

Stabilizer quantum error correction with quantum bus computation

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(Received 24 December 2006; revised manuscript received 19 April 2007; published 5 July 2007)

In this paper we investigate stabilizer quantum error correction codes using controlled phase rotations of strong coherent probe states. We explicitly describe two methods to measure the Pauli operators that generate the stabilizer group of a quantum code. First, we show how to measure a Pauli operator acting on physical qubits using a single coherent state with large average photon number, displacement operations, and photon detection. Second, we show how to measure the stabilizer operators fault-tolerantly by the deterministic preparation of coherent quantum superposition (“cat”) states along with one-bit teleportations between a qubitlike encoding of coherent states and physical qubits.

DOI: [10.1103/PhysRevA.76.012303](https://doi.org/10.1103/PhysRevA.76.012303)

PACS number(s): 03.67.Pp, 03.67.Lx, 42.65.-k

I. INTRODUCTION

The question of which physical system is best suited for quantum-information processing is still open, each implementation proposal having strengths and weaknesses. In some systems (such as optics), it is difficult to make qubits interact, so that the two-qubit gates needed for universal computation are difficult to implement. One scheme, proposed by Gottesman *et al.* [1], circumvents the need to make qubits interact directly by using a modified teleportation protocol. A generalization of this leads to the cluster state proposal of Raussendorf *et al.* [2], where a large entangled state is prepared offline, and computation is performed by a sequence of single-qubit measurements which depend on the outcomes of previous measurements. A different scheme that bypasses the need for qubits to interact directly was proposed by Nemoto *et al.* [3,4]. This scheme showed that, by inducing a phase on a large coherent state bus mode which depends on the logical state of the physical qubits, one can implement a near-deterministic controlled-NOT (CNOT) gate between the qubits. Coherent states are particularly useful because of the ease with which they may be produced, e.g., with lasers or Bose-Einstein condensates. Further developments have shown more direct methods to perform two-qubit gates with bus modes, termed *qubus computation* [5].

If qubus computation is to be seriously considered for physical implementation, a full analysis of the propagation of errors should be undertaken. The starting point for these considerations is whether we can perform quantum error correction (QEC) on qubits efficiently. In particular, can one measure the syndromes for a given stabilizer code directly with controlled rotations (CRs) and strong coherent probe beams efficiently? Recent work by Yamaguchi *et al.* [6] demonstrates how to measure the syndromes for some stabilizer codes using these tools. They show that the stabilizers for the

three-bit-flip code can be measured directly with CRs and a single strong coherent bus mode. The stabilizers for Shor’s nine-qubit code [10] can also be measured, showing that it is possible to correct for any error on a single qubit in an encoded block.

The purpose of this paper is to generalize the results of Yamaguchi *et al.* [6] and demonstrate how CRs can be used to implement quantum error correction with any possible stabilizer code efficiently. The paper is organized as follows. In Sec. II we briefly describe the results by Yamaguchi *et al.* [6] detailing the notation used throughout this paper. Then, we present two schemes to measure the syndromes of an arbitrary weight n Pauli operator, using the stabilizer operators of the seven-qubit code as a concrete example for each one of these schemes. The first scheme, shown in Sec. III, uses a single strong coherent probe beam, a quadratic number of CRs, a linear number of coherent displacements, and a photon number measurement. We also show how this scheme can be modified to use homodyne measurement at the cost of a slightly larger number of CRs and coherent displacements. In Sec. IV we describe the second scheme, which is a fault-tolerant approach to the measurement of the Pauli operators. This scheme requires a linear number of strong coherent pulses, CRs, and detectors. Although we focus on the seven-qubit code—which has stabilizer generators with weight 4—for each of these schemes we describe how to generalize to Pauli operators of weight n .

II. BACKGROUND

In [6] it was shown that the stabilizers for the three-qubit bit-flip code ($|0\rangle \rightarrow |000\rangle$, $|1\rangle \rightarrow |111\rangle$) could be measured with the parity gate depicted in Fig. 1(a).

It can be seen that this circuit is a parity gate when we consider its effect on the input state $|\psi_{\text{in}}\rangle = (a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle)|\alpha\rangle$, where $|\alpha\rangle$ is a coherent bus mode. The effect of the CRs is to apply a phase to the coherent beam if our data qubit is $|1\rangle$ and leave it alone otherwise: $(a|0\rangle + b|1\rangle)|\alpha\rangle \rightarrow a|0\rangle|\alpha\rangle + b|1\rangle|ae^{i\theta}\rangle$. Before the detector D_2 in

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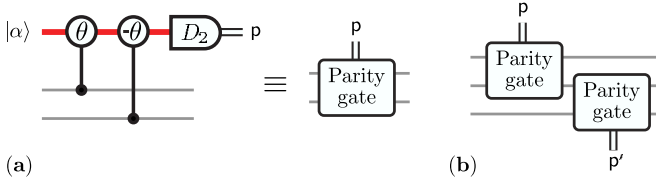


FIG. 1. (Color online) (a) Circuit to measure the parity of two qubits. The CRs are $\pm\theta$. (b) Two parity gates combined to measure the Pauli operators ZZI and IZZ .

Fig. 1(a), the state $|\psi_{in}\rangle$ is $(a_0|00\rangle + a_3|11\rangle)|\alpha\rangle + a_1|01\rangle|\alpha e^{-i\theta}\rangle + a_2|10\rangle|\alpha e^{i\theta}\rangle$. When D_2 is a homodyne detection along the x quadrature we are able to distinguish $a_0|00\rangle + a_3|11\rangle$ from $a_1|01\rangle + a_2|10\rangle$, since a homodyne measurement of $|\alpha\rangle$ along the x quadrature is equivalent to the projection $\langle x|\alpha\rangle$. That is, $|\alpha e^{\pm i\theta}\rangle$ are indistinguishable when we homodyne-detect along the x quadrature. This is the basis of the CNOT gate shown in [3].

With two parity gates we can measure the Pauli operators ZZI and IZZ . That is, one parity gate is applied to qubits 1 and 2 to measure ZZI while the second parity gate is applied to qubits 2 and 3 to measure IZZ , as shown in Fig. 1(b). The state before the application of the parity gates is $|\bar{\psi}_{in}\rangle = (c_0|000\rangle + c_1|111\rangle)|\alpha\rangle|\alpha\rangle$. There are four cases to consider: no error, $|\bar{\psi}_{in}\rangle$; an error on qubit 1, $XI|\bar{\psi}_{in}\rangle$; an error on qubit 2, $IX|\bar{\psi}_{in}\rangle$; an error on qubit 3, $IIX|\bar{\psi}_{in}\rangle$. We can see what the effect of a bit-flip error on each of the modes is by considering the state $|abc\rangle|\alpha\rangle|\alpha\rangle$, where $a, b, c \in \{0, 1\}$. Directly before homodyne detection in Fig. 1(b) $|abc\rangle|\alpha\rangle|\alpha\rangle$ becomes $|abc\rangle|\alpha e^{i(a-b)\theta}\rangle|\alpha e^{i(c-b)\theta}\rangle$. When we measure the probe states to be $|\alpha e^{\pm im\theta}\rangle|\alpha e^{\pm in\theta}\rangle$, where $m, n \in \{0, \pm 1\}$, we know whether there was no error ($m, n=0$) or a one-bit-flip error, the location of the bit flip also being identified by the values of m and n . Similar methods can be applied to measure the stabilizer operators for Shor's nine-qubit code. The natural question that arises is whether we can use techniques similar to those above to measure the syndromes for an arbitrary stabilizer code.

A. Larger codes

As a concrete example, consider the $[[7,1,3]]$ stabilizer code [7]. This code can correct a single arbitrary quantum error in any of the seven qubits, and it has been used extensively in studies of fault tolerance in quantum computers due to the fact that it allows for simple constructions of fault-tolerant encoded gates [8]. In order to detect which error has corrupted the data, one must measure six multiqubit Pauli operators which, up to qubit permutations and local unitaries, are equivalent to the Pauli operator $ZZZZ$, or the measurement of only the parity of four qubits. For an arbitrary stabilizer code, various multiqubit Pauli operator must be measured, each of which is always equivalent to a measurement of only the parity of a subset of qubits; thus it is sufficient to consider only multiqubit parity measurements in order to perform quantum error correction with stabilizer codes.

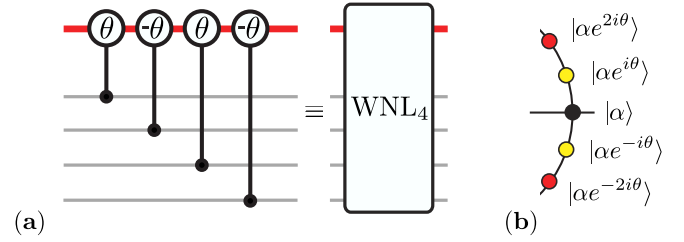


FIG. 2. (Color online) (a) First attempt at using CRs to measure the parity of four qubits with a single strong coherent probe. (b) Position of phase-space peaks corresponding to the state of the coherent probe beam. Light gray (yellow) circles correspond to odd states while dark gray (red) and black circles correspond to even states.

III. SINGLE-COHERENT-STATE PULSE

In order to measure $ZZZZ$ with CRs, we can start with the encoded state $(c_0|0_L\rangle + c_1|1_L\rangle)|\alpha\rangle$, and design a circuit that gives us $|\alpha_1\rangle$ when there was no error (even parity) and $|\alpha_2\rangle$ when there was an error (odd parity), where $\alpha_1 \neq \alpha_2$.

Ideally we would want to do this with just one coherent probe beam, four CRs, and a single homodyne detection, following a direct analogy with the circuit depicted in Fig. 1(a). However, this is not possible. The best we can do is have some even states go to $|\alpha\rangle$ and the rest go to $|\alpha e^{\pm 2i\theta}\rangle$, while the odd states go to $|\alpha e^{\pm i\theta}\rangle$. The circuit that performs this is shown in Fig. 2(a).

Notice that in phase space we would have five points—three for the even states ($|\alpha\rangle, |\alpha e^{\pm 2i\theta}\rangle$) and two for the odd states ($|\alpha e^{\pm i\theta}\rangle$)—as can be seen in Fig. 2(b). If we were to homodyne-detect the probe beam at this stage, we would partially decode our encoded state $c_0|0_L\rangle + c_1|1_L\rangle$ since we can distinguish the state $|\alpha\rangle$ from $|\alpha e^{\pm 2i\theta}\rangle$. The problem now becomes determining what operations must be done before we homodyne detect so that we only distinguish between states of different parity in the first four qubits, and nothing more. It turns out that either homodyne or photon number detection can be used, depending on the operations applied before the measurement.

A. Photon number measurement

If we incorporate displacements along with Fig. 2(a) we can take the five points in phase space to just three. Displacements of a state can be easily implemented by mixing the state with a large coherent state on a weak beam splitter, the size of the coherent state amplitude and beam splitter reflectivity deciding the displacement [9]. If we have three displacements and three applications of Fig. 2(a), as in Fig. 3(a), we find that $|\text{odd}\rangle \rightarrow |-4\alpha \sin^2(\theta/2)[2 \sin^2(\theta) + \cos(\theta)]\rangle$ and $|\text{even}\rangle \rightarrow |\pm 2\alpha \sin^2(\theta)[2 \cos(\theta) - 1]\rangle$, as depicted in Fig. 3(b). The displacements that accomplish this are $D(\beta_1) = D\{-4\alpha \cos^2(\theta/2)[2 \cos(\theta) - 1]\}$, $D(\beta_2) = D\{\alpha[1 + 2 \cos(\theta) + 2 \cos(3\theta)]\}$ and $D(\beta_3) = D\{\alpha[\cos(2\theta) - \cos(3\theta) - \cos(\theta) - 1]\}$.

Notice that the red (gray) and black circles in Fig. 3(b) are equidistant from the p axis. We can thus perform a photon number measurement on the probe beam to determine

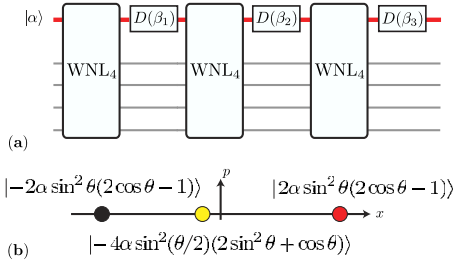


FIG. 3. (Color online) (a) CRs used to measure the parity of four qubits with a photon number detection. (b) Positions of phase-space peaks corresponding to the state of the coherent probe beam.

whether we had an odd or even state. In order for a photon number measurement to distinguish the odd from the even states we require $\alpha\theta^2 \gg 1/\sqrt{3}$.

There will be phases associated with each of the states in Fig. 3(b) due to the displacements [9]: $D(\beta)|\alpha\rangle = e^{i\text{Im}(\beta\alpha^*)}|\alpha+\beta\rangle$. These can easily be taken care of after the photon number measurement with appropriate phase shifts on each mode for the odd and even state cases.

We can use this method to measure the parity for a state of any size. If we have n qubits then we can have at best $n+1$ points in phase space, using the $\theta, -\theta, \theta, -\theta$ pattern for the CRs shown in Fig. 2(a). Using displacements and a photon number detector we are able to measure the parity. In general, if n is even, we need $n-1$ displacements and n^2-n CRs with a photon number measurement. When n is odd, after the application of the circuit analogous to Fig. 2(a) of size n , we will have the point $|\alpha e^{i(2n-1)\theta}\rangle$ in phase space without the point $|\alpha e^{-i(2n-1)\theta}\rangle$. So we need an extra displacement to move the nonsymmetric point. If n is odd we need n displacements and n^2 CRs with a photon number measurement. The phase shifts due to the displacements for the general n case can also be taken care of by phase shifts on each mode for the even and odd state cases.

For this method to work we need the use of a number-discriminating photodetector. In practice, it is well known that homodyne detection is much more precise than number-discriminating photodetectors. For this reason, we describe how to measure a Pauli operator using homodyne detection.

B. Homodyne detection

Consider the ZZZZ case again. After applying Fig. 2(a) we have five points in phase space. Ideally we want $|\alpha\rangle$ and $|\alpha e^{\pm 2i\theta}\rangle$ to become one point in phase space, say R_1+iR_2 , and $|\alpha e^{\pm i\theta}\rangle$ to become one point, say R_3+iR_4 . If this were possible then homodyne detection could be used. This can be done with five displacements and six applications of Fig. 2(a), requiring ten simultaneous equations to be solved. Without loss of generality, we set $R_2=R_4=R_1=0$. The equations to be solved are $e^{4i\beta\theta}A + e^{3i\beta\theta}B + e^{2i\beta\theta}C + e^{i\beta\theta}D + E = e^{-i\beta\theta}\alpha_\beta - e^{5i\beta\theta}$, where $\alpha_{0,\pm 2} = 0$ and $\alpha_{\pm 1} = R_3$.

After solving these equations we find that A, B, C, D , and E scale as $-R_3/\theta^4$. We are free to choose the distance between the origin and R_3 to be arbitrarily large, at the expense of using arbitrarily large displacements. We can also use the above method to distinguish the parity of any given state of

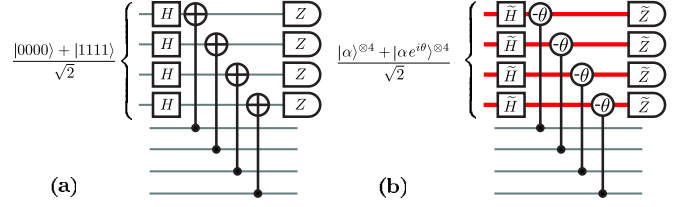


FIG. 4. (Color online) (a) Circuit for the measurement of the parity of four qubits [10]. (b) Same circuit modified to use coherent states and CRs.

n qubits. If we have n qubits, we have $n+1$ points in phase space, using a circuit similar to Fig. 2(a). In order to distinguish the parity we need $n+1$ displacements and $n(n+2)$ CRs. Once again, the phases resulting from the displacements can be taken care of after the homodyne detection.

IV. FAULT TOLERANCE

These two methods to measure weight- n Pauli operators cannot be used for fault-tolerant quantum computation. If there is an error on the coherent probe mode during one of the CRs, say photon loss, it will be transferred to a phase error in each of the physical qubits it interacts with afterward—that is, a single fault can cause a number of errors that is greater than the number of errors the code can correct. For this reason we now look at measuring the syndromes of stabilizers fault-tolerantly.

Shor [10] first described how to fault-tolerantly measure the generators of the stabilizer group of a quantum error correcting code using ancilla Greenberger-Horne-Zeilinger (GHZ) states $(|0\rangle^{\otimes n} + |1\rangle^{\otimes n})/\sqrt{2}$ with CNOT and Hadamard operations. For example, in order to measure the Pauli operator ZZZZ (which is equivalent to measuring the parity of four qubits and nothing else), we would use the circuit shown in Fig. 4(a).

To fault-tolerantly measure the stabilizer group generators of a QEC with CRs we make three modifications to Fig. 4(a). First, instead of using $|0\rangle$ and $|1\rangle$ for the ancilla, we use the coherent states $|\alpha\rangle$ and $|\alpha e^{i\theta}\rangle$, respectively. In that case, the ancilla GHZ state becomes $(|\alpha\rangle^{\otimes n} + |\alpha e^{i\theta}\rangle^{\otimes n})/\sqrt{2}$. Second, we replace the CNOT operations with CRs, which will cause a phase shift if the physical state is $|1\rangle$ and do nothing otherwise, i.e., $|1\rangle|\alpha\rangle \rightarrow |1\rangle|\alpha e^{-i\theta}\rangle$ and $|1\rangle|\alpha e^{i\theta}\rangle \rightarrow |1\rangle|\alpha\rangle$. We also need to replace the Hadamards operations with some quantum operation \tilde{H} that will perform the mapping $\tilde{H}|\alpha\rangle \approx (|\alpha\rangle + |\alpha e^{i\theta}\rangle)/\sqrt{2}$ and $\tilde{H}|\alpha e^{i\theta}\rangle \approx (|\alpha\rangle - |\alpha e^{i\theta}\rangle)/\sqrt{2}$. Third, we replace the qubit measurements with some sort of optical measurement that distinguishes between $|\alpha\rangle$ and $|\alpha e^{\pm i\theta}\rangle$ but not between $|\alpha e^{i\theta}\rangle$ and $|\alpha e^{-i\theta}\rangle$ —this is what we call the \tilde{Z} measurement. This new circuit is depicted in Fig. 4(b). Note that usually the CNOT gate cannot be simulated by a single CR, as we have done here. What allows us to do this substitution is the fact that the coherent mode is measured immediately after the CR. If a sequence of CNOT gates were to be applied, the equivalent circuit using CRs would be more complex [3].

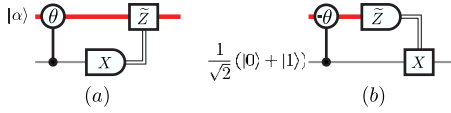


FIG. 5. (Color online) Approximate one-bit teleportation protocols [11] using CRs. The measurement-dependent Pauli corrections need not be performed, as discussed in the body of the paper.

The \tilde{Z} measurements can be performed directly by homodyne detection, or by displacements followed by photon-counting detectors—in both cases, using techniques outlined earlier in this paper. What remains to be specified is the preparation of the coherent quantum superposition “cat” state $(|\alpha\rangle^{\otimes n} + |\alpha e^{i\theta}\rangle^{\otimes n})/\sqrt{2}$ and the implementation of the \tilde{H} operation. One solution for the cat state preparation is to use one bit teleportations [11], which translate states from the $|0\rangle/|1\rangle$ basis to the $|\alpha\rangle/|\alpha e^{i\theta}\rangle$ basis. Preparation of the cat state is done by using the one-bit teleportation in Fig. 5(a) to prepare $|\sqrt{n}\alpha\rangle + |\sqrt{n}\alpha e^{i\theta}\rangle$ from the state $(|0\rangle + |1\rangle)/\sqrt{2}$ and the coherent state $|\sqrt{n}\alpha\rangle$, and then sending this state into an n -port symmetric beam splitter [12,13]. In principle, we are required to correct the state before the beam splitter by applying the transformation \tilde{Z} such that $\tilde{Z}|\alpha\rangle \approx |\alpha\rangle$ while $\tilde{Z}|\alpha e^{i\theta}\rangle \approx -|\alpha e^{i\theta}\rangle$. However, we can avoid explicitly applying this transformation by keeping track of this necessary correction—which is called the *Pauli frame* [14]—and compensating for it in subsequent measurements. Similarly, to perform the \tilde{H} (the approximate Hadamard operation on coherent state logic) we first use Fig. 5(b) to teleport the quantum state from the bus to a qubit, then perform the Hadamard transformation, and finally teleport back to the coherent state logic using the circuit shown in Fig. 5(a). In the case where the \tilde{Z} is implemented as a homodyne measurement on the bus, additional Z rotations on the qubit, with an angle dependent on the measurement outcome, are necessary.

These teleportations, when performed back to back to teleport a qubit state to another qubit, can also be used as *leakage reduction units* to reduce leakage faults to regular faults [15].

A. Noisy ancillas

If the probability of error at each gate is bounded by ϵ , transversal operations and encoding can ensure that the probability of an uncorrectable error is $O(\epsilon^2)$ instead of $O(\epsilon)$. An error during cat state preparation may lead to correlated X -like errors in the cat state with probability $O(\epsilon)$, which can lead to uncorrectable errors in the encoded data during the measurement of the Pauli operator, thus defeating the purpose of encoding the data for fault-tolerant quantum computation. In order to avoid this, one can verify the integrity of the cat state via nondestructive state measurement [16,17]. When using CRs and coherent beam probes, this translates to preparing an extra copy of the cat state, which remains in coherent state logic, interacting with the qubit GHZ state transversally with controlled $-\theta$ rotations, and \tilde{Z} measuring each mode of the ancillary cat state. By performing classical

error correction on the measurement outcomes, one can deduce the locations of X -like errors in either the GHZ state or the ancillary cat state. If the data are encoded in a code that can correct a single error, repeating this procedure with another ancillary cat state allows for the inference of which locations in the qubit GHZ state have X errors with high enough probability to ensure that uncorrectable errors are only introduced into the data with probability $O(\epsilon^2)$ [16], so that Pauli measurements with a verified ancilla can be used for fault-tolerant quantum computation.

Z -like errors (including dephasing of coherent superpositions, one of the consequences of photon loss in the CRs) do not lead to errors in the encoded data, just errors in the outcome of the Pauli operator measurement. If error correction is to be performed, the Pauli operator measurement must be repeated three times, and a majority vote of the outcomes is taken, in order to ensure that the measurement outcome is reliable [16].

Some of the systematic errors in the probe beams, such as phase rotation or attenuation (also consequences of photon loss in the CRs), can be partially compensated for by additional linear-optics elements and by adjusting the \tilde{Z} measurements individually to minimize additional X errors. Moreover, errors in the transversal operations during the preparation of the cat state are independent, and thus do not need special consideration during this verification stage—they do contribute to ϵ , however, and are thus crucial for fault-tolerance threshold calculations.

B. Overhead comparison

The resources required to measure a weight- n Pauli operator are $3n+1$ CRs, $n+1$ ancillary qubits, $2n$ \tilde{Z} measurements, and $n+1$ qubit measurements. If one were to naively implement Shor’s circuit using the general CNOT construction by Nemoto and Munro [3], one would need $4(2n-1)$ CRs, $2(2n-1)$ coherent state measurements, n ancilla qubits, and n qubit measurements.

Similarly, for cat state verification, the overall overhead for each attempt at measuring a weight- n Pauli operator consists of $2(n+1)$ CRs, two ancillary qubit preparations and measurements, and $2n$ \tilde{Z} measurements. A naive implementation making direct use of the general CNOT construction would require $8(2n-1)$ CRs, $4(2n-1)$ coherent state measurements, $2n$ ancilla qubits, and $2n$ qubit measurements.

The approach described here allows for a linear saving in resources, which is significant for physical implementations.

V. DISCUSSION

We have shown two schemes to measure the syndromes of an arbitrary weight- n Pauli operator. The first scheme uses a single strong coherent probe beam, a quadratic number of CRs, a linear number of coherent displacements, and a photon number or homodyne measurement—however, this

scheme is not fault tolerant. The second scheme we described is fault tolerant, and the amount of resources scales linearly with the weight of the Pauli operator. This demonstrates how it is in principle possible to perform general fault-tolerant quantum computation in the qubus architecture. It is worth noting that we could have easily used controlled displacements in the place of CRs in the methods presented here.

ACKNOWLEDGMENTS

We would like to thank P. Aliferis and R. Van Meter for valuable discussions. We are supported in part by NSERC, ARO, CIAR, MITACS, the Institute for Quantum Computing, MEXT in Japan, and the EU project QAP. C.R.M. would like to thank Mike and Ophelia Lazaridis for funding.

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