UNIVERSITY OF THESSALY

Δημιουργία και Οικονομικές Εφαρμογές Μοντέλων Περιβάλλουσας Ανάλυσης Δεδομένων Δύο Σταδίων - Construction of Two-Stage DEA Models and Economic Applications

A thesis presented

by

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Introduction

I. Introduction

Today the process of decision making is of extreme importance. Managers wish to make the best decisions in the smallest amount of time. To this end, managers utilize evaluation procedures. Evaluation procedures assist the managers with decision making by offering consistency and transparency. The constant implementation of evaluation procedures leads to improved performance of organizations which is the desirable objective by every concerned party. Managers wish to monitor the procedures and evaluate the performance of their organizations, government wish to evaluate state services, voters wish to evaluate government, students need to evaluate and compare universities, environmental, healthcare and insurance programs need to be evaluated and the common objective is the improved performance. In order for an organization to achieve improved performance, it should improve the efficiency of its operations. The basic concept of efficiency is to compare similar units, usually called decision making units, and evaluate how efficiently use their inputs to produce outputs.

In economics, production efficiency measures the ability of an economy or an organization to produce the maximum amount of goods while using the least possible amount of resources. In conventional microeconomic theory of production functions, every organization has the ability to optimize its input allocation and operate on the production boundary by generating the maximum amount of output. Alternatively, empirical production approaches investigate the relationships among inputs and outputs based on the available data. On the one hand, empirical regression-based techniques fit a regression line which passes through the middle of the dataset and focuses on average values of production and central tendencies. On the other hand, empirical production frontiers envelope the available dataset and focus on best-practice and benchmarks. Benchmarks are considered as reference points for every unit deviating from the frontier. Evidently, this information is very important for the decision maker because it helps him to specify the changes that need to be made in the input-output combination of the deviating unit in order to reach the benchmark point on the frontier. However, unknown or complex relationships and multiple inputs and outputs usually make difficult the specification of a production frontier.

Data envelopment analysis (DEA) is a non-parametric linear programming approach which handles multiple inputs and outputs measured in different units and evaluates the efficiency of Decision Making Units (DMUs). DEA yields a frontier which envelops all the available dataset, finds the benchmarks units and specifies the necessary changes of inefficient DMUs in order to become efficient. DEA has also a number of desirable advantages from a statistics point of view, such as the lack of requirement for a specific functional form between inputs and outputs (Christopoulos, 2007). DEA has been used extensively across the literature to evaluate various types of organization (firms, non-profit organizations, countries, regions, group of people).

A DEA model consumes inputs to produce outputs without considering the internal structures inside the DMU. Usually this assumption is adequate and DMUs are evaluated without any problem. However, in some cases DMUs may consist of two or more stages and these internal procedures may be important for evaluating the efficiency. Supply chain is a fine example of multistage structure where the supply chain is the DMU and every stage is a decision center. The corporate manager of such a supply chain wishes to maximize the overall efficiency of the supply chain and simultaneously wishes to maximize the efficiency of every decision center. Conventional DEA models are not sufficient in the presence of internal structures. Two-stage and network DEA models are used to accommodate such cases.

II. Contribution of the thesis

There is a wide range of economic applications where the two-stage structures are needed. For example, Fukuyama and Matousek (2011) and Holod and Lewis (2011) evaluate the efficiency of banks where they measure the “value added activity” in the first stage and the “profitability” in the second stage. Another economic example is the case of manufacturing firms (or any other firms) where the first stage measures the “profitability” of the firm and the second stage measures the “marketability” of the firm

(Hung and Wang, 2012). Universities is another interesting case where "teaching" can be considered as the first stage and "research" as the second stage (Kao, 2012; Kao and Lin, 2012).

Two-stage DEA models evaluate the overall efficiency of the DMU while considering the significance of each stage to the whole process. The significance of each stage is usually represented by the assignment of weights, suppose ξ_1 and ξ_2 for the first and the second stage respectively. These weights are usually constant at 0.5 when the models make no specific assumption about the significance of each stage such as the multiplicative model of Kao and Hwang (2008); therefore these models assume that the two stages contribute equally to the whole process. Other models do not assume that the contribution of the two-stages to the whole process is equal, such as the additive two-stage DEA model of Chen et al. (2009a) which assigns variable weights in order to maximize the overall efficiency. However, the additive model does not incorporate a priori information, such as expert opinions and value judgements, regarding the contribution or the significance of each stage to the whole process and there is a problem of infeasibility if one of the two weights ξ_1 and ξ_2 becomes zero.

The research framework of this thesis is the modeling of non-parametric production functions in two stages without assuming any specific functional form. Inside this framework this thesis constructs two-stage DEA models and use them create novel indices which evaluate the efficiency in various economic applications. Specifically, the most significant research contribution of this thesis is the incorporation of a priori information such as expert opinions and value judgements into the modeling process. This objective is achieved with the construction of the Weight Assurance Region (WAR) model which modifies the original additive model in order to incorporate a priori information using assurance region-based weights in the two-stages. Furthermore, WAR model solves the infeasibility problem of the original model. Another research contribution is the mathematical framework for the extension of the original additive model into a time-dependent window-based approach. A third research contribution is the incorporation of metafrontier framework into two-stage DEA analysis in order to treat the heterogeneity of DMUs in different groups (such as firms in different groups or regions in different countries) which experience different technologies. Finally, novel two-stage indices are proposed which evaluate the efficiency in various economic applications. Next, the research contributions of this thesis are analyzed in more details, along with the advantages of the research approach.

Specifically, the production process of Decision Making Units is investigated using Data Envelopment Analysis (DEA) models. DEA is an approach based on linear programming and is used to assess the relative efficiency among a set of DMUs while offering a number of advantages. First, DEA does not use biased and subjective opinions and it is based on the objectivity of the numerical data. In addition, DEA can handle multiple inputs and outputs measured in different units. DEA does not require any assumption regarding the functional form and the distribution of inefficiency. Furthermore, DEA has the ability to identify sources and level of inefficiency in each input and output for each DMU and find the benchmark DMUs which are used as reference points in order to tackle inefficiencies.

DEA makes no assumption about the procedures taking place inside the DMU. On the contrary, DEA treats a DMU as a “black box” which uses inputs to produce outputs without considering the internal procedures, a usually sufficient assumption. However in some cases, like in supply chain systems, DEA models consist of two or more stages and there are intermediate measures which are considered as inputs in one stage and outputs in another stage. Traditional DEA models are not sufficient in these cases. Two-stage and network DEA models are used to accommodate such cases. This thesis classifies two-stage DEA models into four categories which are independent, connected, relational and game theoretic models.

Relational two-stage DEA models assume a multiplicative or additive relationship between the overall and the individual efficiencies. An extreme case of the two-stage additive model of Chen et al. (2009a) is identified where the weight of an individual stage takes the zero value and as a result the individual efficiencies cannot be defined. This thesis constructs a Weight Assurance Region (WAR) model which is a modified version of the additive model with assurance region-based weights. The WAR model is appropriate for policy making in the presence of a priori information such as expert opinion, known information and/or widely accepted beliefs or preferences and other type of information. In addition WAR model is not affected by the aforementioned problem of infeasibility because by construction it restricts the relative weights of each stage to be a non-zero number. Specifically, WAR model restricts the ratio of the weights of each stage inside a region between β and δ which are positive scalars $0 < \beta \leq \delta$. Furthermore when $\delta = 1/\beta$ it yields the same results with the original additive two-stage DEA models. Therefore the WAR model can be considered as a more general case of the original model.

Moreover, the mathematical formulation of the additive two-stage DEA model of Chen et al. (2009a) is extended to window-based LP problem. This approach allows the handling of panel data in a two-stage DEA framework and provides robust efficiency measures. Furthermore, the introduction of metafrontier framework into two-stage DEA analysis allows the treatment of the heterogeneity of DMUs in different groups (such as firms in different groups or regions in different countries) which experience different technologies. DMUs from different groups face different production opportunities; therefore feasible input-output combination in one group may not be feasible in another. These differences among groups may refer to physical, human and financial capital, infrastructures, economic environment, available resources etc; as a result every group has a different frontier. In this framework the metafrontier is an overall frontier which envelopes the groups' specific frontiers so that no point of these frontiers can lie above points on the metafrontier.

Finally, four economic applications are presented where the production processes are examined and novel indices are constructed using two-stage DEA formulations. The economic applications are in educational, banking and environmental sectors. All DEA programs throughout this thesis have been designed and calculated using the R Statistical Package.

III. Structure of the thesis

This Section outlines the structure of the thesis:

Chapter 1 provides the link between production economics and efficiency analysis. Various terms are presented such as the decision making unit, the production process, the production function, technical efficiency and the returns to scale. The graphical presentation of the production frontier, the production possibility curve and the isoquant curve are used in order to find the benchmark frontier and assist the analysis. The benchmark frontier can be specified using either a parametric or a non-parametric approach.

Chapter 2 presents and discusses the basic DEA models which are the multiplier and the envelopment model for input and output orientation. Furthermore, it distinguishes between the CCR models which exhibit constant returns to scale and the BCC models which exhibit variable returns to scale. Simple numerical examples and graphical analysis are employed to aid the analysis.

Chapter 3 classify two-stage DEA models into four categories. Independent two-stage DEA models apply a typical DEA model at each stage separately and evaluate the efficiency without considering the interaction and possible conflicts between the two stages because of the intermediate variables. Connected two-stage DEA models take into account the interaction between the stages. Relational two-stage DEA models assume a multiplicative or additive relationship between the overall and the individual efficiencies. The distinctive feature of this approach is that the multipliers of the intermediate variables are the same regardless of whether the intermediate variables are used as inputs or outputs. The last category is about game theoretic two-stage DEA models.

Chapter 4 presents the principal contribution of this thesis; a newly proposed model, namely the Weight Assurance Region (WAR) DEA model, which is a modification of additive efficiency decomposition model of Chen et al. (2009a) in order to incorporate a priori knowledge and overcome an infeasibility problem of the original model. This Chapter also presents an economic application of the WAR model on cross-country secondary education. The overall efficiency index evaluates how the school environment affects student performance. The first stage measures the "learning environment efficiency" and the second stage measures the "student's performance efficiency".

Chapter 5 demonstrates the mathematical formulation of the window-based LP problem of the relational two-stage DEA model (both the multiplicative and the additive). An economic application about the efficiency of banking systems in OECD countries is presented. The first stage of the efficiency index measures the "value added activity" and the second stage evaluates "profitability".

Chapter 6 creates an environmental sustainability index in order to evaluate countries with advanced economy. Building upon Chapter 5, this economic application also includes the dimension of time. The first stage of the sustainability index measures the "production efficiency" and the second stage measures the "eco-efficiency". ***Chapter 7*** uses the sustainability index as presented in Chapter 6 to measure European regions. The novel approach here is the treatment of heterogeneity among DMUs in different groups using a metafrontier framework.

Finally, ***Chapter 8*** presents the summary and the conclusion of this thesis and provides insights for future research.

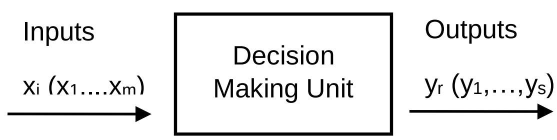
Chapter 1

Production economics and efficiency measurement

1.1. Introduction

Any process which uses a set inputs in order to produce a set of outputs is called *production process*. Typically, a production process takes place inside a *Decision Making Unit* (DMU), which is a unit of organization such as a branch of a company, an educational institution, a government agency, a non-profit organization and a country. Figure 1.1 presents a typical production process inside a DMU. The DMU uses inputs as factors of production in the production process (the actual process is either known or unknown) and produces outputs which are the final goods.

Figure 1.1: Production process



Production function is used to describe a production process. A production function is either used to specify the maximum obtainable output from a given set of inputs or the minimum required input to produce a given amount of outputs. Therefore, a production function describes a frontier which represents the maximum output or the minimum input that can be achieved from a feasible combination of inputs or outputs respectively. Although the idea of production functions dates back to 1767 and the French physiocrat A.R.J. Turgot (Humphrey, 1997)[1], the most famous production function is the Cobb-Douglas function (Cobb and Douglas, 1928) which uses capital and labor as inputs to produce manufacturing output. Cobb and Douglas (1928) estimated their production function using lest squares. Typically, production functions assume *technical efficiency* for all DMUs which implies that all DMUs are able to use their inputs to produce the maximum outputs that are technologically feasible.

The seminal paper of Farrell (1957) who built upon the work of Debreu (1951) and Koopmans (1951) drew the attention from the frontier analysis and pointed it to the

[1] For a detailed review about production functions before the Cobb-Douglas function, see Humphrey (1997).

deviations from the frontier in order to measure the technical efficiency of the DMUs. According to Farrell's (1957) framework a DMU which lie on the frontier is regarded as technical efficient while any deviation from the frontier is regarded as technical inefficiency.

This chapter is structured as follows. Section 1.2 presents the production function and its properties along with the most commonly used production function, the Cobb-Douglas. This section also demonstrates the production frontier and the concept of returns to scale. Section 1.3 describes the production technology using set theory and presents the Production Possibility Curve and the Isoquant Curve. Section 1.4 discusses the efficiency measurement and technical efficiency for the input and output oriented case and for constant and variables returns to scale. Section 1.5 presents the approaches to determine the efficient frontier which are the parametric and the non-parametric approach. Section 1.6 concludes.

1.2. Production functions

The simplest form of production function considers a DMU which uses *M* inputs to produce one output in a single period. The technologically feasible possibilities for the DMU are given by the following production function:

$$y = f(x) \tag{1.1}$$

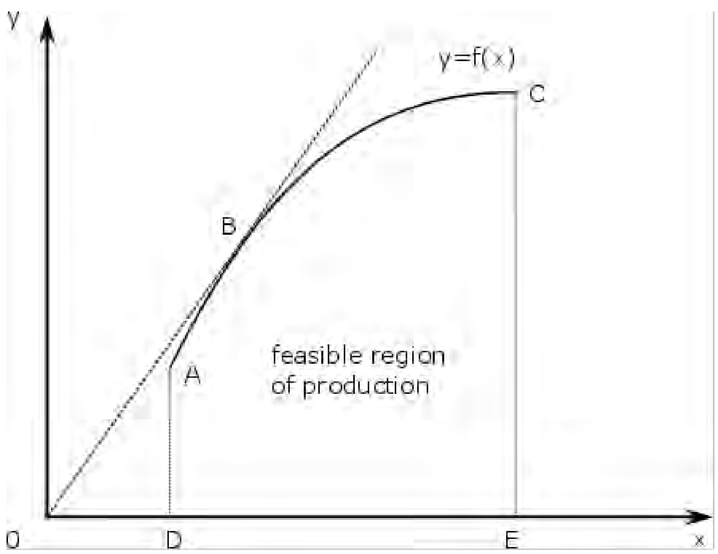
where *y* is the single output and $x = (x_1, x_2, \dots, x_m)$ is a $m \times 1$ vector of inputs (e.g. labor, capital, land, resources). Chambers (1988) presents a number of properties regarding the production function:

- **Non-negativity:** The value of $f(x)$ is a finite, non-negative, real number.
- **Concavity:** $f(\lambda x' + (1-\lambda)\, x) \geq \lambda f(x') + \ (1-\lambda) f(x)$
- **Essentiality:** Weak essentiality implies that a production of positive output is not possible without using at least one input. Strong essentiality implies that a production of positive output is not possible without using every input.
- **Monotonicity:** An additional unit of input will not decrease output. Strong monotonicity implies that an additional unit of input will increase output.

- In addition, $f(x)$ is everywhere **continuous** and it is possible to be twice-continuously differentiable.

Figure 1.2 presents the production function for a single input where all the aforementioned assumptions are satisfied. The production function yields points on the *production frontier* AC. The production frontier show the maximum attainable output for every input level and represents the current technology in the industry. The area DACE which consists of the production frontier, the x axis and every point between them, is called *feasible region of production*. The optimal point of production is B where the slope of the ray that passes through the origin and is tangent to the production frontier, is steeper. The assumption of technical efficiency for every DMU implies that every DMU will operate on the production frontier AC and not beyond it.

Figure 1.2: Production function



The most widely used production function in Economics is the Cobb-Douglas production function. The functional form of Cobb-Douglas is as follows.

$$y = Ax_1^{\alpha}x_2^{\beta} \tag{1.2}$$

where A, α and β are positive constants, y is the total production, x_1 is the labor input, x_2 is the capital input, A is the total factor productivity and α and β are the output elasticities which are determined by the available technology. Output elasticities measure the effect of a change in the input levels on the output level. For example, if *β*=0.6, a 1% increase in capital would lead to a 0.6% increase in output. In addition, $\alpha + \beta$ show the *returns to scale* of the production function. Returns to scale indicate the rate of increase in production level relative to a subsequent proportional increase in the production factors in the long run where all the factors of production are variable.

- if $\alpha + \beta = 1$, the production function exhibits *constant returns to scale* (CRS), that is the output increases by the same proportional change as all inputs, for example if labor and capital increase by 30%, production will increase by 30%
- if $\alpha + \beta > 1$, the production function exhibits *increasing returns to scale* (IRS), that is the output increases by the more than the proportional change in inputs, for example if labor and capital increase by 30%, production will increase by more than 30%
- if $\alpha + \beta < 1$, the production function exhibits *decreasing returns to scale* (DRS), that is the output increases by the less than the proportional change in inputs, for example if labor and capital increase by 30%, production will increase by less than 30%.

1.3. Production technology

An alternative way to describe a production process instead of using functions, is set theory (Färe and Primont, 1995). The two approaches are equivalent. Following Coelli et al. (2005), the term *production technology* is used instead of the term production function for the case of a multiple inputs-outputs production process. Such a production technology can be expressed using set theory and a technology set *T* can be defined. The technology set contains a vector of *m* inputs denoted by $x = (x_1, x_2, \dots, x_m)$ and a vector of *s* outputs denoted by $y = (y_1, y_2, \dots, y_s)$ which contain non-negative real numbers. Then, the technology set *T* will be:

$$T = \{(x, y) \in R_{+}^{m+s}: x\ can\ produce\ y\} \tag{1.3}$$

The set T is called *Production Possibility Set (PPS)* and contains all feasible combinations of *x* and *y* such that *x* can produce *y*. Every input-output combination which is outside the PPS is infeasible. The production technology can also be defined using the output and input sets. The output set *O(x)* contains all outputs y that can be produced by employing inputs *x*.

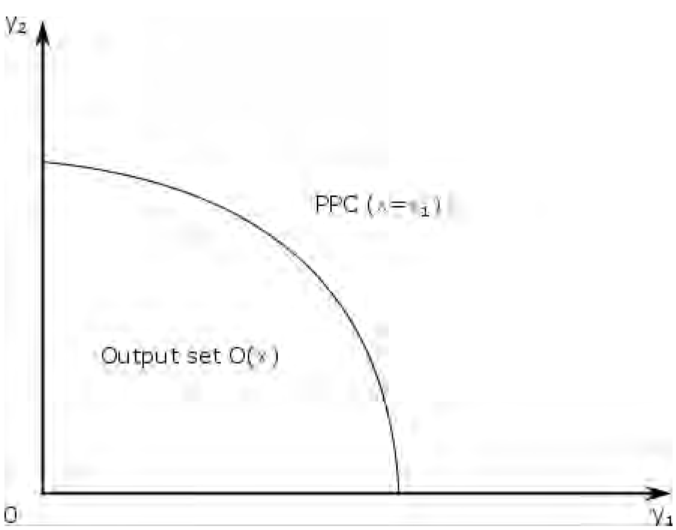
$$O(x) = \{y \in R_{+}^{s}: x\ can\ produce\ y\} = \{y: (x, y) \in T\} \quad (1.4)$$

Coelli et al. (2005) presents the properties associated with the output set:

- Non-zero level of inputs can produce zero level of outputs: $0 \in O(x)$.
- Zero level of inputs cannot produce non-zero level of outputs: if *x*=0 then *y*=0.
- Strong disposability of inputs: if *x* can produce *y*, then *x'* can produce *y* $\forall$ x'≥x.
- Strong disposability of outputs: if $y \in O(x)$, then $y' \in O(x)$ $\forall$ y' ≤ y.
- Closeness, which implies that the set contains all its limit points.
- Convexity, which implies that if a given set of inputs can produce two output combinations, then it can also produce any weighted average combination of them.
- *O(x)* is bounded, which implies the output set is finite.

Using the output set we can create the *Production Possibility Curve* (PPC) as shown in Figure 1.3. PPC depicts the output tradeoffs for a fixed level of inputs.

Figure 1.3: Production Possibility Curve



The input set *I(y)* contains all input *x* which produce a fixed output level *y*.

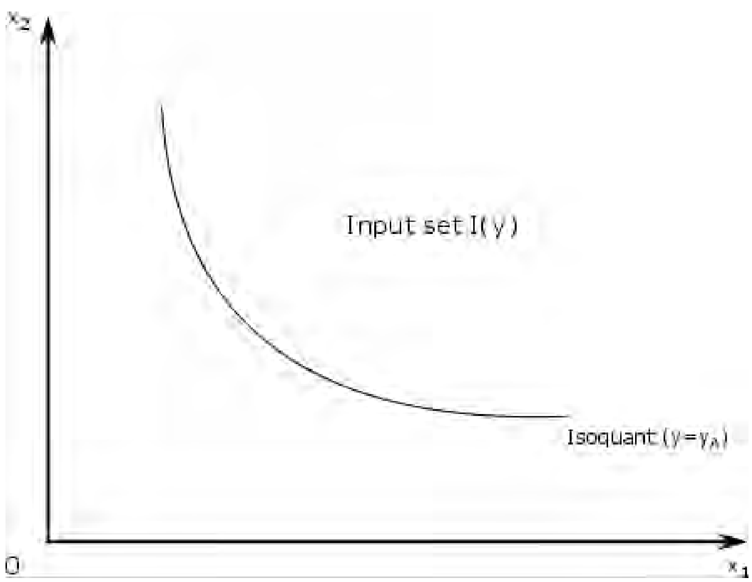
$$I(y) = \{x \in R_{+}^{m}: x\ can\ produce\ y\} = \{x: (x, y) \in T\} \quad (1.5)$$

The input set is associated with the following properties (Coelli et al., 2005):

- Weak disposability of inputs: if $x \in I(y)$, then $\lambda x \in I(y)$ ∀ λ≥1.
- Strong disposability of inputs: if $x \in I(y)$, then $x' \in I(y)$ ∀ x'≥x.
- Closeness, which implies that the set contains all its limit points.
- Convexity, which implies that if a given set of outputs can be produced by two input combinations, then it can also be produced by any weighted average combination of them.

Using the input set the *Isoquant curve* can be created as shown in Figure 1.4. The Isoquant presents all input combinations which can produce a fixed level of outputs.

Figure 1.4: Isoquant curve



1.4. Efficiency Measurement

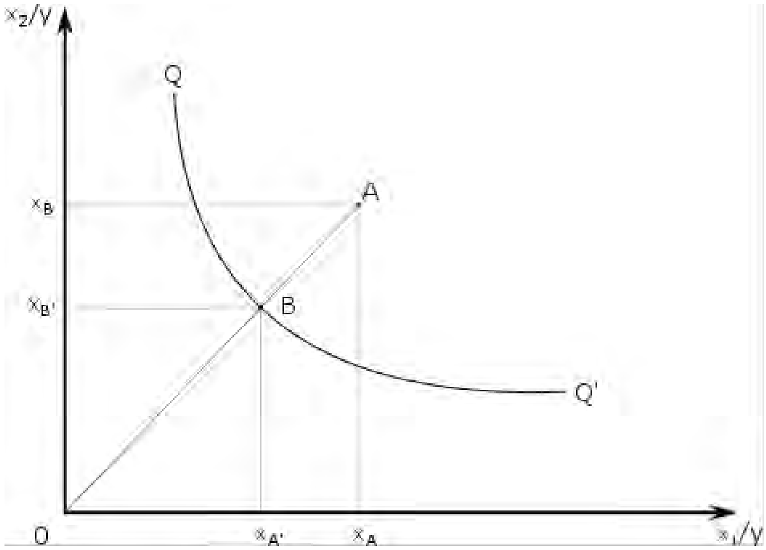
Efficiency and productivity are two cooperative but not identical concepts. *Productivity* of a DMU can be defined as the ratio of the produced outputs to the employed inputs (Lovell, 1993). Koopmans (1951) defined that a DMU can be considered as fully efficient if and only if it cannot increase any output or decease any input without worsening any of the other output or input. This definition of efficiency is in line with the *Pareto* optimal point (Pareto, 1909). Debreu (1951) constructed a radial measure of efficiency which assume proportional change of inputs-outputs. Farrell (1957) relaxed the assumption of the frontier analysis about the ability of all DMUs to use their inputs to produce the maximum outputs that are technologically feasible. Building upon the work of Debreu (1951) and Koopmans (1951) he defined the efficiency relative to the best possible frontier. This measure of efficiency is called *Technical Efficiency*. According to Farrell's (1957) framework a DMU which lie on the frontier is regarded as technical efficient while any deviation from the frontier is regarded as technical inefficiency.

Farrell (1957) demonstrated his ideas using firms which use two inputs to produce a single output while assuming constant returns to scale. Figure 1.5 presents the technical efficiency for the input oriented case where the firm is determined to minimize its inputs to produce an output. The efficient frontier where all firms are technically efficient is depicted by the isoquant curve Q'Q. Suppose a firm which operates at point A using x_A and x_B units of inputs to produce given level of a unit of output. The firm can reduce the input level proportionally to point B where it uses x_A' and x_B' units of inputs to produce the same level of output. The distance AB is the technical inefficiency of the firm and the technical efficiency can be measured as:

$$TE = \frac{0B}{0A} = 1 - \frac{BA}{0A} \qquad (1.6)$$

The ratio ranges from zero to one and a firm with efficiency score of one is rated as fully technical efficient.

Figure 1.5: Input oriented CRS technical efficiency

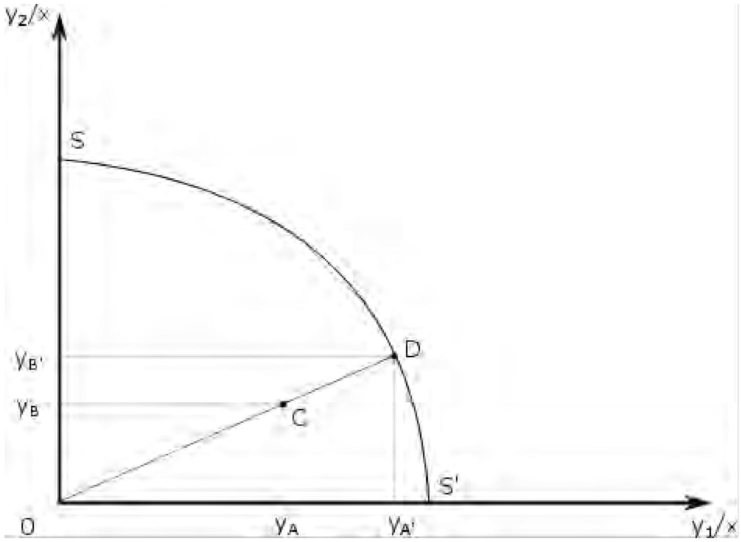


Similarly, suppose that the firms use a single input to produce two outputs while holding the assumption of constant returns to scale. Figure 1.6 presents the technical efficiency for the output oriented case where the firm is determined to maximize its outputs with a given level of input. The efficient frontier where all firms are technically efficient is depicted by the PPC curve S'S. Suppose a firm which operates at point C using y_A and y_B units of outputs using a unit of input. The firm can increase the output level proportionally to point D where it produces y_A' and y_B' units of outputs using the same level of input. The distance CD is the technical inefficiency of the firm and the technical efficiency can be measured as:

$$TE = \frac{0C}{0D} \tag{1.7}$$

Again, the ratio ranges from zero to one and a firm with efficiency score of one is rated as fully technical efficient.

Figure 1.6: Output oriented CRS technical efficiency



Input and output oriented measures of efficiency are exactly the same in the presence of constant returns to scale. Figure 1.7 depicts a production frontier for a firm which uses multiple inputs to produce multiple outputs. The firm operates at point E which is a technically inefficient point because it is not on the production frontier. The firm uses x_A level of inputs and y_A level of outputs. In an input oriented case where the input minimization is the target, the firm can reduce its inputs to x_A' level while holding the production to y_A level of output and move to point F. The technical efficiency of the firm is measured as $\frac{HF}{HE}$. Likewise, in an output oriented case where the target is the output maximization, the firm can increase its outputs to y_A' level while holding the consumption of inputs to x_A level and move to point G. The technical efficiency of the firm is measured as $\frac{IE}{IG}$. It is clear that $\frac{HF}{HE} = \frac{IE}{IG}$ which means that in the presence of constant returns to scale, input and output oriented measures of efficiency are exactly the same.

Figure 1.7: CRS technical efficiency

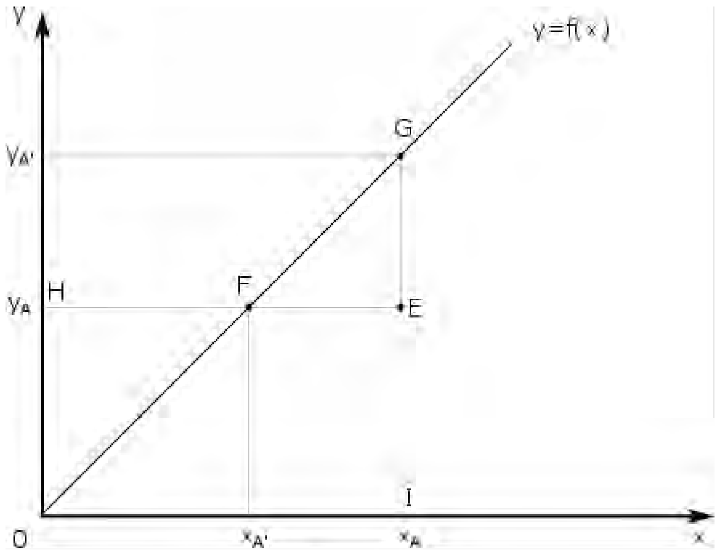
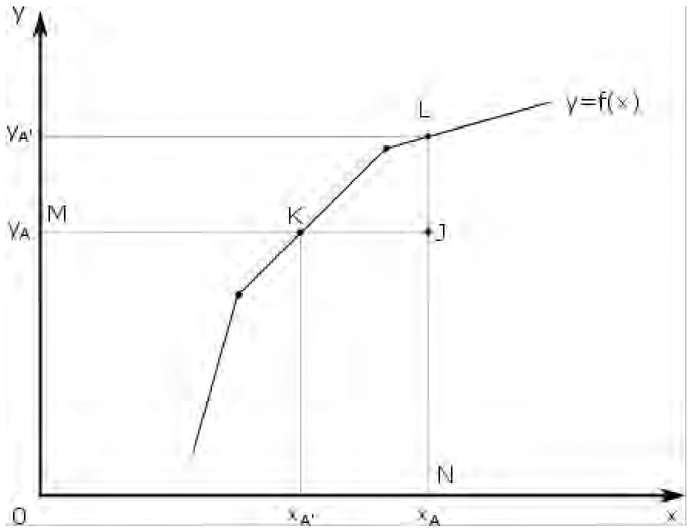


Figure 1.8 shows the production frontier of a similar firm where the assumption of constant returns to scale does not hold. As it has been previously presented constant returns to scale means that a proportional change in inputs results in the same proportional change in outputs. Increasing returns to scale means that the change in outputs is larger than the proportional change in inputs while decreasing returns to scale means that the change in outputs is smaller than the proportional change in inputs. Now, the term *variable returns to scale* (VRS) is introduced for any frontier which does not exhibit CRS. For example, in Figure 1.8 the first segment of the frontier exhibits IRS, the second segment CRS and the last segment DRS. The firm uses multiple inputs and produces multiple outputs. The firm operates at point J which is a technically inefficient point because it is not on the production frontier. The firm uses x_A level of inputs and y_A level of outputs. Similarly with Figure 1.7, for the input oriented case the technical efficiency of the firm is measured as MK/MJ and for the output oriented case the technical efficiency is measured as NJ/NL. It is clear that $MK/MJ \neq NJ/NL$ which means that in the presence of variable returns to scale, input and output oriented measures of efficiency are not the same.

Figure 1.8: VRS technical efficiency



Technical efficiency is not the only measure of efficiency. One can measure the *allocative efficiency* in the presence of information about the input-output prices and also the *scale efficiency* if he is interested about the scale of operations. This thesis uses only the concept of technical efficiency and from this point forward the term "*efficiency*" or "*overall efficiency*" will refer to the technical efficiency of the DMU.

1.5. Parametric and Non-parametric Frontiers

In order to determine the efficient frontier we need to know the exact production function of the fully efficient DMU. However, this is not usually possible. As an alternative the efficient frontier is estimated using the available data from the sample. There are two approaches regarding the construction of the frontier, the parametric and the non-parametric.

1.5.1. Parametric Approach

The use of parametric approach requires the a priori specification of the frontier function. The efficient frontier is called *benchmark* and it shows the best-practice in the industry. Furthermore the frontier can be either deterministic or stochastic. The *deterministic approach* was introduced by Aigner and Chu (1968) who arbitrarily assumed a Cobb-Douglas function of the following form for the *j*-th DMU:

$$\ln y_j = \beta x_j - u_j \tag{1.8}$$

where y_i is the output and x_i is a vector which contains the logarithmic values of inputs, *β* is a vector of unknown parameters and u_i represents the technical inefficiency. Various parametric techniques have been used for the estimation of (1.8) such as maximum likelihood (Afriat, 1972), modified ordinary least squares (Richmond, 1974) and corrected ordinary least squares (Gabrielsen, 1975). The deterministic frontier approach requires a large sample for statistical purposes, it is sensitive to outliers and assumes that every deviation from the frontier is inefficiency therefore it does not allow statistical noise (statistical errors and residuals).

An obvious improvement of the deterministic parametric approach is to introduce a term to account for the statistical noise. Aigner et al. (1977) and Meeusen and van den Broeck (1977) introduced a symmetric random error v_i into (1.8) which accounts for the statistical noise. The resulting model is (1.9) which is known as the *stochastic frontier approach* (SFA).

$$\ln y_j = \beta_j + v_j - u_j \tag{1.9}$$

The SFA approach assumes that any deviation from the frontier could be a result of either inefficiency or statistical noise. The statistical noise contains errors of measurement, other econometric errors such as misspecification of the production function and exogenous effects beyond the control of the DMU (Murillo-Zamorano and Vega-Cervera, 2001). The introduction of statistical noise allows SFA to be less sensitive to outliers and to create confidence intervals. The disadvantages of the SFA approach (and the deterministic approach) are the a priori specification of production function and the distributional assumptions (usually normal or half-normal distribution for the inefficiency term) (Worthington, 2001).

1.5.2. Non-parametric Approach

The nature of non-parametric approach does not require any assumption regarding the functional form and the distribution of inefficiency. The most commonly used non-parametric method is a linear programming tool, namely the *data envelopment analysis* (DEA). Instead of measuring the absolute efficiency of DMUs compared with an a priori specified benchmark which is the case for parametric methods, DEA constructs a piece-wise frontier and measures the relative efficiency of the DMUs in the same industry. Therefore, the benchmark frontier is determined by the best-practice DMUs of the sample data. DEA is a deterministic approach which means that any deviation from the frontier is accounted to inefficiency.

DEA offers a number of desirable strengths however there are some limitations (Ramanathan, 2003). On the one hand, DEA does not use biased and subjective opinions and it is based on the objectivity of the numerical data. In addition, DEA can handle multiple inputs and outputs measured in different units (*unit invariance*). The assumptions regarding the functional form and the distribution of inefficiency are no longer required. Furthermore, DEA has the ability to identify sources and level of inefficiency in each input and output for each DMU and find the benchmark DMUs which are used as reference points in order to tackle inefficiencies. On the other hand, as a deterministic approach DEA is sensitive to outliers and small measurement errors. Furthermore, DEA does not allow for statistical noise and does not directly account for external and environmental factors, omitted variables and measurement errors. Additionally, statistical hypothesis and calculation of confidence intervals are difficult. Atkinson and Wilson (1995) and Simar and Wilson (1998, 2000) proposed a procedure based on bootstrap techniques in order to approach the distribution and calculate confidence intervals.

1.6. Summary

This chapter discussed the fundamental terms for efficiency analysis. Various terms have been laid out such as the decision making unit, the production function and the returns to scale. The graphical presentation of the production frontier, the PPC and the isoquant assisted the analysis. Technical efficiency, which is the basic measure of efficiency that is used throughout this thesis, measures the deviation of a DMU from the best-practice frontier. The best-practice or benchmark frontier can be specified using either a parametric or a non-parametric approach. This thesis uses DEA, a non-parametric linear-programming method which measures the relative efficiency of DMUs. DEA offers a wide array of advantages such as objectivity, the handling of multiple inputs and multiple outputs measured in different units and the “no assumption” requirement regarding the functional form and the distribution of inefficiency. Chapter 2 will further discuss DEA and will present the basic DEA models.

Chapter 2

Data Envelopment Analysis

2.1. Introduction

The previous Chapter discussed the frontier analysis and technical efficiency as it has been presented by Farrell (1957). Farrell (1957) used two inputs and one output in order to demonstrate his analysis, which is easily generalized to multiple inputs case. Charnes et al. (1978a) generalized Farrell's (1957) measure of technical efficiency to a multiple output case and implemented it in a linear programming framework, namely the data envelopment analysis (Murillo-Zamorano, 2004). Initially DEA was constructed to evaluate the results of a U.S. Department of Education named "Program Follow Through" which evaluated educational programs for disadvantaged students in public schools. The results of this effort were presented in Charnes et al. (1978b). The original model was an input-oriented CRS model which was named CCR model from the authors' initials (Charnes, Cooper and Rhodes). Later Banker et al. (1984) proposed the VRS version of the DEA model which was named BCC model (Banker, Charnes and Cooper).

DEA is a data oriented approach which evaluates the efficiency of a DMU relative to other similar DMUs in order to estimate a benchmark frontier which represents the best-practice in the industry. The nature of DEA requires a slightly changed definition of efficiency from Koopmans (1951) definition which was presented in Section 1.4. Koopmans (1951) defined that a DMU can be considered as fully efficient if and only if it cannot increase any output without increasing any input or decease any input without decreasing any output. This definition implies that the theoretical possible level of efficiency is known. Cooper et al. (2011) provided a definition focused on the available dataset and therefore the relative efficiency:

"A DMU is to be rated as fully (100%) efficient on the basis of available evidence if and only if the performances of other DMUs does not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs."

The above definition provides the basis for the subsequent analysis of this thesis.

This chapter is structured as follows. Section 2.2 builds upon the efficiency concept which has been introduced in Chapter 1 and presents simple numerical examples which assist the comprehension of the relative efficiency and frontier analysis. Section 2.3 presents the basic CCR models which are the multiplier and the envelopment models for the input and output orientations. Section 2.4 presents the same models in the VRS form (BCC DEA models) and Section 2.5 concludes.

2.2. Relative efficiency measurement

This section uses the basic concepts of efficiency measurement as presented in Chapter 1 and calculates the relative efficiency in a simple numerical example. Suppose there are five DMUs in an industry which exhibit CRS and they consume capital as the only input (measured in ten thousands) to produce value added as the only output (measured in hundred thousands). The performance for each DMU can be evaluated by calculating the ratio of output to input which is a productivity measure as have been already presented in Section 2.2.

$$Performance = \frac{outputs}{inputs}$$

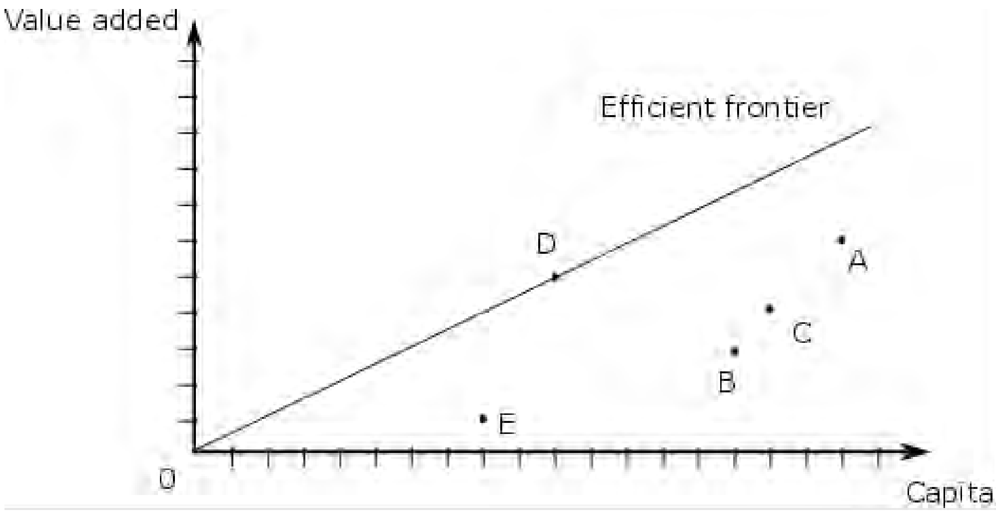
Table 2.1 presents the data for input and output and the calculated measure for performance. DMU *D* has the highest value added per unit of capital (0.500) and DMU *E* has the lowest (0.125).

Table 2.1: Performance of single input-output DMUs

DMU	Capital	Value added	Value added/capital
A	18	6	0.333
B	15	3	0.200
C	16	4	0.250
D	10	5	0.500
E	8	1	0.125

In this single input-output case the ratio of output to input can also be seen as a measure of efficiency. Figure 2.1 plots the data where capital is in *x* axis and value added is in *y* axis. The slope of the line that connects each point with the origin is the ratio of value added to capital. The highest slope is the *efficient frontier* which envelops all the data; a property which gave its name to DEA. The highest slope is the line which passes from point *D* which is also evaluated from Table 2.1 (0.500).

Figure 2.1: Efficient frontier



DMU *D* found to be the most efficient DMU relative to the other DMUs. *D* can be set as a 100% efficient DMU and we can measure the relative efficiency of the other DMUs relative to *D*. Therefore the efficiency of the *j*th DMU (j=1, 2, 3, 4, 5) can be calculated as:

$$Efficiency = \frac{performance\ of\ DMU_j}{performance\ of\ DMU_D} \tag{2.1}$$

The relative efficiency index (2.1) takes values from 0 to 1. For example the efficiency of DMU *A* is calculated as:

$$Efficiency_A = \frac{performance\ of\ DMU_A}{performance\ of\ DMU_D} = \frac{0.333}{0.500} = 0.666$$

DMU *A* is said to be 66.6% efficient. Table 2.2 shows the relative efficiency for the five DMUs.

Table 2.2: Relative efficiency of single input-output DMUs

DMU	Value added/capital	Efficiency
A	0.333	0.666
B	0.200	0.400
C	0.250	0.500
D	0.500	1.000
E	0.125	0.250

An inefficient DMU can either increase its output levels while holding its input levels stable or decrease its input levels while holding its output level stable, in order to become efficient relative to DMU *D* and to operate on the efficient frontier. Input and output targets can be set for the inefficient DMUs (Ramanathan, 2003). The input target for an inefficient DMU is calculate as follows:

$$Input\ Target = Observed\ input \times Efficieny \qquad (2.2)$$

For example the input target for the inefficient DMU *A* is:

$$Input\ Target = 18 \times 0.666 = 11.988$$

Therefore if DMU *A* reduces its input to 11.988 while holding the value added stable at 6 it will be considered as an efficient DMU.

Similarly The output target for an inefficient DMU can be calculated as follows:

$$Output\ Target = \frac{Observed\ Output}{Efficiency} \qquad (2.3)$$

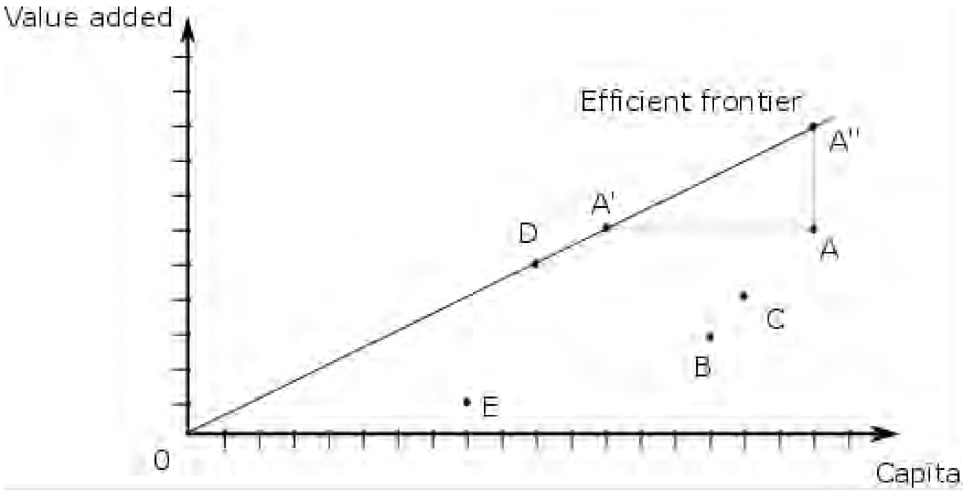
For example the output target for the inefficient DMU *A* is:

$$Output\ Target = \frac{6}{0.666} = 9.009$$

Therefore if DMU *A* increases its output to 9.009 while holding the capital stable at 18 it will be considered as an efficient DMU. Figure 2.2 demonstrates the projections of DMU

A on efficient frontier based on input and output targets. The DMU *A* can become efficient by fulfilling the input target at point *A′* or the output target at point *A′′*.

Figure 2.2: Input and output targets



Now suppose that DMUs use labor as an additional input (measured in hundreds). Table 2.3 demonstrates the data for the two inputs and the single output and the calculated measures of performance. DMU *A* has the highest value added per unit of labor (0.400) and DMU *E* has the lowest (0.143). Note that DMU *D* has the highest performance in terms of value added per unit of capital and DMU *A* has the highest performance in terms of value added per unit of labor. However we cannot determine which DMU is more efficient because the relative importance of each ratio is not known.

Table 2.3: Performance of two inputs and one output DMUs

DMU	Capital	Labor	Value added	Value added/capital	Value added/labor
A	18	15	6	0.333	0.400
B	15	9	3	0.200	0.333
C	16	16	4	0.250	0.250
D	10	15	5	0.500	0.333
E	8	7	1	0.125	0.143

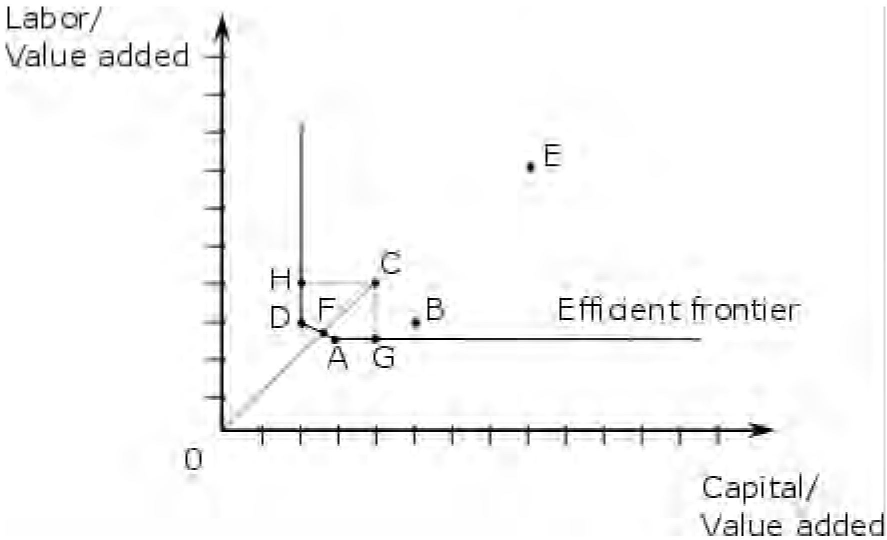
In the case of two inputs and one output, the two inputs can be expressed per unit of output and then draw the efficient frontier. Table 2.4 shows the transformed data.

Table 2.4: Inputs per unit of output

DMU	Capital/value added	Labor/value added	Value added
A	3	2.5	1
B	5	3	1
C	4	4	1
D	2	3	1
E	8	7	1

Figure 2.3 demonstrates the efficient frontier for the case of two inputs and a single output. As expected, DMUs *A* and *D* are considered as efficient and they lie on the efficient frontier. Inefficient DMUs *B*, *C* and *E* can move towards the efficient frontier in order to become efficient. For example DMU *C* can move along the line OC. The best possible performance to achieve is the intersection of line OC with the efficient frontier, at point *F* (2.6, 2.6). Any point beyond the efficient frontier is not possible to achieve. Alternatively, DMU *C* can move to point *G* or point *H* to become efficient.

Figure 2.3: Efficient frontier for two inputs and a single output case



The efficiency of DMU C from equation (1.6) is:

$$TE = \frac{OF}{OC} = \frac{\sqrt{2.6^2 + 2.6^2}}{\sqrt{4^2 + 4^2}} = 0.650$$

Therefore, the efficiency of DMU *C* is 0.650 or 65%. The input target for capital using (2.2) will be:

$$Input\ Target = 16 \times 0.650 = 10.4$$

If the DMU reduces only its capital input to input target, then the DMU will move to point *G*. The input target for labor will also be 10.4 because labor and capital have the same observed value (16). If the DMU reduces only its labor input to input target, then the DMU will move to point *H*. If the DMU reduces both its inputs to input target, then the DMU will move to point *F*. Similarly, in the case of a single input and two outputs, the two outputs can be expressed per unit of input and then draw the efficient frontier.

2.3. CCR DEA model

Now consider the case of two or more inputs and outputs. The above analysis with transformed ratios of inputs to outputs and the graphical analysis is not possible. The calculation of an efficiency index in this case requires the knowledge of the significance of each variable in the total index. One solution is to assign a priori fixed weights to each input and output and then aggregate them in a single index. Alternatively, DEA assigns variable weights to each input-output for every DMU, calculated directly from the data set. Specifically, the best set of weights is assigned to each target DMU which maximizes the efficiency of the DMU relative to the other DMUs. Three conditions are necessary regarding the best efficiency ratio of DEA (Cooper et al., 2007):

- Data and weights are non-negative
- Efficiency scores lies between zero and one
- The same set of weights for the DMU under assessment are applied to all DMUs

The assignment of weights allows the aggregation of inputs and outputs into virtual inputs and virtual outputs. The weighted sum of the *m* inputs *x* for the DMU under assessment can be aggregated as:

$$Virtual\ Input = \sum_{i=1}^{m} v_i x_{i_0} = v_1 x_{1_0} + v_2 x_2 + \cdots + v_m x_{m_0} \tag{2.4}$$

Accordingly the weighted sum of the *s* outputs *y* for the DMU under assessment can be aggregated as:

$$Virtual\ Output = \sum_{r=1}^{s} u_r y_{r_0} = u_1 y_{1_0} + u_2 y_2 + \cdots + u_s y_{s_0} \tag{2.5}$$

Therefore the fractional form of the DEA efficiency can be defined as:

$$Efficiency = \frac{Virtual\ Output}{Virtual\ Input} = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{2.6}$$

2.3.1. Fractional form of the multiplier CCR DEA model

Assume *n* DMUs and x_{ij} (i=1,…,m) and y_{rj} (r=1,…,s) are the *i*th input and the *r*th output respectively, of the *j*th DMU (j=1,…,n). In addition, the weights v_i of inputs and u_r of outputs are called multipliers of the model. Therefore, this type of DEA model which involves multipliers is called multiplier DEA model. The efficiency (2.6) for *DMU₀*, which is the DMU under assessment, is maximized by solving the following fractional model. Note that this model assumes CRS.

$$\max E = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{2.7}$$

Subject to

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \tag{2.8}$$

$$v_i, u_r \geq 0 \tag{2.9}$$

The objective function (2.7) yields the weights v_i and u_r which maximize the ratio of efficiency for the DMU under investigation. The constraint (2.8) restricts the ratio of efficiency for every DMU to be less than or equal to unity. The constraint (2.9) restricts the weights to be non-negative.

2.3.2. Input oriented multiplier CCR DEA model

Fractional model (2.7)-(2.9) can easily be transformed into a linear programming model using Charnes and Cooper (1962) transformation. Normalizing the denominator will result in an input oriented model while normalizing the nominator will result in and output oriented model[2]. The linear form of the input oriented model will be:

$$max \sum_{r=1}^{s} u_r y_{r_0} \tag{2.10}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{i_0} = 1 \tag{2.11}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \tag{2.12}$$

$$v_i, u_r \geq 0 \tag{2.13}$$

[2] Input and output orientation will become clear in a subsequent section of this chapter when we discuss about primal and dual models

The objective function (2.10) is the nominator in (2.7) and reflects the outputs of the DMU under assessment. The constraint (2.11) is the denominator in (2.7) and reflects the inputs of the DMU under assessment. The constraint (2.12) is the linear form of constraint (2.8) and constraint (2.13) is exactly the same with (2.9). According to Cooper et al. (2007) the fractional and the linear form of the model are equivalent. The DMU under assessment is efficient if the objective function (2.10) becomes 1 and there exist at least one optimal solution which yields positive multipliers for both inputs and outputs; otherwise the DMU under assessment is inefficient.

Consider the single input-output case of Table 2.1 and DMU *A* as the DMU under assessment. Then the model takes the following form:

$$max\ 6u_A$$

Subject to

$$18v_A = 1$$

$$6u \leq 18v \quad \text{(for DMU A)}$$

$$3u \leq 15v \quad \text{(for DMU B)}$$

$$4u \leq 16v \quad \text{(for DMU C)}$$

$$5u \leq 10v \quad \text{(for DMU D)}$$

$$u \leq 8v \quad \text{(for DMU E)}$$

$$v_i, u_r \geq 0$$

Solving the simple ratios yields the optimal multipliers $v = 0.056$ and $u = 0.111$ and the efficiency score for DMU *A* is 0.666 or 66.6%. The model needs to be solved four more times (one time for each DMU under evaluation) in order to evaluate the efficiency for all DMUs. Table 2.5 presents the efficiency score for all DMUs. DMU *D* is the most efficient DMU (1.000). The above simple example is easily extended to the case of multiple inputs and outputs.

Table 2.5: Efficiency scores for the five DMUs

DMU	Efficiency
A	0.666
B	0.400
C	0.500
D	1.000
E	0.250

2.3.3. Output oriented multiplier CCR DEA model

Likewise, normalizing the nominator will result in the linear form of the output oriented model:

$$min \sum_{i=1}^{m} v_i x_{i_0} \tag{2.14}$$

Subject to

$$\sum_{r=1}^{s} u_r y_{r_0} = 1 \tag{2.15}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \tag{2.16}$$

$$v_i, u_r \geq 0 \tag{2.17}$$

The objective function (2.14) is the denominator in (2.7) and reflects the inputs of the DMU under assessment. The constraint (2.15) is the nominator in (2.7) and reflects the outputs of the DMU under assessment. The constraint (2.16) is the linear form of constraint (2.8) and constraint (2.17) is exactly the same with (2.9).

2.3.4. Input oriented envelopment CCR DEA model

Every linear programming model has its dual model. The dual of the multiplier DEA model is called the envelopment DEA model. Correspondingly, the dual of the envelopment DEA model is the multiplier DEA model. Generally, envelopment model is considered as the *primal* and the multiplier model is considered as the *dual* in the DEA literature. The optimal values of primal and dual models are equal. The purpose of every linear program is to be as simple as possible. The multiplier DEA model adds constraints as the number of DMUs increases. On the contrary, the envelopment DEA model adds constraints as the number of variables increases. Usually, the number of variables is much smaller than the number of DMUs[3], therefore the envelopment form is much simpler and more efficient linear model than the multiplier form. Keeping the same notation, the input oriented envelopment CCR DEA model is as follows (Zhu, 2009):

$$\min \theta \tag{2.18}$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{i_0} \tag{2.19}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r_0} \tag{2.20}$$

$$\lambda_j \geq 0 \tag{2.21}$$

The objective function (2.18) minimizes θ which is the dual variable corresponding to the equality constraint (2.11) which is the sum of inputs for the DMU under assessment. In addition, λ in (2.19) and (2.20) is the dual variable corresponding to the inequality constraints (2.12). Specifically, the constraint (2.19) restricts the weighted combination of all inputs for all DMUs to be at most equal to the input of the DMU under assessment multiplied by its efficiency. Accordingly, the constraint (2.20) restricts the weighted average of all outputs for all DMUs to be at least equal to the output of the

[3] The reverse creates high level of discrimination (Dyson et al., 2001).

DMU under assessment. The formulation of the single input-output example of Table 2.1 and DMU *A* as the DMU under assessment will be:

$$\min \theta$$

Subject to
$$18\lambda_A + 15\lambda_B + 16\lambda_C + 10\lambda_D + 8\lambda_E \leq 18\theta$$

$$6\lambda_A + 3\lambda_B + 4\lambda_C + 5\lambda_D + \lambda_E \geq 6$$

$$\lambda_j \geq 0$$

or alternatively:

$$\min \theta$$

Subject to
$$18\theta - 18\lambda_A - 15\lambda_B - 16\lambda_C - 10\lambda_D - 8\lambda_E \geq 0$$

$$6\lambda_A + 3\lambda_B + 4\lambda_C + 5\lambda_D + \lambda_E \geq 6$$

$$\lambda_j \geq 0$$

It is already known that the efficiency score for DMU *A* is 0.666. Solving the above model yields the optimal λ values which are $\lambda_D = 1.2$ and all other $\lambda_j = 0$.

2.3.5. Output oriented envelopment CCR DEA model

Likewise, the output oriented envelopment CCR DEA model is as follows:

$$\max \varphi \tag{2.22}$$

Subject to
$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i_0} \tag{2.23}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{r_0} \tag{2.24}$$

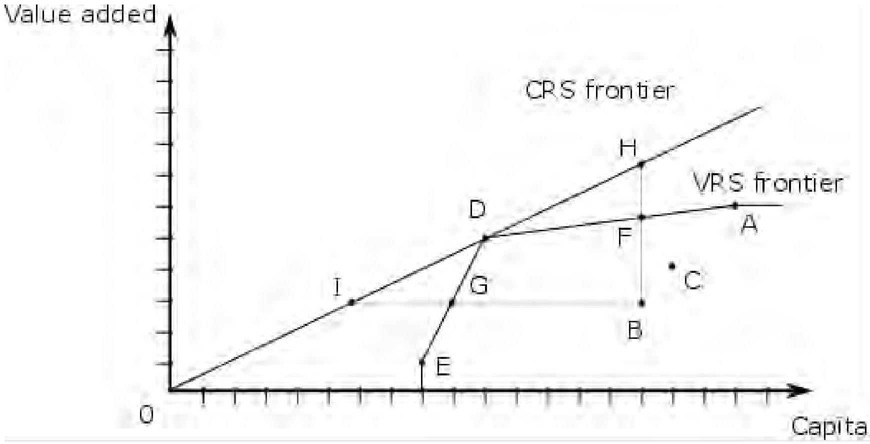
$$\lambda_j \geq 0 \tag{2.25}$$

The objective function (2.22) maximizes φ which is the dual variable corresponding to the equality constraint (2.15) which is the sum of outputs for the DMU under assessment. In addition, λ in (2.23) and (2.24) is the dual variable corresponding to the inequality constraints (2.12). Specifically, the constraint (2.23) restricts the weighted combination of all inputs for all DMUs to be at most equal to the input of the DMU under assessment. Accordingly, the constraint (2.24) restricts the weighted average of all outputs for all DMUs to be at least equal to the output of the DMU under assessment multiplied by its efficiency.

2.4. BCC DEA model

Chapter 1 defined the constant and the variables returns to scale where CRS means that the output increases by the same proportional change as all inputs and VRS is otherwise. Figure 1.8 shown a frontier which exhibit IRS, CRS and DRS at different segments. Now suppose that the single input-output example exhibits VRS. Figure 2.4 demonstrate the VRS frontier along with the CRS frontier which has already been presented in Figure 2.1. The CRS frontier is the line passes through the origin and DMU *D* which is the most efficient DMU (Table 2.5). The VRS frontier is the line which connects DMUs *E*, *D* and *A*. DMU *D* is both CCR-efficient and BCC-efficient while DMUs *E* and *A* are only BCC-efficient. DMUs *B* and *C* are inefficient both in CRS and in VRS, however they are closer to the VRS than the CRS frontier. For example, DMU *B* must increase its outputs to point *F* while holding its inputs stable, in order to become efficient in the case of VRS and output orientation. In the presence of CRS, point *F* would still be an inefficient point and the DMU would further need to increase its outputs to point *H*. Similarly, for input orientation DMU *B* must decrease its inputs to point *G* for VRS and point *I* for CRS. Note that BCC efficiency is at least equal and usually larger than CCR efficiency.

Figure 2.4: CRS and VRS frontiers



2.4.1. Input oriented envelopment BCC DEA model

Computationally, adding the constraint that the sum of λ is equal to unity into the input oriented envelopment CCR model (2.18)-(2.21) creates the input oriented envelopment BBC model.

$$\min \theta \tag{2.26}$$

Subject to
$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{i_0}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r_0}$$

$$\sum_{j=1}^{n} \lambda_j \underset{j}{} = 1$$

$$\lambda_j \geq 0$$

2.4.2. Output oriented envelopment BCC DEA model

Likewise, adding the constraint that the sum of λ is equal to unity into the output oriented envelopment CCR model (2.22)-(2.25) creates the output oriented envelopment BBC model.

$$\max \varphi \tag{2.27}$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i_0}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{r_0}$$

$$\sum_{j=1}^{n} \lambda_{j_j} = 1$$

$$\lambda_j \geq 0$$

2.4.3. Input oriented multiplier BCC DEA model

In multiplier form the dual variable for the constraint $\sum_{j=1}^{n} \lambda_{j_j} = 1$ is a scalar u^1 which is free in sign. Therefore, the input oriented model takes the following form.

$$max \sum_{r=1}^{s} u_r y_{r_0} + u^1 \tag{2.28}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{i_0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u^1 \leq 0$$

$$v_i, u_r \geq 0$$

2.4.4. Output oriented multiplier BCC DEA model

Correspondingly, by adding the free in sign scalar u^1 to the output oriented model it takes the following form.

$$min \sum_{i=1}^{m} v_i x_{i_0} + u^1 \tag{2.29}$$

Subject to
$$\sum_{r=1}^{s} u_r y_{r_0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u^1 \leq 0$$

$$v_i, u_r \geq 0$$

2.5. Summary

This Chapter presented and discussed the basic DEA models which are the multiplier and the envelopment model for input and output orientation. Furthermore, the Chapter distinguished between the CCR models which exhibit constant returns to scale and the BCC models which exhibit variable returns to scale. Simple numerical examples have been employed to aid the analysis.

Up to this point, the basic efficiency concept has been presented along with the advantages of DEA as a technique to evaluate the efficiency of DMUs. Additionally, all the basic DEA models have been presented and discussed. These models use inputs to produce outputs and they are usually sufficient. However in some cases there is a need to investigate the internal structures inside a DMU. Chapter 3 extends the analysis into two-stage and network structures.

Chapter 3

Two-stage DEA models

3.1. Introduction

All DEA models presented in Chapter 2 and every conventional DEA model are single stage models. DMUs in these models consume inputs to produce outputs while making no assumption regarding any possible procedures taking place inside the DMU. Therefore, DEA treats a DMU as a “black box” which uses inputs to produce outputs without considering the internal structures, a usually sufficient assumption (Sexton and Lewis, 2003). However in some cases, like in supply chain systems, DEA models consist of two or more stages and there are intermediate measures which are considered as inputs in one stage and outputs in another stage. Supply chain is a complex system which includes suppliers, manufacturers, distributors and retailers who employ various inputs to produce final or intermediate outputs (Li and Jiang, 2012). The assessment of supply chain performance is one of the most significant problems regarding the long term viability of the supply chain (Xu et al., 2009).

There is a wide range of economic applications where the two-stage structures are needed. For example, Fukuyama and Matousek (2011) and Holod and Lewis (2011) evaluated the efficiency of banks by measuring the “value added activity” in the first stage and the “profitability” in the second stage. Another economic example is the case of manufacturing firms (or any other firms) where the first stage measures the “profitability” of the firm and the second stage measures the “marketability” of the firm (Hung and Wang, 2012). Universities is another interesting case where “teaching” can be considered as the first stage and “research” as the second stage (Kao, 2012; Kao and Lin, 2012). In two-stage models each stage can be considered as a decision center and the overall process is managed by a corporate manager who is the overall decision maker and is willing to improve overall efficiency both internally and externally (Ross, 2000). Internally, the each decision center aims to succeed the best possible allocation of the resources according to its preferences and needs, while externally aims for a bigger market share (Ross and Droge, 2002). The best allocation for each stage refers to higher efficiency in this stage and bigger market share refers to the contribution of this stage to the overall process.

This Chapter surveys the two-stage models which take into account the internal structures inside a DMU and highlight their importance for the decision maker. The general concept of two-stage DEA models is based on the pioneer work of Färe and Grosskopf (1996a) who were the first to study the so-called “black box”. Also the two-stage DEA models can be considered as a special case of network DEA models. Wang et al. (1997) and Seiford and Zhu (1999) were the first to construct a pure two-stage DEA model where all the outputs of the first stage are the only inputs of the second stage.

Two-stage DEA models can be classified into four categories. The classification effort introduced here is inspired by Kao and Hwang (2010) (independent, connected, relational) and Cook et al. (2010a) (standard, relational, network, game theory). The first category includes the independent two-stage DEA approach which apply typical DEA methodology separately to each stage, without considering the interaction between the two stages. The second category is the connected two-stage DEA approach which considers the interaction between the two stages. The third category includes the relational two-stage DEA approach which assumes a mathematical relationship between the overall efficiency and the individual efficiencies. Finally, the last category contains two-stage models which are based on game theoretical approaches.

Furthermore, this Chapter builds upon previous surveys such as the seminal studies of Cook et al. (2010a) and Castelli et al. (2010) by making a distinct contribution in a number of ways. More analytically, following Cook et al. (2010a), the Chapter focus on two-stage and network DEA models however it includes models which allow “exogenous” inputs at the intermediate stages. In addition, the majority of two-stage DEA applications across the literature until early 2015 is presented in a unified manner. Moreover, a more detailed review of network DEA models alongside with a unified classification is provided making therefore easier for the researcher/policy maker to make distinctions among different models. Finally, this survey analyzes the bargaining DEA models which are based on the Nash bargaining game and the network relational two-stage DEA models.

The structure of Chapter 3 is as follows. Section 3.2 presents the independent two-stage DEA approach. Section 3.3 demonstrates the connected two-stage DEA approach and Section 3.4 examines the relational two-stage DEA approach. In Section 3.5 the game theoretic models are presented. The Chapter fosters a continuous discussion and comparison about the connections between the different models. Section 6 provides a detailed table with all the two-stage DEA application along the literature until early 2015 and Section 3.6 concludes.

3.2. Independent two-stage DEA

This type of two-stage model applies the basic DEA approach, as presented in Chapter 2, separately in first and second stage without considering possible conflicts between the two stages (Cook and Zhu, 2014). Such conflicts may arise because of the intermediate measures, which are not treated in a simultaneous manner. Intermediate measures are handled independently in the two stages and it is even possible to increase them in the first stage (when they are considered as outputs) and to decrease them in the second stage (when they are considered as inputs). Also, overall efficiency and individual efficiencies are evaluated separately and as a result a reported efficient DMU does not imply an overall efficiency of the individual stages. Now, suppose a supply chain where the first stage is a manufacturer and the second stage is a retailer. In addition, suppose that the retailer achieves maximum efficiency in contrast with the manufacturer. It is reasonable that the manufacturer would increase his outputs in order to achieve maximum efficiency. However, an increase in the manufacturer's outputs means an increase in the retailers inputs, because the first stage outputs are the second stage inputs, and as a result a decrease in the retailer's efficiency. These conflicts cannot be addressed by these models.

The first who studied these models were Wang et al. (1997) and Seiford and Zhu (1999). Wang et al. (1997) investigated the efficiency of 22 banks, where in the first stage they assessed the IT-related activity and in the second stage they assessed the loan processing system. Seiford and Zhu (1999) applied this approach in order to evaluate the efficiency of the top commercial banks in USA measuring the operational performance and market performance in the first and second stage respectively.

In Seiford and Zhu's (1999) model the DMU is a bank. The bank consumes various inputs and produce profits in the first stage while in the second stage they use profits to create market value. Seiford and Zhu (1999) applied the output oriented CRS DEA model (2.22)-(2.25) with slacks, in order to measure the efficiency of the each stage as follows. Assume *n* DMUs and x_{ij} $(i = 1, \dots, m)$ and y_{rj} $(r = 1, \dots, s)$ are the *i*th input and the *r*th output respectively, of the *j*th DMU $(j = 1, \dots, n)$ for the *t*th stage $(t = 1,2)$.

$$\max \theta_0^t + \varepsilon \cdot \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right) \tag{3.1}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{i_0}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \theta_0 y_{r_0}$$

$$\lambda_j, s_i^-, s_r^+ \geq 0$$

where θ_0^1 and θ_0^2 are the CRS efficiencies from the first and second stage respectively and s_i^- and s_r^+ are the slack variables. If $\theta_0^1 = 1$ and all slack variables are zero, then the *j*th DMU is efficient in the first stage. If $\theta_0^2 = 1$ and all slack variables are zero, then the *j*th DMU is efficient in the second stage. Figure 3.1 presents the two-stage formuation of Seiford and Zhu (1999).

Figure 3.1: Two-stage formulation of Seiford and Zhu (1999)

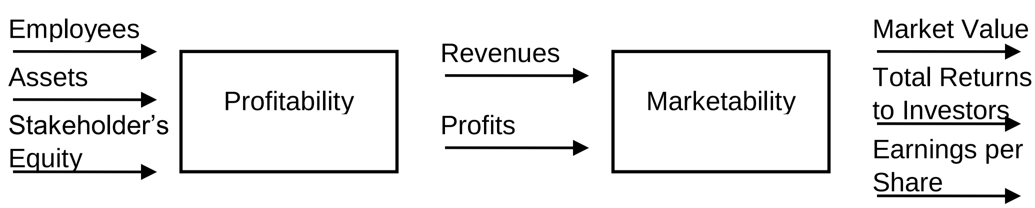
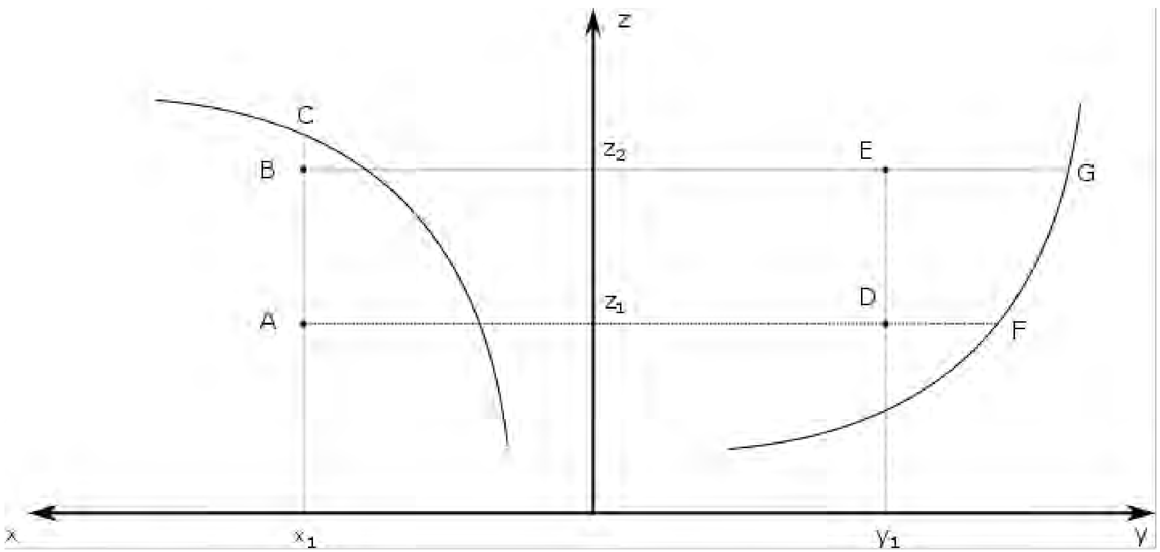


Figure 3.2 presents the graphical presentation of the first and the second stage of an output oriented model. Intermediate variables are treated as outputs in the first stage and inputs in the second stage. In the first stage an increase of intermediate variables z_1 to z_2 while keeping inputs and outputs stable, results in an increase in first stage efficiency because the DMU operates closer to the frontier ($BC < AC$). However, the increase of intermediate variables will result in a decrease in second stage efficiency because the DMU operates further from the frontier ($EG > DF$).

Figure 3.2: The dual role of intermediate variables



3.3. Connected two-stage DEA

In contrast to the independent two-stage DEA, in the connected two-stage DEA approach the interactions between the stages are taken into account for the calculation of the overall efficiency. This approach ensures that in order for a DMU to be overall efficient both the two stages must be fully efficient. In some cases the mathematical model evaluates the overall efficiency and the individual efficiencies simultaneously while in other cases the individual efficiencies are calculated after the overall two-stage model by applying a conventional DEA model such as model (3.1) or they cannot be calculated at all. However, in every case the intermediate measures are treated independently and are allowed to use a different set of multipliers in the two stages. There are various different types of connected two-stage DEA models. Here we divide them into two broad subcategories, the value chain model and the family of network DEA models.

3.3.1. Value Chain model

Chen and Zhu (2004) developed a value-chain model which ensures that in order for the DMU to be overall efficient, all the individual stages must also be efficient. According to Chen and Zhu (2004) the standard CRS DEA model (3.1) or the VRS version of the model are unable to assess the efficiency of a two stage procedure because of the intermediate measures. The authors propose the following VRS model in order to address this problem. Assume *n* DMUs and x_{ij} $(i = 1, \dots, m)$, z_{dj} $(d = 1, \dots, D)$ and y_{rj} $(r = 1, \dots, s)$ are the *i*th input, the *d*th intermediate variable and the *r*th output respectively, of the *j*th DMU $(j = 1, \dots, n)$.

$$\min_{\alpha,\beta,\lambda_j,\mu,\tilde{z}} \xi_1 \cdot \alpha - \xi_2 \cdot \beta \tag{3.2}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \alpha x_{i_0}$$

$$\sum_{j=1}^{n} \lambda_j z_{dj} \geq \tilde{z}_{d_0}$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0$$

$$\sum_{j=1}^{n} \mu_j z_{dj} \leq \tilde{z}_{d_0}$$

$$\sum_{j=1}^{n} \mu_j y_{rj} \geq \beta y_{r_0}$$

$$\sum_{j=1}^{n} \mu_j = 1$$

$$\mu_j \geq 0$$

where ξ_1 and ξ_2 are the weights of the two stages and are defined in an exogenous manner by the decision maker based on the preferences over the two stages and the symbol "~" stands for the unknown decision variables. The first four constraints refer to the first stage and the last four constraints refer to the second stage. The authors pointed out that the inclusion of additional constraints is possible because their model treats intermediate measures as unknown decision variables. Chen and Zhu (2004) applied model (3.2) at the banking sector and measured the indirect impact of information technology on the efficiency of a firm, based on Wang et al. (1997) data set.

According to Zhu (2003) the general case of model (3.2) can be used to determine the efficiency of a supply chain. A supply chain is the most representative case study for this type of models because every single member of the supply chain applies its own strategy in order to become efficient. From a general point of view, the efficiency of a single member does not ensure the efficiency of another member. In fact, it is reasonable that most of the times the inefficiency of a member is caused by someone else's efficiency. Zhu (2003) presented a typical supply chain with four "members"; the supplier, the manufacturer, the distributor and the retailer. Moreover, Zhu (2003) marked the significance to assess the efficiency of the supply chain and its individual members. The evaluation of efficiency helps the decision maker to identify the best practices in order to monitor, manage and improve the performance of the supply chain.

Zhu (2003) proposed the following model to evaluate the efficiency of *j* supply chains, which is the general form of Chen and Zhu's (2004) model (3.2). ξ_i is the weight of each member of the supply chain and is defined in an exogenous manner by the decision maker based on the preferences over the contribution of the stage to the overall process.

$$E^* = \min_{E_i, \lambda_j, \beta_j, \delta_j, \gamma_j, \tilde{z}} \frac{\sum_{i=1}^{4} E_i}{\sum_{i=1}^{4} \xi_i} \tag{3.3}$$

s.t. (supplier)

$$\sum_{j=1}^{n} \lambda_j x_{ij}^{supplier} \leq E_1 x_{ij_0}^{supplier}, \qquad i \in DI^{supplier}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{supplier} \geq y_{rj_0}^{supplier}, \qquad r \in DR^{supplier}$$

$$\sum_{j=1}^{n} \lambda_j z_{tj}^{S-M} \geq \tilde{z}_{tj_0}^{S-M}, \qquad t = 1, \dots, T$$

$$\sum_{j=1}^{n} \lambda_j z_{mj}^{M-S} \leq \tilde{z}_{mj_0}^{M-S}, \qquad m = 1, \dots, M$$

$$\lambda_j \geq 0 \qquad j = 1, \dots, n$$

s.t. (manufacturer)

$$\sum_{j=1}^{n} \mu_j x_{ij}^{manufacturer} \leq E_2 x_{ij_0}^{manufacturer}, \qquad i \in DI^{manufacturer}$$

$$\sum_{j=1}^{n} \mu_j y_{rj}^{manufacturer} \geq y_{rj_0}^{manufacturer}, \qquad r \in DR^{manufacturer}$$

$$\sum_{j=1}^{n} \mu_j z_{tj}^{S-M} \leq \tilde{z}_{tj_0}^{S-M}, \qquad t = 1, \dots, T$$

$$\sum_{j=1}^{n} \mu_j z_{mj}^{M-S} \geq \tilde{z}_{mj_0}^{M-S}, \qquad m = 1, \dots, M$$

$$\sum_{j=1}^{n} \mu_j z_{fj}^{M-D} \geq \tilde{z}_{fj_0}^{M-D}, \qquad f = 1, \dots, F$$

$$\sum_{j=1}^{n} \mu_j z_{gj}^{D-M} \leq \tilde{z}_{gj_0}^{D-M}, \qquad g = 1, \dots, G$$

$$\sum_{j=1}^{n} \mu_j z_{lj}^{M-R} \geq \tilde{z}_{lj_0}^{M-R}, \qquad l = 1, \dots, L$$

$$\sum_{j=1}^{n} \mu_j z_{qj}^{R-M} \leq \tilde{z}_{qj_0}^{R-M}, \qquad q = 1, \dots, Q$$

$$\mu_j \geq 0 \qquad j = 1, \dots, n$$

s.t. (distributor)

$$\sum_{j=1}^{n} \delta_j x_{ij}^{distributor} \leq E_3 x_{ij_0}^{distributor}, \qquad i \in DI^{distributor}$$

$$\sum_{j=1}^{n} \delta_j y_{rj}^{distributor} \geq y_{rj_0}^{distributor}, \qquad r \in DR^{distributor}$$

$$\sum_{j=1}^{n} \delta_j z_{fj}^{M-D} \leq \tilde{z}_{fj_0}^{M-D}, \qquad f = 1, \dots, F$$

$$\sum_{j=1}^{n} \delta_j z_{gj}^{D-M} \geq \tilde{z}_{gj_0}^{D-M}, \qquad g = 1, \dots, G$$

$$\sum_{j=1}^{n} \delta_j z_{ej}^{D-R} \geq \tilde{z}_{ej_0}^{D-R}, \qquad e = 1, \dots, E$$

$$\sum_{j=1}^{n} \delta_j z_{pj}^{D-R} \leq \tilde{z}_{pj_0}^{D-R}, \qquad p = 1, \dots, P$$

$$\delta_j \geq 0 \qquad j = 1, \dots, n$$

s.t. (retailer)

$$\sum_{j=1}^{n}\gamma_j x_{ij}^{retailer} \leq E_4 x_{ij_0}^{retailer}, \qquad i \in DI^{retailer}$$

$$\sum_{j=1}^{n}\gamma_j y_{rj}^{retailer} \geq y_{rj_0}^{retailer}, \qquad r \in DR^{retailer}$$

$$\sum_{j=1}^{n}\gamma_j z_{lj}^{M-R} \leq \tilde{z}_{lj_0}^{M-R}, \qquad l = 1, \dots, L$$

$$\sum_{j=1}^{n}\gamma_j z_{qj}^{R-M} \geq \tilde{z}_{qj_0}^{R-M}, \qquad q = 1, \dots, Q$$

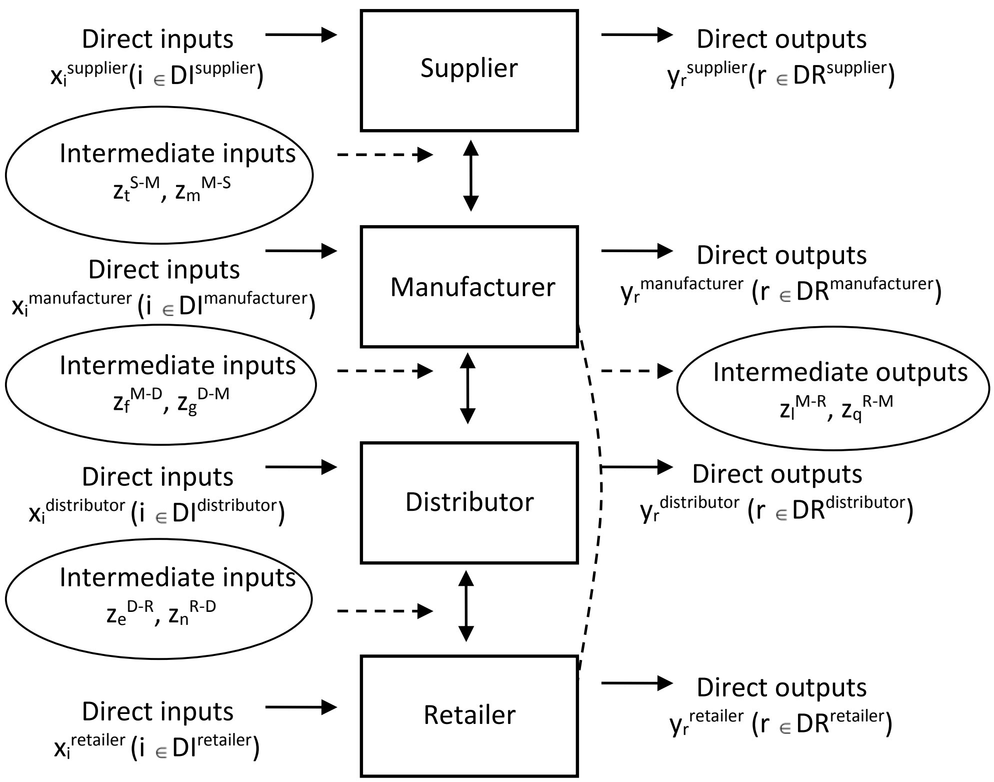
$$\sum_{j=1}^{n}\gamma_j z_{ej}^{D-R} \leq \tilde{z}_{ej_0}^{D-R}, \qquad e = 1, \dots, E$$

$$\sum_{j=1}^{n}\gamma_j z_{pj}^{D-R} \geq \tilde{z}_{pj_0}^{D-R}, \qquad p = 1, \dots, P$$

$$\gamma_j \geq 0 \qquad j = 1, \dots, n$$

where *DI* and *DR* are the direct inputs and direct outputs respectively; the first letter represents its production and the second letter represents its consumption. For example, z^{S-M} represents the intermediate measure which produced by supplier and consumed by manufacturer. Therefore, "*S*" represents the supplier, "*M*" represents the manufacturer, "*D*" represents the distributor and "*R*" represents the retailer. The symbol "~" stands for the unknown decision variables. As noted in model (3.2), the inclusion of additional constraints is possible because the intermediate measures are treated as unknown decision variables. Zhu (2003) pointed out that if $E^* = 1$, then there is an optimal solution that ensures $\lambda_0^* = \beta_0^* = \delta_0^* = \gamma_0^* = 1$, where symbol "*" represents an optimal value in model (3.3). Furthermore, if $E^* = 1$ then the supply chain is rated as efficient and E_i^* is the optimal efficiency for $i = 1,2,3,4$ members of the supply chain. Figure 3.3 presents this supply chain.

Figure 3.3: Supply chain



3.3.2. Network DEA

Network-DEA is not a specific type of model but rather a group of models which share some common features. Färe and Grosskopf (1996a), based on Shephard (1970) and Shephard and Färe (1975), developed a series of models in order to deal with special cases that typical DEA fail to manage.

3.3.2.1. Structure of Network DEA

There are two types of structure in a network DEA model, the serial and the parallel. These two types of network DEA structure are presented in Figure 3.4. More specifically, in subfigures 3.4a and 3.4b introduced by Kao and Hwang (2010), we can see serial and parallel structure respectively. In real life empirical applications usually the structure is not only serial or parallel but rather a mixture of them.

3.3.2.1.1. Serial structure

The two stage models that already have been presented in our paper are in the simple form of a serial network DEA model. Specifically, a serial network DEA model includes DMUs with two or more internal procedures which are linked with intermediate measures. In the simple form, a set of inputs is used by the first stage and a set of intermediate measures is produced, while the second stage uses the intermediate measures that the first stage produced and generates a set of final outputs. In the simple form there are no exogenous inputs in the second stage and the entire intermediate measures are used. Furthermore, final outputs are produced only by the second stage. A general form of a serial network DEA model is presented in subfigure 3.4a.

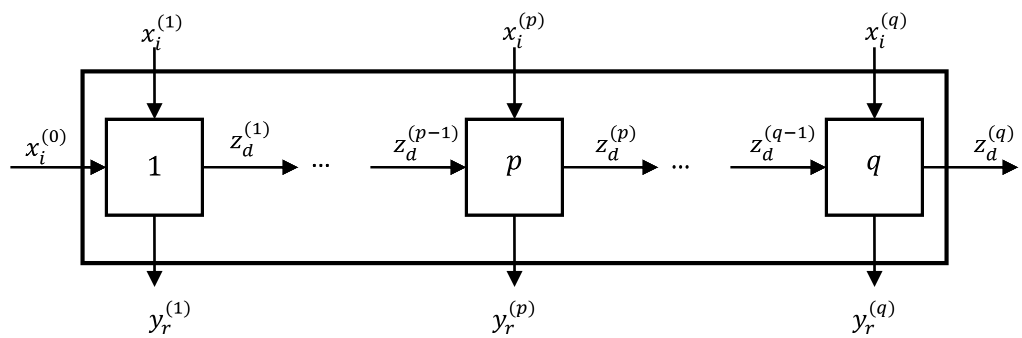
The differences between the simple and the general form lie on the number of internal procedures (in the general form there are more than two stages), inputs may enter in any stage, final outputs may be produced by any stage and intermediate measures may not be consumed entirely.

3.3.2.1.2. Parallel structure

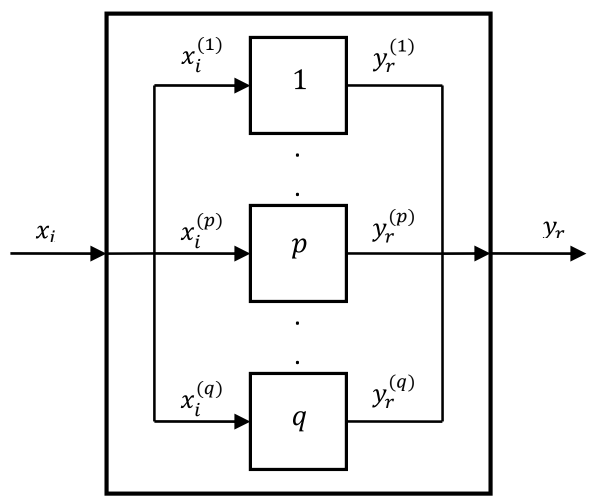
In this type of network DEA models the individual stages operate parallel and separately to each other. An extension of this type of model is the shared flows system where the inputs are shared among the individual stages (Kao and Hwang, 2010). According to Kao and Hwang (2010) a university is a perfect example to describe a parallel system, where the individual stages are the departments which operate parallel and separately inside the university. In addition, the authors pointed out that a parallel model is a special case of a serial model without intermediate measures. Parallel model is presented in subfigure 3.4b.

Figure 3.4: Structure of network DEA models

3.4a: Serial structure.



3.4b: Parallel structure



3.3.2.2. Types of network DEA models

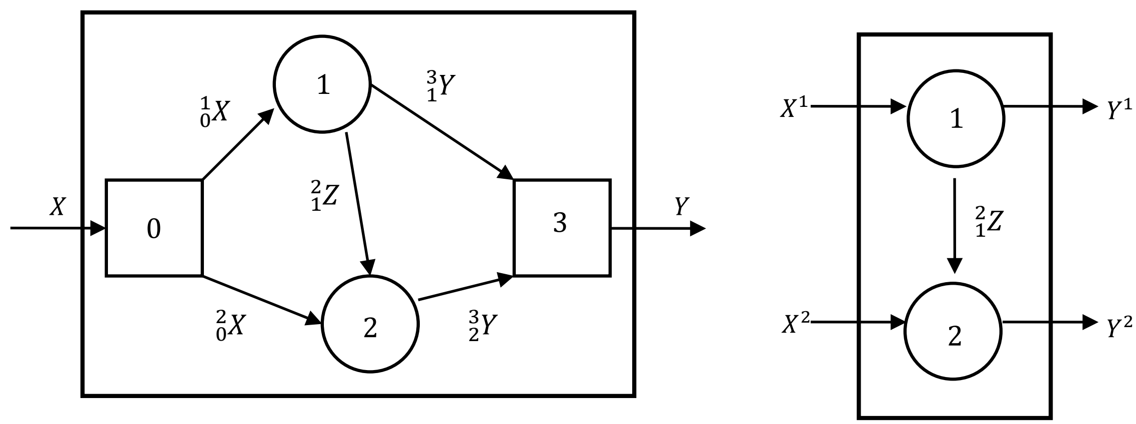
The main types of network DEA models as described by Färe and Grosskopf (2000) and Färe et al. (2007) and presented in Castelli et al. (2010) are static, dynamic and technology adoption or shared resources models. Figure 3.5 presents these three type of models. Specifically, in subfigure 3.5a introduced by Färe and Whittaker (1995), the static network model is presented and in subfigure 3.5b introduced by Färe et al. (2004), there is a static network with externalities. In subfigure 3.5c introduced by Färe and Grosskopf (2000), dynamic network DEA is presented and finally in subfigure 3.5d introduced by Färe et al. (2007), the shared resources model is demonstrated.

All the aforementioned models supposed that the network is owned by a single person. An interesting alternative model is proposed by Chang et al. (2014) where the ownership does not belong to one person only. The authors proposed three different ownership-specified network DEA models. Here, only models with single ownership structures are presented and analyzed.

Figure 3.5: Types of network DEA models

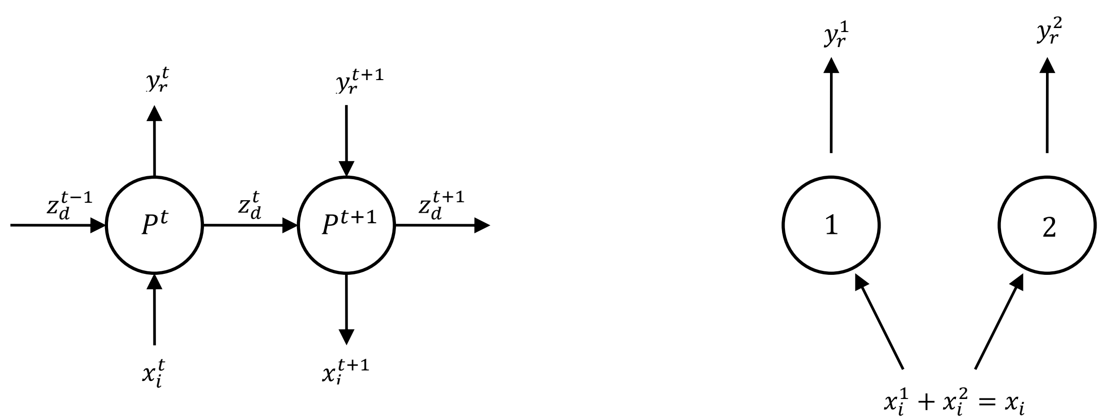
3.5a: Static Network DEA.

3.5b: Static Network DEA with first stage final outputs and bad intermediate outputs.



3.5c: Dynamic Network DEA.

3.5d: Shared flow model.



3.3.2.2.1. Static model

Static model is applied when the individual stages are linked with intermediate measures. Two stage DEA models are special cases of static models. In the general form there may exist multiple stages which are linked with intermediate measures. In addition, exogenous inputs and final outputs may exist in any stage. Färe and Whittaker (1995) investigated a two stage model for rural production, where “1” stands for the first stage and “2” stands for the second stage, “0” is the stage where exogenous inputs enter the system and “3” is the stage where final outputs are produced. This model is presented in subfigure 3.5a.

The vector of inputs is denoted as ${}_{0}^{ic}x$ where “*ic*” stands for individual consumer which is the individual stage which consumes the input and 0 represent the stage where the input enters the system. For example, ${}_{0}^{2}x$ is the vector of inputs for the second stage. Also, overall inputs must be equal or greater than the sum of inputs of individual stages, $X \geq {}_{0}^{1}x + {}_{0}^{2}x$. The vector of outputs is denoted as ${}_{ip}^{ic}y$ where “*ip*” stands for the individual producer which is the individual stage which produces the output and “*ic*” is the individual stage which uses the output. For example, ${}_{1}^{2}y$ is produced in the first stage and consumed by the second stage. Furthermore, this output is the only intermediate measure in subfigure 3.5a and can be denoted as ${}_{1}^{2}z$. Also, overall outputs must be equal with the sum of outputs of individual stages. s^1 is the number of outputs that comes from the first stage and s^2 is the number of outputs that comes from the second stage.

Keeping the same notation as above, suppose *n* DMUs and x_{ij} $(i = 1, \dots, m)$, z_{dj} $(d = 1, \dots, D)$, y_{r1j} $(r1 = 1, \dots, s^1)$ and y_{r2j} $(r2 = 1, \dots, s^2)$ are the *i*th input, the *d*th intermediate variable, the *r1*th first stage output and the *r2*th second stage output respectively, of the *j*th DMU $(j = 1, \dots, n)$. The above network model can be written as a linear problem:

$$Y = ({}_{1}^{3}y, {}_{2}^{3}y) \tag{3.4}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j \cdot {}_{2}^{3}y_{r2j} \geq {}_{2}^{3}y_{r2} \tag{3.5}$$

$$\sum_{j=1}^{n} \lambda_j \cdot {}_{0}^{2}x_{ij} \leq {}_{0}^{2}x_{i} \tag{3.6}$$

$$\sum_{j=1}^{n} \lambda_j \cdot {}_{1}^{2}z_{dj} \leq {}_{1}^{2}z_{d} \tag{3.7}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{3.8}$$

$$\lambda_j \geq 0 \tag{3.9}$$

$$\sum_{j=1}^{n} \mu_j \cdot ({}_{1}^{2}z_{dj} + {}_{1}^{3}y_{rj}) \geq ({}_{1}^{2}z_{d} + {}_{1}^{3}y_{r}) \tag{3.10}$$

$$\sum_{j=1}^{n} \mu_j \cdot {}_{0}^{1}x_{ij} \leq {}_{0}^{1}x_{i} \tag{3.11}$$

$$\sum_{j=1}^{n} \mu_j = 1 \tag{3.12}$$

$$\mu_j \geq 0 \tag{3.13}$$

$${}_{0}^{1}x_{i} + {}_{0}^{2}x_{i} \leq x_i \tag{3.14}$$

where λ_j and μ_j are the weights of DMUs for stages the second and the first stage respectively. From constraints (3.8) and (3.12) it is clear that the model adopts the VRS assumption. Constraint (3.11) is the input constraint for the first stage and constraints (3.6) and (3.7) are the input constraints for the second stage. Constraints (3.5) and (3.10) are the output constraints where the second constraint includes the intermediate measures. Last, constraint (3.14) ensures that the sum of inputs of each stage will not exceed the total available inputs.

An interesting case of the above model is the simple case of the two stages as presented in Figure 1. According to Färe and Grosskopf (1996b) the simple case of the two-stage network DEA is the following:

$$\min_{E,\lambda_j,\mu_j,\tilde{z}} E \tag{3.15}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq E x_{i_0}$$

$$\sum_{j=1}^{n} \lambda_j z_{dj} \geq \tilde{z}_{d_0}$$

$$\lambda_j \geq 0$$

$$\sum_{j=1}^{n} \mu_j z_{dj} \leq \tilde{z}_{d_0}$$

$$\sum_{j=1}^{n} \mu_j y_{rj} \geq y_{r_0}$$

$$\mu_j \geq 0$$

where $\tilde{z}_{d_0}$ are set as unknown decision variables. The first three constraints refer to the first stage and the last three constraints refer to the second stage. Model (3.15) can also be written as follows:

$$\min_{E,\lambda_j,\mu_j,\tilde{z}} E \tag{3.16}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq E x_{i_0}$$

$$\sum_{j=1}^{n} (\lambda_j - \mu_j) z_{dj} \geq 0$$

$$\sum_{j=1}^{n} \mu_j y_{rj} \geq y_{r_0}$$

$$\lambda_j, \mu_j \geq 0$$

According to Cook et al. (2010a) model (3.15) is equivalent to model (3.2) of Chen and Zhu (2004) and model (3.16) is equivalent to the multiplicative model of Kao and Hwang (2008) and the cooperative model of Liang et al. (2008) which will be discussed later.

Another special case is a system with two stages which are linked with intermediate measures but final outputs are generated from both stages. Färe et al. (2004) used this model to study property rights. In their model there are two stages and each stage represents a firm. Firm 1 generates two outputs, a good one and a bad one. The good output is a final output while the bad output is an intermediate output which is used as an input by firm 2. Then, firm 2 converts the bad output into a good final output. This model is presented in subfigure 3.5b.

All the above network models are radial models and Tone and Tsutsui (2009) argued that the assumption of proportional changes which are implied with radial models is not always true. In order to deal with this problem the authors developed a slack-based (SB) network DEA model to study the efficiency of electric power companies with three subdivisions, generation, transmission and distribution.

The static network DEA model can also be used to evaluate the efficiency of a supply chain by incorporating game theory aspects. This alternative method will be presented in a subsequent section of this Chapter.

3.3.2.2.2. Dynamic model

Dynamic network model incorporates the dimension of time in the analysis. Specifically, the outputs of the procedure in a specific time period are used as inputs in the next period and can be treated as intermediate variables in time. There is a variety of applications across the literature. Färe and Grosskopf (1997) investigated countries' inefficiency which occurs from misallocation of resources in time. Nemota and Gota (1999) studied the dynamic inefficiency based on Hamilton-Jacobi-Bellman equation. Jaenicke (2000) applied a dynamic model in rural production while Nemota and Gota (2003) used it in the case of electricity production. Chen (2009) proposed a unified framework for efficiency assessment in a dynamic production network system.

Subfigure 3.5c presents a DMU with two stages, P^t and P^{t+1}, which take place in time t and t+1 respectively. Stage P^t produces y_r^t $(r = 1, \dots, s^t)$ as a final output and z_d^t $(d = 1, \dots, D^t)$ as an intermediate output in time. Inputs x_i^t $(i = 1, \dots, m^t)$ and x_i^{t+1} $(i = 1, \dots, m^{t+1})$ are exogenously entering the system. The terms z_d^{t-1} $(d = 1, \dots, D^{t-1})$ and z_d^{t+1} $(d = 1, \dots, D^{t+1})$ are used to generalize the system into more stages. If only periods *t* and *t+1* are of interest, these terms are excluded. It is obvious that dynamic and static models are both consisted by multiple stages linked with intermediate measures, however in a dynamic model the individual stages operate in a different time period. Tone and Tsutsui (2010) developed a dynamic slack-based network DEA model.

3.3.2.2.3. Shared resources or technology adoption model

This model is used in order to allocate the resources properly among the different stages of production technologies. Färe et al. (1997) applied this model to study the allocation of rural land. Lothgren and Tambour (1999) investigated the allocation of labor time among production and customer service while Färe et al. (2007) examined the use of technology adoption model to allocate pollution permits.

The simple case of technology adoption model is presented in subfigure 3.5d. Inputs x_i are allocated among two production technologies; x_i^1 are the inputs of the first production technology and x_i^2 are the inputs of the second production technology. The sum of individual inputs must not exceed the overall inputs x_i, therefore $x_i \geq x_i^1 + x_i^2$. The two production technologies produce the final outputs y_r^1 and y_r^2 respectively.

As it have been presented previously, Chen and Zhu (2004) studied the impact of information technology on the efficiency of firms. Chen et al. (2006a) argued that the disadvantage of the value chain model (Chen and Zhu, 2004) is that information technology has an impact only in the first stage, ignoring the possible impact in the second stage. Chen et al. (2006a) addressed this problem by proposing a technology adoption model where the impact of information technology is decomposed and allocated among all stages.

A special case of network DEA model of Fare and Grosskopf (2000) is the multistage model of Golany et al. (2006); a two-subsystem series system which computes the aggregate efficiency of the system and the individual efficiencies of each subsystem simultaneously. The authors allowed the inputs to be shared among the subsystems and also allowed the possibility for each subsystem to acquire inputs from the other subsystem. This model can be seen as a combination of static model and shared flow model. Additionally, this model can be considered as a synergy model where the two subsystems try to reach a fair agreement for both of them. The authors proposed three Pareto optimal points in order to reach this agreement.

3.4. Relational two-stage DEA

The relational two-stage DEA approach assumes a mathematical relationship between overall efficiency and individual stage efficiencies which is either multiplicative or additive based on a weighted average. Again the first stage is considered as the manufacturer and the second stage as the retailer, then O'Leary-Kelly and Flores (2002) noted that the decision of the one component of this simple supply chain has a direct impact on the other. Consequently, it is important to incorporate this impact in the model and evaluate the efficiency of the individual stages simultaneously (Xu et al., 2009). The requirement of relational approach is that the intermediate measure must use the same set of multipliers in the two stages. The relational two-stage DEA models across the literature are the multiplicative model of Kao and Hwang (2008) and the additive models of Chen et al. (2009a) and Wang and Chin (2010) which are applied at general insurance companies in Taiwan. Kao (2009a) has extended relational models to network structures.

3.4.1. Multiplicative efficiency decomposition

Next, the multiplicative model of Kao and Hwang (2008) is presented. Model (3.1) calculates the optimal solution in the envelopment CRS DEA problem and apparently it is in linear form. The overall efficiency E_0 and the individual efficiencies E_0^1 and E_0^2 for the first and second stage respectively for the *DMU₀* under assessment, are calculated in the same manner. The efficiency E_0 of the primal problem of model (3.1) in fractional form is calculated below.

$$E_0 = max\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.17}$$

s.t

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1$$

$$u_r, v_i \geq 0$$

$$j = 1, \dots, n;\; i = 1, \dots, m; r = 1, \dots, s$$

The efficiencies E_0^1 and E_0^2 for the first and second stage respectively, are calculated in the same manner.

$$E_0^1 = max\frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.18}$$

s.t

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1$$

$$w_d, v_i \geq 0$$

$$j = 1, \dots, n;\; i = 1, \dots, m; d = 1, \dots, D$$

$$E_0 = max\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0}} \tag{3.19}$$

s.t

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{d=1}^{D} w_d z_{d_j}} \leq 1$$

$$u_r, w_d \geq 0$$

$$j = 1, \dots, n; d = 1, \dots, D; r = 1, \dots, s$$

According to Kao and Hwang (2008) models (3.17), (3.18) and (3.19) calculate the overall and individual efficiencies E_0, E_0^1 and E_0^2 for the DMU under assessment, as follows:

$$E_0 = \frac{\sum_{r=1}^{s} u_r^* y_{r_0}}{\sum_{i=1}^{m} v_i^* x_{i_0}} \leq 1, \qquad E_0^1 = \frac{\sum_{d=1}^{D} w_d^* z_{d_0}}{\sum_{i=1}^{m} v_i^* x_{i_0}} \leq 1, \qquad E_0^2 = \frac{\sum_{r=1}^{s} u_r^* y_{r_0}}{\sum_{d=1}^{D} w_d^* z_{d_0}} \leq 1 \tag{3.20}$$

where v_i^*, w_d^* and u_r^* are the optimal weights. Thus, the overall efficiency is the product of the two individual efficiencies: $E_0 = E_0^1 \times E_0^2$. In order to incorporate the interaction between the two stages, Kao and Hwang (2008) included constraints (3.20) into the overall model (3.17). Also, they considered the weights of intermediate measures as the same regardless if the intermediate measures are considered as outputs in the first stage or as inputs in the second stage. This assumption links the two stages and allows the authors to convert the fractional program into a linear one (Chen et al., 2009a). The fractional form is as follows:

$$E_0 = max \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.21}$$

s.t

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1,$$

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1,$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{d=1}^{D} w_d z_{d_j}} \leq 1$$

$$u_r, w_d, v_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

Kao and Hwang (2008) transform fractional program (3.21) into the linear program (3.22).

$$E_0 = max \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{3.22}$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_0} = 1$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0,$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0,$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \leq 0,$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

Optimal weights in model (3.22) may not be unique and as a result the decomposition of the overall efficiency E_0 into the efficiencies of each stage, E_0^1 and E_0^2 respectively, may not be unique either. Kao and Hwang (2008) proposed the maximization of one of the individual efficiencies, say E_0^1, while maintaining the overall efficiency at E_0 as calculated in model (3.22). The other individual efficiency E_0^2 is calculated as $E_0 = E_0^1 \times E_0^2 \Rightarrow E_0^2 = E_0/E_0^1$. For example, if we wish to maximize the individual efficiency of the second stage E_0^2 while maintaining the overall efficiency at E_0 as calculated in model (3.22), the model will be the following:

$$E_0^2 = max \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{3.23}$$

s.t.

$$\sum_{d=1}^{D} \mu_d z_{d_0} = 1$$

$$\sum_{r=1}^{s} \gamma_r y_{r_0} - E_0 \sum_{i=1}^{m} \omega_i x_{i_0} = 0,$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0,$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0,$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \leq 0,$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n;\; i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

and the other individual efficiency E_0^1 will be $E_0^1 = E_0/E_0^2$. As noted by Cook et al. (2010a) this decomposition is not available either at independent two-stage DEA approach or at network DEA models. In addition, Kao and Hwang (2011) demonstrated a further decomposition into technical and scale efficiencies. Furthermore, Liu (2011) provided an alternative decomposition where the overall and individual efficiencies are calculated simultaneously.

Chen et al. (2009b) proved that Chen and Zhu's (2004) connected value-chain model (3.2) transformed in CRS is equivalent with Kao and Hwang's (2008) relational multiplicative model (3.22). The advantage of model (3.22) is the assessment of individual efficiencies for the two stages. In contrast, model (3.2) of Chen and Zhu (2004) fail to do so, because when transformed in CRS, α and β do not represent the efficiencies of each stage. Therefore, both models yield the same overall efficiency and in addition model (3.22) allows the calculation of the individual efficiencies. Moreover, it has been already presented that Chen and Zhu’s (2004) and Kao and Hwang’s (2008) models are equivalent with network DEA models (3.15) an (3.16) of Färe and Grosskopf (1996b) and the cooperative model of Liang et al. (2008). It is worth noting that Färe and Grosskopf ’s (1996b) models (3.15) and (3.16) do not yield individual efficiencies while Liang et al.’s

(2008) model yields individual efficiencies. In addition, Wang and Chin (2010) demonstrated the extension of the multiplicative model (3.22) under the VRS assumption.

Kao and Hwang (2008) pointed out that due to the intermediate measures, adjusting inputs and outputs in order to achieve efficiency is not sufficient to derive a frontier projection in two-stage DEA models. Chen et al. (2010a) proposed a method to derive frontier projections for inefficient DMUs based on the multiplicative model of Kao and Hwang (2008).

3.4.2. Additive efficiency decomposition

Chen et al. (2009a) proposed a relational two-stage DEA model based on additive decomposition of the overall efficiency which is defined as follows.

$$E_0 = \xi_1 \frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} + \xi_2 \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0}} \tag{3.24}$$

The fractional problem will be expressed as follows.

$$E_0 = max \left[\xi_1 \frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} + \xi_2 \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0}} \right] \tag{3.25}$$

s.t.

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1,$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0}} \leq 1,$$

$$u_r, w_d, v_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

However, this problem cannot be converted into a linear form. In order to surpass this problem, Chen et al. (2009a) tried to find the best possible method to specify the exogenous weights ξ_1 and ξ_2, which represent the contribution of each stage to the overall process and $\xi_1 + \xi_2 = 1$. Instead of an arbitrary specification of these weights, the authors stated that a proper measure for the contribution of each stage is their relative size, which can be proxied by the total inputs of each stage relative to the total inputs of the overall process. Thus, the overall size is $\sum_{i=1}^{m} v_i x_{i_0} + \sum_{d=1}^{D} w_d z_{d_0}$, which is the sum of the first stage size $\sum_{i=1}^{m} v_i x_{i_0}$ and the second stage size $\sum_{d=1}^{D} w_d z_{d_0}$. Therefore, the significance of each stage is calculated as:

$$\xi_1 = \frac{\sum_{i=1}^{m} v_i x_{i_0}}{\sum_{i=1}^{m} v_i x_{i_0} + \sum_{d=1}^{D} w_d z_{d_0}} \quad and \quad \xi_2 = \frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0} + \sum_{d=1}^{D} w_d z_{d_0}} \tag{3.26}$$

so that $\xi_1 + \xi_2 = 1$. Weights ξ_1 and ξ_2 follow the denominator rule of Färe and Karagiannis (2013) which states that when we aggregate ratio-type performance measures we can achieve consistency if we define the weights in terms of the denominator.

The authors included the exogenous weights (3.25) in the overall model and after applying the proper linear transformation, it is as follows.

$$E_0 = max \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{3.27}$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_0} + \sum_{d=1}^{D} \mu_d z_{d_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0,$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \leq 0,$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

The optimal overall efficiency for the process is evaluated by model (3.27). The individual efficiencies are calculated by the authors in a similar manner as in Kao and Hwang (2008) model (3.22). The authors proposed the maximization of one of the individual efficiencies, say E_0^1, while maintaining the overall efficiency at E_0 as calculated in model (3.27). The other individual efficiency E_0^2 is calculated as:

$$E_0^2 = \frac{E_0 - \xi_1^* E_0^1}{\xi_2^*} \tag{3.28}$$

where ξ_1^* and ξ_2^* are the optimal weights calculated in model (3.27) by way of (3.26).

Another additive model is the Wang and Chin (2010) model where the overall efficiency is calculated as the weighted harmonic mean of the individual efficiencies.

$$E_0 = \frac{\xi_1 + \xi_2}{\left(\frac{\xi_1}{E_0^1} + \frac{\xi_2}{E_0^2}\right)} = \frac{1}{\left(\frac{\xi_1}{E_0^1} + \frac{\xi_2}{E_0^2}\right)} \tag{3.29}$$

where ξ_1 and ξ_2 also represent the significance of each stage in the overall process. In contrast with the model of Chen et al. (2009a), ξ_1 and ξ_2 are defined as:

$$\xi_1 = \frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{r=1}^{s} u_r y_{r_0} + \sum_{d=1}^{D} w_d z_{d_0}} \quad and \quad \xi_2 = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{r=1}^{s} u_r y_{r_0} + \sum_{d=1}^{D} w_d z_{d_0}} \tag{3.30}$$

which are the total outputs of each stage. Thus, the model of Wang and Chin (2010) is presented below.

$$E_0^* = max \frac{1}{\left(\xi_1 \frac{\sum_{i=1}^{m} v_i x_{i_0}}{\sum_{d=1}^{D} w_d z_{d_0}} + \xi_2 \frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{r=1}^{s} u_r y_{r_0}}\right)} \tag{3.31}$$

s.t.

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1,$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0}} \leq 1,$$

$$u_r, w_d, v_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

and if ξ_1 and ξ_2 from (3.30) are replaced in (3.31) the result is model (3.25) of Chen et al. (2009a). Also, Wang and Chin (2010) presented how Kao and Hwang's (2008) model (3.22) can be converted in variable returns to scale and how Chen et al. (2009a) model (3.27) can be extended to a more general model.

3.4.3. *Network relational models*

Kao (2009a) developed a relational model which assesses the efficiency of more general systems such as network structures. This is a network DEA model with the principal component that any intermediate measure must use the same set of multipliers whether it is considered as an input or as an output. Kao (2009a) applied this model in the mixed serial/parallel structure of 24 Taiwanese non-life insurance companies. Network relational model has been used in banks (Avrikan, 2009), tourist hotels (Hsieh and Lin, 2010), innovation processes (Guan and Chen, 2010) and printed circuit board manufacturing firms (Lozano, 2011).

Another type of relational network DEA models are those which extend the additive efficiency decomposition approach of Chen et al. (2009a) into more stages or network structures. Cook et al. (2010b) examined 10 vertically integrated power companies in US in three stages, generation, transmission and distribution using a weighted additive network model similar with Chen et al. (2009a). This model holds the assumption of Chen et al. (2009a) that the weight of each stage is a proportion of total resources that are devoted to this stage. Similar models are applied for 66 large mutual funds in US (Premachandra et al., 2012), national innovation systems in 22 OECD countries (Guan and Chen, 2012) and banks (Chen et al., 2010b). Liang et al. (2011) applied an additive model where the weight of each of the two stages is 0.5 and an amount of final outputs returns in the system as feedback. This model still holds the assumption of the relational models that the intermediate measures use the same set of multipliers in the two stages.

3.5. Game theory models

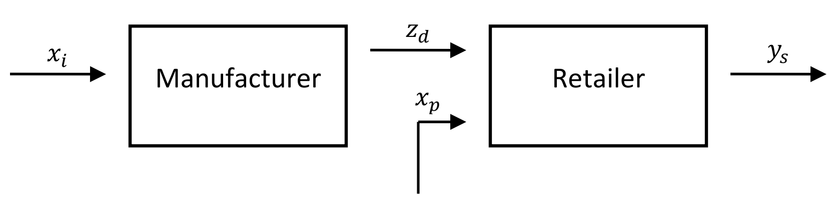
A previous section has presented models which evaluate the efficiency of a supply chain, considering the overall and individual efficiencies of each stage simultaneously (Zhu, 2003; Chen and Zhu, 2004). A typical supply chain is presented in Figure 3.3 which is consisted by a supplier, a manufacturer, a distributor and a retailer. A simpler supply chain may consist of only two members, a manufacturer and a retailer. Liang et al. (2006)

and Li et al. (2012) investigated the supply chain as a seller-buyer game under non-cooperative and cooperative assumptions. A common type of non-cooperative game is the leader-follower model, also known as Stackelberg model. The manufacturer is considered as the leader and the retailer as the follower. In this type of model, the efficiency of the leader (manufacturer) is evaluated first by applying a typical DEA model and then the efficiency of the follower (retailer) is calculated subject to the leader's efficiency. The game considers the maximization of leader's efficiency as more significant for the overall supply chain compared to the follower's efficiency (Liang et al., 2008).

Under the cooperative assumption, both stages are considered as equally important for the overall supply chain. Both parties cooperate with each other and wish to jointly maximize the overall and their individual efficiencies. The key point of the cooperation is found at the intermediate measures. The individual efficiencies are evaluated simultaneously and the overall efficiency is equal with the mean efficiency of the individual stages.

The simple two-stage form of the supply chain is presented in Figure 3.6 as introduced by Zhu (2009), where the first stage is the manufacturer and the second stage is the retailer. The model consists of $j = 1, \dots, n$ DMUs. The manufacturer consumes x_i $(i = 1, \dots, m)$ inputs and generates z_d $(d = 1, \dots, D)$ intermediate outputs. The retailer uses z_d $(d = 1, \dots, D)$ intermediate inputs from the manufacturer and x_p $(p = 1, \dots, P)$ exogenous inputs and produces y_r $(r = 1, \dots, s)$ final outputs.

Figure 3.6: A two-stage supply chain with exogenous inputs



3.5.1. Non-cooperative game

Let's assume a seller-buyer game, where the manufacturer is the seller and the retailer is the buyer. Also, the manufacturer is considered the leader while the retailer is the follower. Then, according to Liang et al. (2006) the leader's efficiency is evaluated by applying a typical DEA model formulated as:

$$\max E_0^1 = \frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.32}$$

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1$$

$$w_d, v_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D$$

which can be easily transformed into a typical CRS DEA model as follows.

$$\max E_0^1 = \sum_{d=1}^{D} \mu_d z_{d_0} \tag{3.33}$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \geq 0$$

$$\sum_{d=1}^{D} \omega_i x_{i_0} = 1$$

$$\mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D$$

Model (3.33) assesses the manufacturer's maximized efficiency E_0^{1*} and the optimal weights μ_d^* and ω_i^*. Subject to these optimal values Liang et al. (2006) evaluated the follower's efficiency as follows.

$$\max E_0^2 = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{Q \times \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{p=1}^{P} v_p x_{p_0}} \tag{3.34}$$

s.t.
$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{Q \times \sum_{d=1}^{D} \mu_d z_{d_j} + \sum_{p=1}^{P} v_p x_{p_j}} \leq 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} = E_1^*$$

$$\sum_{i=1}^{m} \omega_i x_{i_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \geq 0$$

$$\sum_{d=1}^{D} \omega_i x_{i_0} = 1$$

$$\mu_d, \omega_i, u_r, v_p, Q \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

where the first constraint refers to the retailer while next three constraints refer to the manufacturer and ensure his optimal efficiency as calculated in model (3.33). The model (3.34) can be transformed into the following non-linear problem:

$$\max E_0^2 = \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{3.35}$$

s.t.
$$q \times \sum_{d=1}^{D} \mu_d z_{d_j} + \sum_{p=1}^{P} \omega_p x_{p_j} - \sum_{r=1}^{s} \gamma_r y_{r_j} \geq 0$$

$$q \times \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{p=1}^{P} \omega_p x_{p_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} = E_1^*$$

$$\sum_{i=1}^{m} \omega_i x_{i_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \geq 0$$

$$\sum_{d=1}^{D} \omega_i x_{i_0} = 1$$

$$\gamma_r, \mu_d, \omega_i, \omega_p, q \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

Model (3.35) is non-linear because of the “*q*” term. As it can be seen from the constraints:

$$q \times \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{p=1}^{P} \omega_p x_{p_0} = 1 \quad and \quad \sum_{d=1}^{D} \mu_d z_{d_0} = E_0^{1*}$$

Therefore:

$$q = \frac{1 - \sum_{p=1}^{P} \omega_p x_{p_0}}{\sum_{d=1}^{D} \mu_d z_{d_0}} \Rightarrow \frac{1 - \sum_{p=1}^{P} \omega_p x_{p_0}}{E_1^*} \tag{3.36}$$

The constraint $q \times \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{p=1}^{P} \omega_p x_{p_0} = 1$ shows that $\sum_{p=1}^{P} \omega_p x_{p_0}$ can take values from 0 to 1 because both terms, ω_p and x_{p_0}, are non-negative quantities. If $\sum_{p=1}^{P} \omega_p x_{p_0}$ takes zero value, the numerator in (3.36) will become 1 and the overall fraction will become 0, otherwise if $\sum_{p=1}^{P} \omega_p x_{p_0}$ takes unity value, the numerator in (3.36) will become 0 and the overall fraction will become $1/E_0^{1*}$. Therefore, an upper and a lower bound can be determined for *q* term:

$$0 \leq q < \frac{1}{E_0^{1*}} \tag{3.37}$$

Thus, *q* can be treated as a parameter and model (3.35) can be solved as a parametric linear program.

According to Liang et al. (2006) in order to solve the problem, an initial value is being set to *q* term, $q_0 = 1/E_0^{1*}$ and the resulting linear problem is solved. Then, the *q* term is decreased each time by a small number *ε* until the lower bound is reached and the resulting values of *q* are named as q_t. Each resulting linear problem is solved for every q_t and the solutions are named as $E_0^2(q_t)$. The optimal solution is $E_0^{2*} = maxE_0^2(q_t)$ which is the retailer's efficiency and the optimal *q* associated with this solution is q^*.

After the evaluation of individual efficiencies, the overall efficiency of the supply chain can be calculated as follows (Liang et al., 2006).

$$E_0 = \frac{1}{2}(E_0^{1*} + E_0^{2*}) \tag{3.38}$$

In addition, model (3.35) can assess the efficiency of the overall supply chain by considering the retailer as the leader and the manufacturer as the follower, in the same manner.

Li et al. (2012) calculated the leader's efficiency in the same manner as Liang et al. (2006) by applying typical DEA model (3.33), where E_0^{1*} is the leader's maximized efficiency and μ_d^* and ω_i^* are the optimal weights. Then, while maintaining the leader's efficiency fixed, they evaluated the follower's efficiency as follows.

$$\max E_0^{2*} = \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0} + \sum_{p=1}^{P} v_p x_{p_0}} \tag{3.39}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{d=1}^{D} w_d z_{d_j} + \sum_{p=1}^{P} v_p x_{p_j}} \leq 1$$

$$\frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} = E_0^{1*}$$

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1$$

$$u_r, w_d, v_i, v_p \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

which can be transformed into a linear program as follows:

$$\max E_0^{2*} = \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{3.40}$$

s.t.

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{p=1}^{P} \omega_p x_{p_j} \leq 0$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} - \sum_{p=1}^{P} \omega_p x_{p_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{p=1}^{P} \omega_i x_{i_j} \leq 0$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} - E_0^{1*} \sum_{p=1}^{P} \omega_i x_{i_0} = 0$$

$$\gamma_r, \mu_d, \omega_i, \omega_p \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

Therefore, the overall efficiency can be calculated as $E_0 = E_0^{1*} \times E_0^{2*}$.

3.5.2. *Cooperative game*

Non-cooperative model tries to find the optimal weights for intermediate measures which maximize the leader's efficiency. In the cooperative model the seller and the buyer have the same bargaining power and they cooperate to jointly maximize their efficiency. Therefore, they now treat the intermediate measures in a coordinated manner by setting their optimal weights as equal.

The cooperative game of Liang et al. (2006) is the following:

$$\max E_0 = \frac{1}{2}\left[\frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} + \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0} + \sum_{p=1}^{P} v_p x_{p_0}}\right] \tag{3.41}$$

s.t.

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{d=1}^{D} w_d z_{d_j} + \sum_{p=1}^{P} v_p x_{p_j}} \leq 1$$

$$u_r, w_d, v_i, v_p \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

Next, the authors apply the Charnes-Cooper transformation in order to convert model (3.41) into a linear problem. That is:

$$t_1 = \frac{1}{\sum_{i=1}^{m} v_i x_{i_0}}, \qquad t_2 = \frac{1}{\sum_{d=1}^{D} w_d z_{d_0} + \sum_{p=1}^{P} v_p x_{p_0}} \tag{3.42}$$

$$\omega_i = t_1 \cdot v_i, \ \ \omega_p = t_1 \cdot v_p, \ \ \mu_d^1 = t_1 \cdot w_d, \ \ \mu_d^2 = t_2 \cdot w_d, \ \ \gamma_r = t_2 \cdot u_r$$

$$j = 1, \dots, n; \ i = 1, \dots, m; p = 1, \dots, P; \ d = 1, \dots, D; \ r = 1, \dots, s$$

Obviously, there is a linear relation between μ_d^1 and μ_d^2, $\mu_d^2 = k \times \mu_d^1$ where $k = t_2/t_1$ is a positive number. Therefore, the resulting model is:

$$\max E_0 = \frac{1}{2}\left[\sum_{d=1}^{D} \mu_d^1 z_{d_0} + \sum_{r=1}^{s} \gamma_r y_{r_0}\right] \tag{3.43}$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_j} - \sum_{d=1}^{D} \mu_d^1 z_{d_j} \geq 0$$

$$\sum_{d=1}^{D} \mu_d^2 z_{d_j} + \sum_{p=1}^{P} \omega_p x_{p_j} - \sum_{r=1}^{s} \gamma_r y_{r_j} \geq 0$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} = 1$$

$$\sum_{d=1}^{D} \mu_d^2 z_{d_0} + \sum_{p=1}^{P} \omega_p x_{p_0} = 1$$

$$\mu_d^2 = k \times \mu_d^1$$

$$\omega_i, \omega_p, \mu_d^1, \mu_d^2, \gamma_r, k \geq 0$$

$$j = 1, \dots, n; \ i = 1, \dots, m; p = 1, \dots, P; \ d = 1, \dots, D; \ r = 1, \dots, s$$

where the first and the third constraints refer to the manufacturer while the second and the fourth refer to the retailer. Model (3.43) is non-linear because the second constraint contains there is the term μ_d^2 which includes a summation at the denominator as we can see in (3.42). However, this term can be replaced by using the relation $\mu_d^2 = k \times \mu_d^1$.

Therefore:

$$\max E_0 = \frac{1}{2}\left[\sum_{d=1}^{D} \mu_d^1 z_{d_0} + \sum_{r=1}^{s} \gamma_r y_{r_0}\right] \tag{3.44}$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_j} - \sum_{d=1}^{D} \mu_d^1 z_{d_j} \geq 0$$

$$k \times \sum_{d=1}^{D} \mu_d^1 z_{d_j} + \sum_{p=1}^{P} \omega_p x_{p_j} - \sum_{r=1}^{s} \gamma_r y_{r_j} \geq 0$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} = 1$$

$$k \times \sum_{d=1}^{D} \mu_d^1 z_{d_0} + \sum_{p=1}^{P} \omega_p x_{p_0} = 1$$

$$\omega_i, \omega_p, \mu_d^1, \gamma_r, k \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

The term μ_d^1 does not include a summation at the denominator and as a result μ_d^1 does not create a non-linearity problem. Now, only the "*k*" term creates the non-linearity problem. From model (3.44):

$$k \times \sum_{d=1}^{D} \mu_d^1 z_{d_0} + \sum_{p=1}^{P} \omega_p x_{p_0} = 1 \quad and \quad \sum_{d=1}^{D} \mu_d^1 z_{d_0} = E_0^{1*}$$

Therefore:

$$k = \frac{\left(1 - \sum_{p=1}^{P} \omega_p x_{p_0}\right)}{\sum_{d=1}^{D} \mu_d^1 z_{d_0}} \Rightarrow k = \frac{\left(1 - \sum_{p=1}^{P} \omega_p x_{p_0}\right)}{E_1^*} \tag{3.45}$$

The constraint $k \times \sum_{d=1}^{D} \mu_d^1 z_{d_0} + \sum_{p=1}^{P} \omega_p x_{p_0} = 1$ shows that $\sum_{p=1}^{P} \omega_p x_{p_0}$ can take values from 0 to 1 because both terms, ω_p and x_{p_0}, are non-negative quantities. If $\sum_{p=1}^{P} \omega_p x_{p_0}$ takes zero value, the numerator in (3.45) will become 1 and the overall fraction will become 0, otherwise if $\sum_{p=1}^{P} \omega_p x_{p_0}$ takes unity value, the numerator in (3.45) will become 0 and the overall fraction will become $1/E_0^{1*}$. Therefore, an upper and a lower bound can be determined for the *k* term:

$$0 \leq k < \frac{1}{E_0^{1*}}$$

Thus, *k* can be treated as a parameter and model (3.44) can be solved as a parametric linear program, using the same method as in model (3.35).

Liang et al. (2006) proposed the above model in order to assess the overall and the individual efficiencies simultaneously. The individual efficiencies are calculated as $E_0^{1*} = \mu_d^1 z_{d_0}$ and $E_0^{2*} = \gamma_r^1 y_{r_0}$. The authors noted that the cooperative efficiencies are at least equal with the non-cooperative efficiencies. The cooperative model of Liang et al. (2006) evaluates the efficiency of a simple supply chain which consists of two parties. Zhu and Cook (2007) extended the model of Liang et al. (2006) in order to include three or more parties.

Li et al. (2012) presented another approach for the centralized cooperative game where the overall efficiency is the product of the individual efficiencies:

$$E_0 = \max E_0^1 \times E_0^2 = \max \frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \times \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0} + \sum_{p=1}^{P} v_p x_{p_0}} \tag{3.46}$$

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{d=1}^{D} w_d z_{d_j} + \sum_{p=1}^{P} v_p x_{p_j}} \leq 1$$

$$u_r, w_d, v_i, v_p \geq 0$$

$$j = 1, \ldots, n;\ i = 1, \ldots, m; p = 1, \ldots, P;\ d = 1, \ldots, D;\ r = 1, \ldots, s$$

As in Liang et al. (2006), w_d is the same either the intermediate measures are consider as inputs or outputs. The authors proposed a heuristic approach to solve model (3.46) because it is non-linear. First the approach finds the maximum efficiency for the first stage:

$$E_0^{1*} = \max \frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \tag{3.47}$$

s.t.

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}}$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0} + \sum_{p=1}^{P} v_p x_{p_0}}$$

$$u_r, w_d, v_i, v_p \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

The constraints in model (3.47) ensure that the individual efficiencies cannot exceed unity. The objective function evaluates the maximum possible efficiency for the first stage. Thus, the first stage efficiency E_0^1 range from 0 to E_0^{1*}. The above model can be transformed into a linear program as follows.

$$E_0^{1*} = \max \sum_{d=1}^{D} \mu_d z_{d_0} \tag{3.48}$$

s.t.

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{p=1}^{P} \omega_p x_{p_j} \leq 0$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} = 1$$

$$\gamma_r, \mu_d, \omega_i, \omega_p \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

As already mentioned, the first stage efficiency E_0^1 can be treated as a variable and take values from 0 to E_0^{1*}. Therefore the overall efficiency E_0 can be treated as a function of the first stage efficiency E_0^1.

$$E_0 = \max E_0^1 \cdot \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0} + \sum_{p=1}^{P} v_p x_{p_0}} \tag{3.49}$$

s.t.

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{d=1}^{D} w_d z_{d_j} + \sum_{p=1}^{P} v_p x_{p_j}} \leq 1$$

$$\frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} = E_0^1, \quad E_0^1 \in [0, E_0^{1*}]$$

$$u_r, w_d, v_i, v_p \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

Model (3.49) can be converted into a parametric linear program with E_0^1 as a parameter.

$$E_0 = \max E_0^1 \cdot \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{3.50}$$

s.t.

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{p=1}^{P} \omega_p x_{p_j} \leq 0$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{p=1}^{P} \omega_i x_{p_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} + E_0^1 \sum_{i=1}^{m} \omega_i x_{i_0} = 0, \quad E_0^1 \in [0, E_0^{1*}]$$

$$\gamma_r, \mu_d, \omega_i, \omega_p \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; p = 1, \dots, P;\ d = 1, \dots, D;\ r = 1, \dots, s$$

Model (3.50) can be solved as a parametric linear program like model (3.35). An initial value $E_0^1 = E_0^{1*}$ is set and model (3.50) is solved. Then E_0^1 term each time is decreased by a small number ε as follows.

$$E_0^1 = E_0^{1*} - k \cdot \Delta\varepsilon \tag{3.51}$$

where k is an integer ($k = 0,1, \dots, k^{max} + 1$) and k^{max} is the maximal integer which is smaller that $E_0^{1*}/\Delta\varepsilon$. The optimal overall efficiency is $E_0^* = \max E_0$.

3.5.3. Discussion of cooperative and non-cooperative models

The models of Liang et al. (2006) and Li et al. (2012) include exogenous inputs in the second stage. These exogenous inputs create non-linearity which dealt with parametric linear programming. Liang et al. (2008) investigated similar models without exogenous inputs in the second stage. The only inputs in the second stage are the intermediate measures produced in first stage. In Liang et al.'s (2008) models, the overall efficiency is calculated as the product of individual efficiencies, $E_0 = E_0^1 \times E_0^2$ instead of $E_0 = 1/2\,[E_0^1 + E_0^2]$. Exogenous inputs in the second stage do not allow this calculation in Liang et al.'s (2006) models because the transformation into a linear or parametric linear program will not be possible.

Models of Liang et al. (2006), Liang et al. (2008) and Li et al. (2012) have a comparative advantage over other models, like Chen and Zhu (2004), Seiford and Zhu (1999) and network DEA because they assess both overall and individual efficiencies of the supply chain. As we have already noted, this is also true for the model of Kao and Hwang (2008) which according to Cook et al. (2010a) is equivalent to the cooperative model.

Furthermore, Liang et al. (2008) proved that when there is only one intermediate measure in their models, the resulting efficiencies from cooperative and non-cooperative models are exactly the same. Also, the decomposition of the overall efficiency into individuals is unique. Additionally, individual efficiencies are the same as if we apply a typical DEA model at each stage separately. On the other hand, if there are multiple intermediate measures, then the non-cooperative model yields unique efficiency decomposition while efficiency decomposition for the cooperative model is not unique.

3.5.4. Nash bargaining game

Du et al. (2011) applied another form of cooperative model in two-stage DEA, the Nash bargaining game. They adopted a similar supply chain with Liang et al. (2008), where there are no exogenous inputs in the second stage and all the first stage outputs are intermediate measures and consumed entirely by the second stage. Additionally, following the previous cooperation models of Liang et al. (2006), Liang et al. (2008) and Kao and Hwang (2008) they treated the intermediate measures in a coordinated manner by setting their optimal weights as equal treating them either as outputs in the first stage or as inputs in the second stage.

Du et al. (2011) considered the two stages as two players in a Nash bargaining game who bargain for a better payoff. Three main aspects must be defined in a Nash bargaining game, a) the participating players, say a manufacturer and a retailer, $N = \{1,2\}$, b) a feasible set of payoffs, which is the set of DEA efficiencies and c) a breakdown point, which is the payoff if the participating players do not reach an agreement. The authors defined as a breakdown point the efficiencies of the worst possible DMU, which is the DMU with maximum inputs and minimum outputs, thus $\max x_i - \min z_d$ in the first stage and $\max z_d - \min y_r$ in the second stage. These are the worst possible efficiencies and are denoted as $E^1_{0_{min}}$ and $E^2_{0_{min}}$ for the two stages respectively. These efficiencies are set as the breakdown point. In addition, the weights in the two stage model are considered as the possible strategies for the participating players. Nash pointed out that for the bargaining game there is a unique solution which can be found by applying the following maximization problem.

$$\max_{\vec{u}\in S, \vec{u}\geq \vec{b_i}} \prod_{i=1}^{2} (u_i - b_i) \tag{3.52}$$

where $\vec{u}$ is the payoff vector for the two participating players, *S* is the feasible set of payoffs and $\vec{b}$ is the breakdown point.

After defining the above, the bargaining game of Du et al. (2011) is as follows.

$$\max \left[\frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} - E_{0_{min}}^1\right] \times \left[\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0}} - E_{0_{min}}^2\right] \tag{3.53}$$

s.t.

$$\frac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \geq E_{0_{min}}^1$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{d=1}^{D} w_d z_{d_0}} \geq E_{0_{min}}^2$$

$$\frac{\sum_{d=1}^{D} w_d z_{d_j}}{\sum_{i=1}^{m} v_i x_{i_j}} \leq 1$$

$$\frac{\sum_{r=1}^{s} u_r y_{r_j}}{\sum_{d=1}^{D} w_d z_{d_j}} \leq 1$$

$$u_r, w_d, v_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m;\ d = 1, \dots, D;\ r = 1, \dots, s$$

where the objective function is the bargaining problem (3.52). The first two constraints ensure that individual efficiencies will not be less than the worst possible efficiencies $E_{0_{min}}^1$ and $E_{0_{min}}^2$. The next two constraints are the typical constraints of a fractional DEA program.

The authors applied the transformation (3.42) in order to convert the model into a linear one.

$$t_1 = \frac{1}{\sum_{i=1}^{m} v_i x_{i_0}} \quad and \quad t_2 = \frac{1}{\sum_{d=1}^{D} w_d z_{d_0}} \tag{3.54}$$

$$\omega_i = t_1 \cdot v_i,\ \ \mu_d = t_1 \cdot w_d,\ \ \gamma_r^1 = t_1 \cdot u_r,\ \ \gamma_r^2 = t_2 \cdot u_r$$

$$j = 1, \dots, n;\ i = 1, \dots, m;\ d = 1, \dots, D;\ r = 1, \dots, s$$

Obviously, there is a linear relation between γ_r^1 and γ_r^2, $\gamma_r^1 = \alpha \times \gamma_r^2$ where $\alpha = t_1/t_2$ is a positive number. Therefore, the resulting model is:

$$\max \sum_{r=1}^{s} \gamma_r^1 y_{r_0} - E_{0_{min}}^1 \cdot \sum_{r=1}^{s} \gamma_r^2 y_{r_0} - E_{0_{min}}^2 \cdot \sum_{d=1}^{D} \mu_d z_{d_0} + E_{0_{min}}^1 \cdot E_{0_{min}}^2 \tag{3.55}$$

s.t.

$$\sum_{d=1}^{D} \mu_d z_{d_0} \geq E^1_{0_{min}}$$

$$\sum_{r=1}^{s} \gamma_r^2 y_{r_0} \geq E^2_{0_{min}}$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} = a$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0$$

$$\sum_{r=1}^{s} \gamma_r^1 y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \leq 0$$

$$\gamma_r^1 = \alpha \cdot \gamma_r^2$$

$$\mu_d, \omega_i, \gamma_r^1, \gamma_r^2, \alpha > 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m;\ d = 1, \dots, D;\ r = 1, \dots, s$$

The first two constraints ensure that individual efficiencies will not be less than the worst possible efficiencies $E^1_{0_{min}}$ and $E^2_{0_{min}}$.

Model (3.55) can be transformed into the following model by applying the relation $\gamma_r^1 = \alpha \cdot \gamma_r^2$.

$$\max \alpha \cdot \sum_{r=1}^{s} \gamma_r^2 y_{r_0} - E^1_{0_{min}} \cdot \sum_{r=1}^{s} \gamma_r^2 y_{r_0} - E^2_{0_{min}} \cdot \sum_{d=1}^{D} \mu_d z_{d_0} + E^1_{0_{min}} \cdot E^2_{0_{min}} \tag{3.56}$$

s.t.

$$\sum_{d=1}^{D} \mu_d z_{d_0} \geq E^1_{0_{min}}$$

$$\sum_{r=1}^{s} \gamma_r^2 y_{r_0} \geq E_{0_{min}}^2$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} = \alpha$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0$$

$$\alpha \cdot \sum_{r=1}^{s} \gamma_r^2 y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \leq 0$$

$$\mu_d, \omega_i, \gamma_r^2, \alpha > 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m;\ d = 1, \dots, D;\ r = 1, \dots, s$$

Model (3.56) is non-linear because of the "α" term. As we can see from the constraints of model (3.56):

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0 \Rightarrow \sum_{d=1}^{D} \mu_d z_{d_j} \leq \sum_{i=1}^{m} \omega_i x_{i_j}$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} = 1, \qquad \sum_{d=1}^{D} \mu_d z_{d_0} = a \quad and \quad \sum_{d=1}^{D} \mu_d z_{d_0} \geq E_{0_{min}}^1$$

If we combine constraints $\sum_{d=1}^{D} \mu_d z_{d_0} = a$ and $\sum_{d=1}^{D} \mu_d z_{d_0} \geq E_{0_{min}}^1$ then: $\sum_{d=1}^{D} \mu_d z_{d_0} \geq E_{0_{min}}^1 \Rightarrow \alpha \geq E_{0_{min}}^1$. Therefore, constraints $\sum_{i=1}^{m} \omega_i x_{i_0} = 1$ and $\alpha \geq E_{0_{min}}^1$ can be replaced back to the first constraint: $\sum_{d=1}^{D} \mu_d z_{d_j} \leq \sum_{i=1}^{m} \omega_i x_{i_j} \Rightarrow E_{0_{min}}^1 \leq a \leq 1$.

An upper and a lower bound can be determined for the α term. Thus, α can be treated as a parameter and model (3.56) can be solved as a parametric linear program, using the same method as in model (3.35). Thus, according to Du et al. (2011) the efficiency of the first stage from the constraint $\sum_{d=1}^{D} \mu_d z_{d_0} = a$ is $E_0^{1*} = \alpha^*$, the efficiency of the second stage is $E_0^{2*} = \gamma_r^2 \cdot y_{r_0}$ and the efficiency of the entire supply chain can be calculated as $E_0^* = E_0^{1*} \times E_0^{2*}$.

The authors pointed out that if there is only one intermediate measure in the supply chain then the individual efficiencies are the same with applying a typical DEA model at each stage separately. As a result, in this case the efficiencies of the model are equal with the efficiencies of the cooperative model of Liang et al. (2008). In addition, Liang et al. (2008) model is a special case of model (3.56) with zero breakdown point. Finally, the efficiencies of their model are the best feasible efficiencies for model (3.53) as it is not possible to achieve further improvement.

Chen et al. (2006b) proposed another bargaining game DEA model between a supplier and a manufacturer. The authors introduced an efficiency function for each of the two members of the game and showed that multiple Nash equilibria exist in the game. Additionally, the game-model can identify the perfect Nash equilibrium for the two players if it exists.

3.6. Two-stage DEA application across the literature

Table 3.1 is a summary table which presents all the two-stage application in well-known refereed academic journals until early 2015. Papers with more than two stages have been excluded. In the first column of Table 1 there is the name of the authors. In the second column there is a short description of the type of the two-stage DEA model that is used in the study. There is also a note for the category of the two-stage DEA model (independent, connected, relational and game theory). Also, for connected models there is a distinction whether it is a value-chain model or a network model. For network models there is an extra note about the structure (serial or parallel) or the type (static, dynamic or shared resources) of the model. For relational models there is a distinction whether it is multiplicative or additive and if it is a special relational network case. For game theoretic models there is a distinction whether it is cooperative, non-cooperative or bargaining game. Also, for all models there is a note if it uses directional distance functions, fuzzy numbers or if it is slack-based.

After an extensive and detailed study of every two-stage DEA application there is no “perfect” model, on the contrary each model is suitable under specific circumstances. Independent approach does not consider any possible conflicts or connections between the stages however it is the less restrictive approach and yields the largest efficiency scores. Connected approach considers the interactions between the stages while relational approach takes into account any mathematical relationship that exists between them. Game theoretic approach is suitable when we consider the two stages as two players in a cooperative or in a non-cooperative game. The choice of the appropriate model must be made wisely because a choice of a more restrictive model if it is not needed would yield underestimated efficiency scores while a choice of a less restrictive model when more complex relations exist will result in overestimated efficiencies. The choice of the appropriate model is also based on the structure of the overall process, on a priori information and the personal opinion of the researcher.

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Table 3.1: Two-stage DEA applications across the literature

Publication	Type of two-stage DEA study	Application scheme	Individual stages
Akther et al. (2013)	Connected SB NDEA	19 Bangladesh private commercial banks and 2 government owned	Value added activity/profit generation
Amirteimoori (2013)	Connected NDEA shared resources	Iranian car manufacturer	Sale representatives/repair shop
Aviles-Sacoto et al. (2015)	Relational additive	37 Business schools	Student accomplishments in the program/student accomplishments after graduation
Azadi et al. (2014,2015)	Connected NDEA static	Green supply chain management in 24 Iranian bus companies	Operating activity/Profit generation
Bi et al. (2011)	Connected SB NDEA parallel	Taiwanese national forests	Working circles
Bian et al. (2015)	Connected SB NDEA	Chinese regional industrial system	Production/abatement
Chen and Guan (2012)	Connected NDEA static	Chinese regional innovation systems	Technological development/technological commercialization

Chen and Zhu (2004)	Connected value-chain	27 banks	IT-related activity/loan processing system
Chen et al. (2006a)	Connected NDEA shared resources	27 banks	Premium acquisition/profit generation
Chen et al. (2009a)	Relational additive	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Chen et al. (2009b)	Connected value-chain / relational multiplicative	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Chen et al. (2010a)	Relational multiplicative	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Chen et al. (2010b)	Relational NDEA shared resources	27 banks	IT-related activity/loan processing system
Chen et al. (2012)	Game theory C Multiplicative	Sustainable product vehicles performance	Industrial design module/bio-design module
Chilingerian and Sherman (2004)	Independent	Hospitals	Administration/doctors
Chiu and Huang (2011)	Connected value-chain	Taiwanese hotels	Operational efficiency/profitability efficiency
Chiu et al. (2011)	Connected value-chain NDEA	30 Chinese regions	Transit process/economic process

Chiu et al. (2012)	Connected value-chain NDEA	21 Chinese high-tech industries	R&D process/operation process
Chiu et al. (2013)	Connected DDF NDEA	53 International tourist hotels	Productive process/service process
Chun et al. (2015)	Connected Value chain	Korean manufacturing industry	Innovation/commercialization
Cook et al. (2000)	Connected NDEA shared resources	Branches of a major Canadian bank	Sales/service
Despotis et al (2014)	Relational MOLP a posteriori aggregation	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Du et al. (2011)	Game theory BG Multiplicative	1. 30 Top U.S. commercial banks 2. 24 Taiwanese non-life insurance companies	1. Profitability/marketability 2. Premium acquisition/profit generation
Fukuyama and Matousek (2011)	Connected NDEA static	25 Turkish commercial banks	Value added activity/profitability
Fukuyama and Weber (2010)	Connected SB NDEA	Japanese banks	Value added activity/profitability
Färe and Grosskopf (1997)	Connected NDEA dynamic	Dynamic misallocation of resources in APEC countries	Periods

Färe and Whittaker (1995)	Connected NDEA static	137 farms	Dairy production/crop production
Färe et al. (2004)	Connected NDEA static	Property rights	Firm 1/firm 2
Guan and Chen (2010)	Relational NDEA	China's high-tech innovation processes	R&D process/commercialization process
Guan and Chen (2012)	Relational NDEA	National innovation systems in 22 OECD countries	Knowledge production process/knowledge commercialization process
Ho and Oh (2008)	Independent	28 Taiwanese online stockbrokers	Operating efficiency/operating effectiveness
Ho and Zhu (2004)	Independent	41 Taiwanese commercial banks	Operating efficiency/ operating effectiveness
Ho et al. (2014)	Relational additive window analysis	U.S. universities	Research innovation/value creation
Holod and Lewis (2011)	Connected NDEA static	Bank holding companies	Value added activity/profit generation
Hsieh and Lin (2010)	Relational NDEA serial/parallel	International Taiwanese tourist hotels	Service production/service consumption

Huang et al. (2014)	Connected Value chain NDEA serial/parallel	58 Taiwanese international hotels	Production process/service process
Hung and Wang (2012)	Independent	367 Taiwanese manufacturing firms	Profitability/marketability
Jianfeng (2015)	Connected NDEA shared resources	Technological innovation in Chinese large and medium-sized industrial enterprises	Technique innovation/new products innovation
Kao (2009a)	Relational NDEA serial/parallel	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Kao (2009b)	Connected SB NDEA parallel	Taiwanese national forests	Working circles
Kao (2012)	Connected SB NDEA parallel	52 chemistry departments in U.K. universities	Teaching/research
Kao and Hwang (2008)	Relational multiplicative	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Kao and Hwang (2010)	Relational NDEA	27 banks	Fund collection/profit generation
Kao and Hwang (2014)	Relational multiplicative Dynamic	21 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Kao and Lin (2012)	Connected SB NDEA parallel fuzzy data	52 chemistry and physics departments in U.K. universities	Teaching/research

Kao and Liu (2011)	Relational multiplicative fuzzy numbers	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Kao and Liu (2014)	Dynamic NDEA/ Relational NDEA	22 Taiwanese non-life insurance companies	Value added activity/profit generation
Karimi-Ghartemani and Karimi (2014)	Relational multiplicative	Customer relationship management system in bank braches	Customer satisfaction/customer loyalty
Khodakarami et a. (2015)	Connected SB NDEA	Sustainability of supply chain management in 27 Iranian companies	Supplier/manufacturer
Kwon and Lee (2015)	Independent	U.S. banks	Production process/profit earning process
Lewis et al. (2013)	Independent unoriented	Major League Baseball	Front office/on-field competition
Lewis and Sexton (2004)	Connected NDEA serial/parallel	Major League Baseball	Front office/on field
Li et al. (2012)	Game theory NC/C Multiplicative NDEA	Regional R&D process of 30 Provincial Level Regions in China	Premium acquisition/profit generation
Liang et al. (2008)	Game theory NC/C Multiplicative	1. 27 banks 2. 30 Top U.S. commercial banks	1. IT-related activity/loan processing system 2. Profitability/marketability

Liang et al. (2011)	Relational NDEA	50 Chinese universities	Research performance/evaluation performance
Liu (2011)	Relational additive	Taiwanese financial holding companies	Profitability/marketability
Liu (2014)	Relational multiplicative fuzzy numbers	18 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Liu and Lu (2012)	Connected value-chain	27 banks	IT-related activity/loan processing system
Liu and Wang (2009)	Relational multiplicative	Taiwanese printed circuit board (PCB) manufacturing firms	Production acquisition/profit earning
Liu et al.(2015)	Connected SB NDEA	Chinese commercial banks	Profitability/marketability
Lo (2010)	Independent	U.S. S&P 500 firms	Profitability/marketability
Lozano (2011)	Relational NDEA	17 Taiwanese PCB manufacturing firms	Production acquisition/profit earning
Lozano (2014)	Relational multiplicative fuzzy numbers	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Lozano et al. (2013)	Connected DDF NDEA	Spanish airports	Aircraft movement/aircraft loading

Lu (2012)	Relational additive	Taiwanese public universities	Cost efficiency/teaching & research efficiency
Lu et al. (2012)	Relational additive	30 U.S. airline companies	Production efficiency/marketing efficiency
Lu et al. (2010)	Independent	38 Taiwanese fables firms	Capability performance/efficiency performance
Luo (2003)	Independent	245 large banks	Profitability/marketability
Löthren and Tambour (1999)	Connected NDEA shared resources	31 Swedish pharmacies	Production efficiency/customer satisfaction
Meepadung et al. (2009)	Independent	6 segments of a major Thai bank	Operating efficiency/profit efficiency
Mukherjee et al. (2003)	Connected NDEA static	27 Indian public sector banks	Quality efficiency/profitability efficiency
Naini et al. (2013)	Game theory BG multiplicative	35 Iranian bank branches	Profitability/marketability
Narasimhan et al. (2004)	Independent	Manufacturing firms	Flexibility competence/execution competence
Nemoto and Goto (2003)	Connected NDEA dynamic	Japanese electric utilities	Periods

Premachandra et al. (2012)	Relational NDEA	66 large U.S. mutual funds	Operational management/portfolio management
Rho and An (2007)	Connected value-chain	27 banks	IT-related activity/loan processing system
Saranga and Moser (2010)	Connected value-chain	High revenue firms	Operational performance/financial performance
Seiford and Zhu (1999)	Independent	55 U.S. Commercial banks	Profitability/marketability
Sexton and Lewis (2003)	Independent	Major League baseball	Front office/on-field competition
Sheu et al. (2006)	Independent	14 Taiwanese financial holding companies	Profitability/marketability
Song et al. (2014)	CCR and SB independent NDEA structure	Water treatment in Chinese provinces	Production process/pollution treatment session
Toloo et al. (2015)	Relational NDEA Shared resources	1. Banking industry 2. University operations	1. Deposit/loan 2. Researching/teaching
Tsolas (2010)	Independent	Bank branches of a major Greek bank	Profitability efficiency/effectiveness
Tsolas (2011)	Independent	13 commercial banks of Athens stock exchange	Profitability/performance in the stock market

Von Geymueller (2009)	Connected NDEA dynamic	50 of the largest U.S. electric transmission system operators	Periods
Wang and Chin (2010)	Relational additive	24 Taiwanese non-life insurance companies	Premium acquisition/profit generation
Wang et al. (1997)	Independent	22 banks	IT-related activity/loan processing system
Wang et al. (2013)	Connected value-chain NDEA	High-tech technology firms	Operation efficiency/R&D efficiency
Wang et al. (2014a)	Relational additive	16 Chinese commercial banks	Deposit producing/profit earning
Wang et al. (2014b)	Relational multiplicative fuzzy numbers	U.S. banking holding companies	Profitability/value creativity
Wanke (2013)	Game theory C Multiplicative	27 Brazilian ports	Physical infrastructure/shipment consolidation
Wanke and Barros (2013)	Game theory C Multiplicative	Brazilian banks	Cost efficiency/productive efficiency
Xie et al. (2012)	Connected SB NDEA	Vertically integrated power systems in China’s regions	Generation corporations/grid corporations
Yang et al. (2011)	Connected value-chain	17 bank branches of China Construction Bank	Fund collection/profit generation

Yang et al. (2014)	Relational additive	NBA teams	Wage efficiency/on-court efficiency
Zha and Liang (2010)	Game theory C Multiplicative NDEA	30 top U.S. commercial banks	Profitability/marketability
Zha et al. (2015)	Connected NDEA dynamic	25 Chinese banks	Productivity/profitability
Zhou et al. (2013)	Game theory C BG Multiplicative	10 branches of China Construction Bank	Operational efficiency/profitability
Zhu (2011)	Game theory C Multiplicative	21 airlines	Fleet maintenance/revenue generation
Zhu (2000)	Independent	Fortune 500 companies	Profitability/marketability

3.7. Summary

Conventional single-stage DEA approach is a valuable tool for efficiency evaluation, however when there are more complex systems than a simple input-output procedure it fails to address the internal structures. A decision maker needs a tool which can incorporate these interrelations into the model and provide more accurate results in order to monitor the overall and individual procedures more effectively and make better decisions. In order to evaluate these structures there is a need for more complex models such as two-stage DEA models.

This Chapter has provided a thorough survey and a detailed classification of two-stage DEA models. In addition, an analytical summary table was presented with the majority of the two-stage DEA applications across the literature. Along the Chapter we concentrated on two-stage models with intermediate measures between the first and the second stage and some variations such as models with exogenous inputs in the second stage. Some special cases where there are more than two stages or there are no intermediate measures were also included.

The Chapter classifies two-stage DEA models into four broad categories: 1) Independent two-stage DEA approach, which does not consider the possible conflicts between the two stages. 2) Connected two-stage DEA approach, which considers the interaction between the two stages. 3) Relational two-stage DEA models which treat intermediate measures in a coordinated manner and assumes a mathematical relationship between overall efficiency and individual efficiencies. 4) Game theoretic models which are divided in non-cooperative and cooperative models (cooperative models include the Nash bargaining game model).

Various models and their suitability under specific circumstances has been presented. It has been demonstrated the importance of the choice among the appropriate models based on possible conflicts or any mathematical relationship between the stages. After an extensive and detailed study of every two-stage DEA model and application the general conclusion is that there is no “perfect” model, on the contrary each model is suitable under specific circumstances.

The following Chapters use relational two-stage DEA models in order to evaluate not only the overall efficiency of the DMU but also the efficiencies of individual stages. Chen et al. (2014) after an extensive investigation of envelopment and multiplier two-stage DEA models, found that multiplier models (such as all relational models) should be used for the evaluation of the overall and individual efficiencies. Chapter 4 constructs the Weight Assurance Region (WAR) two-stage DEA model which modifies the original additive two-stage DEA model of Chen et al. (2009a) to incorporate assurance region-based weights. The proposed WAR model has the ability to utilize a priori information such as expert opinion and solves an infeasibility problem of the original additive model. WAR model can be considered as a general case of the original additive model.

Chapter 4

Weight Assurance Region model

4.1. Introduction

The previous Chapter presented and discussed every type of two-stage DEA model. This thesis uses relational two-stage DEA models which assume a mathematical relationship (multiplicative or additive) between overall efficiency and individual stage efficiencies. The desirable aspect of relational models is that they yield efficiency scores not only for the overall DMU but also for the individual stages. As already presented in Chapter 3, the two relational approaches are the multiplicative model of Kao and Hwang (2008) and the additive model of Chen et al. (2009a). Apart from the assumed mathematical relationship, the two models have another conceptual difference. The multiplicative model of Kao and Hwang (2008) treats the two stages as equal; therefore each stage contributes to the overall process by 50%. If ξ_1 and ξ_2 are the weights which represent the significance of the first and the second stage respectively, the multiplicative model assumes that $\xi_1 = 0.5$ and $\xi_2 = 0.5$. Alternatively, the additive model of Chen et al. (2009a) does not assign arbitrary a priori weights to the two-stages. The weights are treated as variables and the model assigns the best possible weights to each stage which maximize the overall efficiency. Therefore ξ_1 and ξ_2 are calculated inside the model and they are free to take any value from 0 to 1; $0 \leq \xi_{1,} \xi_2 \leq 1$. Zero value means that this stage does not contribute to the overall process at all and unity value means that the overall process is entirely based on this stage.

Conceptually, assigning a zero weight in to one stage and a unity weight in to the other stage has no meaning. For example if the weight of the first stage is unity and the weight of the second stage is zero, there is no need for a two-stage model; we can use a single-stage DEA model with only the first stage. Furthermore, from a computational point of view such extreme weights result in an infeasibility problem. Again if $\xi_1 = 1$ and $\xi_2 = 0$ then the ratio in (3.28) is not defined and as a result the efficiency of the second stage is not defined. Apart from the extreme case where the weight of one stage is zero and the weight of the other stage is unity, there could be a large debate about the lower acceptable weights. For example is 1%, 2% or 5% contribution of one stage to the overall process acceptable? This debate could easily be answered in the presence of a priori information such as expert opinions, value judgments, known information and/or widely accepted beliefs or preferences and other type of information.

This Chapter proposes the Weight Assurance Region (WAR) two-stage DEA model which is the most significant research contribution of this thesis. Following the novel assurance region concept of Thompson et al. (1990), the WAR model modifies the original additive two-stage DEA model of Chen et al. (2009a) to incorporate assurance region-based weights for the two stages. The proposed WAR model has the ability to utilize a priori information and solves the infeasibility problem of the original additive model. WAR model can be considered as a general case of the original additive model.

This Chapter is structured as follows. Section 4.2 presents a solution for the infeasibility problem proposed by Chen et al. (2008) and Chen et al. (2009a). Section 4.3 constructs the WAR model and presents the necessary definitions, formulations and proofs. Section 4.4 applies the WAR model on a real case study about secondary education across countries and Section 4.5 concludes.

4.2. A solution for the infeasibility problem

Chen et al. (2008) and Chen et al. (2009a) proposed a solution in order to solve the computational problem of infeasibility. They imposed restrictions on ξ_1 and ξ_2 and incorporated them as additional constraints in model (3.27). The idea is to restrict ξ_1 and ξ_2 to be positive which should be sufficient to overcome the infeasibility problem. Specifically, the authors incorporated two additional constraints in model (3.27) which are $\xi_1 > \alpha$ and $\xi_2 > \alpha$. The meaning of these new constraints is that no stage contributes less than α $(0 < \alpha \leq 0.5)$ to the whole process.

Chen et al. (2008) incorporated the two new constraints in model (3.27) which is equivalent with model (4.1):

$$E_0 = max \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{4.1}$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_0} + \sum_{d=1}^{D} \mu_d z_{d_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0,$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \leq 0,$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} \geq \alpha$$

$$\sum_{d=1}^{D} \mu_d z_{d_0} \geq \alpha$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

α is user specified and ($0<\alpha\leq 0.5$)

If we extend the initial idea of Chen et al. (2008) and Chen et al. (2009a), restricting ξ_1 and ξ_2 in a region could be a tool for the decision maker to intervene in the model if he has information about the size of the stages. For example, a decision maker might not know the exact size of the two stages or which stage is larger but he might know that no stage contributes less than α $(0 < \alpha \leq 0.5)$ in the whole process. If that is the case we can restrict ξ_1 and ξ_2 to be larger than α, thus $\xi_1 > \alpha$ and $\xi_2 > \alpha$ which are the newly incorporated constraints in model (3.27). The two restrictions are equivalent with $\alpha \leq \xi_1 \leq 1 - \alpha$ which means that ξ_1 contributes at least α and at most *1-α*. We know that $\xi_1 + \xi_2 = 1$ which implies that if ξ_2 is at least α then ξ_1 is at most *1-α*. Consequently, the above constraint also means that ξ_2 contributes at least α and at most *1-α*. Then model (4.1) can also be written as follows.

$$E_0 = max \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{4.2}$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_0} + \sum_{d=1}^{D} \mu_d z_{d_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0,$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \leq 0,$$

$$-\sum_{i=1}^{m} \omega_i x_{i_0} + \alpha \leq 0$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} - (\alpha - 1) \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

α is user specified and ($0<\alpha\leq 0.5$)

The next section provides an alternative model which is named as Weight Assurance Region model. The new model and the model (4.2) are defined under the scope of Thompson et al.'s (1990) assurance region.

4.3. Weight Assurance Region model

As intuitively has been pointed out by Thanassoulis et al. (2004), assigning a zero weight might not be acceptable from the decision maker or the analyst. In a two-stage process assigning a zero weight means that a stage will not participate in the whole process which is probably not acceptable otherwise a single-stage model would have been used in the first place. In addition, Thanassoulis et al. (2004) indicated the significance of a priori incorporating context value judgments in a model such as known information and/or widely accepted beliefs or preferences. Thanassoulis et al. (2004) described in detail the type of information which might be used in such framework.

From a decision maker's perspective, the advancement proposed by Chen et al. (2008) and Chen et al. (2009a) in models (4.1) and (4.2) is useful in the case where there is prior information that no stage contributes less than α in the whole process. Now, consider a more general case where the decision maker has the information that the ratio of the two stages is inside a region, e.g. among β and δ which are two positive scalars:

$$\beta \leq \frac{\xi_1}{\xi_2} \leq \delta \tag{4.3}$$

Note that β and δ cannot become zero in order to ensure that neither ξ_1 nor ξ_2 are zero.

Restriction (4.3) is more flexible than the restrictions of Chen et al. (2008) and Chen et al. (2009a) because it allows the utilization of every information regarding the relationship among the two stages. Specifically, every information about the relative size of the stages is taken into account by the model such as: if β and δ are smaller than 1 then the first stage is smaller than the second stage while if β and δ are bigger than 1 then the first stage is larger than the second stage. There is a special case where $\delta = 1/\beta$ and the assurance region is symmetric around the ratio of the weights. In this case $\beta = \alpha/(1-\alpha)$ and $\delta = (1-\alpha)/\alpha$. Then, inequality (4.3) ensures that no stage contributes less than α and more than $1\text{-}\alpha$ in the whole process and the WAR model yields the same results with Chen et al. (2008) model. In addition the proposed model can examine any possible asymmetric region around the ratio of the weights. Also note that if $\beta = \delta$ the weights ξ_1 and ξ_2 are not inside a region but they are exactly defined.

If inequality (4.3) is incorporated in model (3.27) the resulting model is as follows.

$$E_0 = max \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{r=1}^{s} \gamma_r y_{r_0} \tag{4.4}$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_0} + \sum_{d=1}^{D} \mu_d z_{d_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} \leq 0,$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} \leq 0,$$

$$-\sum_{i=1}^{m} \omega_i x_{i_0} + \beta \sum_{d=1}^{D} \mu_d z_{d_0} \leq 0$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} - \delta \sum_{d=1}^{D} \mu_d z_{d_0} \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n; \; i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

β and δ are user specified and (0<β≤δ)

Note that the fourth and the fifth constraints in model (4.4) are the new constraints derived from inequality (4.3) and ensure that the ratio of the weights ξ_1 and ξ_2 is between β and δ. These constraints are based on the assurance region model proposed by Thompson et al. (1990), however these are not imposed on the multipliers of the model (such as in the original assurance region approach), but they are imposed on the weights of each stage. For example, suppose a restriction on the ratio of the relative importance of the first stage over the relative importance of the second stage to be between 1/4 and 1/2 which means that the weight of the first stage lies among 0.20 and 0.33 while the weight of the second stage lies among 0.66 and 0.8. Next, the detailed proof of the transformation of the constraint (4.3) into the fourth and fifth constraint in model (4.4) is presented.

Proof: The transformation of the restriction (4.3) into the fourth and fifth constraint in model (4.4) can be obtained as:

$$\beta < \frac{\xi_1}{\xi_2} < \delta \Rightarrow \beta < \frac{\dfrac{\sum_{i=1}^{m} v_i x_{i_0}}{\sum_{i=1}^{m} v_i x_{i_0} + \sum_{d=1}^{D} w_d z_{d_0}}}{\dfrac{\sum_{d=1}^{D} w_d z_{d_0}}{\sum_{i=1}^{m} v_i x_{i_0} + \sum_{d=1}^{D} w_d z_{d_0}}} < \delta \Rightarrow \beta < \frac{\sum_{i=1}^{m} v_i x_{i_0}}{\sum_{d=1}^{D} w_d z_{d_0}} < \delta \Rightarrow$$

$$\beta \cdot \sum_{d=1}^{D} w_d z_{d_0} < \sum_{i=1}^{m} v_i x_{i_0} < \delta \cdot \sum_{d=1}^{D} w_d z_{d_0} \qquad (4.5)$$

From the left hand side of (4.5):

$$\beta \cdot \sum_{d=1}^{D} w_d z_{d_0} < \sum_{i=1}^{m} v_i x_{i_0} \Rightarrow -\sum_{i=1}^{m} v_i x_{i_0} + \beta \cdot \sum_{d=1}^{D} w_d z_{d_0} < 0 \qquad (4.6)$$

and from the right hand side of (4.5):

$$\sum_{i=1}^{m} v_i x_{i_0} - \delta \cdot \sum_{d=1}^{D} w_d z_{d_0} < 0 \qquad (4.7)$$

Then constraints (4.6) and (4.6) are incorporated in model (3.27) resulting in model (4.4). This completes the proof.□

Next, the VRS version of the WAR model is provided which will be used for the needs of the application in the next section.

$$E_0 = max \sum_{d=1}^{D} \mu_d z_{d_0} + \sum_{r=1}^{s} \gamma_r y_{r_0} + u^1 + u^2 \qquad (4.8)$$

s.t.

$$\sum_{i=1}^{m} \omega_i x_{i_0} + \sum_{d=1}^{D} \mu_d z_{d_0} = 1$$

$$\sum_{d=1}^{D} \mu_d z_{d_j} - \sum_{i=1}^{m} \omega_i x_{i_j} + u^1 \leq 0,$$

$$\sum_{r=1}^{s} \gamma_r y_{r_j} - \sum_{d=1}^{D} \mu_d z_{d_j} + u^2 \leq 0,$$

$$-\sum_{i=1}^{m} \omega_i x_{i_0} + \beta \sum_{d=1}^{D} \mu_d z_{d_0} \leq 0$$

$$\sum_{i=1}^{m} \omega_i x_{i_0} - \delta \sum_{d=1}^{D} \mu_d z_{d_0} \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \ldots, n;\ i = 1, \ldots, m; d = 1, \ldots, D; r = 1, \ldots, s$$

β and δ are user specified and (0<β≤δ)

u^1 *and* u^2 *are free in sign*

To sum up, following Thompson et al. (1990) this Chapter applies the assurance region approach in order to impose bounds on ratios of multipliers in the original additive two-stage DEA model. Imposing additional constraints in the traditional DEA model has also been used in other approaches such as the use of regression analysis to restrict weight flexibility in DEA (Dyson and Thanassoulis, 1988), restricting multiplier flexibility with inequalities (Wong and Beasley, 1990; Beasley, 1990, 1995), restricting multipliers to a closed cone (Charnes et al., 1989) and absolute weight restrictions (Podinovski and Athanassopoulos, 1998). Assurance regions have also been used by Zhu (1996) in order to impose bounds on the weights of Analytic Hierarchy Process. For a review of weight restricted DEA models see Thanassoulis et al. (2008). Thanassoulis and Allen (1998) showed that in order to avoid zero or very small weights in a DEA model, unobserved DMUs (UDMUs) can also be used equivalently instead of weight restrictions. This idea is further extended by Allen and Thanassoulis (2004) and Thanassoulis et al. (2012).

The newly proposed WAR model fulfils the strict definition of assurance region and assurance region efficiency definition given by Thompson et al. (1990). These definitions are about virtual multipliers of the traditional DEA model. The definitions have been modified for the needs of the present study in order to be about the relative weights of each individual stage in an additive two-stage DEA model. Note that the vector of the excluded weights is named *q*, the vector of weights inside the region is named *WAR* and the vector of all weights is named W $(WAR \subseteq W \;\; and \;\; q \subseteq W)$. In addition, ξ_p is the weight of the p^{th} stage (*p=1,2*) and *E* has already be set as the vector of overall efficiencies of the whole process.

In the lines of Thompson et al. (1990, pp.100):

Weight assurance region (WAR) definition: For the additive efficiency decomposition approach in two-stage DEA models (Chen et al., 2009a), a *WAR* is a subset of *W* such that vectors *q* excluded from *WAR* are not reasonable weights concerning the relative importance or contribution of the two stages to the overall process.

WAR efficiency definition: A DMU_j in *E* is said to be *WAR-efficient*, relative to a *WAR*, if the intersection of ξ_p (*p=1,2*) and *WAR* is not empty $\xi_p \cup WAR \neq \emptyset$; and it is said to be not *WAR-efficient* otherwise.

From the above definitions the union of *WAR* and the excluded weights *q* is equal with the set of all the weights $(W = WAR \cup q)$. In addition, if a DMU fails to be WAR efficient, then it can be safely assumed that it is an inefficient DMU because all reasonable weights are included in WAR. All weights outside WAR are not satisfying the imposed bounds and are considered unreasonable. Additionally, it is recommend that any further restriction in model (3.27) such as the WAR model should be used carefully and only in the presence of reliable a priori information, otherwise the results may be underestimated. However, it must be noted that neglecting such information might have the opposite effect and the results might be overestimated. This will become clear later when the results of the WAR model will be compared with the typical single stage DEA scores.

In the next section the WAR model is applied in order to construct an efficiency index which evaluates secondary education across 65 countries.

4.4. Application to secondary education across countries

This Section uses the VRS version of the WAR model (4.8) to construct an overall "school efficiency" index for 65 countries. Then, the overall "school efficiency" index will be decomposed into "learning environment efficiency" index in the first stage and "student's performance efficiency" index in the second stage. The principal idea of this application is summarized perfectly in OECD (2010a): better relations between teachers and students create a better disciplinary climate in school which results in higher scores in test subjects.

4.4.1. Efficiency in secondary education

Performance evaluation and efficiency assessment in public organizations have received much attention in recent years. This attention has created the growing demand from governments for efficient operation of the public organizations and goal fulfillment with the minimum resource consumption. If the global economic crisis and the austerity measures are taken into account, the need to achieve the maximum possible outcome while using the minimum resources is more significant than ever.

One of the pillars of every country is education which can be categorized at primary, secondary and tertiary education with many subcategories. Education sector receives a large amount of public and private money every year. In 2008, the public and private expenditure on education in the OECD countries was equivalent to 6.1% of GDP and more than three quarters of this expenditure came from public funding[4]. Therefore, education is a large sector and educational institutions need to be reformed towards a more efficient performance. Some important components of this reform are setting

[4] http://www.oecd-ilibrary.org/sites/factbook-2011-en/10/02/04/index.html;jsessionid=f2xxk9gddf86.delta?contentType=/ns/StatisticalPublication,/ns/Chapter&itemId=/content/chapter/factbook-2011-89-en&containerItemId=/content/serial/18147364&accessItemIds=&mimeType=text/html

performance standards for every party which is involved in the educational process (students, teachers and school environment), granting autonomy to the institutions, rewarding exceptional performance and improving low performance (Bifulco and Bretschneider, 2001).

In order to improve educational efficiency, we need to evaluate it first. On the one hand, the vast majority of the existing studies examine the cost side view of the education. Grosskopf et al. (1997) evaluated the efficiency using a cost distance function and they took into account teaching, administrative and aiding staff costs. Grosskopf et al. (2001) and Haelermans and Ruggiero (2013) also used personnel costs and salaries in their study. Heshmati (2002) applied a cost function in public schools and Banker et al. (2004) employed three expenditure measures. Haelermans and De Witte (2012) used cost per student as a budget constraint while Haelermans et al. (2012) applied a flexible budget constrained output distance function model. On the other hand, there are some studies which highlight the importance of teaching and schooling quality. Ramsden (1991) used the Course Experience Questionnaire in British education to create a performance index of teaching quality. The author highlighted the significance of a uniform questionnaire in order to make cross country evaluation and comparisons. Goldhaber et al. (1999) underlined the importance of teacher's motivation and school climate and Fare et al. (2006) emphasized that quality in schools matters. Hanushek (2013) argued that giving more money to schools do not necessarily guarantee better results but improving quality in school certainly does. This Section concentrates on quality but follows a different approach. The quality of the learning environment in schools is evaluated and it is investigated how this environment is employed to generate student's performance.

An appropriate evaluation approach for measuring the efficiency in schools is DEA which employs multiple inputs to produce multiple outputs and requires only weak assumptions on the underlying technology. There is an extensive literature about DEA and similar techniques which evaluate school efficiency (e.g. Grosskopf et al., 1999; Bifulco and Bretschneider, 2001; Grosskopf and Moutray, 2001; Portela and Thanassoulis, 2001; Heshmati, 2002; Thanassoulis et al., 2002; Banker et al., 2004; Färe et al., 2006; Primont and Domazlicky, 2006; Essid et al., 2010; Haelermans and Ruggiero, 2013; Essid et al., 2014).

Bifulco and Bretschneider (2001) pointed out that education is a complex structure and simple form of DEA is inadequate for its efficiency evaluation. Therefore, educational institutions can be considered as complex structures and single-stage DEA may not be adequate for their efficiency evaluation. Therefore, more sophisticated models are needed and two-stage DEA model is an appropriate solution. Some recent examples of a two-stage DEA model in education are the studies of Lu (2012) and Ho et al. (2013) for Taiwanese and USA universities respectively.

4.4.2. Inputs and outputs

There are several studies across the literature dealing with school performance evaluation. In respect to the input-output specification, there is a consensus about the student's performance as a measure of school output, such as Haelermans and Ruggiero (2013) and Woessmann (2011), and also a lot of different perspectives about the specification of inputs. A lot of studies used expenditures and teachers' characteristics as inputs. However Hanushek (1986) argued that these inputs are not the best indicators for capturing schooling differences. Hanushek (1992) and Woessmann (2011) fount that teacher's quality is a vital determinant of student's performance. Ramsden (1991) used a questionnaire to measure the student's opinion about the quality of the teaching and other aspects of the student'-teacher relationship. Bifulco and Bretschneider (2001) and Haelermans and Ruggiero (2013) signified that any measure of school performance should take into account the learning environment inside the schools. Perhaps the principal idea for the empirical application here is closer to the findings of Goldhaber et al. (1999) who marked the importance of school, teacher and classroom environment on student's performance.

Five inputs, five intermediate variables and three outputs are used in the present study. All the variables have been taken from the fourth cycle of OECD's Programme for International Student Assessment (PISA) in 2009. PISA is an international OECD project which assesses the student performance in reading, mathematics and science every three years since 2000. The target population of the project is 15-years old students from 65 countries, 34 of which are OECD members and 31 are partners. The choice of the students is based only on their age and not on the grade they currently attend. PISA results form an excellent database for cross-country educational comparisons. According to OECD (2010a), in PISA 2009 the participant students are about 470,000, which is a sample taken from 26 million 15-years old students of the 65 participant countries. The students have been tested in a two-hour test in reading, mathematics and science and also they have completed a questionnaire about various school and background aspects.

A number of previous studies use PISA database and they combine scores in the three subjects (reading, mathematics and science) with a variety of inputs. Afonso and Aubyn (2006) examined the educational efficiency of 25 OECD countries and applied a DEA model using the scores in the three subjects from PISA 2003 as outputs, while they used the number of teachers per student and the time spent at school as inputs. Woessmann (2011) studied the teacher's payment relative to their performance. They employed the three subject scores from PISA 2003 along with teaching responsibilities, teacher's qualification and salaries and demographics. Brunello and Rocco (2013) investigated the effect of immigrant students on native student's performance in 19 countries using PISA 2000, 2003, 2006 and 2009.

A different approach is followed here. All the variables in this study have been taken from PISA 2009 and particular focus has been given on educational environment. Specifically, the test scores in reading, mathematics and science serve as the three outputs in this study (detailed definition of scores in reading, mathematics and science is provided by OECD, 2010b, pp.23). The level of students' agreement with five statements about the teacher-student relationship are used as inputs. The relationship between teachers and students is crucial for the establishment of an appropriate learning environment (OECD, 2010a). Level of students' disagreement with five statements of disciplinary climate in the class are used as intermediate measures. Disciplinary climate is a vital factor in the process of learning because a problematic environment distracts students from learning which obviously has an effect on their performance (OECD, 2010a). In fact, according to Jennings and Greenberg (2009) an orderly and cooperative environment inside and outside the class is a vital factor for the effectiveness of the school. Figure 4.1 demonstrates the overall process while all the variables and their descriptive statistics are presented in Table 4.1. Considering the above, the educational system can be studied as a two-stage process where in the first stage the relationship among teachers and students affects the disciplinary climate in the class while in the second stage the learning environment affects the student's performance.

Figure 4.1: Overall two-stage school efficiency process

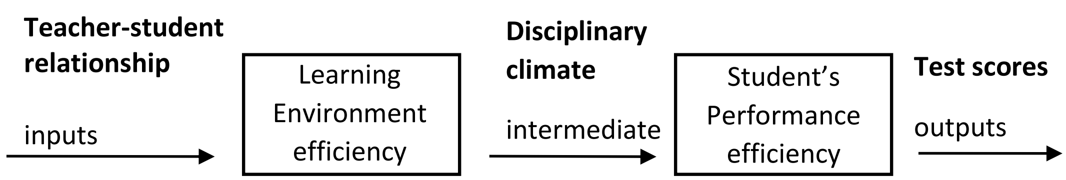

Table 4.1: Descriptive statistics

	Variables	Mean	St.dev.	Min	Max
Inputs $\begin{pmatrix} \text{Teacher} - \\ \text{students} \\ \text{relations} \end{pmatrix}$	I get along well with most of my teachers	85.83	3.88	73.00	94.00
	Most of my teachers are interested in my well-being	69.89	12.99	28.00	89.00
	Most of my teachers really listen to what I have to say	69.91	7.83	53.00	89.00
	If I need extra help, I will receive it from my teachers	80.02	7.24	63.00	93.00
	Most of my teachers treat me fairly	80.11	6.32	65.00	94.00
Intermediate measures $\begin{pmatrix} \text{Disciplinary} \\ \text{climate} \end{pmatrix}$	Students don't listen to what the teacher says	74.78	8.90	55.00	92.00
	There is noise and disorder	72.74	9.82	52.00	93.00
	The teacher has to wait a long time for the students to quieten down	75.02	8.12	62.00	93.00
	Students cannot work well	81.39	5.73	56.00	91.00

	Students don't start working for a long time after the lesson begins	77.05	7.79	55.00	92.00
Outputs	Reading	464.36	51.62	314.02	555.83
$(\boldsymbol{Scores})$	Mathematics	467.62	59.80	331.16	600.08
	Science	471.71	56.07	329.55	574.62

4.4.3. Results

Now, the methodology presented previously is implemented. The VRS version of the WAR model presented in (4.8) is used and it is chosen to give pre-emptive priority to the second stage. All we need is to define β and δ. Following Thompson et al. (1990) the positive scalars β and δ are user specified and they are estimated based on socio-economic and/or environmental data and expert opinion. In the present application, such knowledge is not directly available and the specification of the significance of each stage to the whole process is an open research question. Thanassoulis et al. (2004) provided methods for specifying and incorporating value judgments in such cases. The application here considers three possible scenarios and explains the implications for every scenario chosen. Since the relative importance of each stage to the overall process is an open research question, these three scenarios are chosen in order to provide a robustness measurement of the evaluated educational systems and to observe how the results fluctuate as the scalars β and δ are changed. Note that any possible scenarios could have been chosen for the robustness check.

Next the WAR model is implemented for the three possible scenarios.

1) The first stage is 2-3 times smaller than the second stage:

$$\frac{1}{3} \leq \frac{\xi_1}{\xi_2} \leq \frac{1}{2}$$

2) The first stage is 2-3 times bigger than the second stage:

$$2 \leq \frac{\xi_1}{\xi_2} \leq 3$$

3) The first stage is at least 4 times smaller and at most 4 times bigger the second stage:

$$\frac{1}{4} \leq \frac{\xi_1}{\xi_2} \leq 4$$

Results are presented in Table 4.2 whereas the descriptive statistics for the whole set are presented in Table 4.3.

First, we can see that model (4.5) guarantees that weights ξ_1 and ξ_2 are strictly positive and as a result every individual efficiency can be calculated in every scenario. In specific, the first scenario yields a mean overall “school efficiency” at 0.793 and 27 countries achieve above average scores. The first stage “learning environment” mean efficiency is 0.635 and the second stage “student’s performance” mean efficiency is 0.871. The second scenario yields a significantly increased mean “school efficiency” at 0.881 and 29 countries are above average. The “learning environment” mean efficiency is also increased at 0.883 while the “student’s performance” mean efficiency is at the same level at 0.871. In the third scenario the mean “school efficiency” is further increased at 0.890 and 30 countries achieve above average efficiency scores. The “learning environment” mean efficiency is slightly increased at 0.889 while the “student’s performance” mean efficiency is exactly the same (0.871).

Considering the above analysis, it is clear that the third scenario is less restrictive than the others and as such it yields larger efficiency scores. Note that the third scenario yields the same results with Chen et al. (2008) model if α=0.20. Furthermore, the second scenario yields larger efficiencies than the first scenario and the weights ξ_1 and ξ_2 are examined carefully, it can be seen that second scenario is less restrictive than the first scenario. Consequently, our suggestion about the use of restrictions with caution is supported by the findings. However, given the proper information model (4.5) could be a useful policy making tool. As such model (4.5) is able to examine every possible scenario including the model presented by Chen et al. (2008) where $\delta = 1/\beta$ (third scenario). In addition, the proposed model can examine additional scenarios as the ones presented under the first and the second scenario.

Considering the results in a country level, Korea (0.930, 0.974, 0.979) and Japan (0.915, 0.968, 0.974) achieve high overall efficiency scores across all scenarios. Shanghai, China (0.965) achieves the highest overall efficiency score in the first scenario while

Greece achieves the highest overall efficiency scores in the second and third scenario. Brazil (0.691) in the first scenario and Panama in the second and third scenario achieve the lowest efficiencies. In respect to the “learning environment” stage Korea and Japan achieves perfect efficiency scores across all scenario and also Greece achieves 1.000 efficiency for the third scenario. Considering the “student performance” stage Argentina, Finland, Greece, the Netherlands and Shanghai, China achieve perfect efficiency scores. A careful examination of Table 4.2 reveals that inefficient educational systems should try to improve the performance of their “learning environment” stage, which has generally lower performance than the “student performance” stage. As a result they will improve the overall performance of their educational systems.

Table 4.2: Results of the VRS WAR model (4.5)

	1st scenario					**2nd scenario**					**3rd scenario**				
DMU	E_0	E_0^1	E_0^2	ξ_1	ξ_2	E_0	E_0^1	E_0^2	ξ_1	ξ_2	E_0	E_0^1	E_0^2	ξ_1	ξ_2
Albania	0.732	0.750	0.723	0.33	0.67	0.787	0.808	0.723	0.75	0.25	0.794	0.811	0.723	0.80	0.20
Argentina	0.713	0.138	1.000	0.33	0.67	0.855	0.807	1.000	0.75	0.25	0.873	0.841	1.000	0.80	0.20
Australia	0.833	0.733	0.883	0.33	0.67	0.858	0.849	0.883	0.75	0.25	0.861	0.855	0.883	0.80	0.20
Austria	0.799	0.636	0.880	0.33	0.67	0.926	0.941	0.880	0.75	0.25	0.940	0.956	0.880	0.80	0.20
Azerbaijan	0.732	0.749	0.724	0.33	0.67	0.793	0.817	0.724	0.75	0.25	0.800	0.819	0.724	0.80	0.20
Belgium	0.811	0.589	0.922	0.33	0.67	0.871	0.854	0.922	0.75	0.25	0.878	0.867	0.922	0.80	0.20
Brazil	0.691	0.156	0.958	0.33	0.67	0.816	0.768	0.958	0.75	0.25	0.829	0.796	0.958	0.80	0.20
Bulgaria	0.765	0.597	0.849	0.33	0.67	0.916	0.938	0.849	0.75	0.25	0.925	0.943	0.849	0.80	0.20
Canada	0.832	0.653	0.921	0.33	0.67	0.838	0.800	0.913	0.67	0.33	0.838	0.795	0.912	0.63	0.37
Chile	0.765	0.387	0.954	0.33	0.67	0.932	0.924	0.954	0.75	0.25	0.941	0.938	0.954	0.80	0.20
Chinese Taipei	0.864	0.745	0.923	0.33	0.67	0.884	0.871	0.923	0.75	0.25	0.886	0.877	0.923	0.80	0.20
Colombia	0.721	0.606	0.779	0.33	0.67	0.801	0.808	0.779	0.75	0.25	0.811	0.819	0.779	0.80	0.20
Croatia	0.786	0.460	0.949	0.33	0.67	0.949	0.950	0.949	0.75	0.25	0.959	0.962	0.949	0.80	0.20
Czech Republic	0.798	0.550	0.922	0.33	0.67	0.933	0.937	0.922	0.75	0.25	0.946	0.952	0.922	0.80	0.20

Denmark	0.791	0.719	0.828	0.33	0.67	0.838	0.841	0.828	0.75	0.25	0.842	0.845	0.828	0.80	0.20
Dubai (UAE)	0.747	0.541	0.849	0.33	0.67	0.861	0.865	0.849	0.75	0.25	0.870	0.875	0.849	0.80	0.20
Estonia	0.854	0.750	0.906	0.33	0.67	0.924	0.930	0.906	0.75	0.25	0.929	0.935	0.906	0.80	0.20
Finland	0.909	0.728	1.000	0.33	0.67	0.920	0.881	1.000	0.67	0.33	0.923	0.872	1.000	0.60	0.40
France	0.778	0.345	0.995	0.33	0.67	0.899	0.867	0.995	0.75	0.25	0.913	0.893	0.995	0.80	0.20
Germany	0.848	0.825	0.859	0.33	0.67	0.914	0.933	0.859	0.75	0.25	0.920	0.935	0.859	0.80	0.20
Greece	0.753	0.260	1.000	0.33	0.67	0.991	0.989	1.000	0.75	0.25	1.000	1.000	1.000	0.80	0.20
Hong Kong, China	0.892	0.880	0.898	0.33	0.67	0.896	0.894	0.898	0.67	0.33	0.898	0.899	0.897	0.56	0.44
Hungary	0.808	0.616	0.904	0.33	0.67	0.918	0.923	0.904	0.75	0.25	0.926	0.931	0.904	0.80	0.20
Iceland	0.809	0.714	0.857	0.33	0.67	0.874	0.879	0.857	0.75	0.25	0.879	0.885	0.857	0.80	0.20
Indonesia	0.772	0.747	0.785	0.33	0.67	0.847	0.867	0.785	0.75	0.25	0.853	0.870	0.785	0.80	0.20
Ireland	0.806	0.629	0.895	0.33	0.67	0.890	0.889	0.895	0.75	0.25	0.900	0.901	0.895	0.80	0.20
Israel	0.731	0.489	0.852	0.33	0.67	0.852	0.852	0.852	0.75	0.25	0.866	0.870	0.852	0.80	0.20
Italy	0.770	0.533	0.889	0.33	0.67	0.883	0.880	0.889	0.75	0.25	0.895	0.896	0.889	0.80	0.20
Japan	0.915	1.000	0.872	0.33	0.67	0.968	1.000	0.872	0.75	0.25	0.974	1.000	0.872	0.80	0.20
Jordan	0.769	0.624	0.842	0.33	0.67	0.933	0.963	0.842	0.75	0.25	0.944	0.970	0.842	0.80	0.20
Kazakhstan	0.743	0.853	0.687	0.33	0.67	0.796	0.833	0.687	0.75	0.25	0.801	0.829	0.687	0.80	0.20
Korea	0.930	1.000	0.896	0.33	0.67	0.974	1.000	0.896	0.75	0.25	0.979	1.000	0.896	0.80	0.20
Kyrgyzstan	0.735	0.725	0.740	0.33	0.67	0.806	0.828	0.740	0.75	0.25	0.813	0.831	0.740	0.80	0.20
Latvia	0.777	0.755	0.788	0.33	0.67	0.852	0.873	0.788	0.75	0.25	0.859	0.877	0.788	0.80	0.20
Liechtenstein	0.875	0.739	0.943	0.33	0.67	0.937	0.935	0.943	0.75	0.25	0.942	0.942	0.943	0.80	0.20
Lithuania	0.778	0.815	0.759	0.33	0.67	0.865	0.900	0.759	0.75	0.25	0.874	0.902	0.759	0.80	0.20
Luxembourg	0.752	0.294	0.981	0.33	0.67	0.880	0.846	0.981	0.75	0.25	0.894	0.872	0.981	0.80	0.20
Macao, China	0.892	0.951	0.862	0.33	0.67	0.966	1.000	0.862	0.75	0.25	0.972	1.000	0.862	0.80	0.20
Mexico	0.763	0.696	0.797	0.33	0.67	0.898	0.931	0.797	0.75	0.25	0.906	0.933	0.797	0.80	0.20
Montenegro	0.754	0.688	0.787	0.33	0.67	0.866	0.892	0.787	0.75	0.25	0.871	0.892	0.787	0.80	0.20
Netherlands	0.820	0.460	1.000	0.33	0.67	0.864	0.819	1.000	0.75	0.25	0.869	0.836	1.000	0.80	0.20
New Zealand	0.834	0.657	0.923	0.33	0.67	0.851	0.834	0.900	0.75	0.25	0.851	0.839	0.899	0.80	0.20
Norway	0.815	0.551	0.947	0.33	0.67	0.944	0.942	0.947	0.75	0.25	0.956	0.959	0.947	0.80	0.20
Panama	0.708	0.464	0.829	0.33	0.67	0.785	0.770	0.829	0.75	0.25	0.793	0.784	0.829	0.80	0.20

Peru	0.743	0.718	0.756	0.33	0.67	0.833	0.859	0.756	0.75	0.25	0.841	0.863	0.756	0.80	0.20
Poland	0.838	0.806	0.855	0.33	0.67	0.962	0.997	0.855	0.75	0.25	0.969	0.998	0.855	0.80	0.20
Portugal	0.771	0.752	0.780	0.33	0.67	0.844	0.865	0.780	0.75	0.25	0.849	0.866	0.780	0.80	0.20
Qatar	0.742	0.347	0.939	0.33	0.67	0.901	0.888	0.939	0.75	0.25	0.917	0.912	0.939	0.80	0.20
Romania	0.752	0.829	0.713	0.33	0.67	0.832	0.872	0.713	0.75	0.25	0.838	0.869	0.713	0.80	0.20
Russian Federation	0.767	0.839	0.731	0.33	0.67	0.861	0.905	0.731	0.75	0.25	0.868	0.902	0.731	0.80	0.20
Serbia	0.735	0.459	0.873	0.33	0.67	0.853	0.847	0.873	0.75	0.25	0.864	0.862	0.873	0.80	0.20
Shanghai, China	0.965	0.859	1.000	0.25	0.75	0.910	0.865	1.000	0.67	0.33	0.971	0.855	1.000	0.20	0.80
Singapore	0.885	0.700	0.947	0.25	0.75	0.865	0.824	0.947	0.67	0.33	0.887	0.664	0.943	0.20	0.80
Slovak Republic	0.803	0.679	0.866	0.33	0.67	0.907	0.921	0.866	0.75	0.25	0.916	0.929	0.866	0.80	0.20
Slovenia	0.868	0.657	0.974	0.33	0.67	0.984	0.987	0.974	0.75	0.25	0.992	0.997	0.974	0.80	0.20
Spain	0.773	0.613	0.854	0.33	0.67	0.878	0.886	0.854	0.75	0.25	0.891	0.900	0.854	0.80	0.20
Sweden	0.775	0.569	0.879	0.33	0.67	0.849	0.839	0.879	0.75	0.25	0.855	0.849	0.879	0.80	0.20
Switzerland	0.839	0.652	0.933	0.33	0.67	0.870	0.849	0.933	0.75	0.25	0.873	0.857	0.933	0.80	0.20
Thailand	0.749	0.806	0.721	0.33	0.67	0.812	0.842	0.721	0.75	0.25	0.819	0.844	0.721	0.80	0.20
Trinidad and Tobago	0.745	0.355	0.939	0.33	0.67	0.870	0.848	0.939	0.75	0.25	0.882	0.868	0.939	0.80	0.20
Tunisia	0.695	0.195	0.945	0.33	0.67	0.831	0.794	0.945	0.75	0.25	0.846	0.821	0.945	0.80	0.20
Turkey	0.789	0.691	0.838	0.33	0.67	0.955	0.994	0.838	0.75	0.25	0.961	0.992	0.838	0.80	0.20
United Kingdom	0.806	0.728	0.845	0.33	0.67	0.859	0.863	0.845	0.75	0.25	0.863	0.868	0.845	0.80	0.20
United States	0.769	0.726	0.790	0.33	0.67	0.809	0.815	0.790	0.75	0.25	0.813	0.819	0.790	0.80	0.20
Uruguay	0.763	0.492	0.899	0.33	0.67	0.918	0.925	0.899	0.75	0.25	0.924	0.931	0.899	0.80	0.20

Table 4.3: Summary of results

		Mean	St.dev.	Min	Max
1st scenario	E_0	0.793	0.059	0.691	0.965
	E_0^1	0.635	0.192	0.138	1.000
	E_0^2	0.871	0.083	0.687	1.000
2nd scenario	E_0	0.881	0.052	0.785	0.991
	E_0^1	0.883	0.060	0.768	1.000
	E_0^2	0.871	0.083	0.687	1.000
3rd scenario	E_0	0.890	0.053	0.793	1.000
	E_0^1	0.889	0.064	0.664	1.000
	E_0^2	0.871	0.083	0.687	1.000

Next Table 4.4 compares the results of the WAR model with the results obtain from a) the single-stage input oriented BCC DEA model, b) the original additive two-stage DEA model of Chen et al. (2009a) with the advancement of α=0.05, a very flexible constraint which lets weights ξ_1 and ξ_2 take values from 0.05 to 0.95 and c) the multiplicative two stage DEA model of Kao and Hwang (2008). Single-stage DEA models achieve higher efficiency scores in both stages compared to every other model. The additive model of Chen et al. (2009a), the multiplicative model of Kao and Hwang (2008) and the WAR model are relational two-stage DEA models and as such they take into account the interaction between the stages. As a result these models are more restrictive than the single-stage DEA model and achieve lower results, however they provide a better framework to study a complex system such as in our empirical application.

Next, the WAR model for the three different scenarios is compared with the additive and the multiplicative two-stage DEA models. Chen et al. (2009a) stated that direct comparisons among different models may not yield reliable results and they proposed the comparison of the DMU rankings. Table 4.4 presents the ranking for all the five models. Moreover the Spearman rank correlation is used and the findings show that the ranking of the first scenario for the WAR model is correlated by 0.388 with the additive model of Chen et al. (2009a) and 0.735 with the multiplicative model of Kao and Hwang

(2008). Furthermore, the ranking of the second scenario for the WAR model is correlated by 0.889 with the additive model of Chen et al. (2009a) and 0.956 with the multiplicative model of Kao and Hwang (2008). Last, the ranking of the third scenario for the WAR model is correlated by 0.928 with the additive model of Chen et al. (2009a) and 0.937 with the multiplicative model of Kao and Hwang (2008). As it can be seen, the first scenario is correlated with the multiplicative model of Kao and Hwang (2008) while the other two scenarios are highly correlated with both models.

The comparison of WAR model with the single-stage DEA model allows us to support our suggestion about the careful use of restrictions. Suppose that the correct model to use in the empirical application is the single-stage DEA model. However, the first scenario of the WAR model is mistakenly chosen. In this situation the true efficiency scores would have been underestimated. Similarly, suppose that the correct model is the first scenario of the WAR model and the single-stage DEA model is mistakenly chosen. Now, the results would have been overestimated. Our suggestion is that every model should be used with caution and based on the available information. As a result the proposed WAR model can utilize every available prior information.

Table 4.4: Rankings and comparisons of the efficiency estimates

	Single-stage DEA		**WAR rankings**			**Chen et al. (2009a) with α=0.05**				**Kao and Hwang (2008)**			
DMU	θ^1	θ^2	1st	2nd	3rd	E_0	E_0^1	E_0^2	#	E_0	E_0^1	E_0^2	#
Albania	0.820	0.723	59	64	64	0.814	0.818	0.723	64	0.464	0.806	0.576	63
Argentina	0.936	1.000	62	44	37	0.924	0.920	1.000	27	0.547	0.738	0.741	54
Australia	0.872	0.883	16	43	46	0.869	0.868	0.883	49	0.668	0.774	0.863	31
Austria	0.993	0.880	26	14	15	0.984	0.990	0.880	10	0.733	0.838	0.874	15
Azerbaijan	0.832	0.724	58	63	63	0.818	0.823	0.724	62	0.522	0.808	0.646	59
Belgium	0.907	0.922	20	33	35	0.899	0.898	0.922	33	0.682	0.797	0.856	29
Brazil	0.875	0.958	65	57	57	0.864	0.859	0.958	51	0.524	0.693	0.756	58
Bulgaria	0.956	0.849	42	19	18	0.942	0.947	0.849	19	0.645	0.865	0.746	39
Canada	0.836	0.921	17	52	56	0.838	0.795	0.912	59	0.647	0.735	0.881	38

Chile	0.992	0.954	43	13	14	0.957	0.958	0.954	15	0.676	0.886	0.763	30
Chinese Taipei	0.895	0.923	10	28	32	0.893	0.891	0.923	36	0.721	0.847	0.852	18
Colombia	0.881	0.779	61	61	61	0.839	0.842	0.779	58	0.500	0.770	0.649	60
Croatia	0.995	0.949	30	8	9	0.981	0.983	0.949	11	0.745	0.878	0.849	13
Czech Republic	0.997	0.922	27	11	11	0.984	0.988	0.922	9	0.756	0.847	0.893	12
Denmark	0.885	0.828	28	53	53	0.852	0.854	0.828	55	0.641	0.835	0.767	42
Dubai (UAE)	0.903	0.849	51	40	40	0.883	0.885	0.849	43	0.633	0.840	0.754	43
Estonia	0.945	0.906	11	15	16	0.942	0.944	0.906	18	0.764	0.882	0.866	9
Finland	0.908	1.000	4	16	20	0.923	0.872	1.000	29	0.763	0.770	0.990	10
France	0.971	0.995	31	24	24	0.957	0.955	0.995	16	0.705	0.762	0.926	21
Germany	0.937	0.859	12	20	21	0.921	0.929	0.859	30	0.725	0.879	0.825	17
Greece	1.000	1.000	47	1	1	1.000	1.000	1.000	1	0.784	0.784	1.000	4
Hong Kong, China	0.987	0.898	5	26	27	0.898	0.899	0.897	34	0.734	0.889	0.826	14
Hungary	0.952	0.904	22	17	17	0.935	0.937	0.904	22	0.729	0.886	0.822	16
Iceland	0.899	0.857	21	32	34	0.885	0.886	0.857	40	0.683	0.858	0.796	28
Indonesia	0.891	0.785	36	50	49	0.872	0.877	0.785	48	0.537	0.880	0.611	55
Ireland	0.938	0.895	23	27	26	0.928	0.930	0.895	26	0.704	0.836	0.842	22
Israel	0.918	0.852	60	46	43	0.907	0.910	0.852	32	0.625	0.777	0.805	46
Italy	0.944	0.889	38	29	28	0.932	0.934	0.889	23	0.686	0.840	0.816	27
Japan	1.000	0.872	3	4	4	0.994	1.000	0.872	5	0.782	1.000	0.782	5
Jordan	0.984	0.842	40	12	12	0.966	0.972	0.842	13	0.628	0.877	0.716	45
Kazakhstan	1.000	0.687	54	62	62	0.810	0.816	0.687	65	0.491	0.841	0.584	61
Korea	1.000	0.896	2	3	3	0.995	1.000	0.896	4	0.812	1.000	0.812	3
Kyrgyzstan	0.840	0.740	57	60	59	0.830	0.834	0.740	60	0.411	0.803	0.511	65
Latvia	0.889	0.788	33	47	47	0.879	0.884	0.788	46	0.647	0.873	0.742	37
Liechtenstein	0.951	0.943	8	10	13	0.947	0.947	0.943	17	0.771	0.882	0.874	7
Lithuania	0.906	0.759	32	38	36	0.898	0.905	0.759	35	0.663	0.877	0.755	35
Luxembourg	0.947	0.981	48	30	29	0.936	0.934	0.981	21	0.693	0.738	0.934	25

Macao, China	1.000	0.862	6	5	5	0.993	1.000	0.862	6	0.825	1.000	0.825	1
Mexico	0.948	0.797	44	25	25	0.923	0.929	0.797	28	0.602	0.888	0.678	48
Montenegro	0.907	0.787	46	36	39	0.884	0.889	0.787	42	0.550	0.858	0.640	52
Netherlands	0.889	1.000	18	39	41	0.884	0.878	1.000	41	0.689	0.689	1.000	26
New Zealand	0.850	0.923	15	48	50	0.852	0.846	0.899	56	0.664	0.752	0.883	34
Norway	1.000	0.947	19	9	10	0.995	0.998	0.947	3	0.774	0.860	0.900	6
Panama	0.823	0.829	63	65	65	0.816	0.815	0.829	63	0.453	0.732	0.619	64
Peru	0.875	0.756	53	54	54	0.860	0.866	0.756	52	0.477	0.835	0.572	62
Poland	1.000	0.855	14	6	7	0.993	1.000	0.855	7	0.769	0.932	0.825	8
Portugal	0.876	0.780	37	51	51	0.854	0.871	0.727	54	0.645	0.834	0.773	40
Qatar	0.974	0.939	55	23	22	0.960	0.961	0.939	14	0.550	0.802	0.686	51
Romania	1.000	0.713	49	55	55	0.856	0.864	0.713	53	0.548	0.872	0.629	53
Russian Federation	0.915	0.731	41	41	42	0.881	0.889	0.731	44	0.641	0.904	0.709	41
Serbia	0.899	0.873	56	45	44	0.891	0.892	0.873	37	0.596	0.801	0.745	49
Shanghai, China	0.867	1.000	1	21	6	0.985	0.692	1.000	8	0.761	0.861	0.884	11
Singapore	0.843	0.948	7	37	31	0.889	0.612	0.936	38	0.697	0.813	0.857	23
Slovak Republic	0.947	0.866	25	22	23	0.932	0.935	0.866	24	0.714	0.866	0.825	19
Slovenia	1.000	0.974	9	2	2	0.999	1.000	0.974	2	0.822	0.919	0.895	2
Spain	0.939	0.854	35	31	30	0.929	0.933	0.854	25	0.666	0.822	0.811	33
Sweden	0.878	0.879	34	49	48	0.868	0.867	0.879	50	0.650	0.807	0.806	36
Switzerland	0.882	0.933	13	35	38	0.880	0.877	0.933	45	0.695	0.788	0.882	24
Thailand	1.000	0.721	50	58	58	0.839	0.846	0.721	57	0.524	0.869	0.603	57
Trinidad and Tobago	0.921	0.939	52	34	33	0.910	0.908	0.939	31	0.573	0.832	0.688	50
Tunisia	0.900	0.945	64	56	52	0.887	0.884	0.945	39	0.531	0.686	0.773	56
Turkey	1.000	0.838	29	7	8	0.970	0.977	0.838	12	0.710	0.930	0.763	20
United Kingdom	0.881	0.845	24	42	45	0.876	0.878	0.845	47	0.667	0.838	0.795	32

United States	0.829	0.790	39	59	60	0.825	0.827	0.790	61	0.608	0.791	0.768	47
Uruguay	0.981	0.899	45	18	19	0.941	0.944	0.899	20	0.630	0.878	0.718	44

4.5. Summary

There is an extreme case where the additive two-stage DEA model cannot evaluate the individual efficiencies because either ξ_1 or ξ_2 , which are the optimal weights of the relative importance of each stage, become zero. Chen et al. (2008) and Chen et al. (2009a) proposed an advancement to the model and they restricted the weights to be larger than a positive scalar α. This Chapter constructed a Weight Assurance Region (WAR) model which modifies the original additive two-stage DEA model of Chen et al. (2009a) to incorporate assurance region-based weights for the two stages. The newly proposed model restricts the ratio of ξ_1 and ξ_2 inside a region between β and δ which are positive scalars $(0 < \beta \leq \delta)$. The proposed WAR model deals with the aforementioned problem and when $\delta = 1/\beta$ it yields the same results with Chen et al.'s (2008) model. Furthermore, the proposed WAR model has the ability to incorporate a priori information such as expert opinion, value judgments, known information and/or widely accepted beliefs or preferences and other type of information. WAR model can be considered as a general case of the original additive model.

The WAR model is applied to a real application about cross-country secondary education. It is used to investigate how the school environment affects student performance. This Chapter proposes the construction of an overall "school efficiency" index which consists of two stages. The first stage utilizes teacher-student relationship inputs to create the disciplinary climate which serve as intermediate measures. This stage gives a "learning environment efficiency" index. The second stage uses the disciplinary climate to generate student performance in three subjects, namely reading, mathematics and science. This stage is the "student's performance efficiency" stage. The results revealed that restrictions should be used with caution because it is possible to underestimate the true efficiency scores based on biased assumptions. In the presence of reliable prior information the model is suitable for policy making, however, if this information is neglected the resulting efficiencies may be overestimated.

Chapter 5 cope with another issue of the relational models, the time-dependent efficiency measurement. Specifically, the next chapter provides the mathematical formulation of the window-based LP problem for the multiplicative and the additive two-stage DEA model.

Chapter 5

Relational window-based two-stage DEA approach

5.1. Introduction

Every two-stage model presented so far concerns DMUs in a single time period where the available data is cross-sectional. However, in the presence of panel data the time component must be considered for the evaluation of the efficiency. The examination of the performance of DMUs over multiple periods can assist the decision maker to draw important conclusion. There are two widely used ways to evaluate the efficiency of DMUs over time using DEA models; Malmquist Productivity Index and Window analysis.

Malmquist Productivity Index evaluates the productivity change of a DMU between two time periods and is defined as the product of catch-up effect and technical change. The catch-up effect measures the ability of the DMU to increase its efficiency while the technical change shows the change of the efficient frontier between the two time periods. Window Analysis is based on moving average and compares the efficiency of a DMU with its own efficiency over other periods and the efficiency of other DMUs over the same periods. This Chapter uses the window analysis approach.

Furthermore, the efficiency analysis of the multi-period two-stage DEA models is fairly recent. Ho et al. (2014) and Wang et al. (2014a) applied window analysis in order to measure the efficiency of universities and commercial banks respectively using the additive decomposition approach of Chen et al. (2009a). Kao and Hwang (2014) used the multiplicative decomposition approach of Kao and Hwang (2008) in order to examine the non-life insurance companies in Taiwan and treated different time periods in a parallel network system where each individual period is a different subsystem. According to Kao and Hwang (2014) this approach investigates the effect of each individual period on the overall performance of a two-stage structured DMU.

Similar with the previous Chapter, this Chapter adopts relational two-stage DEA models in order to evaluate both the overall and the individual efficiencies. Specifically, the multiplicative two-stage DEA model (3.22) and the additive two-stage DEA model (3.27) are adopted and they are properly modified for the needs of window analysis. The contribution of this Chapter is the extension of the multiplicative model of Kao and Hwang (2008) to window analysis. In addition, building upon the works of Ho et al. (2014) and

Wang et al. (2014a), this Chapter provides the mathematical formulation of the window-based additive model of Chen et al. (2009a).

This Chapter is structured as follows. Section 5.2 constructs the mathematical formulations for the window-based relational two-stage DEA models (both multiplicative and additive). Section 5.3 applies the window-based relational models on banking systems across OECD countries and Section 5.4 concludes.

5.2. Window analysis in relational two-stage DEA models

Charnes and Cooper (1985) introduced DEA window analysis which based on the principle of moving averages in order to measure efficiency in cross-sectional data over time. Asmild et al. (2004) suggested that by comparing the performance of a DMU against its own performance over other periods and against the performance of the other DMUs provides a useful tool to detect efficiency trends over time. As a moving average procedure it requires a sliding window to be defined which is the number of periods included in the analysis every time. According to Asmild et al. (2004) there are no technical changes within each of the windows because all DMUs in each window are measured against each other. In addition, the authors recommend a narrow window width in order to yield credible results.

This Chapter adopts the notation of Asmild et al. (2004) and after modifying it for the needs of a two-stage analysis, considers *n* DMUs $(j = 1, \dots, n)$ for *T* periods $(t = 1, \dots, T)$ and $x_t^j = (x_{1t}^j, x_{2t}^j, \dots, x_{mt}^j)'$, $z_t^j = (z_{1t}^j, z_{2t}^j, \dots, z_{Dt}^j)'$ and $y_t^j = (y_{1t}^j, y_{2t}^j, \dots, y_{st}^j)'$ are the *i*-dimensional input vector $(i = 1, \dots, m)$, the *d*-dimensional intermediate measure vector $(d = 1, \dots, D)$ and the *r*-dimensional output vector $(r = 1, \dots, s)$ respectively of the *j*th DMU $(j = 1, \dots, n)$ at time t.

Then a window k_w with $n \times w$ observations is denoted starting at time *k*, $1 \leq k \leq T$ width *w*, $1 \leq w \leq T - k$. The matrix of inputs is given as:

$$X_{k_w} = (x_k^1, x_k^2, \dots, x_k^n, x_{k+1}^1, x_{k+1}^2, \dots, x_{k+1}^n, \dots, x_{k+w}^1, x_{k+w}^2, \dots, x_{k+w}^n)$$

the matrix of intermediate variables is given as:

$$Z_{k_w} = (z_k^1, z_k^2, \dots, z_k^n, z_{k+1}^1, z_{k+1}^2, \dots, z_{k+1}^n, \dots, z_{k+w}^1, z_{k+w}^2, \dots, z_{k+w}^n)$$

and the matrix of outputs is given as:

$$Y_{k_w} = (y_k^1, y_k^2, \dots, y_k^n, y_{k+1}^1, y_{k+1}^2, \dots, y_{k+1}^n, \dots, y_{k+w}^1, y_{k+w}^2, \dots, y_{k+w}^n)$$

The multiplicative two-stage window DEA model for the *j*th DMU at time *t* will be the following:

$$E_{k_w t} = \max \gamma \cdot y_t' \tag{5.1}$$

s.t.

$$\omega \cdot x_t' = 1$$

$$\Gamma \cdot Y_{k_w} - \Omega \cdot X_{k_w} \leq 0$$

$$M \cdot Z_{k_w} - \Omega \cdot X_{k_w} \leq 0$$

$$\Gamma \cdot Y_{k_w} - M \cdot Z_{k_w} \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n \times w;\; i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

where γ, μ and ω are the vectors which contains the multipliers γ_r, μ_d and ω_i for the DMU under assessment in period t and Γ, M and Ω are the vectors which contains γ, μ and ω for every DMU in every period in the window k_w. The first stage efficiency of the multiplicative window model is as follows:

$$E_{k_w t}^1 = \max \mu \cdot z_t' \tag{5.2}$$

s.t.

$$\omega \cdot x_t' = 1$$

$$\gamma \cdot y_t' - E_{k_w} \cdot \omega \cdot x_t' = 0$$

$$\Gamma \cdot Y_{k_w} - \Omega \cdot X_{k_w} \leq 0$$

$$M \cdot Z_{k_w} - \Omega \cdot X_{k_w} \leq 0$$

$$\Gamma \cdot Y_{k_w} - M \cdot Z_{k_w} \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n \times w;\; i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

and then the second stage efficiency is:

$$E_{k_w t}^2 = \frac{E_{k_w t}}{E_{k_w t}^1} \tag{5.3}$$

Similarly, the additive two-stage window DEA model for the *j*th DMU at time *t* will be the following:

$$E_{k_w t} = \max \mu \cdot z_t' + \gamma \cdot y_t' \tag{5.4}$$

s.t.

$$\omega \cdot x_t' + \mu \cdot z_t' = 1$$

$$M \cdot Z_{k_w} - \Omega \cdot X_{k_w} \leq 0$$

$$\Gamma \cdot Y_{k_w} - M \cdot Z_{k_w} \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n \times w;\; i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

and the first stage efficiency of the additive window model is as follows:

$$E_{k_w t}^1 = \max \mu \cdot z_t' \tag{5.5}$$

s.t.

$$\omega \cdot x_t' = 1$$

$$(1 - E_{k_w}) \cdot \mu \cdot Z_{k_w} - \gamma \cdot Y_{k_w} = E_{k_w}$$

$$M \cdot Z_{k_w} - \Omega \cdot X_{k_w} \leq 0$$

$$\Gamma \cdot Y_{k_w} - M \cdot Z_{k_w} \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n \times w;\; i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

and then the second stage efficiency is:

$$E_{k_w t}^2 = \frac{E_{k_w t} - \xi_1^* \cdot E_{k_w t}^1}{\xi_2^*} \tag{5.6}$$

where ξ_1^* and ξ_2^* are the optimal weights from model (5.4) computed in a similar manner as in (3.26).

5.3. Application to banking systems across countries

This Section creates a two-stage efficiency index in order to evaluate the banking systems in 17 OECD countries (*n*=17). The first stage measures the "value added activity" and the second stage measures the "profitability" of the banking system. This is the first time that a two-stage DEA model is applied at cross-country banking systems. Relational window-based models (5.1) and (5.4) extend the analysis for the time period 1999—2009 (*T*=11). Following Asmild et al. (2004) a 3-year window has been chosen for the analysis (*w*=3). Specifically, the first window the analysis contains the years 1999, 2000 and 2001 therefore the number of DMUs the model is 51 ($n \times w = 17 \times 3$). Then the second window moves one year forward including 2002 and appending 1999 and the procedure moves on until the last window. The overall procedure includes 9 windows and 459 different DMUs.

5.3.1. Efficiency in banking industry

The assessment of banking efficiency has been a popular issue over the past years. Earlier studies used financial ratios which measure the performance of banks in one dimension at a time; e.g. ROA indicates the level of profitability of a bank relative to its assets. Financial ratios can provide useful information however they do not provide an adequate measure of banking efficiency. An efficiency measure for banking industry should be multi-dimensional since banks are complex organizations employing multiple inputs to produce multiple outputs and DEA is an excellent tool for this purpose (Halkos and Tzeremes, 2013a). Furthermore, Tzeremes (2015) marked the significant managerial implications which can be exploited from measuring bank efficiency in a DEA framework. Berger & Humphrey (1997) provided an extensive literature of 130 studies in banking efficiency measurement, half of which used DEA approach.

Although a lot of studies investigated the efficiency of banking institutions, only a small fraction of these dealt with the efficiency of banking systems across countries. In a novel study, Berg et al. (1993) used DEA to evaluate the efficiency of the banking systems in Norway, Finland and Sweden. Fecher and Pestieau (1993) measured the cross-country banking efficiency in eleven OECD countries. Allen and Rai (1996) and Pastor et al. (1997) used DEA to assess the banking efficiency of fifteen and eight developed countries respectively. The vast majority of the existing studies examined the European banking industry (Bonin et al., 2005; Fries and Taci, 2005; Maudos and de Guevara, 2007; Weill, 2004, 2009).

Two-stage DEA studies are becoming very popular especially for analyzing the efficiency levels of banking institutions. Wang et al. (1997) constructed a model which measures the information technology-related activity in the first stage and the loan processing system in the second stage of 22 banks. A lot of studies have also used the same data set however with different modeling formulations; namely the connected value chain model (Chen and Zhu, 2004; Rho and An, 2007; Liu and Lu, 2012), the network DEA model with shared resources (Chen et al. 2006a), the relational network DEA model (Chen et al., 2010; Kao and Hwang, 2010) and the cooperative and the non-cooperative game theoretic DEA models (Liang et al., 2008). Seiford & Zhu (1999) evaluated the profitability and marketability of 55 US commercial banks. Liang et al. (2008) applied a cooperative and a non-cooperative DEA model at the same data but for only 30 banks. The same reduced data set has also been used by Zha and Liang (2010) for the needs of their cooperative multiplicative network DEA model and from Du et al. (2011) who constructed a Nash bargaining two-stage DEA model.

Alternative two-stage DEA formulations and approaches have been used in order to study banking efficiency in various real life case studies. Luo (2003) applied an independent two-stage DEA model at 245 large banks and measured the profitability and marketability in the two stages respectively. Mukherjee et al. (2003) used a static network DEA model and examined the quality efficiency and the profitability efficiency for the case of 27 Indian public sector banks. Ho and Zhu (2004) studied the case of 41 Taiwanese commercial banks using an independent two-stage DEA model measuring the operating efficiency in the first stage and the operating effectiveness in the second stage. Fukuyama and Weber (2010) constructed a slacks-based network DEA model to measure the value-added activity in the first stage and the profitability in the second stage of Japanese banks.

Fukuyama and Matousek (2011) proposed a static network DEA model in order to examine the value-added activity and the profitability of 25 Turkish commercial banks. . Tsolas (2011) evaluated 13 commercial banks of Athens stock exchange in terms of profitability in the first stage and performance in stock market in the second stage. Akther et al. (2013) investigated 19 private commercial banks and 2 government-owned in Bangladesh. Their model examined the value added activity in the first stage and the profit generation in the second stage. Wanke and Barros (2014) adapted the centralized approach of Liang et al. (2008) to investigate the cost efficiency and the productive efficiency in major Brazilian banks. Wang et al. (2014b) combined a relational model with fuzzy multi-objective approach to study the US bank holding companies. Specifically, the authors assessed the profitability and the value creativity in the first and second stage respectively.

A number of two-stage DEA studies have also examined the efficiency of bank branches. Cook et al. (2000) investigated the efficiency of bank branches in a major Canadian bank measuring the sales efficiency in the first stage and the services efficiency in the second stage. The authors used a network DEA model with shared resources for the needs of their study. Meepadung et al. (2009) used an independent two-stage DEA model to assess the operating and the profit efficiency of 6 branches of a major Taiwanese bank. Tsolas (2010) also used an independent two-stage DEA and studied the case of bank branches in a major Greek bank. Yang et al. (2011) utilized a connected value-chain two-stage DEA model to measure the fund collection and profit generation of 17 branches of China Construction Bank. Zhou et al. (2013) investigated 10 branches of China Construction Bank and measured the operational efficiency and profitability in the first and the second stage respectively. They presented a multiplicative cooperative Nash bargaining two-stage DEA model. Alternatively, Naini et al. (2013) introduced a multiplicative non-cooperative Nash bargaining two-stage DEA model to evaluate the profitability and marketability of 35 Iranian bank branches.

5.3.2. *Inputs and outputs*

One controversial discussion about banking efficiency is the specification of deposits; whether they are inputs or outputs. Berger and Humphrey (1992) presented three approaches about banking efficiency. The asset or intermediation approach considers banks as intermediaries in the financial process which use liabilities (e.g. deposits) in order to produce earning assets (e.g. loans and securities). The value added or production approach considers all financial products with a value added for the bank as outputs (e.g. deposits, loans). The user cost approach considers a financial product as an input or output according to its contribution into bank revenue. If the cost of the financial product (e.g. deposits) is lower that the opportunity cost then it is considered as output while if this is not the case it is considered as input. Berger and Humphrey (1992) argued that deposits have both input and output characteristics.

An interesting alternative is to consider loanable funds (like deposits) as an intermediate variable in a two-stage process; in the first stage the bank consumes inputs to produce deposits and in the second stage the bank uses deposits to produce earning assets (Fukuyama and Weber, 2010; Fukuyama and Matousek, 2011; Holod & Lewis, 2011). This approach insures that the dual role of deposits will be kept intact. This Section adopts the later approach and treats deposits as intermediate variables in a two-stage process. This approach perfectly matches the view of Sealey and Lindley (1977) about banking process where banks are multistage entities which use labor, capital and other inputs to obtain loanable funds which then utilize to produce earning assets.

Furthermore, the Section adopts a similar specification for inputs-outputs with Fukuyama and Matousek (2011) and Holod and Lewis (2011). The first stage measures the "value added activity" and the second stage measures the "profitability" of the banking system. Specifically, the proposed model employs two inputs: total number of employees and total fixed assets. Furthermore, two intermediate variables are considered: interbank deposits and customer deposits. Last, two outputs are used: loans and securities. All variables except labor are measured in millions of dollars and the data has been obtained from the OECD[5]. The model is input-oriented and first stage has been given pre-emptive priority because banks have greater control over their inputs compared to their outputs. In addition, the dataset consists of developed countries which are assumed to experience similar technological framework, therefore the CRS version of the model is adopted. However, the model can easily be extended to VRS. Descriptive statistics are presented in Table 5.1.

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[55]The data have been obtained from the OECD database on 'Bank Profitability' and are available only for the period 1999-2009. The data are available from: https://stats.oecd.org/Index.aspx?DataSetCode=BPF1

Table 5.1: Descriptive statistics

		1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Total number of employees	**Mean**	271587.12	273128.88	276452.65	277165.74	274413.03	275578.68	279992.83	286950.21	291764.24	289678.65	279701.47
	Stdev	500593.90	504417.89	519227.09	530567.68	537407.59	549189.16	563176.28	579383.39	583000.99	568694.08	548286.50
	Min	4462	4663	3949	3934	4280	4455	5025	5681	6286	6132	5693
	Max	2078902	2093973	2158815	2210997	2242872	2299508	2361370	2433386	2450506	2391916	2302628
Total fixed assets	**Mean**	189569.97	203049.56	217032.72	214946.98	211899.36	238595.55	278909.84	296304.16	344634.85	421711.95	361927.15
	Stdev	294701.14	314402.05	341948.55	347586.30	361265.41	417274.00	451435.89	511444.37	576553.39	610851.95	565742.11
	Min	410.18	356.91	278.34	267.18	214.56	212.58	233.34	247.98	384.48	172.82	131.63
	Max	1010651.39	1082632.77	1237903.58	1256759.90	1322716.23	1555176.50	1633236.25	1841127.39	2066776.01	2178808.61	1973712.61
Interbank deposits	**Mean**	346377.37	361885.10	350177.99	348450.85	353627.93	365079.12	407274.45	440330.93	481427.01	488946.93	429338.86
	Stdev	545749.35	564525.52	557719.80	551609.12	528807.52	521758.10	555448.55	593852.57	664766.29	645236.06	564225.02
	min	634.76	609.10	576.07	1035.79	422.12	644.38	642.19	212.36	282.96	79.52	173.84
	max	1757463.26	1903412.67	2006074.15	1961860.50	1838520.14	1904556.70	1885119.00	1898611.97	2243481.20	2300687.07	2018347.13
Customer deposits	**mean**	789058.56	829616.57	913882.14	929697.09	959634.72	1016869.09	1098459.38	1161777.24	1230931.12	1289076.41	1297326.20
	stdev	1196077.73	1266975.56	1448218.41	1501431.95	1553703.75	1661719.21	1751237.16	1837341.17	1918985.17	2021730.02	2049166.39
	min	2693.64	3381.86	3899.88	4257.82	4487.39	5203.35	7210.47	8556.24	8904.47	8869.48	9350.94
	max	4663845.76	4915643.66	5728502.57	6036054.88	6286082.80	6753772.70	7128969.81	7500497.03	7810358.73	8287829.37	8445384.27
Loans	**mean**	884672.17	961235.11	985400.34	1002632.64	1028023.90	1078304.80	1192669.54	1282564.43	1389715.27	1387644.48	1315131.90
	stdev	1338327.80	1439445.76	1453165.34	1481238.32	1526088.70	1639802.79	1753771.88	1840158.73	1952263.66	1899434.28	1751063.31
	min	2638.60	3279.43	3664.03	4314.71	5754.45	7376.93	9456.42	12312.14	14830.41	15250.20	13818.24
	max	5164436.66	5513858.25	5539834.98	5759483.84	6016062.50	6605290.99	7066849.14	7442617.02	7894698.60	7686380.36	7034923.54
Securities	**mean**	378667.78	395211.13	415706.83	428432.94	446422.15	470587.99	506925.97	561757.24	589280.58	541951.58	564011.28
	stdev	525512.11	560224.89	593878.57	641320.38	660195.95	670536.48	688465.57	749833.92	791131.02	783388.34	808214.62
	min	789.83	809.55	988.98	1230.69	804.50	859.37	962.24	1230.45	1270.92	1013.24	1805.31
	max	1814997.22	1882592.88	2004596.56	2282119.48	2386297.65	2384874.99	2325832.00	2463103.28	2595068.02	2759820.27	2980969.52

5.3.3. Results

Tables 5.2 and 5.3 examine the efficiencies over time by applying the window-based relational two-stage DEA models (5.1) and (5.4) for the case of USA as an illustrative example. The results can be read in two ways, by rows and by columns. The rows indicate the trend as well as the behavior across the same data set (the same window), while the column indicate the stability of the efficiency for a specific year across different data sets (different windows). Considering the above, the efficiency scores seem to be stable across different data sets and also appear to slightly decline over the years.

Table 5.2: A three-year window analysis of overall, first stage and second stage efficiencies of the multiplicative model of Kao and Hwang (2008) for the case of USA.

Overall	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
W1	0.310	0.311	0.288								
W2		0.295	0.269	0.287							
W3			0.260	0.277	0.277						
W4				0.282	0.283	0.260					
W5					0.273	0.255	0.249				
W6						0.250	0.248	0.240			
W7							0.243	0.235	0.229		
W8								0.236	0.230	0.226	
W9									0.247	0.246	0.262
Averages	0.310	0.303	0.272	0.282	0.278	0.255	0.247	0.237	0.235	0.236	0.262
1st stage											
W1	0.417	0.417	0.436								
W2		0.404	0.416	0.431							
W3			0.378	0.392	0.391						
W4				0.387	0.335	0.317					
W5					0.285	0.275	0.279				
W6						0.394	0.399	0.384			
W7							0.396	0.382	0.368		
W8								0.384	0.369	0.384	
W9									0.327	0.334	0.385
Averages	0.417	0.411	0.410	0.404	0.337	0.328	0.358	0.383	0.355	0.359	0.385
2nd stage											
W1	0.745	0.745	0.661								
W2		0.731	0.647	0.665							
W3			0.686	0.707	0.710						
W4				0.730	0.845	0.822					
W5					0.958	0.928	0.893				
W6						0.635	0.622	0.624			
W7							0.613	0.614	0.624		
W8								0.614	0.624	0.589	
W9									0.754	0.736	0.680
Averages	0.745	0.738	0.665	0.701	0.837	0.795	0.709	0.617	0.667	0.663	0.680

Table 5.3: A three-year window analysis of overall, first stage and second stage efficiencies of the additive model of Chen et al. (2009a) for the case of USA.

Overall	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
W1	0.514	0.514	0.504								
W2		0.502	0.486	0.502							
W3			0.471	0.487	0.486						
W4				0.487	0.487	0.462					
W5					0.472	0.452	0.453				
W6						0.465	0.466	0.454			
W7							0.459	0.448	0.438		
W8								0.448	0.438	0.441	
W9									0.452	0.454	0.481
Averages	0.514	0.508	0.487	0.492	0.482	0.460	0.459	0.450	0.443	0.448	0.481
1st stage											
W1	0.423	0.417	0.436								
W2		0.415	0.426	0.442							
W3			0.406	0.421	0.419						
W4				0.419	0.418	0.390					
W5					0.401	0.381	0.385				
W6						0.406	0.411	0.395			
W7							0.403	0.389	0.374		
W8								0.386	0.371	0.386	
W9									0.401	0.394	0.433
Averages	0.423	0.416	0.423	0.427	0.413	0.392	0.400	0.390	0.382	0.390	0.433
2nd stage											
W1	0.728	0.745	0.661								
W2		0.711	0.626	0.638							
W3			0.633	0.646	0.648						
W4				0.650	0.652	0.644					
W5					0.649	0.640	0.629				
W6						0.610	0.600	0.601			
W7							0.599	0.601	0.610		
W8								0.609	0.618	0.584	
W9									0.578	0.606	0.592
Averages	0.728	0.728	0.640	0.645	0.650	0.632	0.609	0.604	0.602	0.595	0.592

Tables 5.4 and 5.5 provide the average values of each year for the overall efficiencies, the “value added activity” efficiencies and the “profitability” efficiencies. The efficiency scores for every stage reveal large discrepancies among countries. Regarding the multiplicative model, the overall efficiency ranges from 0.259 in Slovak Republic to 0.939 in Belgium in 1999 and from 0.194 in Italy to 0.871 in Norway in 2009. The “value-added activity” efficiency ranges from 0.291 in Italy to 1.000 in Austria and Switzerland in 1999 and from 0.253 in Italy to 1.000 in Estonia, Norway and Switzerland in 2009. The “profitability efficiency ranges from 0.434 in Slovak Republic to 0.978 in Belgium in 1999

and from 0.594 in Slovak Republic to 1.000 in Denmark in 2009. Regarding the additive model, the overall efficiency ranges from 0.442 in Italy to 0.969 in Belgium in 1999 and from 0.361 in Italy to 0.935 in Norway in 2009. The "value-added activity" efficiency ranges from 0.294 in Italy to 1.000 in Austria and Switzerland in 1999 and from 0.263 in Italy to 1.000 in Norway in 2009. The "profitability efficiency ranges from 0.434 in Slovak Republic to 0.978 in Belgium in 1999 and from 0.523 in Slovak Republic to 1.000 in Denmark in 2009. The gap between the countries appears to slightly widen in respect to the overall and the first stage efficiencies and slightly close in respect to the second stage efficiency. However, a gap nearly up to 70% in some cases is indicative of the large discrepancies. The results are in line with Lozano-Vivas et al. (2001) and Weill (2009) who also found large discrepancies in their studies. Furthermore, the average inefficiency is relatively high which is in accordance with previous studies (Chortareas et al., 2012; Fethi and Pasiouras, 2010).

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Table 5.4: Overall, first and second stage efficiencies (average values obtained by two-stage multiplicative DEA window analysis)

Overall efficiency

	Austria	Belgium	Denmark	Estonia	Finland	France	Germany	Italy	Korea, Rep	Netherlands	Norway	Poland	Slovak Republic	Spain	Sweden	Switzerland	USA
1999	0.836	0.939	0.442	0.294	0.418	0.515	0.640	0.279	0.382	0.845	0.789	0.297	0.259	0.468	0.364	0.684	0.310
2000	0.863	0.899	0.443	0.370	0.458	0.514	0.649	0.283	0.442	0.852	0.801	0.248	0.222	0.446	0.407	0.642	0.303
2001	0.775	0.879	0.496	0.504	0.455	0.483	0.713	0.267	0.448	0.826	0.829	0.198	0.187	0.467	0.405	0.642	0.272
2002	0.801	0.742	0.432	0.547	0.477	0.454	0.801	0.239	0.446	0.865	0.735	0.191	0.192	0.501	0.375	0.563	0.282
2003	0.724	0.670	0.574	0.624	0.243	0.433	0.821	0.207	0.406	0.906	0.612	0.237	0.545	0.544	0.349	0.538	0.278
2004	0.812	0.597	0.444	0.674	0.241	0.399	0.772	0.187	0.463	0.926	0.771	0.165	0.606	0.563	0.372	0.503	0.255
2005	0.772	0.583	0.308	0.671	0.247	0.409	0.808	0.178	0.549	0.856	0.848	0.176	0.541	0.500	0.411	0.512	0.247
2006	0.773	0.550	0.505	0.722	0.251	0.393	0.788	0.178	0.556	1.000	0.850	0.171	0.371	0.506	0.409	0.486	0.237
2007	0.787	0.559	0.461	0.647	0.266	0.415	0.769	0.183	0.527	0.986	0.770	0.155	0.466	0.520	0.444	0.510	0.235
2008	0.660	0.561	0.359	0.799	0.261	0.394	0.631	0.188	0.407	0.664	0.606	0.210	0.423	0.435	0.387	0.358	0.236
2009	0.666	0.570	0.440	0.740	0.212	0.395	0.711	0.194	0.522	0.655	0.871	0.278	0.528	0.536	0.455	0.661	0.262

First stage efficiency

	Austria	Belgium	Denmark	Estonia	Finland	France	Germany	Italy	Korea, Rep	Netherlands	Norway	Poland	Slovak Republic	Spain	Sweden	Switzerland	USA
1999	1.000	0.960	0.556	0.479	0.550	0.803	0.792	0.291	0.601	0.962	0.944	0.492	0.598	0.755	0.501	1.000	0.417
2000	0.994	0.899	0.501	0.650	0.591	0.670	0.768	0.283	0.764	0.971	0.924	0.404	0.513	0.717	0.563	0.898	0.411
2001	0.942	0.887	0.509	0.867	0.567	0.623	0.847	0.267	0.781	0.955	0.963	0.351	0.307	0.754	0.564	0.900	0.410
2002	0.934	0.783	0.447	0.831	0.606	0.599	0.931	0.241	0.783	0.967	0.869	0.285	0.248	0.782	0.528	0.803	0.404
2003	0.850	0.763	0.574	0.836	0.329	0.533	0.963	0.207	0.700	1.000	0.690	0.295	0.545	0.778	0.467	0.778	0.337
2004	0.913	0.770	0.462	0.851	0.312	0.490	0.940	0.190	0.707	1.000	0.894	0.243	0.863	0.779	0.475	0.738	0.328
2005	0.870	0.843	0.382	0.955	0.311	0.513	1.000	0.184	0.847	0.971	0.967	0.289	0.992	0.709	0.499	0.763	0.358
2006	0.862	0.796	0.557	0.978	0.327	0.458	0.993	0.186	0.879	1.000	0.998	0.308	0.669	0.788	0.496	0.678	0.383
2007	0.901	0.746	0.538	0.776	0.339	0.417	0.986	0.183	0.756	1.000	0.884	0.289	0.810	0.783	0.582	0.768	0.355
2008	0.814	0.708	0.385	0.928	0.349	0.447	0.849	0.233	0.447	0.936	0.740	0.346	0.942	0.650	0.436	0.620	0.359
2009	0.897	0.689	0.440	1.000	0.282	0.467	0.946	0.253	0.639	0.872	1.000	0.465	0.889	0.726	0.546	1.000	0.385

Second stage efficiency

	Austria	Belgium	Denmark	Estonia	Finland	France	Germany	Italy	Korea, Rep	Netherlands	Norway	Poland	Slovak Republic	Spain	Sweden	Switzerland	USA
1999	0.836	0.978	0.796	0.613	0.760	0.642	0.808	0.957	0.635	0.879	0.835	0.604	0.434	0.620	0.728	0.684	0.745
2000	0.868	1.000	0.885	0.569	0.775	0.777	0.845	1.000	0.578	0.878	0.866	0.616	0.434	0.621	0.723	0.717	0.738
2001	0.823	0.990	0.975	0.581	0.802	0.777	0.842	1.000	0.576	0.865	0.861	0.567	0.631	0.620	0.717	0.712	0.665
2002	0.858	0.945	0.964	0.665	0.788	0.759	0.861	0.992	0.571	0.895	0.845	0.681	0.775	0.642	0.713	0.703	0.701
2003	0.852	0.878	1.000	0.747	0.740	0.812	0.853	0.997	0.580	0.906	0.886	0.804	1.000	0.699	0.747	0.692	0.837
2004	0.889	0.772	0.961	0.791	0.771	0.817	0.821	0.985	0.657	0.926	0.863	0.691	0.701	0.722	0.782	0.678	0.795
2005	0.887	0.700	0.803	0.701	0.792	0.810	0.808	0.963	0.650	0.880	0.879	0.621	0.545	0.706	0.823	0.671	0.709
2006	0.897	0.711	0.908	0.738	0.768	0.891	0.793	0.955	0.633	1.000	0.852	0.554	0.555	0.642	0.825	0.716	0.617
2007	0.874	0.750	0.858	0.834	0.786	0.995	0.779	1.000	0.708	0.986	0.871	0.535	0.576	0.663	0.762	0.665	0.667
2008	0.817	0.792	0.934	0.861	0.748	0.881	0.744	0.808	0.920	0.709	0.819	0.607	0.450	0.669	0.887	0.578	0.663
2009	0.743	0.827	1.000	0.740	0.754	0.846	0.752	0.768	0.816	0.752	0.871	0.597	0.594	0.738	0.834	0.661	0.680

Table 5.5: Overall, first and second stage efficiencies (average values obtained by two-stage additive DEA window analysis).

Overall efficiency

	Austria	Belgium	Denmark	Estonia	Finland	France	Germany	Italy	Korea, Rep	Netherlands	Norway	Poland	Slovak Republic	Spain	Sweden	Switzerland	USA
1999	0.918	0.969	0.642	0.526	0.624	0.732	0.799	0.442	0.619	0.921	0.891	0.529	0.536	0.697	0.576	0.842	0.514
2000	0.931	0.947	0.629	0.618	0.660	0.722	0.801	0.441	0.685	0.925	0.896	0.468	0.488	0.677	0.621	0.811	0.508
2001	0.884	0.933	0.666	0.733	0.652	0.684	0.844	0.420	0.691	0.910	0.913	0.421	0.407	0.696	0.619	0.809	0.487
2002	0.897	0.849	0.605	0.748	0.674	0.658	0.897	0.385	0.685	0.931	0.858	0.388	0.392	0.721	0.587	0.752	0.492
2003	0.851	0.812	0.730	0.791	0.430	0.636	0.908	0.342	0.650	0.953	0.770	0.431	0.709	0.744	0.556	0.739	0.482
2004	0.901	0.771	0.620	0.820	0.422	0.609	0.882	0.317	0.691	0.963	0.879	0.347	0.786	0.754	0.589	0.713	0.460
2005	0.878	0.782	0.500	0.831	0.427	0.623	0.904	0.307	0.758	0.926	0.923	0.371	0.769	0.707	0.630	0.730	0.459
2006	0.878	0.771	0.682	0.859	0.436	0.616	0.894	0.307	0.766	1.000	0.925	0.371	0.624	0.724	0.635	0.718	0.450
2007	0.888	0.767	0.652	0.801	0.452	0.628	0.884	0.309	0.738	0.993	0.878	0.346	0.707	0.731	0.653	0.728	0.443
2008	0.816	0.746	0.537	0.895	0.453	0.620	0.801	0.350	0.636	0.828	0.773	0.413	0.708	0.657	0.598	0.606	0.448
2009	0.829	0.745	0.611	0.870	0.395	0.607	0.851	0.361	0.719	0.816	0.935	0.507	0.757	0.731	0.648	0.830	0.481

First stage efficiency

	Austria	Belgium	Denmark	Estonia	Finland	France	Germany	Italy	Korea, Rep	Netherlands	Norway	Poland	Slovak Republic	Spain	Sweden	Switzerland	USA
1999	1.000	0.960	0.556	0.492	0.550	0.810	0.792	0.294	0.626	0.962	0.944	0.492	0.598	0.755	0.501	1.000	0.423
2000	0.994	0.899	0.503	0.650	0.591	0.781	0.768	0.283	0.781	0.971	0.924	0.431	0.526	0.717	0.563	0.898	0.416
2001	0.942	0.887	0.511	0.888	0.567	0.672	0.847	0.267	0.807	0.955	0.963	0.393	0.380	0.760	0.566	0.900	0.423
2002	0.939	0.783	0.449	0.836	0.606	0.633	0.931	0.241	0.783	0.967	0.869	0.335	0.341	0.793	0.529	0.803	0.427
2003	0.850	0.763	0.574	0.845	0.329	0.573	0.963	0.207	0.700	1.000	0.690	0.372	0.618	0.785	0.467	0.778	0.413
2004	0.913	0.770	0.462	0.851	0.314	0.543	0.946	0.191	0.751	1.000	0.894	0.289	0.865	0.779	0.535	0.748	0.392
2005	0.870	0.930	0.387	0.955	0.318	0.572	1.000	0.186	0.884	0.971	0.967	0.323	0.992	0.709	0.602	0.815	0.400
2006	0.862	1.000	0.557	0.978	0.327	0.597	0.993	0.186	0.895	1.000	0.998	0.322	0.681	0.790	0.661	0.856	0.390
2007	0.901	1.000	0.555	0.776	0.339	0.621	0.986	0.183	0.854	1.000	0.884	0.294	0.846	0.793	0.611	0.816	0.382
2008	0.915	0.762	0.385	0.928	0.349	0.645	0.849	0.249	0.668	0.970	0.740	0.346	0.979	0.650	0.538	0.631	0.390
2009	0.957	0.689	0.440	1.000	0.303	0.605	0.946	0.263	0.854	0.913	1.000	0.465	0.988	0.726	0.547	1.000	0.433

Second stage efficiency

	Austria	Belgium	Denmark	Estonia	Finland	France	Germany	Italy	Korea, Rep	Netherlands	Norway	Poland	Slovak Republic	Spain	Sweden	Switzerland	USA
1999	0.836	0.978	0.796	0.594	0.760	0.636	0.808	0.942	0.607	0.879	0.835	0.604	0.434	0.620	0.728	0.684	0.728
2000	0.868	1.000	0.880	0.569	0.775	0.647	0.845	1.000	0.561	0.878	0.866	0.555	0.415	0.621	0.723	0.717	0.728
2001	0.823	0.990	0.970	0.562	0.802	0.702	0.842	0.996	0.553	0.865	0.861	0.494	0.478	0.612	0.714	0.712	0.640
2002	0.852	0.945	0.961	0.661	0.788	0.703	0.861	0.992	0.571	0.895	0.845	0.551	0.545	0.632	0.711	0.703	0.645
2003	0.852	0.878	1.000	0.736	0.740	0.747	0.853	0.997	0.580	0.906	0.886	0.592	0.859	0.693	0.747	0.692	0.650
2004	0.889	0.772	0.961	0.791	0.764	0.734	0.815	0.977	0.612	0.926	0.863	0.547	0.699	0.722	0.689	0.666	0.632
2005	0.887	0.622	0.792	0.701	0.770	0.711	0.808	0.950	0.617	0.880	0.879	0.521	0.545	0.706	0.677	0.626	0.609
2006	0.897	0.543	0.908	0.738	0.768	0.648	0.793	0.955	0.621	1.000	0.852	0.522	0.541	0.641	0.597	0.556	0.604
2007	0.874	0.534	0.829	0.834	0.786	0.639	0.779	1.000	0.603	0.986	0.871	0.524	0.543	0.653	0.722	0.622	0.602
2008	0.708	0.730	0.934	0.861	0.748	0.580	0.744	0.754	0.590	0.681	0.819	0.607	0.431	0.669	0.707	0.567	0.595
2009	0.696	0.827	1.000	0.740	0.698	0.611	0.752	0.735	0.561	0.710	0.871	0.597	0.523	0.738	0.833	0.661	0.592

The interpretation of the results for all countries across eleven years is difficult, so in order to facilitate the comprehension of the results the average efficiency over time (1999-2009) for each country along with the average annual growth are provided in Tables 5.6 and 5.7. A careful examination of the average annual growth scores for both models and for every stage reveals relatively stable efficiency scores over time with slightly positive or negative changes. In respect to the multiplicative model in Table 5.6, Netherlands achieves the highest overall efficiency score (0.853) while Poland achieves the lowest score (0.211). In addition, Norway (0.771), Austria (0.770) and Germany (0.737) also achieve high scores. Seven countries experienced a negative average annual growth while ten countries experienced positive growth. The largest percentage change (16.7%) is attributed to Slovak Republic. Similarly, considering the "value added activity" efficiencies, Netherlands (0.967), Germany (0.911), Austria (0.907) and Norway (0.898) achieve the highest scores while Italy achieves the lowest score (0.229). Positive growth is observed for nine countries and negative growth for eight countries. Again, Slovak Republic experienced the largest percentage change (11.9%). The largest efficiency in "profitability" stage is achieved by Italy (0.948) with Denmark (0.917), Netherlands (0.880) and Norway (0.859) also to achieve high scores. Slovak Republic achieved the lowest score (0.609) and also the biggest change in average annual growth (6.1%). Positive growth is observed for ten countries while negative growth for seven countries.

Similarly Table 5.7 presents the results of the window additive two-stage DEA model. Then, they are compared with the results obtained from multiplicative model in Table 5.6. Following Chen et al. (2009a), the rankings of the two models are compared because direct comparisons of the efficiency scores among different models may not yield reliable results. The average annual growth rates are also compared. The rankings appear to be quite similar with in Tables 5.6 and 5.7. In respect to the overall efficiencies for the additive model, the Netherlands achieves the highest overall efficiency score (0.924) while Italy achieves the lowest score (0.362). In addition, Austria (0.879), Germany (0.876) and Norway (0.860) also achieve high scores. Seven countries experienced a negative average annual growth while ten countries experienced positive growth. Similarly, considering the "value added activity" efficiencies, the Netherlands (0.973), Austria (0.922), Germany (0.911) and Norway (0.898) achieve the highest scores while Italy achieves the lowest score (0.232). Positive growth is observed for ten countries and negative growth for seven countries. The largest efficiency in "profitability" stage is achieved by Italy (0.936) with Denmark (0.912), the Netherlands (0.873) and Norway (0.859) also to achieve high scores while Slovak Republic achieved the lowest score (0.547). Positive growth is observed for eight countries while negative growth for nine countries.

The evaluation of the efficiencies for the first and the second stage is an important tool for the decision maker in order to identify the source of the inefficiency of the entire banking system (Wang et al., 2014a). As it is demonstrated in Tables 5.6 and 5.7 the efficiency scores and the rankings of the entire banking system are closer to the first stage which is an indication that the primary source of inefficiency is the "value-added activity" stage. The results are supported by Figures 5.1 and 5.2. Consequently the decision maker should aim to improve the first stage efficiency in order to improve the overall efficiency of the banking system.

Table 5.6: Average efficiencies (1999-2009), average annual growth rates (% change 1999-2009) and rankings of the multiplicative model of Kao and Hwang (2008).

	Overall efficiencies			**1st stage efficiencies**			**2nd stage efficiencies**		
	Average efficiency	**Average annual growth**	**Ranking**	**Average efficiency**	**Average annual growth**	**Ranking**	**Average efficiency**	**Average annual growth**	**Ranking**
Austria	0.770	-0.019	3	0.907	-0.009	3	0.849	-0.011	5
Belgium	0.686	-0.047	5	0.804	-0.031	7	0.849	-0.015	6
Denmark	0.446	0.035	10	0.487	-0.001	13	0.917	0.027	2
Estonia	0.599	0.106	6	0.832	0.089	5	0.713	0.024	11
Finland	0.321	-0.046	14	0.415	-0.048	14	0.771	0.000	10
France	0.437	-0.025	11	0.547	-0.050	11	0.819	0.032	7
Germany	0.737	0.015	4	0.911	0.021	2	0.810	-0.007	8
Italy	0.217	-0.034	16	0.229	-0.008	17	0.948	-0.019	1
Korea, Rep	0.468	0.042	9	0.719	0.032	9	0.666	0.032	15
Netherlands	0.853	-0.017	1	0.967	-0.009	1	0.880	-0.010	3

Norway	0.771	0.026	2	0.898	0.020	4	0.859	0.005	4
Poland	0.211	0.016	17	0.343	0.010	16	0.625	0.006	16
Slovak Republic	0.395	0.167	13	0.670	0.119	10	0.609	0.061	17
Spain	0.499	0.019	8	0.747	0.000	8	0.668	0.019	14
Sweden	0.398	0.027	12	0.514	0.018	12	0.777	0.016	9
Switzerland	0.554	0.027	7	0.813	0.018	6	0.680	-0.001	13
USA	0.265	-0.015	15	0.377	-0.005	15	0.711	-0.005	12

Table 5.7: Average efficiencies (1999-2009), average annual growth rates (% change 1999-2009) and rankings of the additive model of Chen et al. (2009a).

	Overall efficiencies			**1st stage efficiencies**			**2nd stage efficiencies**		
	Average efficiency	**Average annual growth**	**Ranking**	**Average efficiency**	**Average annual growth**	**Ranking**	**Average efficiency**	**Average annual growth**	**Ranking**
Austria	0.879	-0.009	2	0.922	-0.003	2	0.835	-0.016	5
Belgium	0.827	-0.025	5	0.859	-0.026	5	0.802	-0.006	7
Denmark	0.625	0.009	12	0.489	-0.001	13	0.912	0.028	2
Estonia	0.772	0.055	6	0.836	0.086	7	0.708	0.027	10
Finland	0.511	-0.036	14	0.418	-0.042	14	0.764	-0.008	8
France	0.649	-0.018	10	0.641	-0.027	11	0.669	-0.002	11
Germany	0.860	0.007	4	0.911	0.021	3	0.809	-0.007	6
Italy	0.362	-0.018	17	0.232	-0.004	17	0.936	-0.021	1
Korea, Rep	0.694	0.018	9	0.782	0.042	8	0.589	-0.007	15
Netherlands	0.924	-0.010	1	0.973	-0.005	1	0.873	-0.014	3
Norway	0.876	0.009	3	0.898	0.020	4	0.859	0.005	4
Poland	0.418	0.004	16	0.369	0.008	16	0.556	0.002	16
Slovak Republic	0.626	0.062	11	0.710	0.096	10	0.547	0.042	17
Spain	0.713	0.006	8	0.751	0.000	9	0.664	0.019	12
Sweden	0.610	0.013	13	0.556	0.014	12	0.713	0.018	9
Switzerland	0.753	0.006	7	0.841	0.017	6	0.655	0.000	13
USA	0.475	-0.006	15	0.408	0.003	15	0.638	-0.020	14

Furthermore, the Kruskal-Wallis test is applied at the efficiencies for all countries across the period 1999-2009 and the initial findings about the stability of the results over time are confirmed. Specifically, the findings reveal no statistically significant difference across the years. The same conclusion can be reached by looking the first graph in Figures 1 and 2 where the overall efficiencies appear to be stable across all years except 2008 where there is a 4-6% reduction in overall efficiency. This reduction can be attributed to the Global Financial crisis of 2008.

Across the literature, financial stability is considered as a highly desirable but controversial attribute which contributes to the public welfare. Allen and Wood (2006) described the financial stability as a property of a system which experience small fluctuations and returns to equilibrium. The authors stated that financial stability is closely related with the stability of the financial institutions. However, while the lack of financial stability is perfectly observable, financial stability itself is not perfectly observable because it is not possible to know how an economy would react in an intense shock (Allen and Wood, 2006). Based on the above, the results indicate that banking systems across the 17 OECD countries experienced a period of financial stability during 1999-2009. However this finding should be treated with caution and not be extended forward in another period. It is possible that the extension of the study into more years after 2009 where the Global Economic crisis is existent would yield different results. A large discrepancy among countries is another indication of the short-term nature of this financial stability.

Figure 5.1: Average overall and sub-stages efficiencies for the multiplicative model.

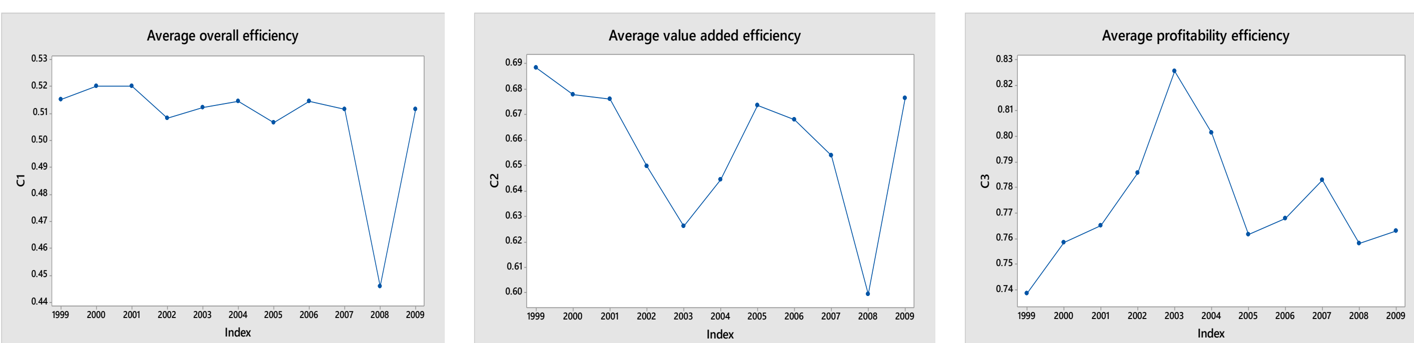
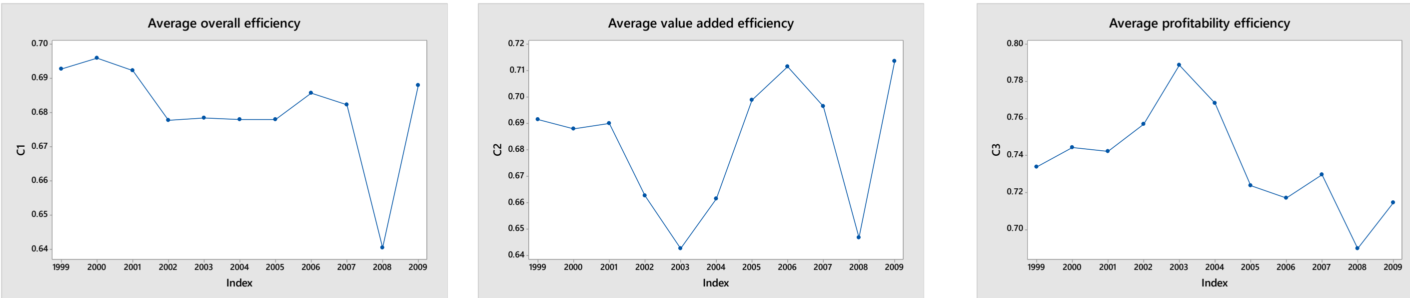


Figure 5.2: Average overall and sub-stages efficiencies for the additive model.



5.4. Summary

The examination of DMU's efficiency over multiple periods is of extreme importance for the decision maker. This Chapter modifies the relational two-stage DEA models in order to incorporate the time component through window-based formulations. Specifically, the multiplicative model of Kao and Hwang (2008) and the additive model of Chen et al. (2009a) are extended to window analysis. In addition, the Chapter provides the mathematical formulation of the window-based version of the two models.

The relational window-based two-stage DEA models are applied to the banking system of 17 OECD countries for eleven years (1999-2009). Deposits have been treated as intermediate variable linking the "value added activity" and the "profitability" of the banking system. The results are relatively stable over time and any positive or negative change is in minor scale. There are large discrepancies among countries which are attributed primarily to the first stage, the "value-added activity" which serves as a valuable information for the decision maker.

Chapter 6 constructs a novel two-stage environmental sustainability index which is decomposed into production efficiency in the first stage and eco-efficiency in the second stage. Then, the newly constructed environmental sustainability index is used in Chapter 7 to demonstrate the metafrontier framework in two-stage DEA analysis.

Chapter 6

Construction of the environmental sustainability index

6.1. Introduction

Environmental degradation and pollution due to human economic activities are in the center of public dialogue in the last few decades. The objective is to achieve economic growth without hampering the environment. Since the United Nations' Earth Summit in Rio in June 1992, a great number of nations have adapted sustainable development and sustainability principals. Sustainability is multidimensional and envelops socio-economic, biological and ecological aspects. Halkos (2012) marked the significance of studying economic development and pollution together towards sustainable development. According to Brundtland's report (1987) sustainable development refers to the "*development that meets the needs of the present without compromising the ability of future generations to meet their own needs*".

An important instrument of sustainable development is eco-efficiency. Kuosmanen and Kortelainen (2005) defined eco-efficiency as the ability to produce the maximum level of economic output while causing the least possible environmental deterioration. It is clear that the notion of eco-efficiency encompasses both economic and ecological aspects. Huppes and Ishikawa (2005) noted that eco-efficiency is a misinterpreted concept and describe four possible types of eco-efficiency which are: environmental productivity, environmental intensity, environmental cost improvement and environmental cost effectiveness. Environmental productivity is the ratio of economic output to environmental pressure while environmental intensity is exactly the opposite ratio, thus environmental pressure to economic output. In addition, environmental cost improvement is the ratio of environmental improvement cost divided by environmental improvement while environmental cost effectiveness is exactly the opposite ratio. This Chapter uses the notion of environmental intensity to assess eco-efficiency.

The contribution of this Chapter is the approach of the environmental sustainability index as a composite index consisting of production efficiency and eco-efficiency. Specifically, the purpose is to provide a framework for constructing environmental sustainability indices using a two-stage DEA model. The newly proposed index is in line with green growth and critical green growth. In addition, the eco-efficiency index of the second stage serves as a decoupling indicator. In addition, this Chapter extends the relational additive window-based model into VRS.

This Chapter is organized as follows. Section 6.2 introduces the terms of sustainable development, green economy and decoupling indicators. Section 6.3 reviews the DEA studies about environmental indices. Section 6.4 constructs the environmental sustainability index and provides the mathematical formulation for the VRS version of the window-based additive two-stage DEA model. Section 6.5 presents the empirical application of the environmental sustainability index for 20 countries with developed economies and Section 6.6 concludes.

6.2. Sustainable development and green economy

Recent economic crisis and major ecological and environmental problems due to anthropogenic activities reveal that traditional growth policies may not lead to the desirable outcome from an economic/social/ecological point of view. According to Jänicke (2012) there are contradicting and questionable views across the literature regarding economic growth. One the one hand is the assumption that economic growth is the solution to financial and social problems, while on the other hand is the assumption that zero growth or de-growth is a necessary condition for solving environmental/ecological problems. UNEP (2009, 2011) proposed the implementation of a green economy which is based on green growth in order to tackle both financial and environmental crisis. The target of green economy is to promote social equity and well-being and simultaneously diminish environmental threats (Chao et al., 2013). Green growth is about a shift of the entire economy towards more efficient and cleaner procedures, and resource saving processes and products (Jänicke, 2012). “Europe 2020” defined the driving force for a green economy which is the smart, sustainable and inclusive growth. Smart growth means that knowledge and innovation fosters the economy; sustainable growth is about resource saving, cleaner procedures, eco-efficiency and competitiveness; and inclusive growth leads towards higher employment for every section of the economy (European Commission, 2010).

An important tool towards green growth is decoupling which refers to breaking the link between environmental pressures and economic goods (OECD, 2002). According to Wursthorn et al. (2011), decoupling indicators measure the ability of an economy to expand without damaging the environment. Essentially a decoupled economy can pursue higher economic growth without damaging the environment. Decoupling can be either absolute or relative (Wang et al., 2013). Absolute decoupling is a state where higher economic growth means stable or less environmental pressures. Relative decoupling is a state where higher economic growth means higher environmental pressures however the increase in economic growth is higher than the increase in environmental pressures. OECD (2002) defined the following indicator to measure the decoupling of an economy:

$$D = 1 - \frac{\left(\frac{EP}{EG}\right)_{end\ of\ period}}{\left(\frac{EP}{EG}\right)_{start\ of\ period}} \tag{6.1}$$

where *D* is the decoupling indicator, *EP* is the environmental pressures and *EG* is the economic goods. If $D \geq 1$ the decoupling is absolute, if $0 \leq D \leq 1$ the decoupling is relative and if $D \leq 0$ there is no decoupling. Decoupling is an important but not a standalone target.

Vazquez-Brust et al. (2014) proposed a more radical approach, the critical green growth. This approach does not aim just the decoupling of environment and economic production but also promotes the synergies among them. It also promotes the investment and growth in smart green sectors and de-growth in brown sectors. The synergies among environment (measured by eco-efficiency) and economic performance have also been marked by Huppes and Ishikawa (2011). In a similar framework this Chapter combines the economic-production efficiency with eco-efficiency which also serves as a decoupling indicator in our case, in order to construct an environmental sustainability index. Clearly the framework presented here is in line with green and critical green growth.

6.3. DEA environmental indices

There are various approaches across the literature regarding the assessment of sustainability. Zhou and Ang (2008) categorized those approaches to non-composite and composite indices. The first category includes simple indices such as energy indicators and integrated indicators such as World Bank's Genuine Savings and Ecological Footprint (Wackernagel and Rees, 1996). The second category includes composite indices such as the United Nation's Human Development Index and the World Economic Forum's Environmental Performance Index. A composite indicator aggregates individual indicators. According to Saisana et al. (2005) the strengths of such an index are the multi-dimensionality, the inclusion of more information and the attraction of public interest due to its summarized form and easy understanding. Composite sustainability indicators can also be constructed using approaches such as DEA.

In order for a model to represent the true production process, the joint production of desirable and undesirable outputs is necessary. Halkos and Tzeremes (2009) marked the significance of simultaneous examination of economic and environmental factors. The most challenging aspect in constructing a DEA environmental index is the incorporation of undesirable outputs. Conventional DEA models cannot deal with undesirable outputs because in such a model inputs can only be decreased and outputs can only be increased, hence an output cannot be decrease if it is not desirable.

Environmental DEA models can be categorized either by their reference technology or by the type of the efficiency measurements (Zhou et al. 2008a). Relatively to the reference technology one can apply a monotone decreasing transformation, such as the use of the outputs' reciprocals (Lovel et al. 1995) and data translation (Seiford and Zhu 2002, 2005). Lovel et al. (1995) proposed the transformation of undesirable outputs into desirable ones using the outputs' reciprocals. This approach has also been used by Ramanathan (2006) who used the reciprocal of the CO_2 output in his study. Seiford and Zhu (2002, 2005) applied data translation at undesirable outputs and assumed strong disposability for all the variables including the newly transformed undesirable outputs.

Data translation has also been used by Lu and Lo (2007) to study the regional development in China and Wang et al. (2014) for the needs of their two-stage DEA model.

Another approach is to apply weak disposability to undesirable outputs (Fare et al. 1989)[6]. Weak disposability allows undesirable outputs to be decreased if the level of production is decreased. Fare et al. (1989) developed the hyperbolic output efficiency measure which compares the performance of a production process with an environmental friendly standard. In their model undesirable outputs can be decreased if also desirable outputs are decreased proportionally. Zaim and Taskin (2000a,b) applied weak disposability and hyberbolic efficiency measure in order to measure the efficiency in OECD countries, using labor and capital as inputs, GDP as desirable output and CO2 as undesirable output. Zofio and Prieto (2001) in a similar framework assessed the environmental efficiency in OECD countries using weak disposability on various F-gases.

A third approach is to treat pollutants as undesirable inputs[7]. Reinhart et al. (2000) employed DEA and stochastic frontier analysis (SFA) and used undesirable inputs, to study Dutch diary firms. Hailu and Veeman (2001) extend Chavas-Cox transformation to DEA approach with the incorporation of undesirable outputs which are treated as inputs. De Koeijer et al. (2002) investigated Dutch sugar beet growers and argue that the incorporation of detrimental inputs supports the construction of a sustainability index. Lansik and Bezlepkin (2003) included CO_2 as undesirable input in their DEA model and examine the environmental efficiency of greenhouse firms in Netherlands. Halkos and Tzeremes (2013b) measured the effect of the national culture on eco-efficiency and included CO_2 and SO_2 as inputs. Halkos and Tzeremes (2014a) investigated the effect of Kyoto protocol on countries' environmental efficiency using CO_2 as input.

Relatively to the type of efficiency, radial efficiency measurements imply proportional increases or decreases for both desirable and undesirable outputs (Zhou et

[6] For an interesting discussion regarding weak disposability see the works by Kuosmanen (2005), Färe and Grosskopf (2009), Kuosmanen and Podinovski (2009) and Kuosmanen and Matin (2011).

[7] This approach has caused some debate about its validity (Seiford and Zhu, 2002; Färe and Grosskopf, 2003; Hailu, 2003).

al. 2008b). Non-radial efficiency measurements imply non-proportional change in both types of outputs (Zhou et al. 2007). Hyperbolic efficiency measurements allow for a simultaneous increase in desirable outputs and decrease in undesirable outputs (Färe et al. 1989; Zaim and Taskin 2000a; Zofio and Prieto 2001; Taskin and Zaim 2001). Directional distance function efficiency measurements allow for a simultaneous increase in desirable outputs and decrease in undesirable outputs based on a predetermined direction vector (Chung et al. 1997; Picazo-Tadeo et al. 2005; Picazo-Tadeo and Prior 2009; Picazo-Tadeo et al. 2012; Halkos and Tzeremes 2013c,d, 2014b, Fukuyama and Weber, 2014).

The vast majority of the above studies constructed the environmental indices in order to measure eco-efficiency and consequently sustainability. Specifically, according to Huppes and Ishikawa's (2005) definition, most of the aforementioned studies used environmental productivity to measure eco-efficiency, which is the ratio of economic output to environmental pressure. Alternatively, Zaim (2004) utilized distance functions to construct an index of desirable outputs and an index of undesirable outputs. The first index reveals the ability of a decision making unit (DMU) to expand the good output while maintaining the level of inputs stable. The second index shows the ability of a DMU to reduce the environmental pressures while maintaining the level of good output stable. The ratio of the second index to the first index gives a pollution intensity index. The author used capital and labor as inputs, gross state product as good output and SOX, NOX and CO as bad outputs. Wursthorn et al. (2011) employed a pollution intensity index to assess the eco-efficiency of German industry. They have stated that an environmental intensity index offers the opportunity of simultaneously being used as a decoupling indicator.

It is clear that sustainability consists of economic-production efficiency and ecological efficiency which can be seen as two different stages. The above studies treated this complex structure inside the single-stage DEA framework. In a similar case, Chen et al. (2012) constructed a two-stage DEA model to assess the sustainable product design performances of automobile industry. In the first stage, the model evaluates the industrial design module efficiency and in the second stage evaluates the bio design efficiency. The first stage is the typical design procedure where the traditional inputs are converted into outputs. This is similar to the production efficiency as it is defined here. The second stage measures the environmental intensity of the design process. This is similar to the eco-efficiency measure as it is defined here.

6.4. Construction of two-stage environmental sustainability index

The index proposed in this Chapter consists of two stages. The first stage efficiency is named as the production efficiency index and the second stage efficiency as the eco-efficiency index. The second stage uses environmental intensity to measure eco-efficiency as defined by Huppes and Ishikawa (2005). Environmental intensity is the ratio of environmental pressure to economic output. The overall efficiency of the two-stage model is a sustainability efficiency index. The eco-efficiency index serves as a decoupling indicator as defined by Wursthorn et al. (2011) because it measures the ability of an economy to expand without damaging the environment and as such it fulfils the concept of sustainability.

The overall environmental sustainability index is constructed using the VRS version of the relational additive two-stage DEA model of Chen et al. (2009a). Furthermore, the window-based model (5.4) which was presented in Chapter 5 is applied to evaluate the results over time. The CRS model (5.4) is modified to the VRS model (6.2) as follows. Pre-emptive priority is given to the eco-efficiency stage 2 because the primal objective is to concentrate on the relation between economic output and environmental pressures.

$$E_{k_w t} = \max \mu \cdot z_t' + \gamma \cdot y_t' + u^1 + u^2 \tag{6.2}$$

s.t.

$$\omega \cdot x_t' + \mu \cdot z_t' = 1$$

$$M \cdot Z_{k_w} - \Omega \cdot X_{k_w} + u^1 \leq 0$$

$$\Gamma \cdot Y_{k_w} - M \cdot Z_{k_w} + u^2 \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

$$j = 1, \dots, n \times w;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

$$u^1 \text{ and } u^2 \text{ are free in sign}$$

and the second stage efficiency of the additive window model is as follows:

$$E_{k_w t}^2 = \max \gamma \cdot y_t' + u^2 \tag{6.3}$$

s.t.

$$\mu \cdot z_t' = 1$$

$$\mu \cdot Z_{k_w} + \gamma \cdot Y_{k_w} - E_{k_w} \cdot \Omega \cdot X_{k_w} + u^1 + u^2 = E_{k_w}$$

$$M \cdot Z_{k_w} - \Omega \cdot X_{k_w} + u^1 \leq 0$$

$$\Gamma \cdot Y_{k_w} - M \cdot Z_{k_w} + u^2 \leq 0$$

$$\gamma_r, \mu_d, \omega_i \geq 0$$

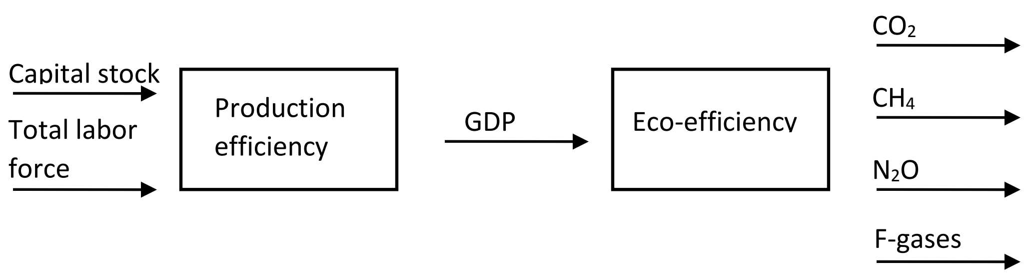
$$j = 1, \dots, n \times w;\ i = 1, \dots, m; d = 1, \dots, D; r = 1, \dots, s$$

and then the first stage efficiency is:

$$E_{k_w t}^1 = \frac{E_{k_w t} - \xi_2^* \cdot E_{k_w t}^2}{\xi_1^*} \tag{6.4}$$

where ξ_1^* and ξ_2^* are the optimal weights from model (6.2) computed in a similar manner as in (3.26). Figure 6.1 is the visual presentation of the proposed model. It is noted that the target is to present a framework for constructing sustainability indices using a relational two-stage DEA model. In that framework we linked as many pollutants we could due to data availability, with the economic-production activity. However, any relevant additional variables could be included into the model.

Figure 6.1: The two-stage framework for the environmental sustainability index



6.5. Application to economically advance countries

Models (6.2)-(6.4) are applied to a group of 20 countries (*n*=20) with advanced economies (IMF, 2014) for the time period of 1990–2011 (*T*=22). Following Webb (2003) a 5-year window has been chosen for the analysis (*w*=5). Specifically, the first window contains the years 1990, 1991, 1992, 1993 and 1994; therefore the number of DMUs in the model is 100 ($n \times w = 20 \times 5 = 100$). Then the second window moves one year forward including 1995 and appending 1990 and the procedure moves on until the last window. The overall procedure includes 18 windows and 1800 different DMUs.

6.5.1. Inputs and outputs

For the needs of the analysis all data was collected from the United Nations Framework Convention on Climate Change (UNFCC)[8] and the Penn World Table (PWT) v8.0 (Feenstra, 2013) for the time period 1990-2011. According to Feenstra (2013) PWT v8.0 address the criticism of previous PWT versions (Johnson et al., 2013) and provides better estimated and more transparent data. Specifically, real GDP measures are based on multiple purchasing power parity (PPP) benchmarks which results to more robust measures. As a result real GDP data of PWT v8.0 is an appropriate measure of output across countries and over time. In addition, PWT v8.0 offers new measures for capital stock and labor. The data referring to a list of 20 countries with advanced economies. As have been already presented, the proposed model consists of two stages. The first stage, which evaluates the production efficiency, utilizes the economic output which is a good output and uses two inputs. The first stage inputs are capital stock and total labor force. Real Gross Domestic Product (GDP) in 2000 prices is the intermediate measure in the model and it is used as a good and the only output in the first stage and as an input in the second stage.

The second stage, which evaluates the eco-efficiency, incorporates the environmental pressures which are bad outputs and uses the real GDP as input. This case

[8] Available from: http://unfccc.int/ghg_data/items/3800.php

study uses the most important greenhouse gases (GHGs) as a measure for environmental pressures which are carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and fluorinated greenhouse gases (F-gases)[9], all measured in gigagrams of CO2 equivalent including land use, land-use change and forestry. GHGs absorb and re-emit thermal radiation which causes a number of dangerous situations such as global warming. According to IPCC (2007), in 2004 the 77% of GHGs was accounted to CO_2, 14% to CH_4, 8% to N_2O and 1% to F-gases. Although it may seems that CO_2 is the primary and only responsible gas for greenhouse gas effect, if we examine the Global Warming Potential[10] (GWP) of each gas we can make a better understanding of the problem in hand. The GWP for 100 years of CO_2 is 1, of CH_4 is 21, of N_2O is 310 and of F-gases ranges from 140 to 23,9006. With this information in mind, one can easily understand the magnitude of the ecological and economic consequences of GHGs.

Finally, undesirable outputs are tackled using Seiford and Zhu (2002) transformation; $f(U) = -U + \beta$. U is the vector of undesirable outputs which is incorporated as a vector of desirable outputs by multiplying it with -1. Then, a proper translation vector β is added in order for the variables to become positive, thus $f(U) > 0$. Table 6.1 gives the descriptive statistics of the data.

Table 6.1: Descriptive statistics

		Total Labour Force (millions)	**Capital Stock (million $)**	**GDP (million $)**	**CO_2 (Gg CO2 eq.)**	**CH_4 (Gg CO2 eq.)**	**N_2O (Gg CO2 eq.)**	**F-gases (Gg CO2 eq.)**
1990	Mean	18.73	2942523	1007383	477384	66054	45758	10483
	St. Dev.	28.89	4960645	1788612	946793	139319	87396	20691
1991	Mean	18.75	3016627	1019866	476397	66152	46954	10219
	St. Dev.	28.76	5009227	1793657	937770	139726	93923	19320
1992	Mean	18.69	3092119	1051100	473989	65774	45912	10342
	St. Dev.	28.86	5068088	1859789	961291	140361	91726	20265

[9] F-gases are a family of three man-made gases HFCs, PFCs and SF_6.

[10] GWP is a relative measure of the heat that a GHG traps in the atmosphere for 20, 100 or 500 years. GWP for CO_2 is 1 and if one gas has GWP of 10 for 100 years it means that this gas traps 10 times more heat in the atmosphere over a period of 100 years.

1993	Mean	18.66	3233524	1072952	477471	65254	46872	10592
	St. Dev.	29.13	5256832	1918203	991147	138269	98117	20685
1994	Mean	18.82	3401106	1117942	483966	65483	46091	11107
	St. Dev.	29.63	5488801	2001320	997940	140704	92735	21685
1995	Mean	19.02	3606323	1158310	500733	65762	47852	12035
	St. Dev.	29.97	5777500	2058187	1019527	138784	97737	25401
1996	Mean	19.19	3673815	1190076	504253	64971	49185	12790
	St. Dev.	30.34	5938175	2134825	1054169	138959	103131	27581
1997	Mean	19.45	3725003	1235965	505415	63611	47990	13340
	St. Dev.	30.92	6118928	2224491	1079991	134833	98631	29096
1998	Mean	19.67	3786348	1275369	521613	63424	45301	13812
	St. Dev.	31.25	6347242	2308381	1098616	132792	90169	31403
1999	Mean	19.91	3846936	1325670	521623	62036	43854	12915
	St. Dev.	31.62	6591397	2409913	1129502	131726	91214	30819
2000	Mean	20.18	3961244	1391881	532066	61530	42527	12718
	St. Dev.	31.97	6868928	2511018	1159262	131432	85827	30988
2001	Mean	20.29	4041850	1408694	526365	60788	42599	11829
	St. Dev.	31.96	7145578	2527872	1123744	129388	88488	28612
2002	Mean	20.31	4094126	1420304	534780	60650	42025	12226
	St. Dev.	31.82	7352617	2557531	1106338	129147	86799	30182
2003	Mean	20.44	4242792	1440931	540950	60176	41553	11999
	St. Dev.	32.04	7616271	2617895	1094141	129597	85796	28754
2004	Mean	20.62	4541433	1482555	538384	58782	42740	12242
	St. Dev.	32.35	8058236	2706760	1112877	127407	90215	30196
2005	Mean	20.89	4908780	1532837	536238	58422	42056	12432
	St. Dev.	32.83	8638638	2785697	1115879	128535	90361	30614
2006	Mean	21.22	5379463	1561195	530206	58792	41557	12600
	St. Dev.	33.35	9268478	2856919	1096305	131046	91095	31210
2007	Mean	21.51	5711002	1596780	541442	58741	42888	13180
	St. Dev.	33.66	9537412	2912065	1124170	131946	97282	33140
2008	Mean	21.59	5957078	1594584	514230	58305	41296	13337
	St. Dev.	33.48	9735190	2896540	1088180	132929	92225	33196
2009	Mean	21.11	5919334	1538389	475121	57254	39912	13140
	St. Dev.	32.35	9506063	2780502	998656	130813	89829	32315
2010	Mean	21.06	5902450	1585699	499842	56469	39425	13956
	St. Dev.	32.18	9346856	2860271	1045235	128468	89085	34403
2011	Mean	21.17	6037471	1604011	483498	55734	39623	14551
	St. Dev.	32.37	9514274	2921631	1018001	126940	90891	36129

6.5.2. Results

Models (6.2) and (6.3) are solved for the time period 1990-2011. The models calculate the overall efficiency which is the environmental sustainability, the first stage efficiency which is the production efficiency and the second stage efficiency which is the eco-efficiency. The resulting overall sustainability index promotes the synergies between economic growth and environment which is in line with critical green growth (Vazquez-Brust et al., 2014). Specifically, if a country succeeds in its decoupling efforts it will achieve high eco-efficiency scores. In order for a country to achieve high sustainability scores it should not just aim the decoupling but also the synergies between economic growth and environment (Vazquez-Brust et al., 2014).

As it has already been mentioned, pre-emptive priority is given at second stage. Table 6.2 examines the efficiencies over time for the case of USA as an illustrative example. The results can be read in two ways, by rows and by columns. The rows indicate the trend as well as the behavior across the same window while the columns indicate the stability of the efficiency for a specific year across different windows.

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Table 6.2: A five-year window analysis of the sustainability efficiency, the production efficiency and the eco-efficiency for the case of USA.

Overall efficiency	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
W1	0.493	0.492	0.501	0.502	0.505																	
W2		0.493	0.501	0.502	0.505	0.505																
W3			0.500	0.501	0.504	0.501	0.505															
W4				0.496	0.499	0.498	0.501	0.504														
W5					0.497	0.496	0.500	0.503	0.505													
W6						0.492	0.496	0.499	0.501	0.505												
W7							0.491	0.495	0.498	0.501	0.504											
W8								0.495	0.497	0.501	0.504	0.504										
W9									0.498	0.502	0.505	0.503	0.505									
W10										0.502	0.505	0.503	0.503	0.505								
W11											0.505	0.503	0.503	0.504	0.505							
W12												0.505	0.505	0.505	0.505	0.505						
W13													0.505	0.505	0.505	0.505	0.505					
W14														0.505	0.505	0.505	0.503	0.505				
W15															0.505	0.505	0.504	0.505	0.505			
W16																0.505	0.504	0.505	0.505	0.505		
W17																	0.503	0.502	0.498	0.498	0.505	
W18																		0.502	0.497	0.495	0.504	0.506
Average	0.493	0.493	0.500	0.500	0.502	0.498	0.499	0.499	0.500	0.502	0.505	0.504	0.504	0.505	0.505	0.505	0.504	0.504	0.501	0.499	0.505	0.506
Production efficiency																						
W1	0.951	0.948	0.981	0.988	1.000																	
W2		0.949	0.982	0.988	1.000	1.000																
W3			0.978	0.985	0.997	0.987	1.000															
W4				0.965	0.979	0.973	0.988	1.000														
W5					0.968	0.964	0.979	0.993	1.000													
W6						0.947	0.963	0.977	0.987	1.000												
W7							0.948	0.963	0.974	0.989	1.000											
W8								0.961	0.973	0.988	1.000	1.000										
W9									0.972	0.988	1.000	0.995	1.000									
W10										0.988	1.000	0.992	0.995	1.000								
W11											1.000	0.991	0.992	0.996	1.000							
W12												1.000	0.998	1.000	1.000	1.000						
W13													0.999	1.000	1.000	1.000	1.000					
W14														1.000	1.000	1.000	0.996	1.000				
W15															1.000	1.000	0.996	1.000	0.999			
W16																1.000	0.996	1.000	0.999	1.000		
W17																	0.992	0.989	0.972	0.970	1.000	

W18																		0.985	0.967	0.959	0.994	1.000
Average	0.951	0.949	0.980	0.982	0.989	0.974	0.976	0.979	0.981	0.991	1.000	0.996	0.997	0.999	1.000	1.000	0.996	0.995	0.984	0.976	0.997	1.000
Eco-efficiency																						
W1	0.012	0.012	0.012	0.012	0.011																	
W2		0.012	0.012	0.012	0.011	0.011																
W3			0.012	0.012	0.011	0.011	0.010															
W4				0.012	0.011	0.011	0.010	0.010														
W5					0.012	0.011	0.011	0.011	0.010													
W6						0.013	0.012	0.012	0.011	0.011												
W7							0.012	0.012	0.011	0.011	0.010											
W8								0.013	0.012	0.012	0.011	0.011										
W9									0.013	0.012	0.012	0.012	0.012									
W10										0.013	0.012	0.012	0.012	0.012								
W11											0.013	0.013	0.013	0.013	0.012							
W12												0.013	0.013	0.013	0.012	0.012						
W13													0.013	0.013	0.012	0.012	0.012					
W14														0.013	0.012	0.012	0.012	0.012				
W15															0.013	0.013	0.012	0.012	0.012			
W16																0.013	0.013	0.012	0.012	0.013		
W17																	0.013	0.012	0.013	0.013	0.013	
W18																		0.013	0.013	0.013	0.013	0.013
Average	0.012	0.012	0.012	0.012	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.012	0.012	0.012	0.012	0.013	0.013	0.013

Table 6.3 provides the average values of each year for the environmental sustainability, the production efficiency and the eco-efficiency. The results reveal large discrepancies among countries for the environmental sustainability, moderate discrepancies for the production efficiency stage and very large discrepancies for the eco-efficiency stage. Efficiency scores for the environmental sustainability ranges from 0.460 for Germany to 0.991 for Portugal in 1990 and from 0.426 for Japan to 0.978 for Finland in 2011. Production efficiency scores range from 0.678 for Austria to 1.000 for Portugal and United Kingdom in 1990 and from 0.632 for Greece to 1.000 for USA in 2011. Eco-efficiency scores range from 0.012 for USA to 0.910 for Finland in 1990 and from 0.013 for USA to 1.000 for Denmark, Finland, Norway, Sweden and Switzerland in 2011. The interpretation of the results for all countries across the entire time period 1990-2011 is difficult. Table 6.4 assists the comprehension of the results by providing the average efficiency over time (1990-2011) for each country along with the average annual growth.

Following Wursthorn et al. (2011), the calculated window-based eco-efficiency scores in second stage provide a time series which provides the opportunity to study a multi-year pattern of eco-efficiency and not just a static pattern. Thus, eco-efficiency can be investigated over time and the results are applied as decoupling indicators. Essentially, an increased eco-efficiency score for a country over the years indicate decreased levels of pollutants without hampering economic growth. This country moves towards to breaking the link between environmental bads and economic goods (OECD, 2002).

Table 6.3: Sustainability efficiency, production efficiency and eco-efficiency (average values obtained by two-stage additive DEA window analysis).

	Sustainability efficiency																						
	1990	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**Average**
Australia	0.542	0.538	0.536	0.531	0.530	0.531	0.529	0.530	0.530	0.526	0.520	0.522	0.527	0.526	0.518	0.514	0.508	0.513	0.500	0.513	0.517	0.511	**0.523**
Austria	0.668	0.645	0.628	0.641	0.615	0.640	0.694	0.648	0.647	0.648	0.634	0.620	0.619	0.613	0.636	0.635	0.622	0.693	0.620	0.610	0.625	0.629	**0.638**
Belgium	0.638	0.631	0.625	0.606	0.614	0.621	0.612	0.615	0.615	0.602	0.611	0.593	0.605	0.601	0.598	0.602	0.593	0.608	0.602	0.595	0.609	0.606	**0.609**
Canada	0.529	0.522	0.520	0.515	0.515	0.515	0.511	0.511	0.510	0.515	0.524	0.524	0.523	0.527	0.528	0.534	0.524	0.529	0.515	0.519	0.526	0.522	**0.521**
Czech Republic	0.807	0.805	0.798	0.778	0.753	0.725	0.703	0.721	0.747	0.751	0.770	0.740	0.741	0.703	0.692	0.698	0.658	0.648	0.631	0.647	0.711	0.720	**0.725**
Denmark	0.970	0.957	0.946	0.917	0.887	0.850	0.847	0.863	0.880	0.895	0.896	0.868	0.885	0.870	0.854	0.856	0.848	0.860	0.817	0.832	0.871	0.895	**0.880**
Finland	0.865	0.850	0.912	0.968	0.988	0.982	0.978	0.965	0.967	0.963	0.960	0.954	0.954	0.952	0.952	0.942	0.934	0.935	0.919	0.948	0.967	0.978	**0.947**
France	0.487	0.484	0.485	0.476	0.474	0.471	0.469	0.475	0.486	0.491	0.500	0.504	0.507	0.498	0.482	0.477	0.466	0.466	0.464	0.462	0.468	0.469	**0.480**
Germany	0.460	0.465	0.467	0.456	0.454	0.450	0.446	0.448	0.452	0.455	0.458	0.460	0.463	0.468	0.468	0.475	0.469	0.467	0.462	0.450	0.470	0.473	**0.461**
Greece	0.745	0.724	0.701	0.665	0.635	0.612	0.618	0.632	0.647	0.654	0.655	0.674	0.673	0.651	0.635	0.642	0.606	0.598	0.583	0.579	0.623	0.635	**0.645**
Italy	0.466	0.463	0.466	0.469	0.480	0.488	0.486	0.487	0.492	0.490	0.490	0.486	0.473	0.467	0.462	0.463	0.463	0.469	0.474	0.465	0.468	0.469	**0.474**
Japan	0.467	0.466	0.459	0.445	0.435	0.433	0.430	0.425	0.417	0.418	0.425	0.423	0.426	0.428	0.428	0.428	0.426	0.427	0.427	0.411	0.425	0.426	**0.431**
Netherlands	0.577	0.573	0.565	0.550	0.542	0.537	0.527	0.524	0.524	0.530	0.547	0.545	0.546	0.539	0.541	0.553	0.540	0.542	0.533	0.528	0.532	0.532	**0.542**
Norway	0.970	0.980	0.983	0.989	1.000	0.971	1.000	0.989	0.972	1.000	0.998	0.976	0.974	0.980	0.987	0.982	0.952	0.969	0.921	0.898	0.946	0.937	**0.972**
Portugal	0.991	0.965	0.932	0.897	0.840	0.795	0.805	0.807	0.805	0.802	0.785	0.750	0.721	0.687	0.673	0.677	0.639	0.674	0.593	0.593	0.575	0.600	**0.755**
Spain	0.510	0.507	0.498	0.483	0.477	0.471	0.467	0.471	0.479	0.477	0.483	0.489	0.491	0.479	0.468	0.465	0.448	0.447	0.453	0.462	0.467	0.472	**0.476**
Sweden	0.710	0.703	0.702	0.740	0.740	0.725	0.783	0.800	0.771	0.882	0.933	0.784	0.779	0.768	0.811	0.909	0.814	0.918	0.725	0.738	0.845	0.952	**0.797**
Switzerland	0.632	0.643	0.614	0.716	0.828	0.647	0.804	0.803	0.793	0.780	0.772	0.650	0.657	0.663	0.731	0.705	0.635	0.821	0.841	0.611	0.748	0.869	**0.726**
United Kingdom	0.542	0.538	0.535	0.527	0.522	0.508	0.507	0.516	0.517	0.529	0.535	0.536	0.540	0.535	0.529	0.523	0.519	0.521	0.503	0.494	0.492	0.491	**0.521**
United States	0.493	0.493	0.500	0.500	0.502	0.498	0.499	0.499	0.500	0.502	0.505	0.504	0.504	0.505	0.505	0.505	0.504	0.504	0.501	0.499	0.505	0.506	**0.501**

	Production efficiency																						
	1990	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**Average**
Australia	0.804	0.795	0.806	0.805	0.784	0.796	0.793	0.794	0.787	0.791	0.777	0.791	0.806	0.805	0.777	0.771	0.748	0.748	0.708	0.761	0.763	0.735	**0.779**
Austria	0.678	0.659	0.668	0.647	0.662	0.698	0.683	0.675	0.678	0.647	0.649	0.623	0.622	0.615	0.610	0.619	0.616	0.619	0.611	0.613	0.637	0.638	**0.644**
Belgium	0.781	0.773	0.792	0.753	0.775	0.772	0.714	0.692	0.669	0.669	0.711	0.690	0.713	0.708	0.675	0.703	0.647	0.643	0.632	0.648	0.678	0.650	**0.704**
Canada	0.899	0.874	0.871	0.859	0.866	0.863	0.848	0.850	0.838	0.867	0.901	0.895	0.884	0.901	0.904	0.929	0.890	0.889	0.840	0.851	0.868	0.844	**0.874**
Czech Republic	0.902	0.815	0.815	0.793	0.770	0.736	0.716	0.721	0.717	0.746	0.729	0.739	0.739	0.699	0.686	0.662	0.619	0.607	0.599	0.641	0.708	0.716	**0.722**
Denmark	0.978	0.965	0.956	0.925	0.891	0.853	0.849	0.866	0.881	0.897	0.898	0.870	0.888	0.870	0.850	0.851	0.844	0.853	0.808	0.828	0.867	0.889	**0.881**
Finland	0.862	0.844	0.908	0.967	0.987	0.982	0.977	0.963	0.965	0.961	0.957	0.954	0.953	0.950	0.949	0.939	0.931	0.932	0.915	0.945	0.966	0.977	**0.945**
France	0.831	0.825	0.832	0.804	0.797	0.783	0.776	0.800	0.833	0.852	0.888	0.903	0.911	0.871	0.820	0.806	0.769	0.766	0.760	0.749	0.767	0.769	**0.814**
Germany	0.774	0.793	0.805	0.772	0.766	0.755	0.743	0.747	0.759	0.769	0.778	0.782	0.790	0.805	0.807	0.831	0.809	0.803	0.786	0.746	0.807	0.815	**0.784**
Greece	0.746	0.728	0.705	0.669	0.638	0.604	0.611	0.629	0.643	0.649	0.654	0.685	0.687	0.656	0.635	0.638	0.596	0.567	0.562	0.569	0.616	0.632	**0.642**
Italy	0.765	0.760	0.772	0.782	0.815	0.835	0.827	0.829	0.843	0.841	0.843	0.835	0.788	0.766	0.751	0.752	0.751	0.766	0.784	0.751	0.756	0.756	**0.789**
Japan	0.828	0.826	0.805	0.766	0.736	0.730	0.722	0.709	0.683	0.683	0.702	0.695	0.704	0.707	0.707	0.707	0.700	0.701	0.703	0.655	0.693	0.695	**0.721**

Netherlands	0.828	0.825	0.809	0.773	0.769	0.745	0.717	0.714	0.705	0.716	0.792	0.796	0.786	0.768	0.764	0.818	0.776	0.768	0.746	0.732	0.735	0.725	**0.764**
Norway	0.985	0.996	1.000	0.997	1.000	0.985	1.000	0.993	0.978	1.000	0.997	0.989	0.993	1.000	0.998	0.993	0.961	0.933	0.849	0.918	0.934	0.932	**0.974**
Portugal	1.000	0.976	0.942	0.891	0.838	0.773	0.769	0.733	0.753	0.742	0.724	0.753	0.720	0.683	0.608	0.613	0.586	0.502	0.551	0.584	0.519	0.487	**0.716**
Spain	0.784	0.785	0.770	0.733	0.720	0.700	0.691	0.709	0.728	0.733	0.754	0.778	0.785	0.747	0.719	0.717	0.675	0.669	0.685	0.708	0.716	0.722	**0.728**
Sweden	0.774	0.756	0.749	0.742	0.770	0.747	0.796	0.790	0.791	0.832	0.875	0.817	0.793	0.793	0.840	0.904	0.831	0.848	0.757	0.747	0.813	0.909	**0.803**
Switzerland	0.731	0.716	0.704	0.682	0.693	0.680	0.657	0.654	0.635	0.612	0.598	0.580	0.645	0.655	0.658	0.655	0.628	0.675	0.708	0.684	0.743	0.756	**0.670**
United Kingdom	1.000	0.985	0.984	0.960	0.947	0.898	0.891	0.918	0.918	0.961	0.985	0.991	1.000	0.978	0.983	0.966	0.948	0.942	0.886	0.851	0.844	0.836	**0.940**
United States	0.951	0.949	0.980	0.982	0.989	0.974	0.976	0.979	0.981	0.991	1.000	0.996	0.997	0.999	1.000	1.000	0.996	0.995	0.984	0.976	0.997	1.000	**0.986**

	Eco-efficiency																						
	1990	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**Average**
Australia	0.226	0.225	0.215	0.208	0.202	0.214	0.213	0.217	0.226	0.218	0.220	0.216	0.217	0.215	0.219	0.214	0.218	0.227	0.231	0.213	0.218	0.224	**0.218**
Austria	0.599	0.558	0.536	0.608	0.522	0.555	0.719	0.614	0.586	0.647	0.593	0.590	0.577	0.582	0.667	0.643	0.624	0.787	0.631	0.604	0.602	0.608	**0.611**
Belgium	0.446	0.443	0.426	0.424	0.429	0.446	0.474	0.494	0.529	0.519	0.496	0.480	0.479	0.484	0.509	0.489	0.526	0.563	0.562	0.524	0.520	0.546	**0.491**
Canada	0.124	0.127	0.126	0.124	0.120	0.126	0.129	0.129	0.139	0.133	0.131	0.136	0.142	0.140	0.140	0.136	0.140	0.144	0.151	0.150	0.151	0.155	**0.136**
Czech Republic	0.535	0.602	0.610	0.481	0.599	0.627	0.619	0.704	0.866	0.835	1.000	0.755	0.780	0.778	0.798	0.822	0.777	0.777	0.783	0.746	0.774	0.790	**0.730**
Denmark	0.828	0.793	0.764	0.761	0.806	0.790	0.806	0.824	0.857	0.858	0.864	0.830	0.841	0.876	0.927	0.933	0.924	1.000	0.992	0.919	0.960	1.000	**0.871**
Finland	0.910	0.961	1.000	1.000	1.000	0.972	1.000	1.000	1.000	1.000	1.000	0.954	0.960	0.983	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	**0.988**
France	0.077	0.076	0.075	0.076	0.075	0.080	0.082	0.081	0.084	0.081	0.080	0.080	0.081	0.087	0.087	0.085	0.086	0.085	0.085	0.088	0.086	0.086	**0.082**
Germany	0.057	0.054	0.052	0.051	0.050	0.050	0.053	0.054	0.056	0.056	0.057	0.059	0.061	0.061	0.061	0.059	0.060	0.059	0.059	0.063	0.060	0.059	**0.057**
Greece	0.713	0.651	0.643	0.628	0.618	0.646	0.659	0.646	0.671	0.680	0.664	0.615	0.595	0.613	0.625	0.629	0.610	0.649	0.616	0.597	0.626	0.686	**0.640**
Italy	0.080	0.078	0.077	0.077	0.076	0.081	0.081	0.084	0.087	0.085	0.084	0.084	0.090	0.093	0.094	0.094	0.093	0.093	0.090	0.095	0.096	0.097	**0.087**
Japan	0.034	0.032	0.031	0.030	0.029	0.028	0.029	0.030	0.033	0.035	0.036	0.038	0.039	0.039	0.039	0.039	0.040	0.041	0.040	0.044	0.041	0.041	**0.036**
Netherlands	0.285	0.279	0.274	0.270	0.263	0.276	0.283	0.278	0.290	0.289	0.275	0.265	0.272	0.281	0.288	0.269	0.273	0.279	0.277	0.276	0.274	0.282	**0.277**
Norway	0.677	0.685	0.666	0.827	1.000	0.713	1.000	0.927	0.826	1.000	1.000	0.730	0.608	0.614	0.787	0.767	0.686	1.000	1.000	0.581	0.817	1.000	**0.814**
Portugal	0.826	0.758	0.742	0.869	0.774	0.797	1.000	1.000	1.000	1.000	1.000	0.723	0.734	0.760	0.876	0.841	0.800	1.000	0.777	0.757	0.761	0.838	**0.847**
Spain	0.167	0.160	0.156	0.154	0.149	0.156	0.158	0.153	0.157	0.150	0.147	0.143	0.143	0.146	0.144	0.137	0.130	0.130	0.127	0.129	0.132	0.135	**0.146**
Sweden	0.506	0.492	0.496	0.634	0.634	0.604	0.756	0.768	0.651	0.942	1.000	0.641	0.542	0.534	0.590	0.740	0.641	1.000	0.572	0.564	0.773	1.000	**0.686**
Switzerland	0.491	0.543	0.491	0.742	1.000	0.602	1.000	1.000	1.000	1.000	1.000	0.729	0.644	0.655	0.805	0.740	0.642	1.000	1.000	0.517	0.753	1.000	**0.789**
United Kingdom	0.081	0.082	0.082	0.079	0.077	0.079	0.079	0.076	0.078	0.078	0.077	0.077	0.080	0.082	0.081	0.081	0.082	0.084	0.087	0.090	0.087	0.088	**0.081**
United States	0.012	0.012	0.012	0.012	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.012	0.012	0.012	0.012	0.013	0.013	0.013	**0.012**

Figure 6.2 provides a visual representation of the results. The classes in Figure 6.2 were chosen based on the nature of the results. At first, most efficient countries were made distinct with bright green color while least efficient countries were marked with red color. A range of 0.10 has been chosen for all classes and starting from the better performing countries the selected colors are dark green, dark teal, blue, turquoise, yellow, rose, pink and orange respectively. Analyzing Table 6.4 and Figure 6.2 together assist the better understanding of the results.

Average annual growth reveals relatively stable efficiency scores over time with slightly positive or negative changes. Only two efficiency scores change more than 5% in average during the time period (the eco-efficiency scores for Switzerland, 7.8%, and Sweden, 6.3%). Regarding environmental sustainability scores, Norway and Finland achieve the highest efficiency scores (0.972 and 0.947 respectively) while Japan achieves the lowest score (0.431). Six countries achieve positive average annual growth while fourteen countries experience negative growth. Subfigure 6.2a provides a visual representation of these results for the average environmental sustainability scores. As it is shown Scandinavian countries appear with bright or dark green color which means they have very high efficiency scores. On the contrary, Central and Southern European countries along with USA appear with yellow color which means they achieve below average results.

USA (0.986), Norway (0.974), Finland (0.945) and United Kingdom (0.940) achieve the highest production efficiency scores while Greece achieves the lowest score (0.642). Again, six countries achieve positive average annual growth while fourteen countries experience negative growth. Subfigure 6.2b shows that the results of production efficiency stage are more balanced and significantly higher than sustainability and eco-efficiency scores. The majority of countries perform very high efficiency scores.

Considering the eco-efficiency stage, Finland achieves the highest average score (0.988) while USA achieves the lowest score (0.012). Denmark (0.871), Portugal (0.847) and Norway (0.814) also achieve very high scores. All countries except Spain and Greece experience positive growth regarding the eco-efficiency stage. USA, Japan, Germany, United Kingdom, France and Italy achieve eco-efficiency below 10%. These countries are responsible for 24.4% of global greenhouse gas (GHG) emissions, while all the other countries of the data set are responsible only for 6.2% of the global GHG emissions[11]. Subfigure 6.2c, demonstrates those results graphically. It is clear that there are large inequalities in eco-efficiency among countries. Again, Scandinavian countries perform very high results while a great number of countries appear with orange or red color.

Table 6.4: Average efficiencies (1990-2011), average annual growth rates (% change 1999-2009) and rankings.

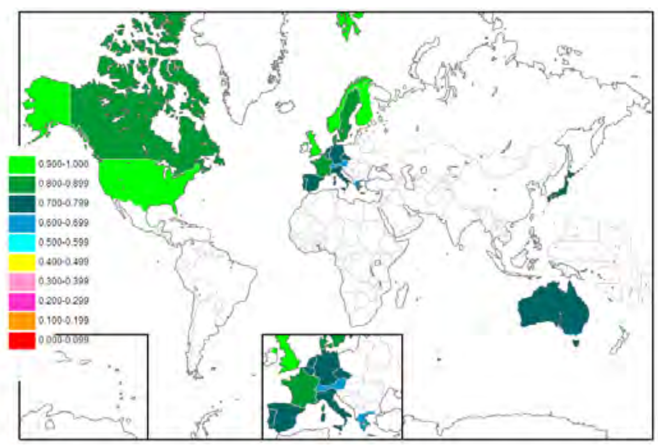
	Environmental sustainability			**Production efficiency**			**Eco-efficiency**		
Countries	Average efficiency	Average annual growth	Ranking	Average efficiency	Average annual growth	Ranking	Average efficiency	Average annual growth	Ranking
Australia	0.523	-0.003	12	0.779	-0.004	11	0.218	0.000	12
Austria	0.638	-0.002	9	0.644	-0.003	19	0.611	0.008	9
Belgium	0.609	-0.002	10	0.704	-0.008	17	0.491	0.011	10
Canada	0.521	-0.001	14	0.874	-0.003	6	0.136	0.011	14
Czech Republic	0.725	-0.005	7	0.722	-0.010	14	0.730	0.026	6
Denmark	0.880	-0.004	3	0.881	-0.004	5	0.871	0.010	2
Finland	0.947	0.006	2	0.945	0.006	3	0.988	0.005	1
France	0.480	-0.002	16	0.814	-0.003	7	0.082	0.006	16
Germany	0.461	0.001	19	0.784	0.003	10	0.057	0.002	18
Greece	0.645	-0.007	8	0.642	-0.007	20	0.640	-0.001	8
Italy	0.474	0.000	18	0.789	0.000	9	0.087	0.009	15
Japan	0.431	-0.004	20	0.721	-0.008	15	0.036	0.011	19
Netherlands	0.542	-0.004	11	0.764	-0.006	12	0.277	0.000	11
Norway	0.972	-0.001	1	0.974	-0.002	2	0.814	0.046	4
Portugal	0.755	-0.023	5	0.716	-0.032	16	0.847	0.009	3
Spain	0.476	-0.004	17	0.728	-0.004	13	0.146	-0.010	13
Sweden	0.797	0.019	4	0.803	0.009	8	0.686	0.063	7
Switzerland	0.726	0.026	6	0.670	0.002	18	0.789	0.078	5
United Kingdom	0.521	-0.005	13	0.940	-0.008	4	0.081	0.004	17
United States	0.501	0.001	15	0.986	0.002	1	0.012	0.001	20

[11] Available from: http://cait2.wri.org/

Figure 6.2: Visual representation of the geographical dispersion of the efficiency scores.

2a

2b



2c

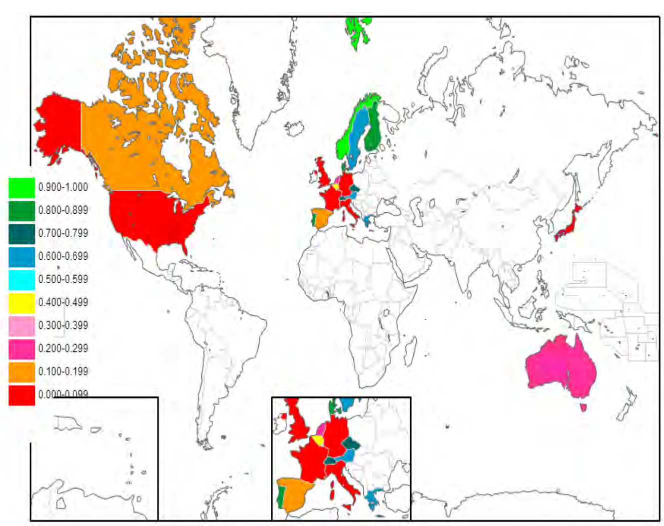


Figure 6.3 presents the environmental sustainability scores over time for each country. Three countries have substantially improved their scores since 1990 and that are Finland, Sweden and Switzerland. The interesting aspect is the reason behind the differentiation between these countries and most of the other countries[12]. According to Vourc'h and Jimenez (2000) the legislative and regulatory framework for environmental conservation in Finland has been greatly improved since 1990, targeting among others climate change and sustainable development. Finland was the first country ever to impose a tax on CO2 emissions in 1990. Since then the country has promoted a number of environmental regulations such as the Nature Conservation Act. Regarding sustainable development, the country establish the National Commission for Sustainable Development in 1993 which is chaired by the Prime Minister and promotes the dialogue about sustainable development policies. This commission aims the cooperation of every concerning party in Finland and is participated by members of the parliament, central public administration, local authorities, business representatives, labor unions, scientists and non-governmental organizations. Another important aspect towards the cooperation in sustainability principals among the concerning parties is the top social infrastructures such as the educational system which is among the best worldwide. Finland also promotes transparency and open governance as tools towards sustainability (OECD, 2010).

In 1960, Sweden realized the problem of depletion of the natural resources and it was among the pioneer members of United Nations which worked towards the organization of the first UN conference on the environment in 1972. Since then Sweden continuously follows sustainability principals, reducing acidification of the lakes from 17% to 10%, increasing the share of renewable energy sources (RES) up to 47% which is the

[12] Sustainability scores for Norway and Denmark were also very high. Here only Finland, Sweden and Switzerland are analyzed because in 1990 their scores were not so high and they improved substantially since then. However, there are similarities in environmental sustainability approaches among these five countries. The purpose here is to highlight these similarities as best practice for other countries which do not perform so well.

highest in the European Union and developing an exemplar water management. Regarding sustainability Sweden established Environmental Objectives Council in 2002 which aims the coordination and monitoring towards the fulfillment of 16 objectives such as clean air and sustainable forest management. The countries efforts resulted in a GHG emissions reduction from approximately 70 to approximately 60 million tCO2 eq. in the time period 1990-2011[13]. Particulate matter and Nitrous oxides emissions have also been decreased significantly. Sweden applies a number of tools towards sustainable development, which are the promotion of dialogue between government and business enterprises, partnerships and investment programs among others (Swedish Ministry of Environment, 2004).

Switzerland declared sustainable development as national target since 1999 (Attah, 2010). Since then Switzerland has managed to be among the countries with the lowest SO_2 and NOx emissions. Also, Switzerland significantly reduced energy consumption and promoted RES. Stringent environmental regulation and management, large financial investments for environmental purposes and a highly modern public transport system are among the key elements towards the country's success. Switzerland also promotes the environmental education from elementary school, the collaboration between government agents and business partners and the implementation of the strategies at sector level. The success of Switzerland is reflected on the results of the Yale's Environmental Performance Index where Switzerland is in the first place (87.67[14] in 2014).

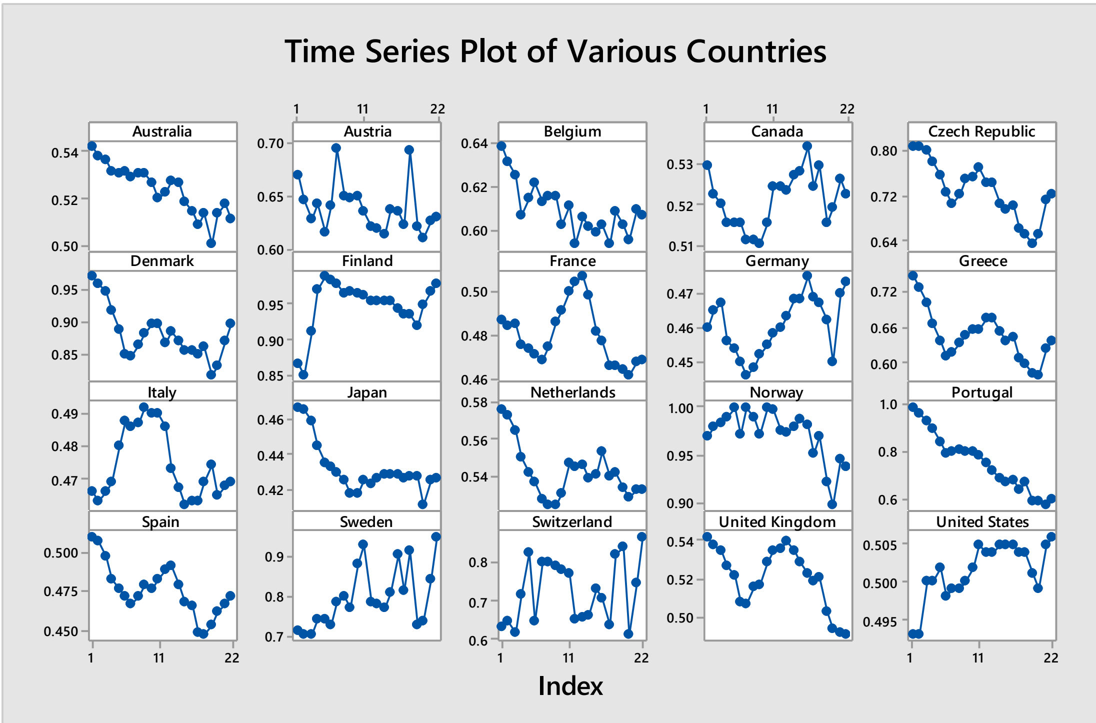
A careful examination of eco-efficiency scores over time reveals a decoupling effect in these countries. Specifically, they achieved decreased levels of pollutants while they increased the level of economic growth. Thus, by the definition of OECD (2002) they broke the link between environmental bads and economic goods. In addition, the above discussion indicates that these countries integrate environmental, economic and social

[13] Available from: http://www.miljomal.se/sv/Environmental-Objectives-Portal/Undre-meny/Publications-and-presentations/

[14] Available from: http://epi.yale.edu/

objectives in order to achieve sustainability. Coordination and collaboration among concerning parties is of extreme importance. In addition, education has an important role in all three countries.

Figure 6.3: Sustainability scores for countries over time



6.6. Summary

This Chapter demonstrated the framework for the construction of an environmental sustainability efficiency index using a two-stage DEA model. The window-based relational additive model was extended to VRS and was used for the construction of the overall index. The first stage measures the production efficiency and the second stage measures the eco-efficiency. The overall efficiency of the model is the proposed sustainability efficiency index. The advantage of this index is that the eco-efficiency index serves as a decoupling indicator as defined by Wursthorn et al. (2011) because it measures the ability of an economy to break the link between environmental pressures and economic goods. Decoupling is considered as an important tool for promoting sustainability (Lu et al., 2014), however it should not be a standalone target. The path towards sustainability requires the synergies between economic growth and environment which is in line with critical green growth (Vazquez-Brust et al., 2014). The nature of the proposed sustainability index requires the synergies between economic growth and environmental targets in order to yield high resulting scores.

The model was applied at a panel of 20 countries with advanced economies for the time period 1990-2011. The results indicated that eco-efficiency stage is characterized by large inequalities among countries and significant lower efficiency scores compared to the overall sustainability and production efficiency levels. In addition, it appears that a country's high production efficiency level does not ensure a high eco-efficiency level. Finally, the results for three high-performing countries were discussed and they indicated that the integration of environmental, economic and social objectives are the key elements towards sustainability. Non-performing countries should follow the path of high-performing countries, motivated by modern growth strategies such as "Europe 2020" which promotes smart, sustainable and inclusive growth (European Commission, 2010). Education and knowledge is at the center of smart growth; resource preservation, cleaner procedures, eco-efficiency and competitiveness are included in sustainable growth; and social targets such as high employment are elements of inclusive growth. In addition, radical growth of green sectors and de-growth of brown sectors appear to be significant targets towards a green sustainable economy.

Chapter 7 builds upon the newly proposed environmental sustainability two-stage DEA index and demonstrates the use of the metafrontier framework into the two-stage DEA analysis.

Chapter 7

Metafrontier framework for two-stage DEA models

7.1. Introduction

One of the few requirements for a DEA model is the homogeneity of DMUs. Specifically, DMUs should perform the same tasks, have similar objectives and use the same inputs to produce the same outputs (Cook et al., 2015). Furthermore, the DMUs should operate under similar technology (Rao et al., 2003). Chapter 6 constructed a novel index which evaluates the environmental sustainability for a group of countries with advanced economies. This group of countries can be considered as a homogenous group.

A metafrontier framework is applied when there is a need to study DMUs in different groups (such as firms in different groups or regions in different countries) having different technologies (Rao et al. 2003). DMUs from different groups face different production opportunities; therefore feasible input-output combination in one group may not be feasible in another. These differences among groups may refer to physical, human and financial capital, infrastructures, economic environment, available resources etc; as a result every group has a different frontier. In this framework the metafrontier is an overall frontier which envelopes the groups' (or countries') specific frontiers so that no point of these frontiers can lie above points on the metafrontier (Rao et al. 2003; Battese et al. 2004). O'Donell et al. (2008) applied this approach on a DEA framework in order to study the agricultural sector in 97 countries. Kounetas et al. (2009), Kontolaimou and Tsekouras (2010) and Kontolaimou et al. (2012) proposed a non-parametric methodology in order to study firms operating under different technologies but under a common metatechnology. Cook et al. (2015) proposed an alternative approach based on Hierarchical models to deal with heterogeneity among different groups of DMUs

This Chapter applies the metafrontier framework to two-stage DEA models in order to treat the heterogeneity among DMUs in different groups which possibly experience different technologies. This framework is introduced into the relational additive model of Chen et al. (2009a), however it can be introduced into every two-stage DEA model in the same way as proposed here. Then, the new two-stage metafrontier framework is applied to European regions and measures the environmental sustainability as it has been proposed by Chapter 6.

This Chapter is structured as follows. Section 7.2 introduces the metafrontier framework into two-stage DEA models. The metafrontier, the group-specific frontiers and the technological gap ratios are defined in this Section. Section 7.3 presents the application to the regional environmental sustainability in Europe and Section 7.4 concludes.

7.2. Metafrontier framework in two-stage DEA models

Following O'Donell et al. (2008), let *x*, *z* and *y* be nonnegative real input, intermediate variable and output vectors of dimension $M \times 1$, $D \times 1$ and $S \times 1$ respectively. The metatechnology set for the overall sustainability index contains all input, intermediate measure and output combinations which are technologically feasible.

$$F = \{(x, z, y): x \geq 0, z \geq 0, y \geq 0; x\ can\ produce\ z, z\ can\ produce\ y\} \quad (7.1)$$

Metatechnology set *F* is applied to the additive two-stage DEA model (3.27) in VRS[15]. This model can be considered as an unrestricted model and the boundary of this model is the metafrontier. The metafrontier overall sustainability efficiency will be denoted as E_0 and a DMU will be overall efficient with respect to metafrontier if $E_0 = 1$.

Regarding the group-specific frontiers, there are *K* different groups with different technologies and different feasible input, intermediate measure and output sets. Accordingly, the input, intermediate measure and output combinations available to the regions in k^{th} group are contained in the following group-specific set.

$$F^k = \begin{Bmatrix} (x, z, y): x \geq 0, z \geq 0, y \geq 0; \\ x\ can\ be\ used\ by\ DMUs\ in\ group\ k\ to\ produce\ z, \\ z\ can\ be\ used\ by\ DMUs\ in\ group\ k\ to\ produce\ y \end{Bmatrix} \quad (7.2)$$

Group-specific sets F^k are applied to the VRS version of model (3.27) which can be considered as a restricted model and the boundaries of this model are the group frontiers. The group overall sustainability efficiency will be denoted as E_0^k and a DMU will be efficient with respect to the country frontier if $E_0^k = 1$.

[15] The VRS version of model (3.27) has been presented in (4.8) and (6.2).

All the group frontiers are contained inside the metafrontier. Therefore, the group efficiency for a DMU can take a value no less than the metafrontier efficiency for the same region. Following O'Donell et al. (2008) the technological gap ratios can be calculated as follows.

$$TGR_0 = \frac{E_0}{E_0^k} \tag{7.3}$$

The technology gap ratio shows the technological gap of a DMU in group *k* relative to the metafrontier due to the reasons which were described in the introductory section (eg. capital, economic environment, etc.).

7.3. Application to European regional environmental sustainability

The metafrontier framework is applied to a group of 157 NUTS2 regions of seven European countries, namely Belgium, Germany, France, Italy, the Netherlands, Spain and United Kingdom. All the seven European countries are among EU-15 countries which have committed to fulfill the Kyoto protocol and reduce GHGs emissions accordingly. The year 2008 is an important year because it marks the beginning of the *first commitment period 2008-2012.* Therefore, there are *K=7* countries which can be seen as seven different groups. Each country has its own group-specific frontier and the overall frontier of the 157 regions of the 7 countries is the metafrontier. The environmental sustainability index is constructed in line with Chapter 6; thus the first stage efficiency is the production efficiency index and the second stage efficiency is the eco-efficiency index. As in Chapter 6, pre-emptive priority is given at eco-efficiency stage because the primal objective is to concentrate on the relation between economic output and environmental pressures. Mickwitz et al. (2006) and Seppälä et al. (2008) marked the significance of regional indicators of eco-efficiency. Both studies provided a framework for calculating eco-efficiency indicators for the region of Kymenlaakso in Finland. Halkos and Tzeremes (2012) evaluated the environmental efficiency for German regions.

7.3.1. Inputs and outputs

All the data was collected from OECD[16] and Eurostat[17] for the year 2008. The first stage which from here on will be referred to as the "*production efficiency*" stage, uses two inputs, namely capital stock and labor and one output, the GDP of each region which serves as an intermediate variable. Capital stock has been calculated following Hall and Jones' (1990) formula:

$$K_t = \frac{GFK_t}{\delta + g} \tag{7.4}$$

where K_t is the gross capital stock in year t, GFK_t is the gross fixed capital formation in year t, δ is the depreciation rate of capital stock which has been set at 6% (Zhang et al. 2011) and g is the rate of growth in gross fixed capital formation.

The second stage which from here on will be referred to as the "*eco-efficiency*" stage, uses the GDP as input which is the only intermediate variable in the model and produces CO_2 and municipal wastes as bad outputs[18]. As has been already presented, conventional DEA models cannot be used because an output expansion cannot be considered as desirable; on the contrary the desirable is an output contraction. Again, data translation is applied to handle bad outputs (Seiford and Zhu, 2002, 2005) exactly as presented in Chapter 6. Descriptive statistics are presented in Table 7.1.

Table 7.1: Descriptive statistics

	Total Labour Force (in thousands)	**Capital Stock (in thousands euros)**	**GDP (in thousands euros)**	**CO_2 (tones)**	**Municipal wastes (tones)**
Mean	951	11,587	62,561	17,826,565	1188.6
St. Dev.	782.2	9,698	63,660	17,337,827	1128.6
Min	22.1	318	1,352	4,205	35
Max	65,453	5,223.1	541,880	104,512,343	9165.5

[16] Available from: http://rag.oecd.org/
[17] Available from: http://epp.eurostat.ec.europa.eu/portal/page/portal/nuts_nomenclature/introduction
[18] Other variables (pollutants) such as SO_2, and NOx emissions can be incorporated in order to for the model to grasp the more aspects on the eco-efficiency stage, however, this was not possible due to data availability.

7.3.2. Results

The model yields the results for the overall “*environmental sustainability*” index, the “*production efficiency*” index and the “*eco-efficiency*” index. The results are presented at Table 7.2 along with the rankings for the common European metafrontier overall sustainability index. Specifically, the first column states the country, the second column states the region; columns 3, 4 and 5 are about the common European metafrontier efficiency of the overall, the first and the second stage respectively and column 6 presents the rankings for the overall metafrontier efficiency. Similarly, columns 7, 8 and 9 are about the country-specific efficiency of the overall, the first and the second stage respectively. In addition, Table 7.3 presents the average results at a country level. For example in Table 7.2, sustainability efficiency for Inner London region with respect to common European metafrontier is 0.791 while relative to United Kingdom’s frontier it is 0.991. These results mean that Inner London region is 99.1% efficient relative to United Kingdom’s technological framework for sustainability. However, Inner London region which uses the United Kingdom’s available technology for sustainability (e.g. more restrictive framework) is only 79.1% efficient relative to the common European metafrontier. This can also be confirmed by the technological gap ratio which is 0.798, indicating that only 79.8% of the efficiency can be achieved using United Kingdom’s available technology for sustainability.

Table 7.3 reveals that technological framework is different across countries. Specifically, United Kingdom faces more restrictive conditions for the achievement of *“sustainability efficiency”* (technological gap ratio is 0.792) than any other country. On the other hand Spain faces the less restrictive conditions (TGR is 0.952). With respect to the *“production efficiency”* all the countries have similar TGRs. However, countries appear to have wide gaps with respect to *“eco-efficiency”*. Germany (0.764), United Kingdom (0.772) and the Netherlands (0.774) have the lower TGRs while Spain faces no restriction relative to the common European metafrontier (TGR is 1.000).

A careful examination of Table 7.2 reveals that European regions achieve high sustainability scores, very high “*production efficiency*” scores and good “*eco-efficiency*” scores relative to the common European metafrontier. Furthermore, small regions tend to achieve better overall sustainability scores than large regions because the former use significantly less inputs (labor force and capital) and produce less environmental pressures (CO_2). Specifically, the results from Tables 7.2 and 7.3 indicate that Belgium achieves the highest average efficiency score (0.810) and three Belgian regions (Luxemburg (BE), Brabant Wallon and Namur) are in the top-ten regions regarding the sustainability scores. Spain and the Netherlands (0.794) are in the second place regarding the sustainability scores. Three Spanish regions (La Roja, Ciudad Autonoma de Melilla and Ciudad Autonoma de Ceuta) and a Dutch region (Zeeland) are in the top-ten achievers in sustainability scores. In the fourth place is France (0.791) and Corse is the French region in the top-ten European regions. Italy and United Kingdom are in the fifth place (0.785) and two Italian regions (Valle d'Aosta, Molise) are in the top-ten regions. Germany achieves the lowest average sustainability score (0.777).

Regarding the "*production efficiency*" relative to the common European metafrontier the highest scores are achieved by large economic centers such as Inner London, Ile-de-France (which is the region of Paris) and Brussels and by small regions which use significantly lower inputs than others such as (Ciudad Autonoma de Melilla and Ciudad Autonoma de Cueta). Specifically, Belgium achieves the highest average production efficiency score (0.900) and it is followed by France (0.894), the Netherlands (0.886), Italy (0.880), United Kingdom (0.876), Germany (0.874) and Spain (0.865). It should be noted that the year under examination is 2008 so any effects from the global economic crisis are not incorporated in the results. It is highly possible that the results could have been changed since 2008. Regarding the "*eco-efficiency*" relative to the common European metafrontier the highest score is achieved by Belgium (0.715) followed by Spain (0.714), the Netherlands (0.692), Italy (0.681), United Kingdom (0.677), Germany (0.662) and France (0.662).

As it is clear, the "*production efficiency*" scores are significantly higher than the "*eco-efficiency*" scores. Consequently, the decision maker should aim to improve the eco-efficiency index in order to improve the overall sustainability index. This can be achieved with an integrated common policy such as the European Sustainable Development

Strategy. The idea of a common European environmental policy is the promotion of economic development with respect to social progress and environmental protection, to address the distortions and to implement common targets in European countries. Kyoto protocol is a fine example of such a common strategy and it has been signed by all the seven countries in the analysis. However, the first commitment period begins in 2008 and ends in 2012. In 2008, the Protocol was not in force yet and the inequalities among countries in eco-efficiency scores are due to national environmental policies. For example, Belgium which achieves the highest eco-efficiency scores had reduced its GHG emissions since 2000 based on measures on climate change, energy efficiency and renewable energy. Such measures include the approval of National Allocation Plan to promote renewable energy and energy efficiency measures, CO_2 allowances, tax reduction to solar panels and other environment-friendly policies.

The general outlook of the results reveals small inequalities among the regions relatively to their production activity and larger inequalities for their polluting activity. Furthermore, the average scores in country level appear to be stable among the seven countries for the sustainability efficiency, the production efficiency and the eco-efficiency indices. A first impression would be that the absence of inequalities among the countries in average scores is due to the successful common European environmental strategies. However, the results should be approached more carefully. In this manner, Figure 7.1 demonstrates the densities of the three efficiency indices.

Table 7.2: Results for the sustainability efficiency, production efficiency and eco-efficiency scores for the common European metafrontier and country-specific frontier, rankings and technological gap ratios.

	NUTS2 Regions	E_0	E_0^1	E_0^2	#	E_0^k	E_0^{k1}	E_0^{k2}	TGR_0	TGR_0^1	TGR_0^2
Belgium	Région de Bruxelles-Capitale	0.800	1.000	0.651	34	0.889	1.000	0.782	0.900	1.000	0.833
	Antwerpen	0.770	0.875	0.650	120	0.875	0.975	0.781	0.881	0.898	0.833
	Limburg	0.805	0.880	0.720	31	0.916	0.967	0.864	0.878	0.910	0.833
	Oost-Vlaanderen	0.781	0.870	0.679	89	0.891	0.963	0.815	0.877	0.904	0.833
	Vlaams-Brabant	0.796	0.894	0.687	40	0.906	0.986	0.824	0.879	0.907	0.833

	West-Vlaanderen	0.787	0.873	0.689	55	0.894	0.959	0.826	0.880	0.910	0.833
	Brabant Wallon	0.847	0.923	0.766	7	0.966	1.000	0.919	0.877	0.923	0.833
	Hainaut	0.805	0.890	0.704	30	0.921	0.996	0.845	0.874	0.893	0.833
	Liège	0.811	0.896	0.711	23	0.927	0.981	0.853	0.875	0.913	0.833
	Luxembourg (BE)	0.871	0.905	0.833	5	1.000	1.000	1.000	0.871	0.905	0.833
	Namur	0.842	0.899	0.778	9	0.964	0.985	0.934	0.874	0.913	0.833
Germany	Stuttgart	0.743	0.861	0.607	147	0.878	0.973	0.795	0.847	0.885	0.764
	Karlsruhe	0.759	0.873	0.629	133	0.895	0.977	0.824	0.848	0.893	0.764
	Freiburg	0.767	0.868	0.651	124	0.905	0.967	0.852	0.847	0.898	0.764
	Tübingen	0.773	0.874	0.657	117	0.910	0.967	0.861	0.849	0.904	0.764
	Oberbayern	0.731	0.845	0.596	153	0.862	0.989	0.780	0.848	0.854	0.764
	Niederbayern	0.794	0.884	0.688	42	0.941	0.951	0.901	0.845	0.930	0.764
	Oberpfalz	0.787	0.872	0.689	57	0.922	0.945	0.902	0.853	0.923	0.764
	Oberfranken	0.810	0.889	0.696	25	0.967	0.981	0.911	0.837	0.906	0.764
	Mittelfranken	0.772	0.873	0.656	118	0.907	0.962	0.859	0.851	0.908	0.764
	Unterfranken	0.792	0.867	0.681	47	0.936	0.947	0.891	0.846	0.916	0.764
	Schwaben	0.771	0.868	0.660	119	0.908	0.958	0.864	0.850	0.906	0.764
	Berlin	0.760	0.872	0.632	132	0.899	0.983	0.828	0.845	0.887	0.764
	Brandenburg	0.767	0.837	0.661	123	0.909	0.958	0.866	0.844	0.874	0.764
	Bremen	0.823	0.933	0.706	15	0.985	1.000	0.925	0.836	0.933	0.764
	Hamburg	0.763	0.922	0.634	131	0.896	1.000	0.831	0.852	0.922	0.764
	Darmstadt	0.746	0.869	0.604	144	0.878	0.980	0.791	0.849	0.887	0.764
	Gießen	0.803	0.890	0.700	32	0.957	0.967	0.917	0.839	0.920	0.764
	Kassel	0.796	0.894	0.686	37	0.942	0.952	0.899	0.845	0.939	0.764
	Mecklenburg-Vorpommern	0.786	0.851	0.688	61	0.927	0.933	0.900	0.847	0.911	0.764
	Braunschweig	0.785	0.887	0.671	66	0.924	0.978	0.878	0.850	0.907	0.764
	Hannover	0.777	0.882	0.652	108	0.916	0.991	0.854	0.848	0.890	0.764
	Lüneburg	0.790	0.860	0.685	52	0.933	0.942	0.898	0.846	0.913	0.764
	Weser-Ems	0.766	0.867	0.650	125	0.904	0.965	0.851	0.847	0.898	0.764
	Düsseldorf	0.745	0.876	0.595	146	0.881	1.000	0.779	0.846	0.876	0.764
	Köln	0.749	0.869	0.611	142	0.886	0.986	0.800	0.845	0.881	0.764
	Münster	0.766	0.867	0.649	126	0.904	0.968	0.849	0.847	0.896	0.764
	Detmold	0.776	0.881	0.657	113	0.914	0.979	0.860	0.848	0.900	0.764
	Arnsberg	0.758	0.876	0.624	135	0.897	0.990	0.817	0.845	0.884	0.764
	Koblenz	0.787	0.871	0.685	58	0.927	0.935	0.896	0.848	0.932	0.764
	Trier	0.825	0.880	0.764	13	1.000	1.000	1.000	0.825	0.880	0.764
	Rheinhessen-Pfalz	0.775	0.877	0.660	115	0.915	0.975	0.865	0.847	0.899	0.764
	Saarland	0.806	0.897	0.698	29	0.960	0.972	0.914	0.839	0.924	0.764
	Dresden	0.777	0.856	0.685	111	0.915	0.936	0.897	0.849	0.914	0.764
	Leipzig	0.807	0.870	0.712	28	0.965	0.973	0.933	0.836	0.895	0.764
	Sachsen-Anhalt	0.776	0.851	0.663	114	0.917	0.973	0.869	0.846	0.875	0.764
	Schleswig-Holstein	0.763	0.867	0.643	129	0.902	0.971	0.842	0.846	0.892	0.764
	Thüringen	0.770	0.852	0.667	121	0.911	0.953	0.874	0.845	0.894	0.764

	Galicia	0.755	0.837	0.657	138	0.803	0.966	0.657	0.940	0.866	1.000
	Principado de Asturias	0.795	0.864	0.716	41	0.836	0.973	0.716	0.951	0.888	1.000
	Cantabria	0.821	0.875	0.759	17	0.856	0.968	0.759	0.959	0.904	1.000
	País Vasco	0.765	0.866	0.649	127	0.812	1.000	0.649	0.942	0.866	1.000
	Comunidad Foral de Navarra	0.807	0.898	0.735	27	0.840	0.959	0.735	0.960	0.936	1.000
	La Rioja	0.843	0.881	0.799	8	0.869	0.949	0.799	0.969	0.928	1.000
	Aragón	0.779	0.855	0.690	102	0.821	0.969	0.690	0.949	0.882	1.000
	Comunidad de Madrid	0.724	0.833	0.592	154	0.776	0.982	0.592	0.932	0.848	1.000
	Castilla y León	0.753	0.832	0.659	141	0.799	0.953	0.659	0.943	0.873	1.000
Spain	Castilla-la Mancha	0.757	0.843	0.682	136	0.800	0.927	0.682	0.946	0.910	1.000
	Extremadura	0.799	0.851	0.737	35	0.839	0.953	0.737	0.952	0.893	1.000
	Cataluña	0.720	0.828	0.590	156	0.774	0.978	0.590	0.930	0.846	1.000
	Comunidad Valenciana	0.733	0.825	0.622	152	0.785	0.966	0.622	0.934	0.855	1.000
	Illes Balears	0.785	0.853	0.706	69	0.826	0.960	0.706	0.951	0.888	1.000
	Andalucía	0.721	0.816	0.605	155	0.775	0.960	0.605	0.931	0.850	1.000
	Región de Murcia	0.774	0.836	0.701	116	0.817	0.944	0.701	0.948	0.885	1.000
	Ciudad Autónoma de Ceuta	0.995	1.000	0.984	2	0.995	1.000	0.984	1.000	1.000	1.000
	Ciudad Autónoma de Melilla	1.000	1.000	1.000	1	1.000	1.000	1.000	1.000	1.000	1.000
	Canarias	0.763	0.834	0.677	130	0.807	0.951	0.677	0.945	0.877	1.000
	Île de France	0.796	1.000	0.546	39	0.986	0.987	0.669	0.807	1.013	0.816
	Champagne-Ardenne	0.783	0.868	0.686	75	0.987	0.987	0.841	0.794	0.879	0.816
	Picardie	0.780	0.873	0.674	96	0.988	0.988	0.825	0.790	0.884	0.816
	Haute-Normandie	0.782	0.884	0.667	86	0.988	0.989	0.818	0.791	0.894	0.816
	Centre	0.781	0.872	0.649	91	0.989	0.989	0.796	0.790	0.881	0.816
	Basse-Normandie	0.783	0.865	0.689	78	0.989	0.990	0.844	0.791	0.874	0.816
	Bourgogne	0.783	0.876	0.676	80	0.990	0.991	0.829	0.791	0.885	0.816
France	Nord - Pas-de-Calais	0.785	0.897	0.628	71	0.991	0.991	0.770	0.792	0.905	0.816
	Lorraine	0.781	0.865	0.659	90	0.991	0.992	0.807	0.788	0.872	0.816
	Alsace	0.782	0.886	0.665	87	0.992	0.993	0.815	0.788	0.892	0.816
	Franche-Comté	0.786	0.858	0.702	59	0.993	0.993	0.861	0.792	0.864	0.816
	Pays de la Loire	0.786	0.897	0.630	64	0.993	0.994	0.772	0.791	0.903	0.816
	Bretagne	0.785	0.889	0.638	70	0.994	0.995	0.782	0.790	0.894	0.816
	Poitou-Charentes	0.783	0.878	0.676	74	0.995	0.995	0.828	0.788	0.883	0.816
	Aquitaine	0.786	0.893	0.635	63	0.995	0.996	0.778	0.790	0.897	0.816
	Midi-Pyrénées	0.786	0.886	0.642	62	0.996	0.997	0.787	0.789	0.889	0.816
	Limousin	0.803	0.886	0.737	33	0.997	0.997	0.903	0.806	0.889	0.816

	Rhône-Alpes	0.793	0.943	0.595	45	0.997	0.998	0.729	0.795	0.945	0.816
	Auvergne	0.787	0.869	0.692	56	0.998	0.998	0.848	0.788	0.871	0.816
	Languedoc-Roussillon	0.785	0.874	0.655	67	0.999	0.999	0.803	0.786	0.875	0.816
	Provence-Alpes-Côte d'Azur	0.792	0.926	0.610	48	0.999	1.000	0.747	0.792	0.926	0.816
	Corse	0.885	0.988	0.816	4	1.000	1.000	1.000	0.885	0.988	0.816
	Piemonte	0.743	0.854	0.613	148	0.835	0.958	0.711	0.890	0.892	0.863
	Valle d'Aosta	0.910	0.980	0.863	3	1.000	1.000	1.000	0.910	0.980	0.863
	Liguria	0.791	0.898	0.672	49	0.888	1.000	0.779	0.891	0.898	0.863
	Lombardia	0.718	0.845	0.567	157	0.808	1.000	0.657	0.888	0.845	0.863
	Provincia Autonoma Bolzano/Bozen	0.819	0.927	0.735	19	0.907	1.000	0.852	0.903	0.927	0.863
	Provincia Autonoma Trento	0.820	0.918	0.745	18	0.909	0.980	0.863	0.902	0.937	0.863
	Veneto	0.737	0.849	0.606	151	0.828	0.952	0.702	0.890	0.892	0.863
	Friuli-Venezia Giulia	0.789	0.881	0.686	53	0.882	0.969	0.795	0.895	0.909	0.863
Italy	Emilia-Romagna	0.741	0.855	0.608	149	0.832	0.958	0.705	0.890	0.892	0.863
	Toscana	0.754	0.867	0.624	139	0.848	0.975	0.723	0.889	0.890	0.863
	Umbria	0.813	0.890	0.721	22	0.910	0.987	0.835	0.893	0.902	0.863
	Marche	0.788	0.885	0.678	54	0.884	0.984	0.786	0.891	0.899	0.863
	Lazio	0.739	0.861	0.599	150	0.831	0.968	0.694	0.890	0.889	0.863
	Abruzzo	0.794	0.875	0.701	43	0.890	0.967	0.812	0.892	0.905	0.863
	Molise	0.867	0.911	0.819	6	0.961	0.974	0.949	0.902	0.935	0.863
	Campania	0.748	0.850	0.628	143	0.841	0.953	0.727	0.889	0.892	0.863
	Puglia	0.764	0.865	0.646	128	0.861	0.974	0.749	0.887	0.889	0.863
	Basilicata	0.836	0.892	0.774	11	0.931	0.965	0.897	0.898	0.924	0.863
	Calabria	0.784	0.865	0.691	72	0.878	0.954	0.801	0.893	0.906	0.863
	Sicilia	0.753	0.855	0.634	140	0.847	0.959	0.735	0.889	0.892	0.863
	Sardegna	0.784	0.863	0.692	73	0.878	0.953	0.802	0.892	0.905	0.863
	Groningen	0.824	0.942	0.700	14	0.972	1.000	0.921	0.848	0.942	0.760
	Friesland	0.816	0.890	0.733	20	0.965	0.969	0.957	0.846	0.918	0.766
	Drenthe	0.833	0.898	0.757	12	0.996	0.994	1.000	0.837	0.903	0.757
	Overijssel	0.792	0.884	0.689	46	0.926	0.944	0.892	0.855	0.936	0.772
	Gelderland	0.769	0.868	0.655	122	0.892	0.970	0.838	0.861	0.894	0.781
The Netherlands	Flevoland	0.823	0.875	0.781	16	0.988	0.988	1.000	0.833	0.886	0.781
	Utrecht	0.780	0.881	0.666	93	0.902	0.964	0.859	0.865	0.914	0.775
	Noord-Holland	0.756	0.873	0.623	137	0.880	1.000	0.797	0.860	0.873	0.781
	Zuid-Holland	0.745	0.858	0.614	145	0.869	0.989	0.785	0.857	0.868	0.781
	Zeeland	0.841	0.913	0.762	10	1.000	1.000	0.986	0.841	0.913	0.773
	Noord-Brabant	0.759	0.868	0.634	134	0.882	0.985	0.811	0.860	0.881	0.781
	Limburg	0.794	0.888	0.688	44	0.925	0.947	0.884	0.858	0.938	0.778

	Tees Valley and Durham	0.778	0.864	0.706	106	0.980	0.980	0.912	0.794	0.882	0.774
	Northumberland and Tyne and Wear	0.778	0.862	0.681	105	0.981	0.981	0.880	0.793	0.879	0.774
	Cumbria	0.809	0.870	0.761	26	0.993	0.993	1.000	0.814	0.877	0.761
	Greater Manchester	0.777	0.871	0.640	112	0.982	0.982	0.827	0.791	0.887	0.774
	Lancashire	0.777	0.859	0.683	107	0.983	0.983	0.883	0.791	0.874	0.774
	East Yorkshire and Northern Lincolnshire	0.785	0.871	0.715	68	0.984	0.984	0.924	0.798	0.885	0.774
	North Yorkshire	0.791	0.883	0.716	50	0.985	0.985	0.926	0.803	0.897	0.774
	South Yorkshire	0.780	0.856	0.691	95	0.985	0.985	0.894	0.791	0.869	0.774
	West Yorkshire	0.777	0.863	0.650	110	0.986	0.986	0.841	0.788	0.876	0.774
	Derbyshire and Nottinghamshire	0.777	0.878	0.656	109	0.986	0.986	0.848	0.788	0.890	0.774
	Leicestershire. Rutland and Northamptonshire	0.779	0.883	0.661	103	0.987	0.987	0.855	0.789	0.894	0.774
	Lincolnshire	0.796	0.861	0.743	38	0.988	0.988	0.960	0.806	0.871	0.774
United Kingdom	Herefordshire. Worcestershire and Warwickshire	0.780	0.861	0.686	94	0.988	0.988	0.887	0.789	0.871	0.774
	Shropshire and Staffordshire	0.778	0.860	0.683	104	0.989	0.989	0.883	0.787	0.870	0.774
	West Midlands	0.780	0.878	0.639	98	0.989	0.989	0.826	0.789	0.888	0.774
	East Anglia	0.780	0.874	0.644	99	0.990	0.990	0.832	0.788	0.883	0.774
	Bedfordshire and Hertfordshire	0.781	0.895	0.655	88	0.990	0.990	0.846	0.789	0.904	0.774
	Essex	0.779	0.875	0.669	101	0.991	0.991	0.865	0.786	0.883	0.774
	Inner London	0.791	0.961	0.573	51	0.991	0.991	0.741	0.798	0.969	0.774
	Outer London	0.786	0.916	0.608	65	0.992	0.992	0.786	0.792	0.924	0.774
	Berkshire. Buckinghamshire and Oxfordshire	0.783	0.894	0.627	83	0.992	0.992	0.811	0.789	0.901	0.774
	Surrey. East and West Sussex	0.783	0.891	0.630	84	0.993	0.993	0.815	0.789	0.898	0.774
	Hampshire and Isle of Wight	0.780	0.888	0.653	92	0.993	0.993	0.844	0.786	0.894	0.774
	Kent	0.780	0.873	0.674	97	0.994	0.994	0.871	0.785	0.878	0.774
	Gloucestershire. Wiltshire and Bristol/Bath area	0.783	0.887	0.636	82	0.994	0.994	0.822	0.787	0.892	0.774

Dorset and Somerset	0.783	0.864	0.690	77	0.995	0.995	0.892	0.787	0.868	0.773
Cornwall and Isles of Scilly	0.811	0.860	0.771	24	1.000	1.000	1.000	0.811	0.860	0.771
Devon	0.783	0.852	0.702	76	0.996	0.996	0.909	0.786	0.855	0.772
West Wales and The Valleys	0.779	0.865	0.680	100	0.997	0.997	0.879	0.782	0.867	0.774
East Wales	0.786	0.870	0.689	60	0.998	0.998	0.891	0.788	0.872	0.774
Eastern Scotland	0.783	0.877	0.648	79	0.998	0.998	0.837	0.785	0.879	0.774
South Western Scotland	0.783	0.878	0.647	81	0.998	0.998	0.837	0.784	0.879	0.774
North Eastern Scotland	0.815	0.927	0.729	21	1.000	1.000	1.000	0.815	0.927	0.729
Highlands and Islands	0.799	0.829	0.774	36	1.000	1.000	1.000	0.799	0.829	0.774
Northern Ireland (UK)	0.782	0.882	0.670	85	1.000	1.000	0.866	0.782	0.882	0.774

Table 7.3: Average scores at a country level

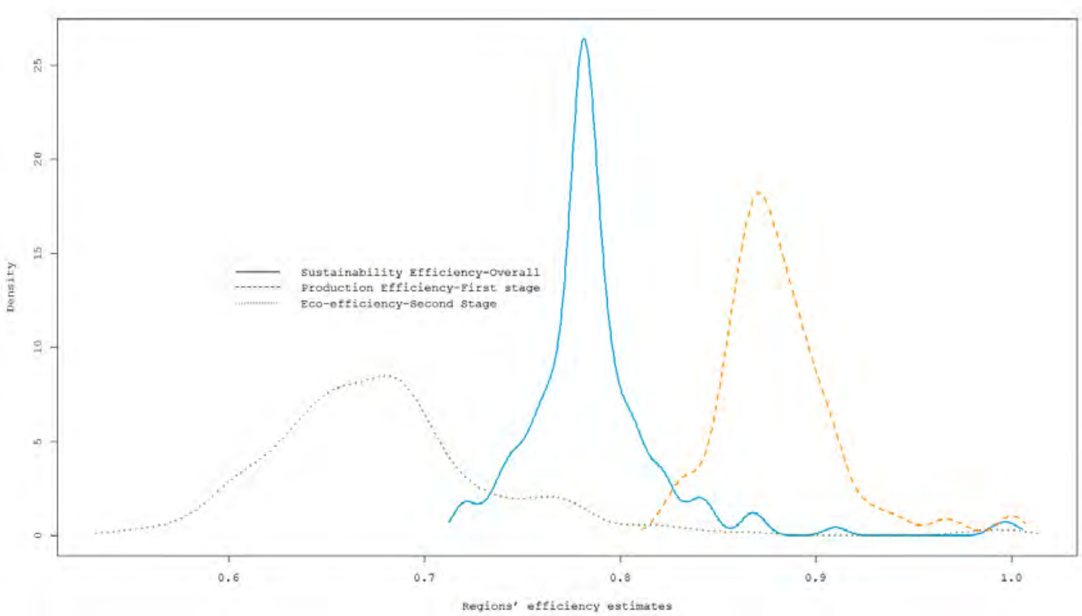
Countries	E_0	E_0^1	E_0^2	E_0^k	E_0^{k1}	E_0^{k2}	TGR_0	TGR_0^1	TGR_0^2
Belgium	0.810	0.900	0.715	0.923	0.983	0.858	0.879	0.916	0.833
Germany	0.777	0.874	0.662	0.919	0.970	0.867	0.846	0.901	0.764
Spain	0.794	0.865	0.714	0.833	0.966	0.714	0.952	0.895	1.000
France	0.791	0.894	0.662	0.993	0.994	0.811	0.796	0.900	0.816
Italy	0.785	0.880	0.681	0.879	0.973	0.789	0.894	0.905	0.863
The Netherlands	0.794	0.886	0.692	0.933	0.979	0.894	0.852	0.906	0.774
United Kingdom	0.785	0.876	0.677	0.991	0.991	0.877	0.792	0.884	0.772

Figure 7.1 confirms the initial findings about overall sustainability efficiency scores and production efficiency scores. Specifically, the distribution of the sustainability scores is leptokurtic and the efficiency scores are clustered around the mean (0.787) which is the highest peak among the three distributions. Furthermore, most of the efficiency scores lie between 0.720 and 0.830 and only a small fraction lies above 0.830. The high peak and the fat tails of the distribution reveal the small standard deviation of sustainability scores. Similarly for production efficiency scores, the distribution is leptokurtic with high peak at 0.880. The peak for production efficiency is a slightly lower than the peak for sustainability

efficiency and most of the scores lie between 0.810 and 0.930. The standard deviation is again low due to the high peak and the fat tails. Considering the eco-efficiency stage, the distribution is platykurtic with a much lower peak and thinner tails. The efficiency scores are less clustered around the mean (0.680) and they are more dispersed relative to the other two distributions which results into much higher standard deviation.

The analysis of Figure 7.1 reveals that the standard deviation in production efficiency stage is significantly lower than in eco-efficiency stage. On the one hand, the clustered scores around the mean and low standard deviation for the production efficiency stage is an indication of the common economic strategies among the European countries. On the other hand, the dispersed scores and the high standard deviation for the eco-efficiency stage is an indication of different environmental policies in the European countries or different level of implementation of the common strategies due to national administrative arrangements (Knill and Lenschow 1998). The results confirm previous findings about the lack of convergence of environmental policies across countries. Holzinger and Knill (2005) argued that the results about policy convergence are rather ambiguous. Furthermore, similar to our findings Nicolli et al. (2012) found heterogeneity across countries about the level of implementation of other common European environmental policies, the waste-related policies. As has been stated, year 2008 marks the beginning of the first commitment period of Kyoto protocol. It is likely that a future study after year 2012, when the first commitment period ends, might yield different results. Considering the empirical findings, the suggestion to the decision maker about the improvement of the eco-efficiency index, rather than the production efficiency index, in order to improve the overall sustainability index, seems more realistic.

Figure 7.1: Densities of sustainability efficiency, production efficiency and eco-efficiency indices.



7.4. Summary

This Chapter incorporated a metafrontier framework into two-stage DEA models. A metafrontier framework is applied when there is a need to study DMUs in different groups which possibly experience different technologies. Metafrontier framework is able to handle the heterogeneity of DMUs in different groups by calculating different group-specific frontiers for every group and also a common metafrontier which envelops every group frontier. Therefore, a DMU is compared relative to its group-specific frontier and also relative to the overall metafrontier.

The two-stage metafrontier framework is applied to evaluate the environmental sustainability at NUTS 2 regions in seven European countries for the year 2008. The environmental sustainability index is adopted from Chapter 6 and consists of "*production efficiency*" and "*eco-efficiency*". The results reveal different technological frameworks among countries for *"eco-efficiency"* and slightly different for *"sustainability efficiency"*. Furthermore, metafrontier results show high scores for the overall sustainability index, very high scores for the "*production efficiency*" index and good scores for the "*eco-efficiency*" index.

From a decision maker's point of view, the regions have greater potential to improve their "*eco-efficiency*" scores in order to improve the overall sustainability scores. Furthermore, the results indicate small inequalities among the regions relatively to their production activity and larger inequalities for their polluting activity. However, in an average country level the results seem to be relatively stable. The densities of the indices further reveal the inequalities in the eco-efficiency stage which might be a result of different environmental policies among different European countries or different level of implementation of common environmental strategies.

Chapter 8 provides a summary of the thesis, marks the research contributions and the most significant findings and proposes a number of aspects for future research.

Chapter 8

Summary, conclusions and future research

8.1. Summary and major conclusions

This thesis proposes a research framework for the modeling of non-parametric production functions in two stages without assuming any specific functional form. Inside this research framework, this thesis makes a number of distinct contributions by constructing two-stage DEA models which are able to handle special cases (such as the incorporation of expert opinion, the introduction of time component and the heterogeneity of DMUs in different groups) and use them to create novel indices which evaluate the efficiency in various economic applications.

Specifically, ***Chapter 1*** gives the outline of technical efficiency and provides the link between production economics and efficiency analysis. Furthermore, Chapter 1 presents the basic terminology and graphical presentation which are needed for the rest of the thesis. Finally, Chapter 1 presents the advantages of data envelopment analysis which is the method used in this thesis. DEA does not use biased and subjective opinions and it is based on the objectivity of the numerical data. In addition, DEA can handle multiple inputs and outputs measured in different units. DEA does not require any specific assumptions regarding the functional form and the distribution of inefficiency. Furthermore, DEA has the ability to identify sources and level of inefficiency in each input and output for each DMU and find the benchmark DMUs which are used as reference points in order to tackle inefficiencies.

Chapter 2 presents and discusses the basic DEA models which are the multiplier and the envelopment model for input and output orientation. Furthermore, Chapter 2 presents the CCR model which exhibit constant returns to scale and the BCC model which exhibit variable returns to scale. ***Chapter 3*** provides the methodological framework for this thesis which is the two-stage DEA models. Conventional DEA models, such as the ones presented in Chapter 2, assume that the DMU is a “black box” which consumes inputs to produce outputs without considering any possible internal procedures. Network and two-stage (which are a special case of network) models are necessary in the presence of such internal procedures. Chapter 3 classifies two-stage DEA models into four categories. Independent two-stage DEA models apply a typical DEA model at each stage separately and evaluate the efficiency without considering the interaction and possible conflicts between the two stages because of the intermediate variables. Connected two-stage DEA models take into account the interaction between the stages. Relational two-stage DEA models assume a multiplicative or additive relationship between the overall and the individual efficiencies. The distinctive feature of this approach is that the multipliers of the intermediate variables are the same regardless of whether the intermediate variables are used as inputs or outputs. The last category is about game theoretic two-stage DEA models. Chen et al. (2014) after an extensive investigation of envelopment and multiplier two-stage DEA models, found that multiplier models (such as all relational models) should be used for the evaluation of the overall and individual efficiencies. Following Chen et al. (2014) this thesis uses relational two-stage DEA models (the additive and the multiplicative) as the basic models for the subsequent chapters. Finally, Chapter 3 presents and categorize every two-stage DEA application which has been published in well-known refereed academic journals until early 2015.

Chapter 4 presents the principal contribution of this thesis, which is the **Weight Assurance Region (WAR)** DEA model. WAR model is a modification of additive efficiency decomposition model of Chen et al. (2009a) in order to incorporate a priori knowledge, such as expert opinion, value judgements, known information and/or widely accepted beliefs or preferences and other type of information. Specifically, WAR model restricts the ratio of the weights of each stage inside a region between β and δ which are positive scalars $0 < \beta \leq \delta$. Furthermore, when $\delta = 1/\beta$ it yields the same results with the original additive two-stage DEA models. Therefore the WAR model can be considered as a more general case of the original model. Moreover, WAR model overcomes an infeasibility problem of the original additive model when the weight of the first or the second stage takes the zero value. Conceptually, it is not reasonable for a stage to have no contribution to the overall process because the need for a two-stage model would no longer stand. Computationally, assigning zero weight to one stage makes the calculation of the other stage impossible. The proposed WAR model overcomes this drawback by construction.

Then, the WAR model is applied at an economic application about the cross-country efficiency evaluation of secondary education in 65 countries. The overall efficiency index evaluates how the school environment affects student performance. The first stage measures the “learning environment efficiency” and the second stage measures the “student’s performance efficiency”. The results reveal an interesting conclusion about restrictions in general which is also true for the WAR model. Neglecting an important restriction in a model results in overestimation of the true results.

Chapter 5 examines the performance of DMUs over multiple periods by introducing the time component into relational DEA models. The contribution of Chapter 6 is the extension of the multiplicative two-stage DEA model into **window-based** approach and the **mathematical formulation** of the window-based LP problem of the relational two-stage DEA model (both the multiplicative and the additive). The window-based relational models are applied to the banking system of 17 OECD countries for eleven years (1999-2009). The first stage of the efficiency index measures the “value added activity” and the second stage evaluates “profitability”. The results are relatively stable over time and any positive or negative change is in minor scale. There are large discrepancies among countries which are attributed primarily to the “value added activity” stage.

Chapter 6 creates an **environmental sustainability index** which measures the “production efficiency” in the first stage and the “eco-efficiency” in the second stage. The proposed index offers a number of advantages. Specifically, the overall sustainability index promotes the synergies between economic growth and environmental objectives which is in line with green growth. Furthermore, the eco-efficiency index in the second stage serves as a decoupling indicator as defined by Wursthorn et al. (2011) because it measures the ability of an economy to break the link between environmental pressures and economic goods. The proposed index is applied at 20 countries with advanced economy for the time period 1990-2011. The time component is handled as proposed in Chapter 5. Furthermore, Chapter 6 provides the VRS version of the window-based additive model. The results indicated that eco-efficiency stage is characterized by large inequalities among countries and significant lower efficiency scores compared to the overall sustainability and production efficiency levels. The results indicated that the integration of environmental, economic and social objectives are the key elements towards sustainability. Non-performing countries should promote smart (education and knowledge), sustainable (resource preservation, cleaner procedures, eco-efficiency and competitiveness) and (social targets such as higher employment) inclusive growth.

Chapter 7 presents a novel approach which introduces **metafrontier** framework into two-stage DEA models. This approach is used in order to treat of heterogeneity among DMUs in different groups (such as firms in different groups or regions in different countries) which experience possibly different technologies. DMUs from different groups face different production opportunities; therefore feasible input-output combination in one group may not be feasible in another. These differences among groups may refer to physical, human and financial capital, infrastructures, economic environment, available resources etc; as a result every group has a different frontier. The overall frontier which envelops all the group frontiers is the metafrontier. The two-stage metafrontier framework is applied at 157 regions in 7 countries and evaluates the regional sustainability efficiency as it has been presented in Chapter 6. The results reveal a greater potential for improvements in "*eco-efficiency*" stage, in order to improve the overall sustainability scores. Furthermore, the results indicate small inequalities among the regions relatively to their production activity and larger inequalities for their polluting activity. The "eco-efficiency" results might be an outcome of different environmental policies among different European countries or different level of implementation of common environmental strategies.

8.2. Future perspectives

The research framework of two-stage DEA formulations which was used throughout this thesis allows for a number of aspects to be investigated in future research. One way for future research is to extend the models proposed in this thesis into more stages via network approaches. More stages would allow for more complex economic applications to be studied. Furthermore, regarding the proposed indices (school efficiency, banking system efficiency and sustainability efficiency) this thesis provides the general framework for their construction. Specifically, the general framework for the overall sustainability index is that it consists of “production efficiency” and “eco-efficiency” indices in the first and second stage respectively. The specific input-output datasets which have been used here are in some cases constrained by the availability of the data. For example, regarding the regional environmental sustainability application presented in Chapter 7, one can include more pollutants such as SO_2, and NOx emissions. Therefore, additional variables could be used inside the proposed framework.

Regarding the applications over time periods in Chapters 5 and 6, adding more years might alter the results. This is more likely to be true especially for the application on banking systems because the years after 2009 would incorporate the effects of the Global Economic crisis. Another direction for future research is to incorporate the time component into the WAR model. An additional interesting field for the WAR model is to empirically investigate the relations among the two-stages (in any economic application) and find the proper regions for the weights of the model.

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