

Quantum measurement theory and the  
quantum Zeno effect

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## 0.1 Acknowledgements

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I wish to express my deep appreciation to my supervisor for the guidance offered during this thesis. Surely, one of the more difficult of the objectives of supervision would be the encouragement of creativity in a student - and made all the more so by the student's ignorance and lack of confidence. I have now had the very real pleasure of participating in, and, in part, initiating some rare moments of creative invention in my physics research. For this, I am deeply grateful.

I would also like to extend that same deep appreciation to H.M.Wiseman, a fellow student who delights in tossing ideas around and who contributed enormously to these moments of creativity.

The reader of this thesis will also share my appreciation for the assistance of S.J.Carter, who contributed the plotting routines used to produce most of the figures in this thesis.

## 0.2 Abstract

This is a theoretical thesis in the area of quantum measurement theory. Due to the extensive breadth of this field we choose to narrow our focus to examine a particular problem - the quantum Zeno effect (defined below).

Quantum measurement theory is introduced in Chap. (1), using the terminology of effects and operations. This approach allows an operational definition of such terms as a state vector, an ensemble, and a measurement device (for instance), and a consideration of interactions between quantum systems and inaccurate measurement devices. We further introduce the quantum trajectories approach to consider the evolution of an individual quantum system subject to measurement.

The quantum Zeno effect is introduced in Chap. (2). Any quantum treatment of a measurement interaction must consider the measurement backaction onto the measured system and this backaction will disrupt the free evolution of the system. The quantum Zeno effect occurs in the strong measurement limit where the measurement backaction totally freezes the evolution of the system, thus rendering the measurement useless. The effect is introduced via projective measurements of two level systems subject to measurement of level populations. At this stage we are able to discuss the main questions addressed by this thesis, and present its structure in Chap. (2).

We then develop a new measurement model for the interaction between a system and a measurement device in Chap. (3). Our motivation in doing this is to better model the usual laboratory meter, and in our approach the meter dynamics are such that it relaxes towards an appropriate readout of the system parameter of interest. The irreducible quantum noise of the meter introduces fluctuations that drive the stochastic dynamical collapse of the system wavefunction. In our model, the measured system dynamics (if treated selectively) are described by a stochastic, nonlinear Schrödinger equation.

A double well system subject to position measurement provides a natural first application for this model. This is done in Chap. (4) where we monitor the coherent tunnelling of a particle from one well to the other. The advantage afforded by considering this system is that it displays differing regimes where the measurement observable (position) is approximated as possessing either, respectively, a continuous or a discrete eigenvalue structure. Thus, we use this one model to explore the quantum Zeno effect in both measurement regimes.

The above treatment is of a theoretical measurement model. In Chap. (5) we turn to consider a recent experimental test of the quantum Zeno effect which examined the dynamics of a two level atom subject to pulsed measurements of atomic level populations. We treat a slightly modified experiment in a fully continuous measurement regime. By first unravelling the optical Bloch equations, and second, using the quantum trajectories approach we demonstrate the existence of certain measurement regimes where there is a quantum Zeno effect, and other regimes where no measurement of the atomic populations is being effected at all. Through these results we demonstrate the importance of making a full analysis of the system-detector interaction before any conclusions can be made.

In the remainder of the thesis we propose further possible tests of the quantum Zeno effect. In Chap. (6) the evolution of a Rydberg atom exchanging one photon with a single cavity mode subject to measurement is examined. The measurement is made by monitoring the photon number occupancy of the cavity mode using a beam of Rydberg atoms configured so as to perform phase sensitive detection. In the limit of frequent monitoring we show that the free oscillation of the atomic inversion is disrupted, and the atom is trapped close to its initial state. This is the quantum Zeno effect.

In Chap. (7) we realize the Zeno effect on two possible systems. We consider first, a two level Jaynes-Cummings atom interacting with a cavity mode, and second, two electromagnetic modes configured as a multi-level parametric frequency converter. These systems interact with another cavity mode via a quadratic coupling system based on four wave mixing, and constructed to be a quantum nondemolition measurement of the photon number. This mode is damped to the environment thus effecting a measurement of the system populations. Again we show that this interaction, can manifest the quantum Zeno effect. Our explicit modelling of the system-detector interaction enables us to show how the effect depends on the resolution time of the detector.

Finally, we consider a proposed measurement of the square of the quadrature phase of an electromagnetic mode in Chap. (8). Here, a three mode interaction mediated by a second order nonlinear susceptibility is considered. One mode, the pump, is prepared in a feedback generated photon number state to give insight into the role of pump noise. The other two modes are treated as an angular momentum system, and we show that photon counting on the two mode rotation system effects the above mentioned measurement. In addition, this measurement provides a direct measure of the second order squeezing of the signal.

With that we finish our investigation of the quantum Zeno effect using the techniques of quantum measurement theory. However, in the epilogue [Chap. (9)] we note that no thesis in quantum measurement theory would be complete without some consideration of the “meaning” attributed to the theory. In the epilogue we take a novel historical approach and examine the method by which metaphysical theories are formed to draw conclusions regarding quantum metaphysics.

## 0.3 List of Publications

Please note : \* indicates which publications were included in this thesis.

- 1 *Emission at the Rabi frequency in stark coupled driven three level systems*, B.J.Dalton, M.J.Gagen, Coherence and Quantum Optics V. Editor L.Mandel, E.Wolf, (1984)
- 2 *Strongly driven stark coupled three level systems and transitions at the Rabi frequency*, B.J.Dalton, M.J.Gagen, J.Phys.B:At.Mol.Phys., **18**, 4403-23 (1985)
- 3 *Correlated photons in parametric frequency conversion with initial two-mode squeezed states*, M.J.Gagen, G.J.Milburn, Optics Comms., **76**, 253, (1990)
- 4 \* *Quantum nondemolition measurements using a fully quantised parametric interaction*, M.J.Gagen, G.J.Milburn, Phys.Rev.A, **43**, 6177 (1991)
- 5 \* *The quantum Zeno effect induced by quantum nondemolition measurement of photon number*, M.J.Gagen, G.J.Milburn, Phys.Rev.A, **45**, 5228 (1992)
- 6 \* *Rydberg-atom phase-sensitive detection and the quantum Zeno effect*, G.J.Milburn, M.J.Gagen, Phys.Rev.A., **46** (1992)
- 7 \* *Atomic tests of the Zeno effect*, M.J.Gagen, G.J.Milburn, Phys.Rev.A, **47**, 1467 (1993)
- 8 \* *Continuous position measurements and the quantum Zeno effect*, M.J.Gagen, H.M.Wiseman, G.J.Milburn, accepted by Phys.Rev.A, (1993)



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