JEAN-BAPTISTE BÉLANGER, HYDRAULIC ENGINEER, RESEARCHER AND ACADEMIC

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Abstract: Jean-Baptiste BÉLANGER (1790-1874) worked as a hydraulic engineer at the beginning of his career. He developed the backwater equation to calculate gradually-varied open channel flow properties for steady flow conditions. Later, as an academic at the leading French engineering schools (Ecole Centrale des Arts et Manufactures, Ecole des Ponts et Chaussées, and Ecole Polytechnique), he developed a new university curriculum in mechanics and several textbooks including a seminal text in hydraulic engineering. His influence on his contemporaries was considerable, and his name is written on the border of one of the four facades of the Eiffel Tower. BÉLANGER's leading role demonstrated the dynamism of practicing engineers at the time, and his contributions paved the way to many significant works in hydraulics.

Keywords: Jean-Baptiste BÉLANGER, Open channel flows, Backwater equation, Gradually-varied flows, Critical flow conditions, Hydraulic jumps, Momentum equation, Mechanics curriculum, Hydraulic engineering, Fluid dynamics.

INTRODUCTION

The application of the momentum principle to the hydraulic jump is commonly called the Bélanger equation. It is named after Jean-Baptiste BÉLANGER (1790-1874) (Fig. 1) who first derived the correct relationship between the flow properties upstream and downstream of a hydraulic jump in a horizontal rectangular prismatic channel (BÉLANGER 1841):

$$\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1 + 8\,Fr_1^{\,2}} - 1\right) \qquad (1)$$

where d and V are the flow depth and velocity respectively, the subscripts 1 and 2 refer to the upstream and downstream flow conditions respectively, Fr is the Froude number: $Fr = V / \sqrt{g\,d}$, and g is the gravity acceleration.

Jean-Baptiste BÉLANGER's considerable influence on his contemporaries is too often lost. In this paper, the contribution of Jean-Baptiste BÉLANGER to hydraulic engineering and fluid mechanics is re-considered broadly. In hydraulic engineering, he developed the backwater equation, the concept of critical flow conditions and the basic equation of the broad-crested weir overflow (Table 1). As an academic at the French leading engineering schools (Ecole Centrale des Arts et Manufactures, Ecole des Ponts et Chaussées, and Ecole Polytechnique), he developed a new university curriculum in statics and dynamics.

Life of Jean-Baptiste BÉLANGER (1790-1874)

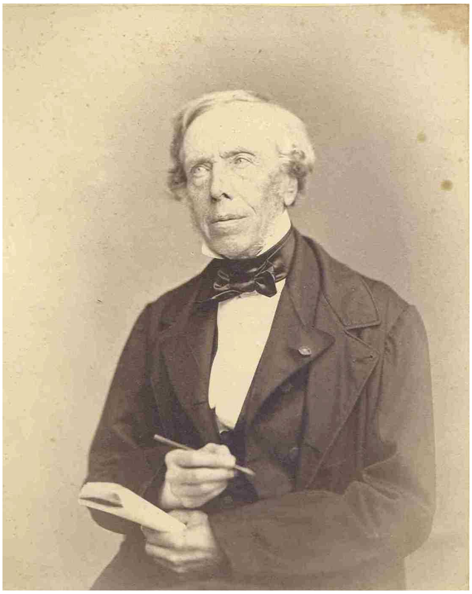
Jean-Baptiste Charles Joseph BÉLANGER was born in Valenciennes (Parish of St Vaast en Ville) in northern France, on 4 April 1790. He was the son of Charles Antoine Aimé Joseph BÉLANGER, master locksmith, and of Jeanne Françoise Joseph FAUCONNIER who were married on 21 April 1789 (CHANSON 2009a).

He studied in Paris at the Ecole Polytechnique in the 1808 cohort (*promotion 1808*) together with Gustave Gaspard CORIOLIS (1792-1843) and Jean-Victor PONCELET (1788-1867) (Journal de l'Ecole Polytechnique 1931), graduating second in his cohort. He studied from 1812 at the Ecole des Ponts et Chaussées (TARBÉ de St HARDOUIN 1884). BÉLANGER started his professional life as Ingénieur du Corps des Ponts et Chaussée in 1816. He was promoted to Ingénieur en Chef du Corps des Ponts in 1843. From 1838, he became a Lecturer at the Ecole

33rd IAHR Congress: Water Engineering for a Sustainable Environment

ISBN: 978-94-90365-01-1

Centrale des Arts et Manufactures, Ecole des Ponts et Chaussées and Ecole Polytechnique. BÉLANGER married Louise Aimée DUMAS (1797-1877). He retired in 1864. He died on 8 May 1874 at Neuilly-sur-Seine, and he was buried in the old cemetery together with his wife (*cimetière ancien, 5ème division*) (Fig. 2).



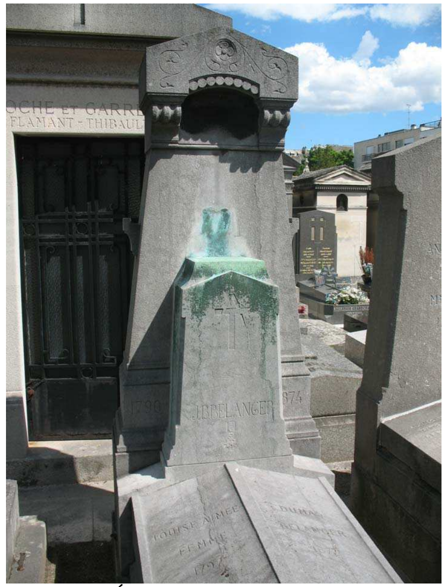


Fig. 1 - Photograph of Jean-Baptiste BÉLANGER
Fig. 2 - Tombstone of J.B. BÉLANGER at Neuilly-sur-Seine(*cimetière ancien, 5ème division*) - Note his wife's engraved name in foreground and the destroyed bust

Table 1 - Leading contributions of Jean-Baptiste BÉLANGER in open channel hydraulics

Period (1)	Reference (2)	Edition (s) (3)	Contribution (4)	Comment (5)
1816-1838	BÉLANGER (1828)	1828	Backwater equation, Direct step method, Froude number, Critical flow, GVF vs RVF	Incorrect treatment of hydraulic jump
1838-1864	BÉLANGER (1841)	1841, 1849	Hydraulic jump equation, Composite channel calculations, Broad-crested weir, Maximum discharge	

JEAN-BAPTISTE BÉLANGER: THE ENGINEER (1816-1838)

As Ingénieur du Corps des Ponts et Chaussées, Jean-Baptiste Charles Joseph BÉLANGER began his engineering career in 1816 at La Réole. From 1821, he moved to work on the Somme navigation canal and after 1826 on the Ardennes navigation canal. During these two missions, he designed navigation canals, their water supplies and water resource management (CHANSON 2009a). Later Jean-Baptiste BÉLANGER worked with Antoine-Rémi POLONCEAU (1778-1847) in the Seine-et-Oise department. His contribution included the first design of the Paris-Rouen-Le Havre railway, opened in 1837 between Paris-Saint-Lazare and La Garenne-Colombes, extended to Rouen in 1843 and completed in 1847 to Le Havre (POLONCEAU and BÉLANGER 1837). The study integrated also the design of the Paris-Dieppe railway.

BÉLANGER's (1828) memoir in open channel hydraulics

In 1826, Jean-Baptiste BÉLANGER developed new ideas in open channel hydraulics and he submitted a detailed memoir to the Commission des Ponts et Chaussées et des Mines in 1827. It was successfully peer-reviewed and published in 1828 (BÉLANGER 1828). BÉLANGER (1828) developed a new equation for the prediction of the free-surface profile in gradually-varied flows: i.e., the backwater equation (CHANSON 2009a). The basic assumptions were: (a) a steady flow, (b) an one-dimensional flow motion, (c) a gradual variation of the wetted surface with distance x along the channel, (d) friction losses that are the same as for an uniform equilibrium flow for the same depth and discharge, and (e) a hydrostatic pressure distribution. BÉLANGER (1828) derived the backwater equation from momentum considerations as:

$$\sin\theta\,\partial x - \cos\theta\,\partial d - \frac{P_w}{A}(a\,V + b\,V^2) + \frac{Q^2}{g\,A^3}\partial A = 0 \qquad (2)$$

where θ is the angle between the bed and the horizontal, x is the longitudinal distance positive downstream, d is the flow depth measured normal to the invert, A is the cross-section area, P_w is the wetted perimeter, Q is the discharge. In Equation (2), BÉLANGER estimated the friction losses using the Prony formula :

$$-\frac{\partial H}{\partial x} = \frac{4}{D_H}\left(a\,V + b\,V^2\right) = \frac{f}{D_H}\frac{V^2}{2\,g} \qquad (3)$$

where H is the total head, D_H is the hydraulic diameter: $D_H = 4\ A/P_w$, and a and b are constant ($a = 4.44499\ 10^{-5}$, $b = 3.093140\ 10^{-4}$). Equation (3) is compared with modern expressions in terms of the Darcy-Weisbach friction factor on the right handside. Interestingly BÉLANGER (1849) was aware of the work of Henry DARCY (1803-1858) in pipe flows, but he continued to use the Prony formula for its simplicity. Denoting S_f the friction slope: $S_f = -\partial H/\partial x$, and S_o the bed slope: $S_o = \sin\theta$, Equation (2) may be combined with the continuity equation to yield:

$$\frac{\partial}{\partial x}\left(d\cos\theta + \frac{V^2}{2\,g}\right) = S_o - S_f \qquad (4)$$

Equation (4) is identical to modern expressions of the backwater equation (HENDERSON 1966, MONTES 1998). Equation (2) was tested for two configurations (CHANSON 2008,2009a). The results showed basically very little differences between data and calculations, despite the challenging geometry and the crude nature of the Prony formula. Note that BÉLANGER performed his calculations manually and this justified the usage of PRONY's simplified formula at the time (BROWN 2002).

Jean-Baptiste BÉLANGER integrated the backwater equation by selecting known water depths and calculating manually the distance in between. This technique is called the step method distance calculated from depth (HENDERSON 1966, CHANSON 2004) or the direct step method. BÉLANGER introduced also the concept of critical flow conditions as one of two singularities of the backwater equation. He also recognised the importance of the ratio $V/\sqrt{g\,d}$ now called the Reech-Froude number after the works of Ferdinand REECH (1852) and William FROUDE (1872), 24 and 44 years respectively later. Further BÉLANGER understood the concepts of gradually-varied flows (GVF) and rapidly-varied flows (RVF). His 1828 treatise marked a seminal advance in open channel hydraulics.

JEAN-BAPTISTE BÉLANGER: A LEADING ACADEMIC (1838-1864)

In 1838, Jean-Baptiste BÉLANGER became a lecturer at the Ecole Centrale des Arts et Manufactures and he lectured there until 1864 (Fig. 3). He lectured also at the Ecole des Ponts et Chaussées from 1841 to 1855, and at the Ecole Polytechnique from 1851 to 1860 (Table 2).

From 1851, as a Full Professor at the Ecole Polytechnique, he developed a new university curriculum in mechanics (cours de Mécanique) in response to a re-structure of the engineering programme at Ecole Polytechnique (CHATZIS 1995). Linking kinematics and dynamics, he argued that the mechanics is based upon three principles: inertia, action-reaction and constant ratio force to acceleration at any point. Among the innovations, he considered statics as a limited case of dynamics which was most innovative in France at the time. His basic ideas were first developed in his lecture notes (BÉLANGER 1847), although criticised by Adhémar Jean Claude BARRÉ de SAINT-VENANT (1797-1886) and Ferdinand REECH (1805-1880) without much success (CHATZIS 1985). BARRÉ de SAINT-VENANT believed that kinematics could be taught without the concept of force. Instead REECH argued that BÉLANGER's definition of force was erroneous. Jean-Baptiste BÉLANGER wrote further textbooks, including a series of treatise in Mechanics and Applied Mechanics: BÉLANGER (1864a,1864b,1866). His ideas influenced strongly the teaching of statics and dynamics, and mechanics in France and Europe during the 19th century as well as into the 20th century.

Table 2 - Lecturing responsibilities of Jean-Baptiste BÉLANGER

Engineering school:	Ecole Centrale des Arts et Manufactures	Ecole Polytechnique	Ecole des Ponts et Chaussées
Start:	1838: Chair in Geometry (analyse géometrique) and Mechanics (mécanique générale), & Director of Studies in replacement of Jean-Baptiste LIOUVILLE	1851: Chair in Mechanics	1841: Chair in Applied mechanics (mécanique appliquée) Lectures in Hydraulic Engineering
End:	1864 (or 1866)	1861	1855 (or 1853)

Fig. 3 - Photograph of Jean-Baptiste BÉLANGER among his colleagues of the Ecole Centrale des Arts et Manufactures - Jean-Baptiste BÉLANGER is on the second row, third from the left (with white hairs)

Jean-Baptiste BÉLANGER's contribution in hydraulics

Twenty one years after this original essay, BÉLANGER (1841) expanded his hydraulics memoir into the form of a series of lecture notes for the Ecole des Ponts et Chaussées. The lecture notes were used at the Ecole des Ponts et Chaussées and Ecole Centrale des Arts et Manufactures, and available at the Ecole Polytechnique et Ecole des Mines de Paris. This comprehensive treatise in hydraulic engineering was re-edited at least five times with relatively small to moderate differences (Table 3). The textbook has had a profound influence in leading hydraulic engineering scholars including BOUSSINESQ (1877), FORCHHEIMER (1914), BAKHMETEFF (1932), JAEGER (1956), MONTES (1998).

Table 3 - Editions of BÉLANGER's lecture notes in hydraulic engineering

Date	Title	University year	Engineering school
(1)	(2)	(3)	(4)
1841	Notes sur l'Hydraulique	1841-42	Ecole des Ponts et Chaussées
1845	Notes sur l'Hydraulique	1845-1846	Ecole des Ponts et Chaussées
1846	Résumé du cours d'hydraulique	1846-47	Ecole Centrale des Arts et Manufactures
1849	Notes sur le Cours d'Hydraulique	1849-50	Ecole des Ponts et Chaussées
1850	Notes sur le cours d'hydraulique	1850-51	Ecole Centrale des Arts et Manufactures
1850	Notes sur l'Hydraulique		

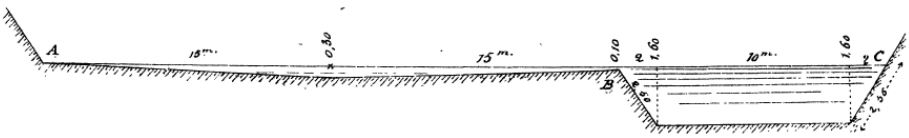


Fig. 4 - Composite open channel cross-section

Jean-Baptiste BÉLANGER corrected his treatment of the hydraulic jump in 1838 (BÉLANGER 1841). He solved the momentum equation for a hydraulic jump in a flat channel. For a rectangular channel and neglecting the friction force, he obtained:

$$\frac{d_2}{d_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2\,\alpha' \mathrm{Fr}_1^{\;2}} \qquad (5)$$

where α' is a velocity correction coefficient. This reasoning became commonly accepted thereafter (BÉLANGER 1849, BRESSE 1860). For a hydraulic jump in a horizontal, rectangular, prismatic channel, BÉLANGER (1849) calculated the loss in kinetic energy head:

$$\frac{V_1^{\;2}}{2\,g} - \frac{V_2^{\;2}}{2\,g} = \frac{(d_1 + d_2)^2}{4\,d_1 d_2}(d_2 - d_1) \qquad (6)$$

that may be rewritten in terms of the head loss in the hydraulic jump:

$$\Delta H = \frac{(d_2 - d_1)^3}{4\,d_1 d_2} \qquad (7)$$

Equation (7) is a well-known result for a hydraulic jump in a horizontal rectangular channel (HENDERSON 1966, CHANSON 2004).

In the same treatise, BÉLANGER (1841,1849) presented explicitly a number of basic features of open channel flows. He developed an expression of the uniform equilibrium flow depth (normal depth) that was derived from energy considerations. He further developed the calculations of the normal depth for a composite channel (Fig. 4), showing accurately that the total discharge is the sum of the flow rates in the main channel and in the flood plain, and that the friction slope is identical for both channel sections, but with different friction coefficients.

BÉLANGER (1841,1849) showed that, in a rectangular channel, the discharge per unit width is maximum at critical flow conditions for a given specific energy E. This result is sometimes called BÉLANGER's principle (JAEGER 1956). BÉLANGER derived the expression of the critical depth d_c:

$$d_c = \frac{2}{3} E \tag{8}$$

In the same section, he analysed the overflow on a broad-crested weir (Fig. 5). His treatment yielded further the classical expression of the flow rate Q:

$$\frac{Q}{B} = \sqrt{g}\left(\frac{2}{3} H\right)^{3/2} \tag{9}$$

for a rectangular channel of width B, where H is the total head above the crest invert. All these results are common knowledge today (HENDERSON 1966, CHANSON 2004), but were new and important developments in the early 1840s.

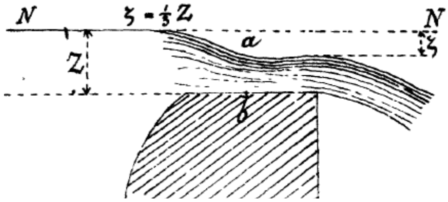


Fig. 5 - Flow over a broad-crested weir

DISCUSSION

Between 1816 and 1864, the work of Jean-Baptiste BÉLANGER demonstrated the dynamism of practicing engineers in France. His own contributions were remarkable and influenced many leading hydraulic engineers including H. DARCY, E. MACH, J. BOUSSINESQ, B. BAKHMETEFF, and very likely R. MANNING, R. FREEMAN, L. PRANDTL, and R. von MISES as illustrated in Figure 6. Henry DARCY (1805-1858) had a high respect for Jean-Baptiste BÉLANGER's hydraulic engineering notes. He also praised J.V. PONCELET and J.B. BÉLANGER for their support and inputs into his research on flow resistance in pipes and channels (DARCY 1858). Both attended in person several experiments conducted by H. DARCY. Franz REULEAUX (1829-1905) respected highly BÉLANGER's (1847) text (REULEAUX (1877). Ernst MACH (1838-1916) listed BÉLANGER's 1847 treatise among a few basic references in Mechanics (MACH 1883). Joseph BOUSSINESQ (1842-1929) included ample references of BÉLANGER' works in his papers (BOUSSINESQ 1877).

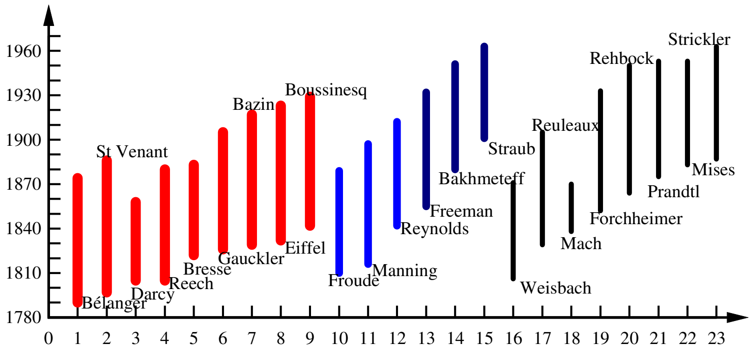


Fig. 6 - Lifespan of leading hydraulic engineers including J.B. BÉLANGER (far left) - Legend: Red = French; Blue = English/American; Black = German speakers.

In hydraulic engineering, BÉLANGER's work has been commonly cited in classical textbooks during the 19th and 20th centuries (BRESSE 1860, FORCHHEIMER 1914, BAKHMETEFF 1932, JAEGER 1956). Modern open channel hydraulic texts continue to acknowledge his seminal contributions: e.g., MONTES (1998), CHANSON (2004).

Jean-Baptiste BÉLANGER lectured for more than 26 years the best engineering students in France. He was highly respected by his former students and peers. One of his students at the Ecole Centrale, was Gustave EIFFEL (1832-1923) who built the Eiffel tower and engraved his name around the first floor together with the names of 71 other scientists (Fig. 7). Altogether 14 hydraulic engineers and scholars had their name engraved (CHANSON 2009b).

Fig. 7 - Inscription BÉLANGER on the Eiffel Tower (Tour Eiffel) between LAGRANGE and CUVIER, with PONCELET and BRESSE on the left - Photograph taken on 25 July 2008

CONCLUSION

Jean-Baptiste BÉLANGER (1790-1874) is a forgotten hero of hydraulic engineering, but his engraved name on the Eiffel tower is a reminder of his praiseworthy contributions. His life spanned over 6 major political regimes in France. For nearly forty five years, he generated a major stimulus in the development of rationale mechanics, fluid dynamics and open channel hydraulics. His works influenced leading fluid mechanics researchers ranging from H. DARCY and J. BOUSSINESQ, to E. MACH, P. FORCHHEIMER, B. BAKHMETEFF and many more.

The unusual feature of BÉLANGER's contribution was his successful involvement in both professional engineering (1816-1838) and academic teaching (1838-1864), and his abilities to develop new fundamental textbook materials. Another aspect was the longevity of his career.

Considering his contribution to hydraulics and fluid mechanics, the backwater equation should be renamed the "Bélanger equation" (CHANSON 2009a), the maximum discharge equation for a given specific energy should be called the "Bélanger principle" (JAEGER 1956) and the application of the momentum principle to the hydraulic jump should be renamed the "Bélanger method" (CHANSON 2009a).

ACKNOWLEDGEMENTS

The author acknowledges the helpful assistance of many people, including the Bibliothèque de l'Ecole Centrale de Paris. the Bibliothèque de l'Ecole Nationale des Ponts et Chaussées, MM. Jean-Yves JOUAN, Philippe LANDRU, and Guillaume SAQUET.

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