve days so that an upper limit of 7,200 bags was placed on the annual wheat harvest.

R_{13}, R_{14}, and R_{15}. Tractor hour restraints

The plant capacity only becomes limiting from December to March. Restraints R_{13} to R_{15} constitute the level of available tractor hours in the January, February–March, and December periods respectively.

R_{16}. Supplementary sheep units

It is the general consensus of opinion of graziers in the Goondiwindi district that during the summer flush, from October to April, approximately five breeding cows can be run to every 100 breeding ewes (or equivalent dry sheep) without active competition for feed. During the winter months, when less tall feed is available, this ratio falls to 3 per cent. This relationship was expressed in the R_{16} restraint.

R_{17}. Arable type 2 wheat supply

This restraint was specified so that adequate wheat would be available to act as a zero cost cover crop for lucerne on this land type.

R_{18} to R_{27}. Feed restraints

The supply of forage by crops and pastures and the demands for forage by livestock were all expressed in Dry Merino Ewe equivalents (D.M.E.s).[3] The feed year was divided into four equal periods which roughly coincided with the four seasons of the pasture year. In addition three feed pools were established in order to differentiate crops and pastures according to the characteristics of forage supplied for livestock production.

The first or so-called "transferable" feed pool collects all forage from perennial crops and pastures. A characteristic of forage supplied to this pool is that it is freely transferable at the cost of some loss in nutritional value from the period in which it is produced to a future period. Livestock do not consume directly from this pool but all feed, after inter-period transfers, is supplied to a separate pool called the "consumption" pool.

As far as this study is concerned, the consumption requirements of livestock have been broken up into two components:

(i) a requirement of oats forage for special-purpose grazing;

(ii) a requirement of forage of at least maintenance quality for general-purpose grazing.

[3] One D.M.E. is defined as the energy requirement of an adult merino ewe, neither pregnant nor lactating, for normal maintenance and wool growth over a one-month period.

Separate consumption pools have been established for these two feed components. The "oats" feed pool collects all forage from grazing oats crops and supplies the need of livestock activities requiring special-purpose grazing.

The general-purpose consumption requirements of livestock are met from the so-called "consumption" pool. This pool collects forage supplies from annual crops (excepting grazing oats during the May–September period) as well as transfers from the "transferable" feed pool. No transfer activities operate within this pool as forage from annual crops usually has zero substitutability with respect to time.

The restraints R_{18} to R_{21} relate to the levels of supply of forage in the "transferable" feed pool in the four periods of the feed year. R_{22} and R_{23} relate to the levels of supply of special-purpose forage in the "oats" feed pool in the May–June and July–September periods respectively. R_{24} and R_{27} relate to the levels of supply of forage in the "consumption" pool in the feed-year periods.

R_{28}. The grazing sorghum supply restraint

This restraint ensures an adequate supply of sorghum forage for the "crop wether" activity which requires four months of crop forage during the April to September period.

The activities considered

A range of no more than six alternative crops was considered for each of the five soil types under cultivation on the property in 1964. The relevant crops were wheat, early grazing oats, late grazing oats, early grazing sorghum, late grazing sorghum, and lucerne.

The first three activities considered, X_1 to X_3, were grain wheat activities on soil types 1, 2, and 3 respectively.

Activities X_4 to X_8 represent early-season grazing oats on soil types 1 to 5 respectively while activities X_9 to X_{13} represent late-season grazing oats on the same soil types.

October-planted forage sorghum or so-called "early grazing sorghum" on soil types 1, 2, 4, and 5 respectively is represented by activities X_{14} to X_{17}. This crop has a three-year cycle on soil types 1, 2, and 4, on which it readily produces ratoon growth, but only a two-year cycle on soil type 5 where the ratoon stand is not successful.

The next four activities X_{18} to X_{21} represent January-planted or "late grazing sorghum" on soil types 1, 2, 4, and 5 respectively. This crop is normally planted with the auxiliary tractor so that activities X_{18} to X_{21} are not competitive for tractor hours in January.

Activities X_{22} to X_{26} represent lucerne-growing activities on soil types 1 to 5 respectively. A preliminary investigation showed that wheat production on soil types 1 and 2 is optimum in the programmed solution and hence wheat is considered to provide a zero cost cover crop for lucerne activities on these two soil types. In contrast, wheat production alone is not economically justified on soil types 3 and 4. On these soils the lucerne activities (X_{24} and X_{25}), by definition, include wheat as an initial cover crop. This practice seems reasonable as it allows the operator to realize a net profit instead of incurring a cost in the year of sowing. A cover crop is not specified for lucerne on soil type 5 because of the unsuitability of this soil for wheat production.

Land under cleared native pasture was subdivided into three types, namely X, Y, and Z, on the basis of pasture productivity. Activities X_{27} to X_{29} respectively refer to pasture activities on these three land types. The production coefficients used for each pasture type represent the seasonal feed productivity of the pastures and not a measure of annual production under some specific form of pasture management.

The next ten activities, X_{30} to X_{39}, are feed-transfer activities and relate to transfer of forage within and among the three feed pools previously defined. The first two of these activities, X_{30} and X_{31}, allow transfer of oats forage from the "oats" feed pool to the "consumption" pool. The remaining feed-transfer activities were specified so that the computing routine would be enabled to select the optimum form of pasture management for the property.

Activities X_{32} to X_{35} allow transfer of feed within the "transferable" feed pool from each of the four periods of the feed year to the following period at a loss of nutrient value. A nutrient decline of 35 per cent was assumed for transfers into a frost-free period and a decline of 50 per cent assumed for transfers into a period of frost incidence.[4]

In order to relate the "transferable" feed pool, with its associated inter-period transfer activities, to the consumption requirements of livestock, four additional activities (X_{36} to X_{39}) were specified to allow the transfer of feed from any period in the "transferable" feed pool to the corresponding period in the "consumption" pool.

Matrix A was completed by specifying nine livestock activities. The nutrient requirements of all livestock were expressed in Dry Merino Ewe equivalents, one D.M.E. being set at 36 lb of total digestible nutrients.[5]

The first livestock activity, X_{40}, represents flock breeding sheep and one unit of this activity is taken to be a Poll merino breeding ewe and her normal supporting stock—approximately 3 per cent rams, all lambs, and 2-tooths. It is implicit in this vector that ewes and lambs are grazed on oats from May until October, that ewes and weaners cut 12 lb of wool, and that lambs cut 4 lb.

X_{41} represents the stud Poll merino ewe activity which is defined similarly to the flock ewe activity.

The next two activities X_{42} and X_{43} are Poll merino wether activities. In X_{42} it is assumed that wethers are grazed on natural pastures throughout the year and cut an average of 12.5 lb of wool per head. In contrast, X_{43} assumes that wethers are given access to forage sorghum crops for four months during the winter and consequently cut an average of 15 lb of wool per head.

X_{44} represents breeding cattle which are fully competitive with sheep as regards feed requirements. One unit of this activity is taken as a Hereford breeding cow and her normal supporting stock—approximately 4 per cent bulls, all calves, weaners, steers, and heifers up to twenty-four months. It is assumed that all steers and 70 per cent of heifers are fattened on oats and sold at twenty-four months. The remaining heifers replace cast-for-age breeders.

X_{45} represents vealer production which is also fully competitive with sheep as regards feed. One unit of this activity is assumed to be a Hereford breeding cow plus 4 per cent bulls, and all calves, weaners, and carryover stock up to twenty-four months of age. It is assumed that all weaners are fattened on oats but that only 75 per cent reach sale condition in twelve months. The carryover stock, with the exception of replacement heifers, are sold fat at twenty-four months.

The majority of graziers interviewed in the Goondiwindi district estimated that the border between supplementary and competitive range of cattle grazing in

[4]For a discussion on the nutritional value of subtropical pastures see R. Milford, "Nutritional values for 17 subtropical grasses", *Australian Journal of Agricultural Research*, XI (1960), 138.

[5]The nutrient requirements for cattle and sheep activities were taken from: Committee on Animal Nutrition, *Beef Cattle* and *Sheep* (*Nutrient Requirements of Domestic Animals*, Nos. 4 and 5 [Washington D.C.: National Academy of Sciences—National Research Council, 1959]).

R	Activities X	B. Column	Wheat Type 1	Wheat Ty. 2	Wheat Ty. 3	Early Oats Ty. 1	E. Oats Ty. 2	E. Oats Ty. 3	E. Oats Ty. 4	E. Oats Ty. 5	Late Oats Ty. 1	L. Oats Ty. 2	L. Oats Ty. 3	L. Oats Ty. 4	L. Oats Ty. 5	E. Grz. Sorgh. Ty. 1	E. Grz. Sorgh. Ty. 2	E. Grz. Sorgh. Ty. 4	E. Grz. Sorgh. Ty. 5	L. Grz. Sorgh. Ty. 1	L. Grz. Sorgh. Ty. 2	L. Grz. Sorgh. Ty. 4	L. Grz. Sorgh. Ty. 5	Lucerne Ty. 1	Lucerne Ty. 2	Lucerne Ty. 3
	Activities X		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	Revenue C		12·74	9·68	8·17	−1·86	−1·86	−1·86	−1·86	−1·86	−2·1	−2·1	−2·1	−2·1	−2·1	−·42	−·42	−·42	−·5	−·54	−·54	−·54	−·56	−·14	−·14	1·87
1	Arable type 1	519	1			1					1					1				1				1		
2	Arable type 2	582		1			1					1					1				1				1	
3	Arable type 3	74			1			1					1													1
4	Arable type 4	730							1					1				1				1				
5	Arable type 5	169								1					1				1				1			
6	Pasture X	1875																								
7	Pasture Y	762																								
8	Pasture Z	2857																								
9	Maximum stud ewes	500																								
10	Lucerne restraint	0																						−24·1	−24·1	−14·3
11	Max. late grz. sorgh.	160																		·33	·33	·33	·5			
12	Wheat max.	7200	10·0	8·0	7·0																					1·75
13	January tractor	240	·143	·143	·143	·143	·143	·143	·143	·143	·286	·286	·286	·286	·286											·036
14	Feb.-Mar. tractor	480	·528	·528	·528	·235	·235	·235	·235	·235	·256	·256	·256	·256	·256											·132
15	December tractor	240	·16	·16	·16	·303	·303	·303	·303	·303	·16	·16	·16	·16	·16					·048	·048	·048	·072			·04
16	Supplementary sheep unit	0																								
17	Arable 2 wheat supply	0		−1																					0·2	
	"Transferable" feed																									
18	Jan.-Mar. D. M. E.	0																						−12·7	−12·7	−6·3
19	Apr.-June D. M. E.	0																						−6·4	−6·4	−3·2
20	July-Sept. D. M. E.	0																						−5·1	−5·1	−2·5
21	Oct.-Dec. D. M. E.	0																						−5·5	−5·5	−2·3
	"Oats" feed																									
22	May-June D. M. E.	0				−17·0	−17·0	−15·7	−11·4	−8·5																
23	July-Sept. D. M. E.	0				−25·5	−25·5	−23·0	−17·1	−12·8	−27·0	−27·0	−24·3	−18·1	−13·5											
	"Consumption" feed																									
24	Jan.-Mar. D. M. E.	0														−7·3	−7·3	−3·7	−1·8	−3·7	−3·7	−1·8				
25	Apr.-June D. M. E.	0														−18·6	−18·6	−9·3	−4·7	−21·7	−21·7	−10·9	−6·2			
26	July-Sept. D. M. E.	0																		−6·2	−6·2	−3·1	−3·1			
27	Oct.-Dec. D. M. E.	0	−6·0	−4·8	−4·2						−9·3	−9·3	−8·4	−6·2	−4·7	−7·3	−7·3	−3·7	−1·8	−7·3	−7·3	−3·7	−1·8			
28	Grz. sorgh. supply	0														−18·6	−18·6	−9·3	−4·7	−27·9	−27·9	−14	−9·3			

Fig. 2—Matrix A

R		Activities X	25 Lucerne Ty. 4	26 Lucerne Ty. 5	27 Native Pasture X	28 Nat. Pasture Y	29 Nat. Pasture Z	30 May-June Oat Transfer	31 July-Sept. Oat Tf.	32 Oct.-Dec. Feed Tf.	33 Jan.-Mar. Feed Tf.	34 Apr.-June Feed Tf.	35 July-Sept. Feed Tf.	36 Oct.-Dec. Consump. Tf.	37 Jan.-Mar. Consump. Tf.	38 Apr.-June Consump. Tf.	39 July-Sept. Consump. Tf.	40 Flock Breed. Ewes	41 Stud Br. Ewes	42 Wethers	43 Crop Wethers	44 Fully Compet. Br. Cows	45 Fully Compet. Vealers	46 Part. Compet. Br. Cows	47 Part. Compet. Vealers	48 Crop Fatteners
		Revenue C	1·1	–·57	–·11	–·11	–·11	0	0	0	0	0	0	0	0	0	0	7·23	12·88	2·19	2·87	29·7	26·38	30·05	26·73	11·25
R		B. Column																								
1	Arable type 1	519																								
2	Arable type 2	582																								
3	Arable type 3	74																								
4	Arable type 4	730	1																							
5	Arable type 5	169		1																						
6	Pasture X	1875			1																					
7	Pasture Y	762				1																				
8	Pasture Z	2857					1																			
9	Maximum stud ewes	500																	1							
10	Lucerne restraint	0	–14·3	–9·5														5·1	5·1							
11	Max. late grz. sorgh.	160																								
12	Wheat max.	7200	1·25																							
13	January tractor	240	·036	·036																						
14	Feb.-Mar. tractor	480	·132	·063																						
15	December tractor	240	·04	·036																						
16	Supplementary sheep unit	0																–1	–1	–·33	–·33			20	20	
17	Arable 2 wheat supply	0																								
	"Transferable" feed																									
18	Jan.-Mar. D. M. E.	0	–6·3	–5·0	–4·9	–4·9	–4·9			–·65	1				1											
19	Apr.-June D. M. E.	0	–3·2	–2·5	–3·3	–1·7	–1·3				–·65	1				1										
20	July-Sept. D. M. E.	0	–2·5	–2·0	–1·7	–1·3	–1·1					–·5	1				1									
21	Oct.-Dec. D. M. E.	0	–2·3	–1·8	–2·2	–1·7	–1·7			1			–·65	1												
	"Oats" feed																									
22	May-June D. M. E.	0						1										1·7	1·5			12·8	13·6	12·8	13·6	9·3
23	July-Sept. D. M. E.	0							1									5·9	5·3			29·6	25·6	29·6	25·6	27·9
	"Consumption" feed																									
24	Jan.-Mar. D. M. E.	0													–1			7·7	7·3	3·3	3·3	60·0	46·6			
25	Apr.-June D. M. E.	0						–1								–1		7·4	7·0	3·3	3·3	48·4	36·9	19·4	14·8	
26	July-Sept. D. M. E.	0							–1								–1	5·4	5·2	3·3	3·3	32·1	32·0	12·8	12·8	
27	Oct.-Dec. D. M. E.	0												–1				9·9	9·3	3·3	3·3	58·3	46·4			
28	Grz. sorgh. supply	0																			4·4					

Fig. 2—Matrix A (*Continued*)

association with sheep is 5 per cent during October to March and 3 per cent during April to September. Consequently, it would be feasible to run a limited number of either breeding cows or vealer mothers in a supplementary relationship with sheep in addition to X_{44} and X_{45} which were assumed to be fully competitive activities. X_{46} and X_{47} representing "partially competitive" breeding cows and "partially competitive" vealers respectively were specified such that the upper limit of these activities was set at 5 per cent of the breeding ewe numbers (or 1.67 per cent of the wether numbers). At this level X_{46} and X_{47} only become competitive for pasture feed in the April–September period and even then 60 per cent of their pasture feed requirements can be supplied without diminishing the amount of feed available for sheep activities.

The final activity X_{48} represents crop fattening. This activity entails the purchase of thirty-month-old store cattle during May, June, and July, followed by intensive grazing on oats and sale of fat cattle in September and early October.

An arithmetic description of Matrix A is included in Figure 2. Only non-zero elements are shown in the body of the matrix.

TABLE 1

PLAN 1—THE PROGRAMMED SOLUTION FROM MATRIX A*

Activity	*Unit*	*Activity Level*
X_1 Wheat type 1	Acre	519.0
X_2 Wheat type 2	Acre	101.5
X_6 Early oats type 3	Acre	74.0
X_7 Early oats type 4	Acre	149.9
X_{12} Late oats type 4	Acre	68.6
X_{13} Late oats type 5	Acre	169.0
X_{16} Early grazing sorghum type 4	Acre	216.6
X_{19} Late grazing sorghum type 2	Acre	480.5
X_{25} Wheat sown lucerne type 4	Acre	294.9
X_{27} Native pasture X	Acre	1,875.0
X_{28} Native pasture Y	Acre	762.0
X_{29} Native pasture Z	Acre	2,857.0
X_{30} May–June oat transfer	D.M.E.	191.3
X_{33} January–March feed transfer	D.M.E.	13,268.4
X_{34} April–June feed transfer	D.M.E.	13,491.6
X_{36} October–December consumption transfer	D.M.E.	10,768.7
X_{37} January–March consumption transfer	D.M.E.	15,641.8
X_{38} April–June consumption transfer	D.M.E.	7,277.8
X_{39} July–September consumption transfer	D.M.E.	14,731.2
X_{40} Flock sheep	Breeding ewe	321.0
X_{41} Stud sheep	Breeding ewe	500.0
X_{42} Wethers	Wether	155.5
X_{43} Crop wethers	Wether	3,504.5
X_{46} Partially competitive cattle	Breeding cow	101.4
X_{48} Crop fatteners	Steer	8.7
Surplus Resources		
R_{12} Wheat maximum	Bag	829.2
R_{15} January tractor	Hour	40.7
R_{17} Arable type 2 wheat supply	Acre	101.5
Revenue†	£28,355.1	

* Matrix A was submitted for computation to the G.E. 225 electronic computer at the University of Queensland. Solution was reached in approximately six minutes.

† The revenue from plan is found by multiplying together the "revenue" of each activity with the level to which that activity is represented in the plan and then summing over all activities. The "revenue" from any activity does not include an allowance for fixed costs such as rents, rates, insurance, depreciation, or any other charges which are unaffected by the level at which the activity is carried out.

THE PLAN FROM MATRIX A

The programmed solution to Matrix A, plan 1, is included in Table 1. This plan represents an optimum farm plan for the case study property, given that the location of crop-production activities need only be restrained by soil-type distribution. If plan 1 was fitted to the property, according to the soil-type boundaries marked on Figure 1, insurmountable difficulties would arise in attempting to put it into operation.

This can be well illustrated by considering activity X_1, which represents wheat production on the 519 acres of soil type 1 under cultivation. This soil type occurs in parts of paddocks B, C, F, G, H, I, K, and L. Should wheat be grown on these areas it would be impractical to use these paddocks for grazing purposes. In other words, land under wheat should be fenced separately from that used for grazing. To erect subdivision fences on each of the eight paddocks on which soil type 1 occurs would scarcely be practicable from a managerial viewpoint and, furthermore, the cost of this fencing would most certainly negate the economic advantage of the computed crop distribution.

Similar problems arise if an attempt is made to fit any of the other crop activities into the property organization in the manner specified in plan 1. Clearly the inclusion of soil-type restraints will not result in practicable farm plans except where soil-type boundaries and fencelines coincide. This is an unlikely situation in practice.

Hence, in order to generate practicable farm plans, it seems necessary that crop-production activities should be restrained by the size and location of cultivation paddocks as well as by soil-type distribution. In the present study this step necessitated the construction of a new matrix (Matrix B), in which acreage restraints from crop-production activities were specified for *each* cultivation paddock. The way in which this was done is now described in some detail. In addition, an arithmetic description of Matrix B is included in Figure 3.

CONSTRUCTION OF MATRIX B

The programmed restraints

Twelve cultivation paddocks, distinguished by letters A to L in Figure 1, were available for cropping in the 1964 planning period. Restraints R_1 to R_{12} represent the acreages of arable land in these paddocks.

Restraints R_{13} to R_{23} in Matrix B are defined identically with restraints R_6 to R_{16} of Matrix A. In addition the feed restraints R_{24} to R_{34} of Matrix B are identical with the corresponding feed restraints, R_{18} to R_{28} of Matrix A.

The activities considered

Six alternative crops were considered for each cultivation paddock. These crops were wheat, early grazing oats, late grazing oats, early grazing sorghum, late grazing sorghum, and lucerne.

The vector of a particular crop activity on a particular paddock represents the production process that would be operative if the whole of the paddock was committed to that crop. This vector is derived from weighting the relevant crop-production process for each soil type in the paddock, according to the proportion of the paddock which is made up of that soil type, and then summing over all soil types in the paddock. In other words the new paddock-crop production processes are weighted linear combinations of the soil-type processes of Matrix A. The procedure used to determine these vectors is elaborated by Rickards & Musgrave (1965) in a recent article.

R		B Column	Early Oats Pdck. A	Late Oats Pdck. A	Wheat Pdck B	E. Oats Pdck. B	L. Oats Pdck. B	Lucerne Pdck. B	E. Oats Pdck. C	L. Oats Pdck. C	E. Grz. Sorgh. Pdck. C	L. Grz. Sorgh. Pdck. C	Lucerne Pdck. C	Wheat Pdck. D	E. Oats Pdck. D	L. Oats Pdck. D	E. Grz. Sorgh. Pdck. D	L. Grz. Sorgh. Pdck. D	Lucerne Pdck. D	E. Oats Pdck. E	L. Oats Pdck. E	E. Grz. Sorgh. Pdck. E	L. Grz. Sorgh. Pdck. E
		Activities P	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
		Revenue C	-1·86	-2·1	8·58	-1·86	-2·1	-·38	-1·86	-2·1	-·42	-·54	-·46	5·09	-1·86	-2·1	-·42	-·54	-·57	-1·86	-2·1	-·5	-·56
1	Paddock A	25	1	1																			
2	Paddock B	108·6			1	1	1	1															
3	Paddock C	90·6							1	1	1	1	1										
4	Paddock D	95·2												1	1	1	1	1	1				
5	Paddock E	144																		1	1	1	1
6	Paddock F	70·3																					
7	Paddock G	73·8																					
8	Paddock H	71·3																					
9	Paddock I	434·5																					
10	Paddock J	107·1																					
11	Paddock K	505·9																					
12	Paddock L	347·6																					
13	Pasture X	1875																					
14	Pasture Y	762																					
15	Pasture Z	2857																					
16	Maximum stud ewes	500																					
17	Lucerne restraint	0						-20·1					-16·8						-12·0				
18	Max. late grz. sorgh.	160										·33						·33					·5
19	Wheat max.	7200			7·27									5									
20	January tractor	240	·143	·286	·143	·143	·286	·024	·143	·286			·029	·143	·143	·286			·036	·143	·286		
21	Feb.-Mar. tractor	480	·235	·256	·528	·235	·256	·042	·235	·256			·051	·528	·235	·256			·063	·235	·256		
22	December tractor	240	·303	·16	·16	·303	·16	·024	·303	·16		·048	·029	·16	·303	·16		·048	·036	·303	·16		·072
23	Supplementary sheep unit	0																					
	"Transferable feed"																						
24	Jan.-Mar. D. M. E.	0						-10·6					-8·7						-6·3				
25	Apr.-June D. M. E.	0						-5·3					-4·5						-3·2				
26	July-Sept. D. M. E.	0						-4·2					-3·6						-2·5				
27	Oct.-Dec. D. M. E.	0						-4·6					-3·7						-2·3				
	"Oats feed"																						
28	May-June D. M. E.	0	-8·5			-16·1			-14·7						-11·4					-8·5			
29	July-Sept. D. M. E.	0	-12·8	-13·5		-23·7	-25		-22	-23·3					-17·1	-18·1				-12·8	-13·5		
	"Consumption feed"																						
30	Jan.-Mar. D. M. E.	0									-5·8	-2·9					-3·7	-1·8				-1·8	
31	Apr.-June D. M. E.	0									-14·9	-17·4					-9·3	-10·9				-4·7	-6·2
32	July-Sept. D. M. E.	0										-5·0						-3·1					-3·1
33	Oct.-Dec. D. M. E.	0		-4·7	-4·4		-8·6			-8	-5·8	-5·8		-3·0		-6·2	-3·7	-3·7			-4·7	-1·8	-1·8
34	Grz. sorgh. supply	0									-14·9	-22·4					-9·3	-14				-4·7	-9·3

Fig. 3—Matrix B

R		B Column	Lucerne Pdck. E	Wheat Pdck. F	E. Oats Pdck. F	L. Oats Pdck. F	E. Grz. Sorgh. Pdck. F	L. Grz. Sorgh. Pdck. F	Lucerne Pdck. F	E. Oats Pdck. G	L. Oats Pdck. G	E. Grz. Sorgh. Pdck. G	L. Grz. Sorgh. Pdck. G	Lucerne Pdck. G	Wheat Pdck. H	E. Oats Pdck. H	L. Oats Pdck. H	E. Grz. Sorgh. Pdck. H	L. Grz. Sorgh. Pdck. H	Lucerne Pdck. H	Wheat Pdck. I	E. Oats Pdck. I	L. Oats Pdck. I
		Activities P	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
		Revenue C	-·57	9·73	-1·86	-2·1	-·42	-·54	-·38	-1·86	-2·1	-·42	-·54	-·38	8·91	-1·86	-2·1	-·42	-·54	-·46	6·51	-1·86	-2·1
1	Paddock A	25																					
2	Paddock B	108·6																					
3	Paddock C	90·6																					
4	Paddock D	95·2																					
5	Paddock E	144	1																				
6	Paddock F	70·3		1	1	1	1	1	1														
7	Paddock G	73·8								1	1	1	1	1									
8	Paddock H	71·3													1	1	1	1	1	1			
9	Paddock I	434·5																			1	1	1
10	Paddock J	107·1																					
11	Paddock K	505·9																					
12	Paddock L	347·6																					
13	Pasture X	1875																					
14	Pasture Y	762																					
15	Pasture Z	2857																					
16	Maximum stud ewes	500																					
17	Lucerne restraint	0	-9·5						-19·0					-20·1						-16·2			
18	Max. late grz. sorgh.	160						·33					·33						·33				
19	Wheat max.	7200		7·8											7·5						5·9		
20	January tractor	240	·036	·143	·143	·286			·024	·143	·286			·024	·143	·143	·286			·029	·143	·143	·286
21	Feb.-Mar. tractor	480	·063	·528	·235	·256			·042	·235	·256			·042	·528	·235	·256			·051	·528	·235	·256
22	December tractor	240	·036	·16	·303	·16		·048	·024	·303	·16		·048	·024	·16	·303	·16		·048	·029	·16	·303	·16
23	Supplementary sheep unit	0																					
	"Transferable feed"																						
24	Jan.-Mar. D. M. E.	0	-5·0						-18·0					-10·6						-8·3			
25	Apr.-June D. M. E.	0	-2·5						-5·0					-5·3						-4·4			
26	July-Sept. D. M. E.	0	-2·0						-4·0					-4·2						-3·5			
27	Oct.-Dec. D. M. E.	0	-1·8						-4·4					-4·6						-3·6			
	"Oats feed"																						
28	May-June D. M. E.	0			-16·4					-17·0						-14·4						-13·1	
29	July-Sept. D. M. E.	0			-24·6	-26·0				-25·5	-27·0					-21·6	-22·5					-19·7	-20·9
	"Consumption feed"																						
30	Jan.-Mar. D. M. E.	0					-6·9	-3·5				-7·3	-3·7					-5·6	-2·8				
31	Apr.-June D. M. E.	0					-17·6	-20·6				-18·6	-21·7					-14·3	-16·7				
32	July-Sept. D. M. E.	0						-5·9					-6·2						-4·7				
33	Oct.-Dec. D. M. E.	0		-4·7		-9·0	-6·9	-6·9			-9·3	-7·3	-7·3		-5·0		-7·9	-5·6	-5·6		-3·6		-7·2
34	Grz. sorgh. supply	0					-17·6	-26·5				-18·6	-27·9					-14·3	-21·4				

Fig. 3—Matrix B (*Continued*)

(*Continued on next two pages*)

R		B Column	E. Grz. Sorgh. Pdck. I	L. Grz. Sorgh. Pdck. I	Lucerne Pdck. I	E. Oats Pdck. J	L. Oats Pdck. J	E. Grz. Sorgh. Pdck. J	L. Grz. Sorgh. Pdck. J	Lucerne Pdck. J	Wheat Pdck. K	E. Oats Pdck. K	L. Oats Pdck. K	E. Grz. Sorgh. Pdck. K	L. Grz. Sorgh. Pdck. K	Lucerne Pdck. K	Wheat Pdck. L	L. Oats Pdck. L	E. Grz. Sorgh. Pdck. L	L. Grz. Sorgh. Pdck. L	Lucerne Pdck. L	Native Pasture X	Nat. Pasture Y
	Activities P		43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
	Revenue C		−·42	−·54	−·57	−1·86	−2·1	−·42	−·54	−·38	10·32	−1·86	−2·1	−·42	−·54	−·38	8·9	−2·1	−·42	−·54	−·38	−·11	−·11
1	Paddock A	25																					
2	Paddock B	108·6																					
3	Paddock C	90·6																					
4	Paddock D	95·2																					
5	Paddock E	144																					
6	Paddock F	70·3																					
7	Paddock G	73·8																					
8	Paddock H	71·3																					
9	Paddock I	434·5	1	1	1																		
10	Paddock J	107·1				1	1	1	1	1													
11	Paddock K	505·9									1	1	1	1	1	1							
12	Paddock L	347·6															1	1	1	1	1		
13	Pasture X	1875																				1	
14	Pasture Y	762																					1
15	Pasture Z	2837																					
16	Maximum stud ewes	500																					
17	Lucerne restraint	0			−15·3					−20·1						−17·3					−17·3		
18	Max. late grz. sorgh.	160		·33					·33						·33					·33			
19	Wheat max.	7200									8·4						7·5						
20	January tractor	240			·036	·143	·286			·024	·143	·143	·286			·024	·143	·286			·024		
21	Feb.-Mar. tractor	480			·063	·235	·256			·042	·528	·235	·256			·042	·528	·256			·042		
22	December tractor	240		·048	·036	·303	·16		·048	·024	·16	·303	·16		·048	·024	·16	·16		·048	·024		
23	Supplementary sheep unit	0																					
	"Transferable feed"																						
24	Jan.-Mar. D. M. E.	0			−17·9					−10·6						−9·0					−3·9	−4·9	−4·9
25	Apr.-June D. M. E.	0			−3·9					−5·3						−4·6					−4·6	−3·3	−1·7
26	July-Sept. D. M. E.	0			−3·1					−4·2						−3·7					−3·8	−1·7	−1·3
27	Oct.-Dec. D. M. E.	0			−2·7					−4·6						−4·0					−3·9	−2·2	−1·7
	"Oats feed"																						
28	May-June D. M. E.	0				−17·0						−15·4											
29	July-Sept. D. M. E.	0				−25·5	−27·0					−23·0	−24·4					−24·2					
	"Consumption feed"																						
30	Jan.-Mar. D. M. E.	0	−4·9	−2·4				−7·3	−3·7					−6·3	−3·1				−6·1	−3·1			
31	Apr.-June D. M. E.	0	−12·2	−14·2				−18·6	−21·7					−15·9	−18·5				−15·6	−18·2			
32	July-Sept. D. M. E.	0		−4·1					−6·2						−5·3					−5·2			
33	Oct.-Dec. D. M. E.	0	−4·9	−4·9			−9·3	−7·3	−7·3		−5·0		−8·4	−6·3	−6·3		−4·5	−8·3	−6·1	−6·1			
34	Grz. sorgh. supply	0	−12·2	−18·3				−18·6	−27·9					−15·9	−23·8				−15·6	−23·4			

R		B Column	Nat. Pasture Z	May-June Oat Transfer	July-Sept. Oat Tf.	Oct.-Dec. Feed Tf.	Jan.-Mar. Feed Tf.	Apr.-June Feed Tf.	July-Sept. Feed Tf.	Oct.-Dec. Consump. Tf.	Jan.-Mar. Consump. Tf.	Apr.-June Consump. Tf.	July-Sept. Consump. Tf.	Flock Breed. Ewes	Stud Breed. Ewes	Wethers	Crop Wethers	Fully Compet. Breed. Cows	Fully Compet. Vealers	Part. Compet. Breed. Cows	Part. Compet. Vealers	Crop Fatterners
		Activities P	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83
		Revenue C	-·11	0	0	0	0	0	0	0	0	0	0	7·23	12·88	2·19	2·87	29·7	26·38	30·05	26·73	11·25
1	Paddock A	25																				
2	Paddock B	108·6																				
3	Paddock C	90·6																				
4	Paddock D	95·2																				
5	Paddock E	144																				
6	Paddock F	70·3																				
7	Paddock G	73·8																				
8	Paddock H	71·3																				
9	Paddock I	434·5																				
10	Paddock J	107·1																				
11	Paddock K	505·9																				
12	Paddock L	347·6																				
13	Pasture X	1875																				
14	Pasture Y	762																				
15	Pasture Z	2857	1																			
16	Maximum stud ewes	500													1							
17	Lucerne restraint	0												5·1	5·1							
18	Max. late grz. sorgh.	160																				
19	Wheat max.	7200																				
20	January tractor	240																				
21	Feb.-Mar. tractor	480																				
22	December tractor	240																				
23	Supplementary sheep unit	0												-1	-1	-·33	-·33			20	20	
	"Transferable feed"																					
24	Jan.-Mar. D. M. E.	[illegible]	-4·9			-·65	1				1											
25	Apr.-June D. M. E.	0	-1·3				-·65	1				1										
26	July-Sept. D. M. E.	0	-1·1					-0·5	1				1									
27	Oct.-Dec. D. M. E.	0	-1·7			1			-·65	1												
	"Oats feed"																					
28	May-June D. M. E.	0		1										1·7	1·5			12·8	13·6	12·8	13·6	9·3
29	July-Sept. D. M. E.	0			1									5·9	5·3			29·6	25·6	29·6	25·6	27·9
	"Consumption feed																					
30	Jan.-Mar. D. M. E.	0									-1			7·7	7·3	3·3	3·3	6[illegible]·0	46·6			
31	Apr.-June D. M. E.	0		-1								-1		7·4	7·[illegible]	3·3	3·3	48·4	36·9	19·4	14·8	
32	July-Sept. D. M. E.	0			-1								-1	5·4	5·2	3·3	3·3	32·1	32·0	12·8	12·8	
33	Oct.-Dec. D. M. E.	0								-1				9·9	9·3	3·3	3·3	58·3	46·4			
34	Grz. sorgh. supply	0															4·4					

Fig. 3—Matrix B (*Continued*)

While a full complement of alternative crops was initially considered for each cultivation paddock, discussions with the farm operator revealed that eleven of these alternatives were agronomically infeasible. After deleting these, sixty-one crop-paddock alternatives remained and these were specified in activities P_1 to P_{61}.

Matrix B was completed by specifying three native pasture activities P_{62} to P_{64}, ten feed transfer activities P_{65} to P_{74} and nine livestock activities P_{75} to P_{83}. These 22 activities, P_{62} to P_{83}, are defined as for activities X_{27} to X_{48} in Matrix A.

THE PLAN FROM MATRIX B

The programmed solution to Matrix B is included in Table 2. A comparison of plans 1 and 2 shows that the grossly impracticable placement of crops in the former plan has largely been corrected in plan 2 in which the placement of crops is restrained by paddock boundaries. On the other hand, inspection of the latter plan reveals that four of the twelve paddocks, namely B, C, I, and L, contain more than one crop. Such a solution would not normally be acceptable to the farm operator.[6] Thus, while the construction and subsequent solution of Matrix

TABLE 2

PLAN 2—THE PROGRAMMED SOLUTION FROM MATRIX B

Activity	*Unit*	*Activity Level*
P_1 Early oats paddock A	Acre	25.0
P_4 Early oats paddock B	Acre	49.8
P_6 Lucerne paddock B	Acre	58.8
P_7 Early oats paddock C	Acre	30.2
P_{10} Late grazing sorghum paddock C	Acre	60.4
P_{15} Early grazing sorghum paddock D	Acre	95.2
P_{21} Late grazing sorghum paddock E	Acre	144.0
P_{25} Late oats paddock F	Acre	70.3
P_{31} Early grazing sorghum paddock G	Acre	73.8
P_{34} Wheat paddock H	Acre	71.3
P_{41} Early oats paddock I	Acre	190.0
P_{43} Early grazing sorghum paddock I	Acre	244.5
P_{49} Late grazing sorghum paddock J	Acre	107.1
P_{51} Wheat paddock K	Acre	505.9
P_{57} Wheat paddock L	Acre	178.7
P_{60} Late grazing sorghum paddock L	Acre	168.9
P_{62} Native pasture X	Acre	1,875.0
P_{63} Native pasture Y	Acre	762.0
P_{64} Native pasture Z	Acre	2,857.0
P_{65} May–June oat transfer	D.M.E.	1,835.2
P_{66} July–September oat transfer	D.M.E.	32.8
P_{68} January–March feed transfer	D.M.E.	13,194.6
P_{69} April–June feed transfer	D.M.E.	15,663.4
P_{71} October–December consumption transfer	D.M.E.	10,807.0
P_{72} January–March consumption transfer	D.M.E.	15,069.3
P_{73} April–June consumption transfer	D.M.E.	4,781.8
P_{74} July–September consumption transfer	D.M.E.	15,687.5
P_{76} Stud sheep	Breeding ewe	500.0
P_{77} Wethers	Wether	764.1
P_{78} Crop wethers	Wether	3,655.1
P_{81} Partially competitive cattle	Breeding cow	97.9
P_{83} Crop fatteners	Steer	11.7
Surplus Resources		
R_{19} Wheat maximum	Bag	1,075.3
R_{20} January tractor	Hour	83.1

Revenue £27,465.1

[6] As far as this study is concerned a necessary condition for an acceptable farm plan would be that (with the exception of Paddocks G, I, and L, for which temporary subdivision fences are available) the whole of each cultivation paddock should be placed either under a single crop or under a combination of agronomically compatible crops.

B has resulted in an improvement on reality, it appears likely that a rigorous optimum which is also acceptable to the farm operator could only be reached using integer programming techniques. This would involve specifying integer restraints for alternative crop activities on each cultivation paddock so that each paddock is forced to accept one crop alone in the final plan. Unfortunately, if the experience of Bennett & Dakin (1961) can be taken as a guide, problems of slow convergence to an optimum can be expected with matrices of the size used in this study.

Alternatively, if the conventional static programming solution contains only minor infeasibilities, then the technique discussed by Rickards & Musgrave (1965) for examining border plan[7] information can be applied. Systematic use of this technique in such cases allows the programmer to reach a practicable solution which is not significantly different from the "true" optimum.

A closer look at the paddocks containing more than one crop shows that the misallocation problem in plan 2 is not serious. In fact the crop combinations in paddocks I and L are quite acceptable as the use of temporary subdivision fences in these paddocks is normal management practice. Paddock B is divided in its use between early oats and lucerne in the ratio 4:5. This is an acceptable combination, however, since both crops would be required to provide winter grazing for the 500 stud ewes of plan 2. On the other hand, the recommended combination of crops for paddock C, namely early oats (P_7) and late grazing sorghum (P_{10}), is unacceptable since it is implicit that these crops provide forage for different classes of livestock.

Clearly either P_7 or P_{10} must be excluded from plan 2 in order to reach an acceptable farm plan. Thus the border plans at both the upper and lower border prices for each of these two activities were analyzed. The plan corresponding to the lower border price of P_7 was selected as the practicable solution to farm planning and is referred to as *plan 3* in further discussion. The only difference in the basis variables of plan 3 is the replacement of P_7 (early oats paddock C) by P_9 (early grazing sorghum paddock C). Thus, a compatible pair of crops is introduced to paddock C. Practicability in the programmed solution has been achieved in plan 3 at a revenue decrement of only £10.22, which is smaller than the decrement resulting from other border plans. In the interest of brevity, plan 3 will not be enumerated since it is almost identical with plan 2.

While it is reassuring to the theorist that a precise solution to a farm planning problem can be obtained within the programming framework by the use of sensitivity analysis, the practitioner may argue that such techniques only serve to introduce an unwarranted air of accuracy into farm planning. For instance, an extension worker, having arrived at plan 2, may feel that it is so close to a practicable solution that he is prepared to make the final adjustment arbitrarily after discussion with the farm operator. Where the required adjustments are only small in magnitude, as in the present study, this is probably an acceptable approach. Only experience with the use of matrices which include paddock restraints will tell whether an orthodox programming technique can usually be expected to provide a satisfactory approximation to the "true" optimum. Such experience should also indicate the usefulness of the border-plan techniques used in this study.

[7] Most computer routines for linear programming problems are able to calculate the minimum change in the objective coefficients of each current basis activity which would be necessary to induce a change in basis variables. The value of an objective coefficient after adding this change to its original value is called the "border price" while the new plan that becomes optimum at a "border price" of an activity is known as the "border plan". Rickards & Musgrave suggest an examination of alternative border plans and selection of that plan which overcomes the problem of crop incompatibility with the least decrement in revenue.

EVALUATION OF THE PROGRAMMED SOLUTION

Normative application of the programmed solution

Traditionally, the normative value of the computed solution is gauged by comparing the revenue from the computed plan with that for the observed plan. In order to estimate the latter value, the production activities included in the observed plan for 1964 were identified, as closely as possible, with corresponding activities in Matrix B. The anticipated revenue from the observed plan was estimated to be £27,237.0, which falls short of that from plan 3 by £3,217.9. This value, of course, does not include an allowance for the costs of changing enterprise combinations and is therefore an upwardly biased estimate of the financial superiority of the computed solution when such costs exist. Other differences between the plans are summarized in Table 3.

TABLE 3

A COMPARISON OF THE LEVELS OF MAJOR ACTIVITIES IN PLAN 3 AND THE OBSERVED PLAN FOR 1964

Major Crop or Livestock Activity	Unit	Level in Plan 3	Level in Observed Plan	Difference in Level
Wheat	Acre	755.9	692.6	+ 63.3
Grazing oats	Acre	259.0	516.6	— 221.6
Forage sorghum	Acre	820.2	60.0	+ 740.2
Lucerne	Acre	202.8	545.2	— 342.4
Flock ewes	Breeding ewe		1,200.0	— 1,200.0
Stud ewes	Breeding ewe	500.0	500.0	
Wethers	Wether	754.0	500.0	+ 254.0
Crop wethers	Wether	3,674.0		+ 3,674.0
Partially competitive cattle	Breeding cow	98.0	93.0	+ 5.0
Fully competitive cattle	Breeding cow		17.0	— 17.0
Competitive vealers	Breeding cow		30.0	— 30.0
Crop fatteners	Steer	8.0		+ 8.0
Revenue		£27,454.9	£24,237.0	£3,217.9

Differences of some importance are that all the flock ewes and part of the lucerne and grazing oats aereages included in the observed plan are replaced by wethers and grazing sorghum in plan 3. This raises a pertinent question of whether plan 3 could have been put into operation in 1964, given that it was available in advance. Certainly it seems that there would be little difficulty in adjusting the acreages of annual crops in the observed plan to those indicated in plan 3. On the other hand the major reorganization required for sheep activities could only be completed in the short run by selling all flock ewes and replacing these by purchased wethers.

Livestock transactions such as this would inevitably result in long-run financial and stock management difficulties. In the first instance the difference between the sale price and book value of flock ewes would be considered as property income subject to taxation. Secondly, in purchasing wethers there would be a possibility of reintroducing certain internal parasites which had been eradicated from the property.

Hence in this particular study the normative application of the programmed solution, while being feasible, is not a convenient plan for management to adopt immediately. The apparent revenue advantage of the computed solution is upwardly biased because of costs involved in short-run changes in the level of livestock activities. This problem could be overcome by introducing time as a variable in the linear programming model. Examples of this type of analysis are provided by Throsby (1962) and Pearse (1963), who used "dynamic" linear programming to determine optimum patterns of pasture improvement.

While the model used in this study failed to provide a pat solution to farm planning for the case property, the analysis of the farm firm is still of value. A characteristic of the linear programming routine is that beside solving the *allocation problem* it also solves the *valuation problem*. That is, it calculates an optimum set of values for the resources available to the farm operator. These values are referred to as shadow prices. It will be shown below that, even in cases where strict application of the programmed solution is undesirable, the combined use of both the solution itself and the shadow prices permits the analyst to determine the desirable avenue of property reorganization in both the planning period considered and in future periods.

Application of shadow prices to farm planning

Shadow prices are calculated for all non-basis activities in the final solution. In the case of resources which are limiting in the final plan, these prices indicate the marginal value products (M.V.P.s) of these resources. In contrast, the shadow prices for original non-basis or "real" activities indicate the marginal opportunity costs (M.O.C.s) of including these activities in the basis of the final

TABLE 4

MARGINAL VALUE PRODUCTS OF RESTRAINTS IN LIMITED SUPPLY IN PLAN 3

Restraint	*Unit*	*M.V.P. in £'s*
R_1 Paddock A	Acre	1.34
R_2 Paddock B	Acre	6.43
R_3 Paddock C	Acre	5.60
R_4 Paddock D	Acre	3.34
R_5 Paddock E	Acre	2.29
R_6 Paddock F	Acre	7.25
R_7 Paddock G	Acre	7.82
R_8 Paddock H	Acre	6.06
R_9 Paddock I	Acre	4.53
R_{10} Paddock J	Acre	7.82
R_{11} Paddock K	Acre	7.47
R_{12} Paddock L	Acre	6.00
R_{13} Pasture X	Acre	1.78
R_{14} Pasture Y	Acre	1.32
R_{15} Pasture Z	Acre	1.18
R_{16} Maximum stud ewes	Breeding ewe	5.88
R_{17} Lucerne restraint	D.M.E.	0.158
R_{18} Maximum late grazing sorghum	Planting acre/year	7.82
R_{21} February–March tractor	Hour	4.56
R_{22} December tractor	D.M.E.	5.33
R_{23} Supplementary sheep units	D.M.E.	0.491
R_{24} January–March transferable D.M.E.	D.M.E.	0.112
R_{25} April–June transferable D.M.E.	D.M.E.	0.173
R_{26} July–September transferable D.M.E.	D.M.E.	0.345
R_{27} October–December transferable D.M.E.	D.M.E.	0.082
R_{28} May–June oat D.M.E.	D.M.E.	0.173
R_{29} July–September oat D.M.E.	D.M.E.	0.346
R_{30} January–March consumption D.M.E.	D.M.E.	0.112
R_{31} April–June consumption D.M.E.	D.M.E.	0.172
R_{32} July–September consumption D.M.E.	D.M.E.	0.346
R_{33} October–December consumption D.M.E.	D.M.E.	0.082
R_{34} Grazing sorghum supply	D.M.E.	0.155

plan in the least sub-optimal way. These values can be used to gauge the efficiency of resource use within the farm firm.

Table 4 includes the M.V.P.s of restraints specified in Matrix B which are limiting in the final plan.

The M.V.P.s of cultivation land vary from £7.82 on paddocks G and J to £1.34 in paddock A. These values indicate the per acre economic rent earned by paddocks of varying soil-type distribution and provide a guide to predicting the relative profitability of expanding cultivation activities into new paddocks of known soil-type distribution. The M.V.P.s of grazing land vary from £1.78 on cleared brigalow scrub country to £1.18 on cleared box-sandalwood country.

The late grazing sorghum restraint, R_{18}, has an imputed value of £7.82, indicating that farm revenue could be substantially improved by increasing the acreage of this crop. On the other hand, the static linear programme does not take into account the high variability of outcomes from this crop. Thus the operator's preference for restricting the area of forage sorghum could well be an expression of risk preference, in which case the restriction is realistic.

The tractor hours available for cultivation operations became limiting in both the December and February–March periods and M.V.P.s of £5.33 and £4.56 respectively were imputed to these resources. These values are far in excess of the cost of increasing the cultivating capacity by either hiring more labour or operating a larger plant with the current labour supply. This indicates a major weakness in property organization.

The M.V.P.s of limiting general-purpose feed resources indicate that the marginal value of a D.M.E. unit of "transferable" feed varies from a minimum of £0.082 in the October–December to a maximum of £0.346 in the July–September period. An appraisal of techniques for alleviating the acute feed shortage in late winter and early spring seems warranted. Two alternatives are examined, namely the use of purchased grain as opposed to home-produced silage for supplementary feeding during the winter months.

There seems to be little likelihood that supplementary grain feeding would ever be justified in a normal season. For instance, the cost of feed oats would have to fall to 4s. 6d. per bushel or feed wheat to 8s. per bushel before grain supplements could be provided at less than £0.346 per D.M.E. unit. In contrast, the profitable use of silage for supplementary feeding seems plausible. It is estimated that with a standing forage crop marginally valued at £0.112 per D.M.E. during January to March and allowing 15s. per ton harvesting and ensiling costs and 25 per cent deterioration in feed value during storage, then silage could be fed out for £0.206 per D.M.E. in mid and late winter. This cost is £0.140 per D.M.E. less than the value imputed to general purpose feed during this period.

The M.O.C.s of all sub-optimal activities will not be included here due to the restricted space available. Instead, only the values imputed to sub-optimal livestock activities are listed.

Activity	*M.O.C.*
P_{75} Flock ewes	£0.24 per breeding ewe
P_{79} Fully competitive cattle	£13.72 per breeding cow
P_{80} Fully competitive vealers	£11.30 per breeding cow
P_{81} Partially competitive vealers	£1.28 per breeding cow

While no flock ewes are included in plan 3, the M.O.C. of this activity (£0.24 per ewe) is sufficiently low for the programmed solution to be sensitive to changes in the input-output relationships of flock sheep. On the other hand, stud sheep have been restrained to 500 in plan 3 by R_{16} which is marginally valued at £5.88 in this solution. These values indicate a disequilibrium with the market and a need to expand the stud enterprise at the expense of flock sheep. Finally, the

M.O.C.s of P_{79} and P_{80} are so high that plan 3 would be completely insensitive to any likely changes in either production relationships or product prices of these fully competitive cattle activities.

At this stage, having examined both the programmed solution and the available shadow prices, we are in a position to determine the direction in which the operator should reorganize production in order to reach long-run stability with the market. In brief, it appears that the following steps should be taken:

i. The stud ewe enterprise should be increased at the expense of flock ewes.
ii. The wether flock should be increased by retaining the annual culling of 2-tooth males from the stud flock.
iii. The acreage of grazing sorghum should be increased at the expense of grazing oats and lucerne.
iv. The possibility of producing silage from summer crops should be further investigated with a view to supplementary winter feeding.
v. The working capacity of the cultivating plant should be increased.

Extension of experience from an individual study

Experience gained in this study indicates that linear programming offers a useful approach to individual farm planning. The limiting factor to its application in isolated studies will probably be the high cost involved. For instance, given full-time devotion to the task, the present study would have taken the analyst approximately three months while cash expenses amounted to £100. Thus the total cost of the study could be set at approximately £600. Given the turnover of the property, this is scarcely an exorbitant figure but it would certainly fall outside the budget of any publicly financed extension authority. Hence, as far as isolated studies are concerned, it would seem that the farm operator would have to bear most of the expense.

In contrast, the use of programming techniques for farm planning becomes more attractive as the number of farms in a district which require this service is increased. For instance, the authors consider that the cost of programming additional properties in the Goondiwindi district would rapidly diminish. While it would still be necessary to interview each farm operator in order to establish the unique characteristics of his property, many of the input-output coefficients necessary for matrix construction could be drawn from the pool of information already available. Thus it is estimated that a practicable farm plan could be achieved for properties similar in size and complexity to "Trevanna Downs" in approximately three weeks. In addition, cash costs (including data preparation and processing) could be expected to approach £100.

Costs of this order may still place linear programming analysis of individual farms out of reach for public extension agencies unless the farm operators involved are prepared to bear part of the cost. On the other hand, farm management club advisers and private consultants should find that the cost economies forthcoming as the number of programming studies increases are such as to make this form of farm planning financially attractive to their clients.

SUMMARY

While the literature on the application of linear programming methods to agricultural problems is extensive, little attention has been given to the use of programming for production planning on case study properties. Furthermore, most authors who have concentrated on this latter approach have applied it to farms with soil types which are, or which are assumed to be, homogeneous as regards

production. In practice, in Australia for example, such homogeneity is true of only a small proportion of the total universe of farms. Thus, the literature is particularly deficient in applications of linear programming methods to real farm situations of this type.

Where a number of soil types are represented on a property, each type having a unique set of production functions for feasible agricultural cropping, management is faced with an extremely complex problem of determining the optimum allocation of resources among alternative production activities. Nevertheless, this study on a property in the Goondiwindi district has indicated that even under such circumstances a practicable farm plan can be developed. This can be done by progressively revising and reprogramming the original matrix with the help of the farm operator.

Normative application of the solution derived in this way may not always be advisable, particularly if it involves major short-run adjustments to existing livestock enterprises. This does not necessarily deny the value of programmed solutions. Rather it means that the programmed solution cannot be used merely as a blue print for farm planning but that a more skilful interpretation has to be placed on the programmed results. Combined use of both the solution itself and also of shadow prices imputed to final non-basis activities permits the analyst to determine the direction in which the farm operator should reorganize his production pattern in future periods.

If farm management workers are prepared to adopt such an approach, as well as to specify the production planning problem more precisely than hitherto, then linear programming can be of much greater use as a farm planning aid. Under these circumstances the limiting factor to its application will probably be the expense involved rather than imperfect appreciation of the method as at present.

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