Mon. Not. R. Astron. Soc. **408,** L16–L20 (2010) doi:10.1111/j.1745-3933.2010.00919.x

On the mass–radius relation of hot stellar systems

Mark Gieles,[1,2,3★] Holger Baumgardt,[4,5] Douglas C. Heggie[2] and Henny J. G. L. M. Lamers[6]

[1] *Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA*
[2] *School of Mathematics and Maxwell Institute for Mathematical Sciences, University of Edinburgh, King's Buildings, Edinburgh EH9 3JZ*
[3] *European Southern Observatory, Casilla 19001, Santiago 19, Chile*
[4] *Argelander Astronomical Institute, University of Bonn, Auf dem Hügel 71, 53121 Bonn, Germany*
[5] *School of Mathematics and Physics, The University of Queensland, Brisbane, QLD 4072, Australia*
[6] *Astronomical Institute, Utrecht University, Princetonplein 5, NL-3584CC Utrecht, the Netherlands*

Accepted 2010 July 14. Received 2010 July 13; in original form 2010 June 24

ABSTRACT

Most globular clusters have half-mass radii of a few pc with no apparent correlation with their masses. This is different from elliptical galaxies, for which the Faber–Jackson relation suggests a strong positive correlation between mass and radius. Objects that are somewhat in between globular clusters and low-mass galaxies, such as ultracompact dwarf galaxies, have a mass–radius relation consistent with the extension of the relation for bright ellipticals. Here we show that at an age of 10 Gyr a break in the mass–radius relation at $\sim 10^6\,\mathrm{M}_\odot$ is established because objects below this mass, i.e. globular clusters, have undergone expansion driven by stellar evolution and hard binaries. From numerical simulations we find that the combined energy production of these two effects in the core comes into balance with the flux of energy that is conducted across the half-mass radius by relaxation. An important property of this 'balanced' evolution is that the cluster half-mass radius is independent of its initial value and is a function of the number of bound stars and the age only. It is therefore not possible to infer the initial mass–radius relation of globular clusters, and we can only conclude that the present day properties are consistent with the hypothesis that all hot stellar systems formed with the same mass–radius relation and that globular clusters have moved away from this relation because of a Hubble time of stellar and dynamical evolution.

Key words: methods: numerical – globular clusters: general – galaxies: fundamental parameters – galaxies: star clusters.

1 INTRODUCTION

The half-mass radius of old globular clusters in the Milky Way depends only weakly on mass (e.g. van den Bergh, Morbey & Pazder 1991). If anything, a negative correlation between radius and mass is found for the clusters in the outer halo (van den Bergh & Mackey 2004). Because this is also found for extragalactic globular clusters (Jordán et al. 2005; Barmby et al. 2007; Georgiev et al. 2009), the mass–radius relation, or lack thereof, is an important aspect of the Fundamental Plane relations of globular clusters (Djorgovski 1995; McLaughlin 2000).

Objects more massive than typical globular clusters, such as the recently discovered ultracompact dwarf galaxies (UCDs, Hilker et al. 1999; Drinkwater et al. 2003), and also the most massive globular clusters, do exhibit a positive correlation between radius and mass (Haşegan et al. 2005; Rejkuba et al. 2007; Mieske et al. 2008). Interestingly, the position of systems more massive than $\sim 10^6\,\mathrm{M}_\odot$ in the mass–radius diagram coincides with the extension to low masses of the Faber & Jackson (1976) relation for bright elliptical galaxies (Haşegan et al. 2005). The mass–radius relation of stellar systems more massive than $\sim 10^6\,\mathrm{M}_\odot$ has been explained by the details of their formation (Murray 2009), where this was considered a deviation from the near constant radius of less massive systems. In this study we test the hypothesis that all hot stellar systems (globular clusters, UCDs and elliptical galaxies) had the same mass–radius relation initially and that the globular clusters ($\lesssim 10^6\,\mathrm{M}_\odot$) are the deviators because they have moved away from this relation because of dynamical evolution.

Intuitively we can expect that low-mass stellar systems are dynamically more evolved than massive systems because of their shorter relaxation time-scale. This evolutionary time-scale is often expressed in terms of the half-mass properties of the system (Spitzer 1987):

$$T_{\mathrm{rh}} = 0.138\frac{N^{1/2}R_{\mathrm{h}}^{3/2}}{G^{1/2}\bar{m}^{1/2}\ln\Lambda}, \tag{1}$$

★E-mail: mgieles@ast.cam.ac.uk

where N is the number of stars, $R_{\rm h}$ is the half-mass radius, G is the gravitational constant, $\bar{m}$ is the mean stellar mass and Λ is the argument of the Coulomb logarithm and equals $0.02N \lesssim \Lambda \lesssim 0.11N$ depending on the stellar mass function in the cluster (Giersz & Heggie 1994a). If we take the initial mass–radius relation to be of the form $R_{\rm h0} \propto M_0^{\lambda}$, then $T_{\rm rh0}$ is an increasing function of M_0 for all $\lambda > -1/3$. Although the value of λ is poorly constrained from observations, it is unlikely to be negative and we can, therefore, safely say that low-mass stellar systems have shorter relaxation times than massive systems immediately after formation.

Here we consider the expansion of star clusters driven by mass loss due to stellar evolution and hard binaries, and we present a description for the radius evolution including both effects, based on results of N-body simulations (Section 2). In Section 3 we show that at an age of 10 Gyr, a Faber–Jackson type initial mass–radius relation has been erased because of the expansion of stellar systems with $M \lesssim 10^6\,{\rm M}_\odot$. A summary and discussion is presented in Section 4.

2 EXPANSION OF STELLAR SYSTEMS

We want to understand the evolution of the radius of a stellar system with a realistic stellar mass function in which the stars evolve and lose mass in time. This evolution is distinct from the well-studied and well-understood behaviour of an equal-mass cluster (e.g. Hénon 1965; Goodman 1984). Because we are mainly interested in the expansion we ignore the effect of a tidal cut-off. As we will show in Section 3, the results explain the mass–radius relation of objects with $M \gtrsim 10^5\,{\rm M}_\odot$, suggesting that tides are not very important in shaping the mass–radius relation of these objects. We first consider various stellar mass functions, ignoring the effect of stellar evolution (Section 2.1), and then add the effect of stellar evolution in Section 2.2.

2.1 Expansion driven by hard binaries

The evolution of equal-mass clusters has been studied in quite some detail (e.g. Giersz & Heggie 1994a; Baumgardt, Hut & Heggie 2002). To first order their entire evolution follows from the fact that gravitational systems have negative total energy, which causes them to always evolve away from thermal equilibrium. In the early evolution, this results in a contraction of the core and this inevitably leads to the gravothermal catastrophe, or core collapse (Lynden-Bell & Eggleton 1980). For an equal-mass Plummer (1911) model the time of core collapse is at $T_{\rm cc} \approx 17\,T_{\rm rh0}$ (e.g. Spitzer 1987). After core collapse the evolution is driven by binaries in the core that release energy to the rest of the cluster when they form and harden in three-body interactions. This energy is conducted outwards by two-body relaxation and in the absence of a tidal field this results in an expansion of the cluster as a whole, because escape of stars is inefficient. This increase of $R_{\rm h}$ happens on a relaxation time-scale such that we can say $\dot{R}_{\rm h} = \zeta\,R_{\rm h}/T_{\rm rh}$. If we integrate this relation from $T_{\rm cc}$ to T, taking into account the $R_{\rm h}$ dependence in $T_{\rm rh}$ (equation 1), we find

$$R_{\rm h} = R_{\rm h0}\left[1 + \frac{\chi(T - T_{\rm cc})}{T_{\rm rh0}}\right]^{2/3}, \tag{2}$$

$$\approx R_{\rm h0}\left(\frac{\chi T}{T_{\rm rh0}}\right)^{2/3}, \tag{3}$$

where χ is a constant that relates to ζ as $\chi \equiv (3/2)\zeta$. In the last step we have used $T_{\rm cc} \approx T_{\rm rh0}/\chi$ as the integration boundary, which is not strictly true. For an equal-mass cluster $\chi \approx 0.14$ (Hénon 1965; Heggie & Hut 2003) and, therefore, $1/\chi \approx 7.2$, whereas for the Plummer model we have $T_{\rm cc}/T_{\rm rh0} \approx 17$ (see also Giersz & Heggie 1994b). But equation (3) describes the asymptotic behaviour of $R_{\rm h}$ for $T \gg T_{\rm cc}$ and is therefore a useful approximation. It also follows from equation (3) that after $T_{\rm cc}$ the evolution of $R_{\rm h}$ is independent of $R_{\rm h0}$; if we assume that N and $\bar{m}$ do not change in time then we can say $T_{\rm rh0} = T_{\rm rh}R_{\rm h0}^{3/2}/R_{\rm h}^{3/2}$ (equation 1), and equation (3) is equivalent to $T_{\rm rh} = \chi T$. This means that after some time clusters evolve towards a mass–radius relation of the form $R_{\rm h} \propto T^{2/3}M^{-1/3}$, independent of the initial mass–radius relation. This is an important result, and in Section 3 we will show that it applies to globular clusters.

The presence of a mass function speeds up the dynamical evolution (Inagaki & Saslaw 1985), in the sense that core collapse happens earlier (Gürkan, Freitag & Rasio 2004) and the escape rate of clusters in a tidal field is higher (Lee & Goodman 1995). Here we establish by means of direct N-body simulations how the rate of expansion, i.e. the value of χ, depends on the mass function of the stars. We consider a Kroupa (2001) stellar mass function and vary $\mu \equiv m_{\rm max}/m_{\rm min}$, where $m_{\rm max}$ and $m_{\rm min} = 0.1\,{\rm M}_\odot$ are the maximum and minimum stellar mass, respectively.[1] We consider values from $\mu = 1$ (= equal mass) to 10^3 (= full mass function) in steps of a factor of 10. These values cover the relevant values of μ for real clusters. We model clusters with N = 4096, 8192, 16 384 and 32 768 particles, and multiple runs are done for clusters with low N and/or high μ to average out statistical fluctuations due to the low number of (massive) stars. The number of simulations was chosen to be max $[1, (32768/N)/(5 - \log\mu)]$. The initial density profile of all clusters is described by Plummer models in virial equilibrium and during the simulation, stars are taken out of the simulation when they reach $20\,R_{\rm v}$, where $R_{\rm v}$ is the virial radius. The models are all scaled to the usual N-body units ($G = R_{\rm v0} = -4\,E_0 = 1$, where E_0 is the total initial energy, Heggie & Mathieu 1986), and we use the KIRA integrator which is part of the STARLAB software (Portegies Zwart et al. 2001) to numerically solve the N-body problem in time. At each time the values for $R_{\rm h}$, $\bar{m}$ and N are recorded and $T_{\rm rh}$ is calculated using equation (1). For $\mu = 1$ we use $\gamma = 0.11$, while for $\mu > 1$ we use $\gamma = 0.02$ as recommended by Giersz & Heggie (1994a).

In Fig. 1 we show the (average) resulting evolution of $T_{\rm rh}$ for all 16 different initial conditions, specified by the number of stars N and the width of the stellar mass function, μ. The increase of $T_{\rm rh}$ after $T \approx T_{\rm rh0}/\chi$ is dominated by expansion, because $\bar{m}$ remains constant (no stellar evolution) and the number of bound stars does not change much. The dashed lines indicate different values of χ and it can be seen that the dependence of χ on the mass function can to first order be approximated by $\chi \approx 0.1\mu^{1/2}$ [or $\chi \approx 0.1(m_{\rm max}/\bar{m})^{0.7}$]. This scaling roughly recovers Hénon's result for equal-mass models. In summary, we see from Fig. 1 that $T/T_{\rm rh}$ increases until $T \approx T_{\rm rh0}/\chi$, after which $T/T_{\rm rh} \approx$ constant.

In Section 2.2 we repeat the simulations with $\mu = 10^3$ and turn stellar evolution on such that μ naturally decreases from 10^3 at $T = 0$ to $\mu \approx 10$ at $T \approx 10$ Gyr during the simulation because of stellar evolution.

[1] We use the ratio $m_{\rm max}/m_{\rm min}$ because it is easy to relate to real clusters. Gürkan et al. (2004) show that the relevant parameter is $m_{\rm max}/\bar{m}$ which captures variations in $m_{\rm max}$ and the slope of the mass function.

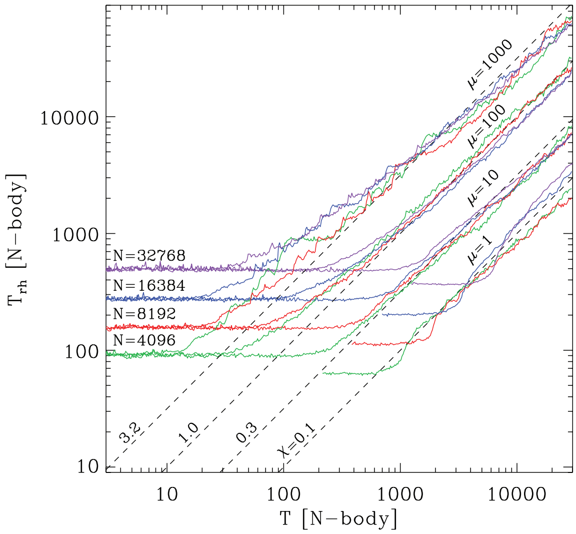

Figure 1. Evolution of the half-mass relaxation time, $T_{\rm rh}$, for clusters with different N and different μ. The N-body unit of time, $T_{\rm dyn}$, can be related to physical units through $T_{\rm dyn} = (GM/R_{\rm v}^3)^{-1/2}$. A Kroupa (2001) mass function is used for the stars in the range $0.1 \le m/{\rm M}_{\odot} \le 0.1\mu$. Clusters of different N and the same μ evolve to the same $T_{\rm rh} \approx \chi T$ after core collapse (equation 3). This asymptotic behaviour is roughly matched by the relation $\chi \approx 0.1\mu^{1/2}$, shown as dashed lines. The $T_{\rm rh}$ values of the equal-mass clusters are calculated using a slightly different argument in the Coulomb logarithm ($\Lambda = 0.11N$, equation 1) as compared to the multi-mass clusters ($\Lambda = 0.02N$). For clarity the $\mu = 1$ curves are only plotted for $T \gtrsim 3T_{\rm rh0}$.

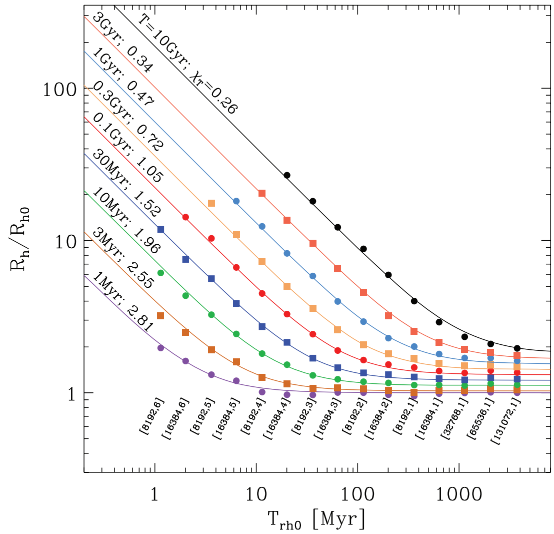

Figure 2. Expansion from the N-body runs including the effect of stellar evolution together with the functional fits (equation 6, full lines).

2.2 The combined effect of stellar evolution and binaries

The time-scales of stellar evolution are set by the stellar interiors and are independent of the relaxation time-scale of the cluster wherein the stars evolve. We thus expect the details of the evolution to depend on a combination of the stellar evolution time-scale and the relaxation time of the cluster. Here we show that the resulting expansion still depends in a simple way on the dynamical properties of the cluster as a whole.

We want to consider a large range of $T_{\rm rh0}$ with our simulations to cover a parameter space that is relevant for real globular clusters. Because computing times limit us to $N \lesssim 10^5$ with direct N-body simulations, we vary both N and the initial half-mass density, $\rho_{\rm h} \equiv 3M/(8\pi R_{\rm h}^3)$. We consider 15 different values of $T_{\rm rh0}$ ranging from $T_{\rm rh0} \approx 1$ Myr ([N, $\log \rho_{\rm h}$] = [8192, 6]) to $T_{\rm rh0} \approx 4$ Gyr ([N, $\log \rho_{\rm h}$] = [131072, 1]), with $\rho_{\rm h}$ in ${\rm M}_{\odot}\,{\rm pc}^{-3}$. Here $T_{\rm rh0}$ is increased by increasing N by factors of 2 and by decreasing $\rho_{\rm h}$ by factors of 10. We again use the KIRA integrator and the stellar evolution package SEBA for solar metallicity (Portegies Zwart et al. 2001). We use a Kroupa (2001) initial mass function between 0.1 ${\rm M}_{\odot}$ and 100 ${\rm M}_{\odot}$, which has $\bar{m} \approx 0.64\,{\rm M}_{\odot}$. The retention fraction of black holes and neutron stars was set to zero.

In Fig. 2 we show the resulting expansion in the form of $R_{\rm h}/R_{\rm h0}$ as a function of $T_{\rm rh0}$ at different ages. The asymptotic behaviour of these runs can easily be understood by considering the extremes. Clusters that are dynamically young (low $T/T_{\rm rh0}$) expand adiabatically in order to retain virial equilibrium after stellar mass loss. The continuous loss of mass from a Kroupa (2001) mass function together with the stellar evolution prescription of STARLAB (appendix B2 of Portegies Zwart et al. 2001) leads to a reduction of the total cluster mass:

$$M \approx M_0 \left(\frac{T}{T_*}\right)^{-\delta}, \quad T \ge T_*, \quad \delta \approx 0.07, \quad T_* \approx 2\,{\rm Myr}. \tag{4}$$

In this regime the radius thus evolves as (e.g. Hills 1980)

$$R_{\rm h} \approx R_{\rm h0} \left(\frac{T}{T_*}\right)^{\delta}. \tag{5}$$

This adiabatic expansion is slow in time and gives a maximum increase of $R_{\rm h}/R_{\rm h0} \approx 2$ after a Hubble time. At the other extreme we have clusters that are dynamically old (high $T/T_{\rm rh0}$) and they expand quickly in a way that is comparable to what we have seen in Section 2.1. We propose a function that stitches together these two extremes in an attempt to match $R_{\rm h}/R_{\rm h0}$ for all values of $T/T_{\rm rh0}$:

$$R_{\rm h} = R_{\rm h0} \left[\left(\frac{T}{T_*}\right)^{2\delta} + \left(\frac{\chi_T T}{T_{\rm rh0}}\right)^{4/3}\right]^{1/2}, \quad T \ge T_*. \tag{6}$$

Here χ_T is a parameter comparable to χ of Section 2.1, but now time-dependent due to the variation of the mass function, and its value at an age T is found from a fit of equation (6) to the results of the N-body runs. In Fig. 2 we show the fit results as full lines and the resulting values of χ_T are indicated. It shows that equation (6) provides a good description of the evolution of $R_{\rm h}/R_{\rm h0}$. The relation between χ_T and T is well approximated by a simple power-law function

$$\chi_T \approx 3\left(\frac{T}{T_*}\right)^{-0.3}, \quad T_* \le T \lesssim 20\,{\rm Gyr}. \tag{7}$$

If we now define $T_* \equiv \min([2\,{\rm Myr}, T])$ we have a continuous function for $R_{\rm h}(T_{\rm rh0}, T)$, or $R_{\rm h}(M_0, R_{\rm h0}, T)$ for all T. For high $T/T_{\rm rh0}$ we find from equation (6) that $T_{\rm rh} = (\bar{m}_0/\bar{m})^{1/2}\,\chi_T T \propto T^{0.74}$ (equations 4 and 7). We indicate below how the small deviation from a linear scaling with T (as found in Section 2.1) can be interpreted in terms of the evolution of the mass function. In Section 2.1 we found $T_{\rm rh} \propto \mu^{1/2} T$. If we approximate the main-sequence lifetime of stars by $t_{\rm ms} \propto m^{-2.5}$ (Bressan et al. 1993) and thus $\mu \propto T^{-1/2.5}$, then from the changing mass function we expect $T_{\rm rh} \propto T^{0.8}$, very close to what we find from the numerical simulations. We conclude that the evolution of the cluster is 'balanced' when the second term on the right-hand side of equation (6) dominates (high $T/T_{\rm rh0}$), in the sense that the energy flux at the half-mass boundary that drives the expansion is provided by the production of energy in the core by binaries and stellar evolution combined. Stellar evolution in fact slows down the expansion rate, because $\dot{R}_{\rm h}$ is determined by the instantaneous width of the stellar mass function, resulting in a smaller $\dot{R}_{\rm h}$ at old ages than if μ had stayed constant at $\mu = 10^3$ (Section 2.1). When the first term on the right-hand side of equation (6) dominates (low $T/T_{\rm rh0}$) the evolution is unbalanced and we have the usual adiabatic expansion.

The fact that the interplay between dynamical evolution and stellar evolution is in fact quite simple can be understood from the energy budget. The total energy of a stellar system depends on M and $R_{\rm h}$ as $E \propto -M^2/R_{\rm h}$. Together with equations (4) and (5) we find that E evolves in time as $E \propto -(T/T_*)^{-3\delta}$ because of mass loss and (adiabatic) expansion. The rate of energy change as a result of stellar evolution is then $\dot{E}_{\rm SEV} \propto |E|/T$, where the constant of proportionality depends on the degree of mass segregation; it will be higher when stars lose mass from the centre and/or when the density profile is centrally concentrated. The rate of energy increase due to binaries and relaxation is similar. This is because $\dot{E} \propto |E|/T_{\rm rh}$ and $T_{\rm rh} \propto T$ (Section 2.1). After some dynamical relaxation mass loss by stellar evolution will be predominantly from the core because of mass segregation and this will boost $\dot{E}_{\rm SEV}$. We tentatively pose the idea that $\dot{E}_{\rm SEV}$ acts as a central energy source that is subject to a feedback mechanism comparable to what happens with binaries; if $\dot{E}_{\rm SEV}$ is too high, the core expands and the central potential decreases and $\dot{E}_{\rm SEV}$ drops. If $\dot{E}_{\rm SEV}$ is too low, the core contracts, thereby increasing the depth of the central potential and increasing $\dot{E}_{\rm SEV}$. This fits in the view of Hénon (1975) that 'the rate of flow of energy is controlled by the system as a whole, not by the singularity'. One of the consequences is that there is no sharp transition between a stellar evolution dominated phase and a relaxation dominated phase.

3 APPLICATION TO OLD STELLAR SYSTEMS

With the expression for the evolution of the radius as a function of $T_{\rm rh0}$ at hand, we can easily calculate the evolution of $R_{\rm h}$ for any initial mass–radius relation. We apply our result to the mass–radius relation of old and hot stellar systems in the mass range $\sim10^4$–$10^8\,{\rm M}_\odot$.

The original Faber–Jackson relation relates the central velocity dispersion of (bright) elliptical galaxies to their total luminosity. Haşegan et al. (2005) have converted this result into relations between M, $R_{\rm h}$ and surface density. The resulting mass–radius relation (their equation 15) with an additional log 4/3 to correct for projection is $\log(R_{\rm h}/{\rm pc}) = -3.142 + 0.615\log(M/{\rm M}_\odot)$. They show that this relation matches the objects with $M \gtrsim 10^6\,{\rm M}_\odot$ (UCDs, massive globular clusters and their dwarf-globular transition objects, DGTOs) in the mass–radius diagram. Because this concerns collisionless systems, we can safely assume that two-body relaxation has not affected this relation and it should, therefore, reflect the initial relation. To get an expression for the initial mass–radius relation, we only need to correct for mass loss by stellar evolution and the subsequent adiabatic expansion. For $T = 10$ Gyr, we find $M/M_0 = R_{\rm h0}/R_{\rm h} \approx 0.55$ (equations 4 and 5) and thus

$$\log\left(\frac{R_{\rm h0}}{\rm pc}\right) = -3.560 + 0.615\log\left(\frac{M_0}{{\rm M}_\odot}\right). \qquad (8)$$

In Fig. 3 we show how this initial mass–radius relation evolves using our result from equation (6) together with data points that cover the

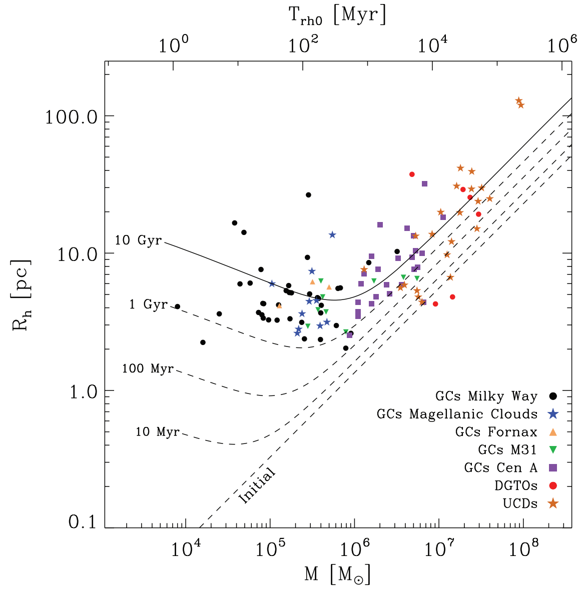

Figure 3. Mass–radius values for hot stellar systems. The values for globular clusters in the Milky Way, the Magellanic Clouds and Fornax are taken from McLaughlin & van der Marel (2005). The clusters in M31 are from Dubath & Grillmair (1997). The values for globular clusters in NGC 5128 (Cen A), UCDs and DGTOs are from the compilation presented in Mieske et al. (2008). The lines show the evolution of the mass–radius relation using the Faber–Jackson relation, corrected for stellar evolution, as initial conditions. The break at $\sim10^6\,{\rm M}_\odot$ at $T \approx 10$ Gyr is because lower mass objects have expanded.

mass regime we are interested in. For high $T/T_{\rm rh0}$ the radius is set by M_0, independent of $R_{\rm h0}$, while for low $T/T_{\rm rh0}$ we are seeing roughly the initial mass–radius relation. By construction the right-hand side of the 10-Gyr line coincides with the representation of the Faber–Jackson relation of Haşegan et al. (2005). From solving ${\rm d}R_{\rm h}/{\rm d}M = 0$ in equation (6), we find that at an age of 10 Gyr the break between the two regimes occurs at $M_0 \approx 1.1 \times 10^6\,{\rm M}_\odot$ and at that age systems with this mass have $T_{\rm rh}/T \approx 0.8$. Mieske et al. (2008) noted already that the break occurs at systems with $T_{\rm rh}$ roughly equal to a Hubble time. In this Letter we give a quantitative explanation for it.

4 SUMMARY AND DISCUSSION

In this study we provide the arguments that explain why there is a break in the mass–radius relation of hot stellar systems at $\sim10^6\,{\rm M}_\odot$. We show that the mass–radius relation of the massive systems ($\gtrsim10^6\,{\rm M}_\odot$) is only slightly affected by stellar evolution and represents, therefore, approximately the initial mass–radius relation. The origin of this relation needs to be searched for in the details of their formation and is not discussed here (see e.g. Murray 2009). Combining scaling relations for the (adiabatic) expansion of clusters because of stellar evolution with relations for expansion due to two-body relaxation we present a simple formula for the radius evolution as a function of initial mass, radius and time. Applying this result to a Faber–Jackson type initial mass–radius relation (the representation in units of mass and radius are taken from Haşegan et al. 2005), we show that at an age of 10 Gyr a break occurs at $\sim10^6\,{\rm M}_\odot$. This break can be thought of as the boundary between collisional systems ($T_{\rm rh} \lesssim$ age) and collisionless systems ($T_{\rm rh} \gtrsim$ age).

For young massive clusters there is also no obvious correlation between radius and mass/luminosity (Zepf et al. 1999; Larsen 2004;

Scheepmaker et al. 2007; Portegies Zwart, McMillan & Gieles 2010). From Fig. 3, it can be seen that for clusters with an age of ∼10–100 Myr there has already been significant expansion of clusters with masses $\lesssim 10^5\,\mathrm{M}_{\odot}$. Although this break mass depends on the initial mass–radius relation, it at least qualitatively shows that at young ages most clusters[2] are affected by the expansion we consider here. It is worthwhile to compare the theory to the parameters of young, well-resolved star clusters (e.g. Mackey & Gilmore 2003). We emphasize that the balanced evolution provides a lower limit to cluster radii. If clusters form above the relation marked initial in Fig. 3, then they expand only slightly because of stellar evolution at young ages until T/T_{rh0} is high enough for the balanced evolution/expansion to start. The mass–radius relation of young clusters is important in the evolution of cluster populations. This is because in the early evolution clusters suffer from encounters with the molecular gas clouds from which they form. The time-scale of disruption due to such encounters scales with the density of the cluster (Spitzer 1958). If all cluster have the same density, their disruption time-scale is independent of their mass. For a constant radius the time-scale of disruption becomes strongly mass-dependent because then $\rho_{\mathrm{h}} \propto M$, and for a constant T_{rh} we have $\rho_{\mathrm{h}} \propto M^2$. The mass–radius relation, therefore, determines the properties of the clusters that survive continuous encounters with massive clouds (Gieles et al. 2006; Elmegreen 2010).

We have ignored the tidal limitation due to the host galaxy. Once the density of a cluster drops below a critical value, depending on the tidal field strength, our result will overestimate the radius of such clusters because the presence of a tidal limitation will prevent further growth. The good agreement between the simple model presented here and the data points suggest that at least to first order the positions in the mass–radius diagram of objects with $M \gtrsim$ few $\times$ $10^4\,\mathrm{M}_{\odot}$ is not much affected by tides. Including a tidal field would bend down the curves at low masses. The transition from expansion-dominated evolution to Roche lobe filling evolution is considered in more detail in a follow-up study (Gieles, Heggie & Zhao, in preparation).

ACKNOWLEDGMENTS

MG thanks the Royal Society for financial support. The simulations were done on the GRAPE-6 BLX64 boards of the European Southern Observatory in Garching. This research was supported by the DFG cluster of excellence Origin and Structure of the Universe (http://www.universe-cluster.de). HB and HJGLML thank ESO for a Visiting Scientist Fellowship in Santiago in 2009 where this project was started.

[2] The luminosity function of young clusters is a power-law distribution with index ~ -2 such that a typical young cluster population only has a small fraction of its clusters in the massive ($\gtrsim 10^5\,\mathrm{M}_{\odot}$) tail.

REFERENCES

Barmby P. et al., 2007, AJ, 133, 2764
Baumgardt H., Hut P., Heggie D. C., 2002, MNRAS, 336, 1069
Bressan A., Fagotto F., Bertelli G., Chiosi C., 1993, A&AS, 100, 647
Djorgovski S., 1995, ApJ, 438, L29
Drinkwater M. J. et al., 2003, Nat, 423, 519
Dubath P., Grillmair C. J., 1997, A&A, 321, 379
Elmegreen B. G., 2010, ApJ, 712, L184
Faber S. M., Jackson R. E., 1976, ApJ, 204, 668
Georgiev I. Y., Puzia T. H., Hilker M., Goudfrooij P., 2009, MNRAS, 392, 879
Gieles M. et al., 2006, MNRAS, 371, 793
Giersz M., Heggie D. C., 1994a, MNRAS, 268, 257
Giersz M., Heggie D. C., 1994b, MNRAS, 270, 298
Goodman J., 1984, ApJ, 280, 298
Gürkan M. A., Freitag M., Rasio F. A., 2004, ApJ, 604, 632
Haşegan M. et al., 2005, ApJ, 627, 203
Heggie D., Hut P., 2003, in Heggie D. and Hut P. eds, The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics. Cambridge Univ. Press, Cambridge
Heggie D. C., Mathieu R. D., 1986, in Hut P., McMillan S., eds, Lecture Notes in Physics Vol. 267, The Use of Supercomputers in Stellar Dynamics, Springer-Verlag, Berlin, p. 233
Hénon M., 1965, Ann. Astrophys., 28, 62
Hénon M., 1975, in A. Hayli, ed., Proc. IAU Symp. 69, Dynamics of the Solar Systems. Reidel, Dordrecht, p. 133
Hilker M. et al., 1999, A&AS, 134, 75
Hills J. G., 1980, ApJ, 235, 986
Inagaki S., Saslaw W. C., 1985, ApJ, 292, 339
Jordán A. et al., 2005, ApJ, 634, 1002
Kroupa P., 2001, MNRAS, 322, 231
Larsen S. S., 2004, A&A, 416, 537
Lee H. M., Goodman J., 1995, ApJ, 443, 109
Lynden-Bell D., Eggleton P. P., 1980, MNRAS, 191, 483
Mackey A. D., Gilmore G. F., 2003, MNRAS, 338, 85
McLaughlin D. E., 2000, ApJ, 539, 618
McLaughlin D. E., van der Marel R. P., 2005, ApJS, 161, 304
Mieske S. et al., 2008, A&A, 487, 921
Murray N., 2009, ApJ, 691, 946
Plummer H. C., 1911, MNRAS, 71, 460
Portegies Zwart S., McMillan S. L. W., Hut P., Makino J., 2001, MNRAS, 321, 199
Portegies Zwart S., McMillan S., Gieles M., 2010, ARA&A, in press (arXiv:1002.1961)
Rejkuba M., Dubath P., Minniti D., Meylan G., 2007, A&A, 469, 147
Scheepmaker R. A. et al., 2007, A&A, 469, 925
Spitzer L. J., 1958, ApJ, 127, 17
Spitzer L., 1987, Dynamical evolution of globular clusters. Princeton Univ. Press, Princeton, NJ, p. 191
van den Bergh S., Mackey A. D., 2004, MNRAS, 354, 713
van den Bergh S., Morbey C., Pazder J., 1991, ApJ, 375, 594
Zepf S. E. et al., 1999, AJ, 118, 752

This paper has been typeset from a TEX/LATEX file prepared by the author.