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# Reliability Evaluation of Power System Considering Voltage Stability and Continuation Power Flow

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***This article describes the methodology for evaluation of the reliability of an composite electrical power system considering voltage stability and continuation power flow, which takes into account the peak load and steady state stability limit. The voltage stability is obtained for the probable outage of transmission lines and removal of generators along with the combined state probabilities. The loss of load probabilities (LOLP) index is evaluated by merging the capacity probability with load model. State space is truncated by assuming the limits on total numbers of outages of generators and transmission lines. A prediction correction technique has been used along with one dimensional search method to get optimized stability limit for each outage states. The algorithm has been implemented on a six-bus test system.***

**Keywords**: Continuation power flow, loss of load probability, optimization, peak load, voltage stability limit.

## NOMENCLATURE

$\alpha$ : load participation factor

$\beta$ : vector of $\tan\theta_p$ , and $\theta_p$ being power factor angle at p$^{th}$ bus

$\gamma$ : a vector of bus voltage angles

$\eta$ : predicted value of continuation variable

$f_i$ : $i^{th}$ line real power (MW) flow

$f_i^{\text{limit}}$ : line real power flow limit

$k$ : vector of generation participation

$NC$ : total number of reactive power control variables

$P_d^{\text{limit}}$ : static voltage stability limit

$P_i^{\min}, P_i^{\max}$ : lower and upper limits of real power generation at i$^{th}$ bus

$Q_p^{\min}, Q_p^{\max}$ : lower and upper limits of reactive power generation limits at p$^{th}$ bus

$U_p$ : $p^{th}$ reactive power control variables

$U_p^{\min}, U_p^{\max}$ : lower and upper limits of reactive power control variables at p$^{th}$ bus

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$V_i^{\min}, V_i^{\max}$ : lower and upper limits of bus voltage at i[th] bus

$V$ : a vector of bus voltage magnitudes

$t$ : load parameter

$X_k$ : continuation variable

## 1. INTRODUCTION

In recent years reliability evaluation of the combined generation-transmission (bulk power system) has been a major concern in power system planning. Recent papers [1]-[4] present an extensive bibliographical survey for evaluation of reliability of power system. A composite system can be divided in many operating states in terms of the capacity available to fulfill demand subject to the satisfaction of security limits (line flows and voltage limit). Hence, the evaluation of a reliability index for a composite system is very much computationally demanding. Power system reliability is usually categorized into the regions of adequacy and security. System adequacy is defined as the ability of the system to supply its load accounting line flow constraints and accounting outages of generators and branches whereas system security (dynamic) is defined as the ability of the power system to withstand disturbances arising from faults or unscheduled removal of bulk power supply equipment. This means that adequacy assessment is the steady state post outage analysis of the composite power system while security assessment (in reliability evaluation aspect) involves dynamic condition analysis. This paper focuses attention on adequacy assessment.

A linear programming model accounting voltage and line flow constraints for adequacy assessment of bulk power system has been used in [5]. Pereira et al. [6] developed a reliability evaluation methodology for composite system based on Monte-Carlo sampling with a variance reduction scheme, which permits the incorporation of planner's experience or analytical models as 'Regression' variables. Deng et al. [7] proposed an efficient new approach for power system reliability evaluation using the decomposition simulation approach. The interconnected systems in this approach have been modeled by a probabilistic flow network with capacitated areas. Each area is denoted by a node in the network. Source and load are represented by additional nodes. Billinton et al. [8] developed a system state transition sampling method for composite system reliability evaluation using Monte Carlo simulation technique. Meio et al. [9] presented the effects of voltage collapse problems in the reliability evaluation of composite system and described an approach to calculate voltage collapse related bulk reliability indices as well as their impact on the adequacy reliability indices. The adequacy analysis of each selected system state is carried out in two steps. A system state transition sequence is utilized to calculate frequency index.

Mitra et al. [10] incorporated dc load flow model in the decomposition-simulation method for evaluating multi-area reliability evaluation. State enumeration approach using topological analysis has been used to evaluate bulk power system reliability in [11]. System frequency, duration and availability indices have been obtained using topological enumeration. The method requires the use of ac or dc load flow to test the condition of contingency state. Singh et al. [12] have used state space pruning for evaluation of bulk power system reliability by performing Monte Carlo simulation selectively on those regions of the state space where loss of load states are more likely to occur. Khan [13] used security-based model to evaluate reliability of a composite power system and presented an approach to quantify a power network into several operating states in terms of degree of adequacy and security (static) constraints.

A methodology for reliability assessment at a restructured power system using reliability network equivalent techniques has been presented in [14]. The main objective of power system restructuring and deregulation is to introduce competition in the power industry and to allow customers to select their suppliers based on price and reliability. Rios et al. [15] presented a methodology to evaluate the reliability and to calculate interruption costs at the load bus level in the bulk power system. The methodology is based on a non-sequential Monte-Carlo simulation combined with a linear optimization model in which the load at every bus is represented by two components i.e. a firm and non-firm portion. Expected values of not served energy, not served demand, and LOLP are computed for the whole system. Billinton et al. [16] developed a system for unreliability cost assessment of an electric power system using reliability network equivalent approaches. Unreliability cost evaluation of an entire power system provides a set of indexes, which can be used by a system planner to balance the investments in different segments of the system in order to provide acceptable load point reliability.

Nowadays voltage stability is a serious problem that power utilities usually explore in the planning stage. It is essential that the capacity state of combined generation and transmission system must be evaluated based on static voltage stability limit. It has become important because this limit in power network is approaching much earlier than thermal or angle stability constrained limit due to network limitations or reactive power deficiency. Meio et al. [9] presented an approach to calculate voltage collapse related bulk reliability indices as well as their impact on composite power system adequacy indices based on restoring system solvability by load shedding. A methodology has been presented in [17], which induces voltage stability consideration in adequacy assessment of bulk power system. The voltage stability indicator is calculated for all possible system contingencies. A bisectional algorithm is then used to determine the amount of load, which is required to be shed to alleviate all voltage stability violations.

In this article, a methodology has been developed to evaluate probability of failure based on peak load for the composite system accounting voltage stability considerations. Steady state voltage stability limit for possible line outages and generation outages has been calculated along with the combined generation and transmission state probabilities. Thus the capacity probability may be merged with suitable load model (peak load/load duration curve) to evaluate the LOLP index. State space is truncated by assuming the limits on total number of component (generation and transmission line) failure.

## 2. STEADY STATE VOLTAGE STABILITY LIMIT USING PREDICTOR-CORRECTOR TECHNIQUE

Predictor-corrector technique overcomes the non-convergence of conventional Newton-Raphson (N-R) method of load flow analysis near the voltage stability limit [18]. This technique uses an iterative process involving predictor and corrector steps. Using a known initial solution and the tangent vector, a new solution is predicted for a specified pattern or load increases. Using these estimated solution corrector step converges to exact solution point. Continuation power flow equations are similar to those of conventional power flow analysis except that the increase in total load is added as a load parameter ‘t’. The general form of equations is given as

$$F(V, \delta, t, k, \beta, \alpha) = 0 \tag{1}$$

### 2.1 Predictor Step

It is assumed that initial load flow solution is available. For predicting the next step solution tangent vector is obtained by setting differential of eqn. (1) equals to zero as follows

$$\begin{bmatrix} F_\delta & F_v & F_t \end{bmatrix} \begin{bmatrix} d\delta \\ dv \\ dt \end{bmatrix} = 0 \qquad (2)$$

where $F_\delta = \dfrac{\partial F}{\partial \delta}, F_v = \dfrac{\partial F}{\partial v}, F_t = \dfrac{\partial F}{\partial t}$

The appearance of load parameter '$t$' adds one more equation. To solve eqn. (2) one of the components of the tangent vector is set to +1 or –1. This also removes ill conditioning of the equations. This component is called continuation parameter. Now eqn. (2) is written as

$$\begin{bmatrix} F_\delta & F_v & F_t \\ e_k^T & 0 & 0 \end{bmatrix} \begin{bmatrix} d\delta \\ dv \\ dt \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 1 \end{bmatrix} \qquad (3)$$

Initially the load parameter is chosen as continuation parameter and the corresponding component of tangent vector is set to +1. When the system is heavily stressed, then the continuation parameter is chosen to be the state variable being the greatest rate of change near the given solution and sign of its slope determines the sign of the corresponding component of tangent vector.

After solving for tangent vector the prediction for the next solution is given as

$$\begin{bmatrix} \delta \\ v \\ t \end{bmatrix} = \begin{bmatrix} \delta^0 \\ v^0 \\ t^0 \end{bmatrix} + \sigma \begin{bmatrix} d\delta \\ dv \\ dt \end{bmatrix} \qquad (4)$$

where $\left\{\delta^0, v^0, t^0\right\}$ is the initial solution vector. Convergence of the corrector step solution mainly depends on the step size $\sigma$, if convergence is not obtained in corrector step that predicted solution should be obtained again, using eqn. (4), with reduced value of step size $\sigma$ and again corrector step is repeated with new predicted solution.

### 2.2 Corrector Step

In corrector step the continuation power eqn. (1) is augmented by one more equation that specifies the continuation parameter as follows:

$$\begin{bmatrix} F(v, \delta, t, k, \alpha, \beta) \\ (X_k - \eta) \end{bmatrix} = 0 \qquad (5)$$

Eqn. (5) is solved by N-R method using the initial condition as given by eqn. (5). The introduction of one additional equation specifying $X_k$ models the Jacobian on N-R method, non-singular even at collapse point. Thus, it is possible to obtain steady state voltage stability limit.

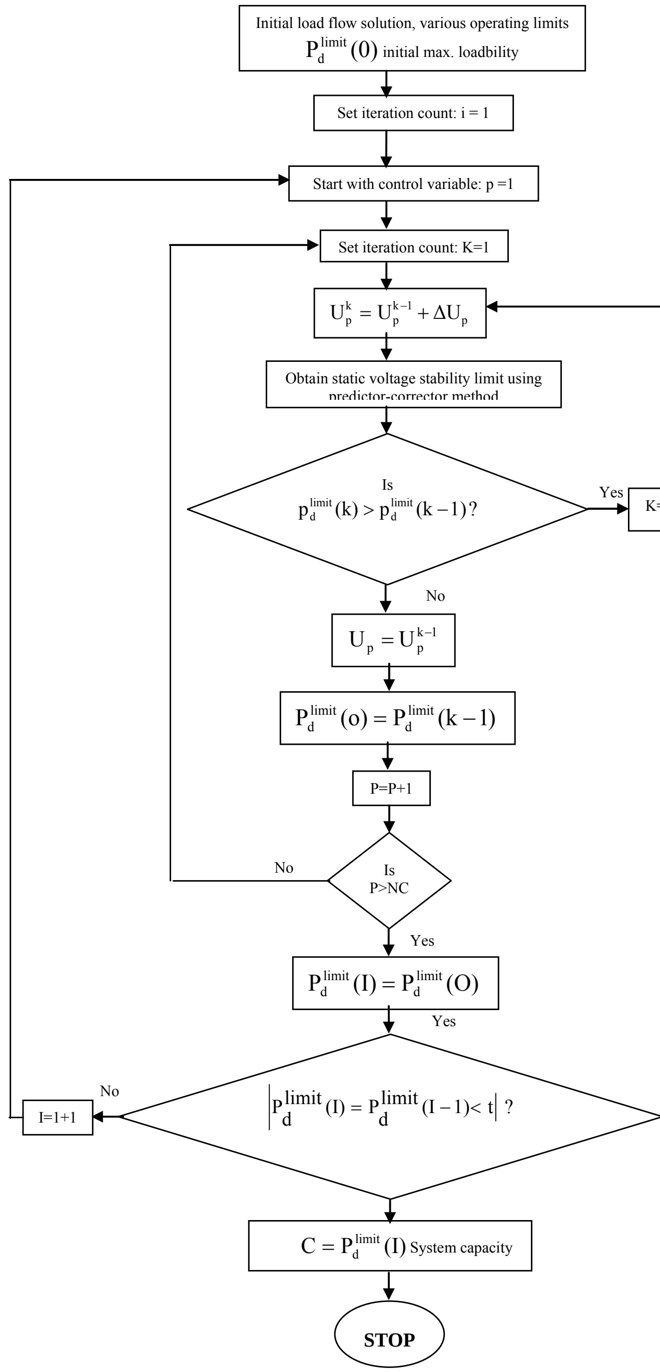


Figure 1: Flow chart for obtaining maximum loadability using method of local variation (MLV) and predictor-corrector method.

## 3. OPTIMIZATION OF STEADY STATE VOLTAGE STABILITY LIMIT

For each generating capacity state the objective is to obtain the maximum static voltage stability limit accounting to real and reactive power generation limits. The complete formulation can be written as

$$J = \max\{P_d^{\text{limit}}\} \tag{6}$$

Subject to following constraints

$$U_p^{\min} \leq U_p \leq U_p^{\max},\ p = 1, NC \tag{7}$$

$$Q_p^{\min} \leq Q_p \leq Q_p^{\max},\ p = 1, NC \tag{8}$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, p = 1, NC \tag{9}$$

$$V_i^{\min} \leq V_i \leq V_i^{\max}, p = 1, NC \tag{10}$$

$$f_i \leq f_i^{\text{limit}} \tag{11}$$

The method used in optimizing the objective function of eqn. (6) is a local variation or direct search method. In this each iteration, one control variable is varied and via maximum continuation power flow algorithm maximum of eqn. (6) is obtained. This is repeated for all control variables within limits till no change in objective function is observed. In each iteration it is also observed that the control is varied till no violation of the operating constraint takes place. These constraints are represented in relations to equations (7)-(11). The computational steps are shown in the flowchart of Fig. 1. Starting point of the algorithm is to compute initial load flow solution and initial maximum loadability limit $[P_d^{\text{limit}}(0)]$ within specified operating constraints. Inner iteration loop obtain maximum loadability limit by varying individual control variables $U_p$, for $p = 1,\ NC$ in sequence. Such sequence is repeated by outer iteration loop. This outer loop is repeated till there is no change in static voltage stability limit. $\Delta U_p$ denotes the change in $p^{th}$ control variables in $k^{th}$ iteration. It is worth mentioning at this stage that for determining static voltage stability limit, load is increased in specified direction at each bus till there is no violation in operating constraints.

## 4. EVALUATION OF PROBABILITY OF FAILURE FROM PEAK LOAD CONSIDERATION

Discrete probabilities for various generating states are obtained using Markov modeling based on constant failure and repair rate. Then for each capacity states, using the continuation power flow (predictor-corrector method of Section 2) the static voltage stability is obtained for base case and for different line outage conditions. For evaluating such limits total outage components are normally considered less than five. Hence, for individual line outages and at the most double line outages are considered. Outages of more number of transmission lines may not be significant as probability of occurrence of such conditions are small and may be neglected. In the present case static security limits have been considered. This means that after the outage of the components synchronism is maintained. Probabilities of line outage states are again evaluated using Markov modeling as availability and unavailability functions. Generation and transmission line states are merged and corresponding to each combined states probabilities are calculated. States then can be partitioned and probability of success and failure, i.e., availability and unavailability of bulk power systems are evaluated. The steps are shown in the flow chart of Fig. 2. Specifically using failure rate ($\lambda$) and repair rate ($\mu$), the availability ($A_i$) and unavailability ($\overline{A}_i$) of each generating unit is calculated as

$$A_i = \frac{\mu_i}{(\lambda_i + \mu_i)} \tag{12}$$

$$\overline{A}_i = \frac{\lambda_i}{(\lambda_i + \mu_i)} \tag{13}$$

State probabilities $P(X_i)$ can be calculated as

$$P(X_i) = \prod_k A_k \prod_n \overline{A}_i \tag{14}$$

where $k$ lies in the sets of generation available and *n* lies in sets of alternator not available in $i^{th}$ state. Hence, generation capacity and capacity probabilities are obtained as $X_i$, $C_i$ and $P(X_i)$.

Similarly, the transmission network state $Y_i$ and associated probabilities $P(Y_i)$ are obtained. Combined state space is obtained by merging the generation and transmission system as follows

$$Z_k = (X_i, Y_j) \tag{15}$$

and

$$P(Z_k) = (P(X_i), P(Y_j)) \tag{16}$$

Capacity corresponding to each combined state is obtained by solving the optimization problem. Composite system capacity and probabilities $C(Z_i)$ and $P(Z_i)$ and load models are merged to evaluate success and failure probability.

## 5. RESULTS AND DISCUSSIONS

The algorithm developed in the paper is based on IEEE six-bus [Appendix A] test system. Bus No 1 and 2 are generator buses. It is assumed that generator bus No 1 is connected with 4 generators having real power generation capacity 0.5. pu each. Reactive power limit of each generator is also assumed 0.5 pu.

Similarly generator Bus No 2 is connected with 4 generator having 0.25 pu real power generation capacity each. A reactive power limit of each generator is taken of equal capacity, i.e., 0.25 pu. Shunts are provided at load Bus No. 4 and 5 of magnitude 0.05 pu each.

The failure and repair rate of each generator has been assumed to be 0.4/year and 9.6/year, respectively. Similarly, failure and repair rate of each transmission line has been assumed to be 0.02/year and 0.25/year. Hence, availability and unavailability of each generator are given as 0.96 and 0.04. Similarly, availability and unavailability of each transmission line are given as 0.9259 and 0.074074.

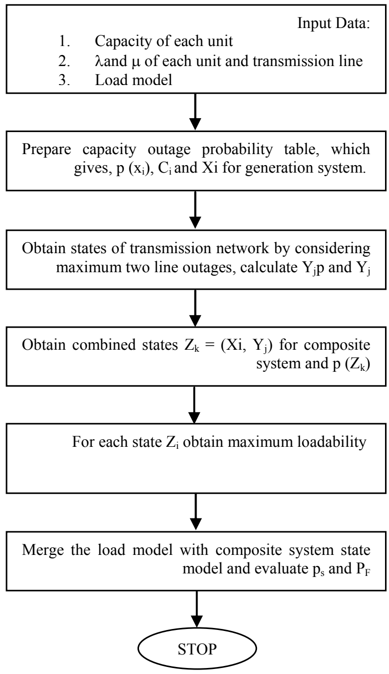


Figure 2: Flow chart for evaluating failure and success probability of composite power system.

Table I: Generation capacity outage probability table

| State $x_i$ | Capacity at State 1 (pu) | Capacity at State 2 (pu) | Total Capacity $C_i$ (pu) | Probability State p ($x_i$) | State ($x_i$) |
|---|---|---|---|---|---|
| 1 | 2.00 | 1.00 | 3.00 | 0.721389579 | $X_0 Y_0$ |
| 2 | 2.00 | 0.75 | 2.75 | 0.030057899 | $X_0 Y_1$ |
| 3 | 2.00 | 0.50 | 2.50 | 0.0012541246 | $X_0 Y_2$ |
| 4 | 1.50 | 1.00 | 2.50 | 0.030057899 | $X_1 Y_0$ |
| 5 | 1.50 | 0.75 | 2.25 | 0.0012541296 | $X_1 Y_1$ |
| 6 | 1.50 | 0.50 | 2.00 | $0.521838 \times 10^{-4}$ | $X_1 Y_2$ |
| 7 | 1.00 | 1.00 | 2.00 | $0.12524 \times 10^{-2}$ | $X_2 Y_0$ |
| 8 | 1.00 | 0.75 | 1.75 | $0.521838 \times 10^{-4}$ | $X_2 Y_1$ |
| 9 | 1.00 | 0.50 | 1.50 | $0.21743 \times 10^{-5}$ | $X_2 Y_2$ |

The probability of event when all lines are working is given as. $A^7$ = 0.583376. The probability of event one line faulty and all other working is given as $\overline{A}A^6$ = 0.0460788, where $A$ stand for availability and $\overline{A}$ stand for unavailability of that particular transmission line/generator. The probability of two lines faulty and five lines working is given as follows : $\overline{A}^2A^5$ = $3.7338083\times10^{-3}$. The probability of working all generators at Station 1 or station 2 is $A^4$ = 0.8493465. The probability of failing one generator at each of station is given as $\overline{A}^1A^3$ = 0.0353894. The probability of two generators working successfully considering outage of two generators at either station is given as follows: $\overline{A}^2A^2$ = $1.47456\times10^{-3}$.

The outage of two generators at each bus has been considered for reliability evaluation. Similarly, outage of maximum two transmission lines has been considered. The combined generating capacity outage probability table is given in Table I where, X and Y are notations for generator buses 1 and 2, respectively. Subscript indicates number of unavailability of generator at that bus as given below:

0 = all generators are available at specified bus.

1 = one generators is unavailable at specified bus.

2 = two generators are unavailable at specified bus.

For each capacity state of Table 1 continuation power flow solution was made and overall composite static voltage stability limits were evaluated in base case condition, as well as single and double line outage conditions. For the safe limits operation the actual working limit has been assumed to be 80% of critical loading points in each case.

Table 2: Peak loadability with capacity and line outage states of electrical power system

| | $\bar{L}_0$ | $\bar{L}_1$ | $\bar{L}_3$ | $\bar{L}_5$ | $\bar{L}_1\bar{L}_2$ |
|---|---|---|---|---|---|
| $Z_0$ $(X_0, Y_0)$ | $\bar{C}(X_i)$ = 2.17768 | 1.16472 | 1.93184 | 0.78176 | -- |
| | $\bar{P}(X_i)$ = 0.40242 | 0.03062 | 0.03062 | 0.03062 | 2.3297×10$^{-3}$ |
| $Z_4$ $(X_1, Y_1)$ | $\bar{C}(X_i)$ = .59192 | 1.14392 | 1.58128 | 0.60552 | -- |
| | $\bar{P}(X_i)$ = 6.99×10$^{-4}$ | 5.32356×10$^{-5}$ | 5.32356×10$^{-5}$ | 5.32356×10$^{-5}$ | 4.0502×10$^{-6}$ |
| | $\bar{P}(X_i)$ = 2.860×10$^{-5}$ | 2.17693×10$^{-6}$ | 2.17693×10$^{-6}$ | 2.17693×10$^{-6}$ | 1.65622×10$^{-7}$ |
| $Z_8$ $(X_2, Y_2)$ | $\bar{C}(X_i)$ = 0.87304 | 0.78432 | 0.86888 | 0.73112 | -- |

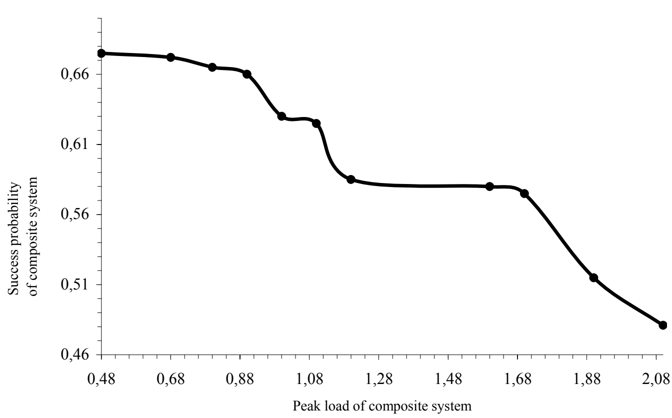


Figure 3: Peak load v/s success probability of interconnected composite electrical power system based on voltage stability unit

Table 3: Composite electrical power system capacity outage probability

| Line outage state | Generation capacity outage state | Peek loadability of composite state | Probability of composite state P ($x_i$) |
|---|---|---|---|
| No outage | $X_0Y_0$ | 2.17870 | 0.4024 |
| 7 | $X_0Y_0$ | 2.17810 | 0.0306 |
| 4,7 | $X_0Y_1$ | 2.17800 | $9.7072 \times 10^{-3}$ |
| 3,4 | $X_0Y_1$ | 1.9359 | $9.7071 \times 10^{-5}$ |
| 6,7 | $X_0Y_0$ | 1.7565 | $2.3297 \times 10^{-3}$ |
| 3,7 | $X_1Y_0$ | 1.7547 | $9.7071 \times 10^{-5}$ |
| 3 | $X_1Y_0$ | 1.7513 | $1.2759 \times 10^{-3}$ |
| 1 | $X_0Y_0$ | 1.1647 | 0.0306 |
| 6 | $X_2Y_0$ | 1.1405 | $5.3236 \times 10^{-5}$ |
| 4 | $X_2Y_1$ | 1.0978 | $2.1769 \times 10^{-6}$ |
| 2 | $X_0Y_1$ | 0.9279 | $1.2759 \times 10^{-3}$ |
| 2,7 | $X_1Y_0$ | 0.9094 | $9.7071 \times 10^{-6}$ |
| 5,2 | $X_0Y_2$ | 0.7912 | $4.0501 \times 10^{-6}$ |
| 2 | $X_0Y_2$ | 0.7910 | $5.3235 \times 10^{-5}$ |
| 5 | $X_0Y_2$ | 0.7866 | $5.3235 \times 10^{-5}$ |
| 1,4 | $X_0Y_2$ | 0.7847 | $7.0241 \times 10^{-9}$ |
| 1 | $X_2Y_2$ | 0.7843 | $9.2325 \times 10^{-8}$ |
| 1,7 | $X_2Y_2$ | 0.7842 | $7.0241 \times 10^{-9}$ |
| 4,5 | $X_0Y_2$ | 0.7839 | $4.0501 \times 10^{-6}$ |
| 5,7 | $X_0Y_0$ | 0.7836 | $2.3297 \times 10^{-3}$ |
| 4,5 | $X_0Y_0$ | 0.7817 | $2.3297 \times 10^{-3}$ |
| 5 | $X_0Y_0$ | 0.7817 | 0.0306 |
| 2,3 | $X_2Y_1$ | 0.7728 | $1.6562 \times 10^{-7}$ |
| 1,6 | $X_2Y_2$ | 0.7638 | $7.0241 \times 10^{-9}$ |
| 2,7 | $X_2Y_2$ | 0.7159 | $7.0241 \times 10^{-9}$ |
| 2,4 | $X_2Y_2$ | 0.7159 | $7.0241 \times 10^{-9}$ |
| 3,5 | $X_2Y_2$ | 0.6348 | $7.0241 \times 10^{-9}$ |
| 2,5 | $X_2Y_2$ | 0.4399 | $7.0241 \times 10^{-9}$ |
| 2,6 | $X_2Y_2$ | 0.3748 | $7.0241 \times 10^{-9}$ |
| 1,3 | $X_1Y_2$ | 0.2311 | $1.6852 \times 10^{-7}$ |
| 1,5 | $X_2Y_2$ | 0.1460 | $7.0241 \times 10^{-9}$ |

The probability of each transmission network has also been evaluated and peak loadability with all possible capacity and probability states of composite electrical power system are given in Table 2. The combined states (transmission and generation) and corresponding capacity (reduced to 80%) were obtained. The system availability and unavailability were calculated for different values of peak loads. The probability and capacity for composite system state was arranged in descending order of loadability given in Table 3. From Table 3 for different peak load consideration, success and failure probabilities obtained are given in Table 4. The graphical plots of success and failure probability against peak load are shown in the Fig. 3. The algorithm developed in this paper has been implemented on 6-bus test system [14]. From Fig. 3 it is observed that as the peak load of the system increases, the success probability of the system automatically decreases.

Table 4: Peak load and success/failure probability

| Peak load | Success probability ($P_s$) | Failure probability ($P_f$) |
|---|---|---|
| 2.1 | 0.48663 | 0.51336 |
| 2.0 | 0.48663 | 0.51336 |
| 1.9 | 0.52389 | 0.47611 |
| 1.8 | 0.52389 | 0.47611 |
| 1.7 | 0.58159 | 0.41841 |
| 1.6 | 0.58529 | 0.41471 |
| 1.5 | 0.58652 | 0.81348 |
| 1.4 | 0.58652 | 0.41348 |
| 1.3 | 0.58652 | 0.41358 |
| 1.2 | 0.58746 | 0.41254 |
| 1.1 | 0.62583 | 0.37417 |
| 1.0 | 0.62586 | 0.37414 |
| 0.9 | 0.66651 | 0.33349 |
| 0.8 | 0.66712 | 0.33288 |
| 0.7 | 0.67319 | 0.32681 |
| 0.6 | 0.67558 | 0.32442 |
| 0.5 | 0.67788 | 0.32212 |
| 0.4 | 0.67797 | 0.32203 |

## 6. CONCLUSIONS

A methodology has been developed for calculating the failure probability of composite generation and transmission system based on voltage stability consideration. Since the issue of reactive power deficiency has become a prime importance for heavily stressed modern power network. The consideration of voltage stability in reliability evaluation will gain more and more importance. Failure probability has been evaluated by merging together (1) generation, (2) transmission and (3) load models in sequence.

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## Appendix: Six-Bus System Data

**System data**

| No. of Bus | No. of Shunt | No. of Lines | No. of generators |
|---|---|---|---|
| 6 | 2 | 7 | 4 |

## Line data

| Line No. | Bus No. From | Bus No. To | Resistance in per unit (pu) | Reactance pu | $B_{line}$ in pu | Tap |
|---|---|---|---|---|---|---|
| 1 | 1 | 6 | 0.1230 | 0.5180 | 0.0000 | 1.0000 |
| 2 | 1 | 4 | 0.0800 | 0.3700 | 0.0000 | 1.0000 |
| 3 | 4 | 6 | 0.0970 | 0.4070 | 0.0000 | 1.0000 |
| 4 | 6 | 5 | 0.0000 | 0.3000 | 0.0000 | 1.0000 |
| 5 | 5 | 2 | 0.2820 | 0.6400 | 0.0000 | 1.0000 |
| 6 | 2 | 3 | 0.7230 | 1.0500 | 0.0000 | 1.0000 |
| 7 | 4 | 3 | 0.0000 | 0.1330 | 0.0000 | 1.0000 |

## Bus data

| Bus | Voltage $\|V\|$ | $\angle\delta$ | $P_G$ | $Q_G$ | $P_L$ | $Q_L$ |
|---|---|---|---|---|---|---|
| 1 (Slack) | 1.1500 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 (PV) | 1.1500 | 0.0000 | 0.5000 | 0.0000 | 0.0000 | 0.0000 |
| 3 (PQ) | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2750 | 0.0650 |
| 4 (PQ) | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 (PQ) | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1500 | 0.0900 |
| 6 (PQ) | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2500 | 0.0250 |

## Shunt data

| Bus | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Shunt | 0.0000 | 0.0000 | 0.0000 | 0.0500 | 0.0500 | 0.0000 |

## Maximum reactive generation

| Bus | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $Q_{limit}$ | 2.0000 | 1.5000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |