LEARNING MATHEMATICS WITH TECHNOLOGY FROM A SOCIAL PERSPECTIVE:
A STUDY OF SECONDARY STUDENTS' INDIVIDUAL AND COLLABORATIVE PRACTICES IN A TECHNOLOGICALLY RICH MATHEMATICS CLASSROOM

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The University of Queensland in August 2008

School of Education

Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

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Galbraith, P., Goos, M., Renshaw, P., & Geiger, V. (2001). Integrating technology in mathematics learning: What some students say. In J. Bobis, B. Perry & M. M. (Eds.), *Numeracy and Beyond* (Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia, Sydney, pp. 225-232). Sydney: MERGA. All authors contributed equally to conception and design. Geiger and Goos contributed equally to data collection. Galbraith, Goos and Geiger shared responsibility for analysis, drafting and writing.

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Abstract

The role of collaborative classroom practices, and of technology, in students' learning of mathematics, is now receiving increased attention in curriculum and policy documents in Australia and internationally. The implementation of pedagogical reforms associated with these areas, however, has been limited due to a range of theoretical and practical concerns. An issue which has emerged from separate interests in technology and collaborative practice is the role of digital tools in enhancing meaningful learning in both individual and collaborative group settings. While the corpus of research literature in the area of technology mediated learning in mathematics is now significant, much of the focus of studies in this area has been concerned with the effect of technology on individual learning outcomes within specific topic domains. Research is needed into the role of digital tools within collaborative classroom environments.

This study investigated the various roles of technology in mediating students' learning, from both individual and collective perspectives, within an authentic, senior secondary classroom setting and sought to identify patterns of student behaviour within this environment. The theoretical framework for the study drew on two bodies of educational research. Firstly, social theories of learning, including Vygotskian socio-cultural ideas, the field of learning discourses, and distributed cognition, were considered. Secondly, research literature associated with approaches to learning mathematics with the assistance of digital technologies, including ideas related to tool co-construction and instrumental genesis, are examined.

Because the focus of this study is on authentic systems of activity rather than individual student outcomes, a naturalistic approach to data collection and analysis was employed. Research was conducted in two senior secondary classrooms over a two and a half year period (1997 – 1999). This involved a pilot study (1997 – 1998) and overlapping main study (1998 – 1999) which were conducted with two different cohorts of students. The investigation was carried out by a teacher-researcher with the support of a research assistant. Qualitative and quantitative data were collected using: student surveys; individual and whole class student interviews; stimulated recall procedures; videotaping of episodes of students working as individuals, in small groups and in whole class settings; and longitudinal participant observation. Data analysis techniques were chosen to match the form and nature of available data and were sensitive to the generation and confirmation of categories of emergent student behaviour. This process was iterative and included an additional phase devoted to category refinement and eventually to theory development.

Patterns of behaviour for students working with digital technologies were identified and the metaphors of *Master*, *Servant*, *Partner* and *Extension-of-self* were chosen to describe the categories that emerged. These categories were further developed into a framework which describes students' interaction with technology while learning mathematics in individual, small group and whole class settings.

The theoretical and practical implications of this study include: the identification of the role of digital technologies in mediating the social practices within authentic mathematics classrooms; the potential of technology to empower students as individuals and as collectives of co-dependent learners; and the potential shift in power structures between teachers and students within mainstream classroom when students are so empowered.

Keywords

technology, learning, computer, calculator, collaboration, community, inquiry, practice, digital, tools

Australian and New Zealand Standard Research Classifications (ANZSRC)

130208 Mathematics and Numeracy Curriculum and Pedagogy 50%, 930102 Learner and Learning Processes 30%, 130212 Science, Technology and Engineering Curriculum and Pedagogy 20%

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Chapter 1

Introduction

1.1. Background, Aims and Significance of the Study

The origin of this study lies in a curiosity aroused by three influences: the author's involvement in a previous study which investigated student interaction within a classroom community of inquiry; the need to extend research knowledge of how technology influences students' learning in collaborative classroom settings; and the need to develop theoretical underpinnings for a world wide perception that technology has the potential to enhance curriculum. These influences are discussed in sections 1.1.1 to 1.1.3 below.

1.1.1. The Perspective of the Teacher-Researcher

This thesis grew out of a desire to extend the findings of a previous study conducted in 1994 – 1996 by Goos (2004) that used a socio-cultural framework to investigate how a teacher established norms and practices that emphasised mathematical sense making, collective reasoning and argumentation, and standards for establishing mathematical "truth" (Goos, 2004). I was the teacher at the centre of Goos's (2004) study. While not a focus of the original investigation, technology was used freely in my classroom due to my belief that technology has great potential to enhance the teaching and learning of mathematics. Questions about the role technology played in assisting me, the teacher, to establish and maintain the norms and practices of my classroom, including the different types of student-student-technology interactions observed, led to the follow-up study on which this thesis reports.

The aim of this study is to identify and understand the role of technology in mediating learning within my own classroom, which is conducted as a community of inquiry. My commitment, as the teacher-researcher, to working with students in collaborative contexts and my capacity to promote student learning through collaborative processes were documented in the study referred to above (Goos, 2004). The current study seeks to build upon the findings of the earlier investigation by exploring the role of technology in the process of knowledge creation via collective reasoning and argumentation through a focus on student-student-technology interaction.

1.1.2. Literature Related to Teaching and Learning

Two bodies of literature are drawn upon in order to establish a theoretical framework for the present study. As this thesis is concerned with student-student interaction, studies that view learning from a social perspective are considered. The role of digital tools is also central to this study and so the corpus of literature devoted to the role of technology in student learning is examined.

Socio-cultural theory is chosen as the lens through which to view students' collaborative practices in the classroom under investigation because this theoretical perspective identifies social interaction as the point from which learning originates and assigns a central role to tools in the mediation of intellectual development. Because the study is situated in a classroom where collective modes of reasoning lead to the co-construction of knowledge, research into communities of inquiry (e.g., Boaler, 1999;

Forman, 2003; Lerman, 2000) and the complementary field of discourse (e.g., Sfard, 2002) are used to gain insight into the different types of student-student interaction that are documented. It must be noted, however, that few social-cultural studies have investigated the influence of digital tools upon social interaction – the origin of intellectual development from a social perspective.

The use of technology to enhance teaching and learning in mathematics has received considerable attention over the past two decades, although this interest has produced only a limited number of theoretical frameworks that describe the role of digital tools in mediating learning. Research into the transformation of artefacts into tools (e.g. Artigue, 2002; Doerr & Zangor, 2000; D. Guin & Trouche, 1999) describes the role of a user in the co-construction of digital devices, such as a computer, into a tool with a specific purpose. Thus, it is the interaction between the user and the device that turns the device into, for example, a mathematical instrument. This process, however, focuses on the interaction between only the individual and the artefact during the formation of a tool and does not consider other possible interactions, such as those between groups of students and a digital device.

Studies into Computer Supported Collaborative Learning (CSCL) (e.g., Stahl, 2006) have documented the potential of digital technologies to facilitate collaborative knowledge creation in both synchronous and asynchronous contexts. While this research demonstrates that technology can mediate collaborative learning, CSCL studies focus on virtual communities rather than mainstream classroom settings.

In summary, research studies have documented the potential of digital technologies to mediate learning through collaborative processes and also argue that the user has a role in shaping the formation of a tool. While findings in this area now appear well established, there is a gap related to the role of technology in mediating learning, both individually and collectively, through social interaction within mainstream classrooms. Further investigation is also required into how social interaction influences the formation of digital devices into mathematical tools.

1.1.3. Curriculum Influence

The use of technology has also been advocated as a means of enriching mathematics curricula (Australian Association of Mathematics Teachers, 1996, 2000; National Council of Teachers of Mathematics, 2000). Areas that have attracted attention include number (Kieran & Guzmán, 2005), algebra and calculus (Ferrara, Pratt, & Robutta, 2006) and geometry (Laborde, Kynigos, Hollebrands, & Strasser, 2006). Findings in relation to the advantages offered by technology to teaching and learning mathematics, however, are equivocal as there are many examples of conflicting results. It has been argued that the lack of clear trends in findings is due to the limited integration of digital technologies into teaching practice (Mariotti, 2002) and the lack of mathematics classrooms available as research sites (Zbiek, Heid, Blume, & Dick, 2007). In addition, much research to date has been based on quasi-experimental designs which limit the applicability of findings to mainstream classrooms (Dunham & Dick, 1994). This situation signals a need for studies in authentic classroom settings where technology is freely available and where digital tools are integrated into the teaching and learning practice.

1.2. Research Questions

In order to investigate the use of digital technologies as both individual and collaborative learning tools within authentic, mainstream classrooms, the following research questions are used to guide the collection and analysis of data.

1. What are the dispositions and preference of students towards using technology in learning mathematics?

2. What are the perceptions of students with respect to their global facility and confidence with digital technologies as a personal resource?

In order to profile students' choice and use of digital technologies the study sought to ascertain their dispositions and preferences for working with technology and how these changed over time. While there is a considerable corpus of research literature devoted to students' mathematical achievement while using technology (e.g., Barton, 2000; Dunham & Dick, 1994) there is little which has considered students' own perceptions of the benefits, or otherwise, of using technology to mediate the learning of mathematics. This study extends previous research in that it chronicles the benefits of using technology from the perspective of the learner rather than through the lens of students' performance of mathematical tasks alone.

3. What choices of specific forms of technology use are favoured by students?

As the students in this study had access to a range of digital technologies, including graphing calculators, computers equipped with both generic and mathematics specific applications, and on-line resources available via the internet, this study considered students' preferences for available technologies, especially in relation to the potential of these technologies to mediate individual and group learning.

4. What choices of general strategic purposes for technology use are favoured by students?

The potential for technology to promote exploratory approaches to learning mathematics (Burrill, G., A. J., Breaux, G., Kastberg, S., Leatham, K., & Sanchez, W., 2002; Templer, R., Klug, D., Gould, I., Kent, P., Ramsden, P., & James, M., 1998) means that there is need to understand strategic choices students make during investigation based classroom inquiry. Current typographies that document students' use of technology in these circumstances are too coarse grained to guide classroom practice and so further research is required in order to develop the descriptive detail necessary for authentic implementation in mathematics classrooms.

5. What roles can be identified for technology in mediating individual student learning?

Much of the existing research has focused on the potential of digital technologies to improve students' achievement in mathematics (Zbiek et al., 2007), and less attention has been given to developing a detailed explanation of how technology transforms the ways that students can learn. While frameworks for students' adoption and use of digital tools are emerging (e.g., Doerr & Zangor, 2000; Dominique Guin, Ruthven, & Trouche, 2005), no common understanding has yet developed of how interaction between learner and technology transforms the artefact into a tool with specific purpose or how these tools mediate mathematical learning. This is a gap specifically addressed by the present study.

6. What roles can be identified for technology in mediating collaborative student learning?

This question captures the distinct contribution of the thesis as it signals an investigation of a relatively unexplored area – how the integration of digital technologies into mathematics classrooms is played out in productive student-student-technology interactions. In doing so, it builds upon an earlier study (Geiger & Goos, 1996) of a single class as students worked on mathematical problem solving tasks. The findings of Geiger and Goo's (1996) study indicated that both the task and the technology were influential in mediating collaborative student interaction and suggested that further research was required to develop a well defined understanding of the role of technology in classrooms which encourage collaborative learning.

1.3. Research Methods

This thesis seeks to understand and document interactions between individual students and technology and between groups of students and technology within everyday classroom settings which are inevitably framed by complex social and cultural influences. Because the focus of this study is on authentic systems of activity rather than individual student achievement, methodologies which employ experimental and control groupings of "subjects" in order to control unwanted variation and to isolate variables of interest are inappropriate (Greeno & MMAP, 1998). Instead a naturalistic research strategy was implemented in order to study systems of interaction between students and technology while embracing all the complexities of a classroom *in situ*, including the accommodation of unexpected emergent phenomena (Lincoln & Guba, 1985).

Research was conducted in two senior secondary classrooms over a two and a half year period (1997 – 1999) as part of a larger study supported by an Australian Research Council grant. This involved a pilot study (1997 – 1998), and overlapping main study (1998 – 1999), which were conducted with different cohorts of students. The investigation was carried out by a teacher-researcher with the support of a research assistant. Qualitative and quantitative data collection instruments were used including: student surveys; individual and whole class student interviews; stimulated recall procedures; videotaping of students working as individuals, in small groups and in whole class settings; and longitudinal participant observation. Data analysis techniques were chosen to match the form and nature of available data but which always remained sensitive to the generation and confirmation of categories of emergent student behaviour. This process was iterative and included an additional phase devoted to category refinement and eventually to theory development. While the specifics of this process are unique and driven by the data collected, the general approach was based primarily on procedures adapted from Galbraith and Haines (1998) and Goos (2004).

1.4. Structure of the Thesis

The theoretical framework of this study is developed in Chapters 2 and 3 which draw on two distinct fields of inquiry in mathematics education. Chapter 2 begins by examining the theories of learning that have had the greatest influence on the forms of pedagogy implemented in school classrooms over the last century. The theoretical underpinnings of behaviourism, constructivism and socio-culturalism are outlined and a rationale provided for the choice of socio-culturalism as the theoretical model most appropriate for framing the current study. The second part of the chapter expands on socio-cultural fields of research related to the development of modes of discourse and reasoning within distinct communities of inquiry. Attention is paid to how learners are initiated into such communities and how they appropriate the cultural tools accepted by the community as mediators of knowledge creation and validation. Further, the role of physical artefacts as mediators of learning and thinking is also addressed.

Chapter 3 offers a critical analysis of research into the use of technology to enhance the teaching and learning of mathematics. A broad overview of studies related to student learning and technology, including factors such as achievement and attitude, is presented. Frameworks that typify students' use of technology when doing mathematics are also considered. The chapter concludes by noting increasing recognition of the potential for technology to mediate social interaction and reviewing studies that take a social perspective on the role of digital technology in promoting students' learning and thinking.

The design and methodological approach to the study are described in Chapter 4. The opening sections of this chapter justify the choice of a naturalistic approach and identify and describe a number of contexts in which the study is embedded. The sections which follow outline the chronology of the investigation and describe the participants as well as the methods of data collection, interpretation and analysis. Finally the trustworthiness of findings is discussed and a framework for presenting findings overviewed.

Chapters 5 to 8 present the findings of the study. Chapter 5 considers research questions concerning students' dispositions, preferences and choices in relation to learning mathematics with technology. Analysis of survey data, video taped classroom episodes of small groups of students working together and a whole class student interview indicated that students in both the main and pilot studies exhibited a positive disposition towards the use of technology in studying mathematics. Further, students identified a number of advantages in relation to working with technology including the capacity to perform difficult or tedious numerical calculations as well as the facility to engage more readily in exploratory approaches to learning mathematics. A number of concerns on the part of the students were also identified, such as the permanence of learning that takes place with the assistance of technology.

Chapter 6 deals with the role of technology in mediating individual learning. Data from questionnaires were considered in developing four metaphors for describing categories of students' technology use during mathematics classes. These categories were confirmed via a process of self-identification of students against the proposed categories.

In Chapter 7 students' collaborative preferences and the nature of small group interaction in technologically rich classroom settings are analysed. Small group behaviours, as identified via survey and video taped classroom data, are categorised by building on the framework developed in Chapter 6. Student support for the use of technology was particularly strong in relation to its potential to facilitate sharing of multiple student perspectives on a mathematical idea through viewing a common display.

The framework of student-technology interactions is further extended in Chapter 8 by considering the role of technology when learning mathematics in whole class settings. Categories were identified and confirmed through the analysis of survey data and video recordings of whole classroom activity. Behaviours were found to be consistent with the metaphors for technology use developed through this thesis. The capacity for technology to act as a mediator for the fuller participation of marginalized students was also identified.

Finally, Chapter 9 summarises and synthesises the findings of the study and considers the contribution to theory and practical implications of these findings. Implications for practice include the identification of forms of collective inquiry which can be facilitated through the use of technology. The final chapter also reflects on the limitations of the study's findings and identifies areas of potential further research.

Chapter 2

Perspectives on Learning and Thinking – Where is My Mind?

In this chapter, theories of learning are examined in order to establish a foundation for discussion about how students think and learn in school mathematics classrooms. As this thesis is concerned with the role of technological tools in students' mathematical thinking and learning, it is critical to develop a theoretical framework that places physical artefacts and cultural tools as central to the process of learning.

The chapter is divided into two sections. The first section, 2.1, Models of Learning, presents a broad overview of theories of intellectual development. The literature considered in this section explores theoretical orientations that attempt to explain the development of consciousness and how we think and learn. While the literature reviewed here draws primarily from the field of mathematics education, broader theories of intellectual development are also considered. Despite criticism from supporters of other theoretical perspectives, it is argued that socio-cultural theory offers a view of teaching and learning that is consistent with the intent of the research agenda embodied in this study.

The next section of this chapter, 2.2, Socio-cultural Theory and School Classrooms, provides further elaboration of the fundamental ideas of socio-cultural theory and, in particular, how these are enacted in school classroom settings. A particular focus here is research into classroom cultures that have been described as communities of inquiry or communities of practice.

The literature selected for review in this chapter provides a theoretical backdrop for an investigation into classroom practices which are supportive of collaborative learning, thinking and problem solving. Consideration of this body of knowledge leads to the identification of a theoretical framework for exploring classroom practices in which the responsibilities of learning and thinking are distributed more evenly among participants than is the case in traditional mathematics classrooms.

2.1. Models of Learning

Of the more influential theories of intellectual development that have emerged since the turn of last century, those that fall under the general umbrellas of behaviourism, constructivism and socio-culturalism have had the greatest impact on the research community and ultimately on school classrooms. An overview of each of these theories of learning is now presented.

2.1.1. Behaviourism

In traditional classrooms, mathematics is considered to be a fixed body of knowledge passed from a sole, expert authority, in the form of a teacher, to students whose role is to "absorb" what is presented to them (Romberg & Carpenter, 1986). This represents an epistemological stance that supports methods of teaching and learning in which students are considered to be empty vessels or blank slates, tabula rasa, waiting to be filled with knowledge and that implies the ability to effectively recall this knowledge is developed through repetition of the very process of recall (Schoenfeld, 1987). While this stance could arguably be attributed to Aristotle, who believed that ideas are associated in memory because they are similar or opposite (Fetherston, 2007), this view of learning is generally described as behaviourism. This movement is often traced back to J. B. Watson's *Psychology as the Behaviourist Views It*, originally published in 1913 (reprinted in 1994). The following excerpt clearly declares his position.

> Psychology as the behaviorist views it is a purely objective experimental branch of natural science. Its theoretical goal is the prediction and control of behavior. Introspection forms no essential part of its methods, nor is the scientific value of its data dependent upon the readiness with which they lend themselves to interpretation in terms of consciousness.
>
> (Watson, 1994, p. 248)

Watson and other influential behaviourists such as B. F. Skinner held that any consideration of "mentalism", which they referred to as the study of mental structures, was irrelevant. All that mattered was observable behaviours or changes in behaviour. In this view, behaviour was controllable and malleable, and was determined by the internalization of responses to stimuli embedded in a subject's environment. This view portrays human learning as essentially passive and reactive. The central place of the environment in behaviourist theory resulted in a focus on the management of the environment in order to effect learning. Investigations into the effectiveness of "programmed instruction" approaches to teaching (e.g. Skinner, 1958), carefully sequenced modules of instruction in which ideas and concepts were broken down into elemental steps, sought to promote a position that the faultless acquisition of knowledge could be achieved through exclusive exposure to a well designed learning environment. These programs were to be engaged with individually as any interference from the ambient environment increased the risk of the contamination of the learning sequence. Thus, in this view, learning takes place when the new concepts are acquired via a process of transmission from one point, in this case from the learning program, to another point, the learner. Interaction with any other sources was to be avoided at all costs as any interference would risk the corruption of the fact, idea or concept being transmitted.

While not strictly part of the behaviourist tradition, the movement in psychology known as associationism paralleled the development of behaviourism and proposed principles of learning that were integrated into what were considered behaviourist approaches to instruction. While associationists considered the acquisition of knowledge was reliant on the organisation of mental structures, the theory of instruction they proposed to achieve this organisation was fundamentally behaviourist in nature. Associationist psychologists such as Thorndike (1922) described a theory of learning in which associations were developed in individuals, between stimulus and the response to this stimulus. Repeated association between stimulus and response resulted in the strengthening of the relationship of the response to the stimulus. Failure to provide the association with reinforcement resulted in the decay or extinction of the association. This approach found favour with those who considered the stance of the radical behaviourists too extreme and was incorporated into classroom practice via the “drill and practice” modes of instruction that predominated during the first half of the twentieth century and then again during the back to basics movement which followed the “new mathematics” curricular reform of the 1960s and 1970s (Schoenfeld, 1987).

Critics of behaviourism identify the inability of a theory that places the idea of a stimulus response bond as its corner-stone to adequately explain activities such as strategic behaviour, problem solving, selective attention and language acquisition (Reynolds, Sinatra & Jetton, 1996). As the shortcomings of behaviourism became more apparent, especially in a societal context where higher order mental processes such as strategic thinking and problem solving were increasingly valued, new ways of describing how knowledge is acquired and then used were developed.

2.1.2. Constructivism

In response to the perceived shortcomings of behaviourism, a new class of theories was developed that collectively became known as cognitive theories of learning (Reynolds, Sinatra & Jetton, 1996). These new theories sought to go beyond the behaviourists’ simplistic stimulus and response paradigm to explain the complexity of human thinking and cognitive development. Of the many theories which sit under the umbrella of cognitivism (e.g., schema theory, connectionism), it was the range of psychological theories that became known as constructivism that had the greatest influence on mathematics education (Confrey & Kazak, 2006). The constructivist position holds that learning is a process whereby the learner actively constructs symbolic representations of the world and uses interpretations of these representations to interact with the world (Noddings, 1990).

The seminal influence of Jean Piaget is generally accepted to have laid the foundations for constructivist theories. Piaget viewed learning and understanding as the active construction of meaning in which schemata were created via the processes of accommodation and assimilation. This represents a departure from the behaviourists' perspective as an understanding of the complexity of mental processes was considered paramount in promoting learning and understanding. Piaget described learning as a process whereby individuals constructed meaning and understanding from interaction with the world. These interactions either consolidated what was already known or produced perturbations or contradictions that required the individual to reorganise existing cognitive structures. Thus, any interaction is interpreted through perceptual filters developed from previous experience which results in understandings and meanings that are unique to the individual.

Building on this foundation, the neo-Piagetian theory of radical constructivism advanced by von Glasersfeld (e.g., 1984, 1990, 1991, 1995) developed and has been highly influential on mathematics education research and pedagogy. Lerman (1989), citing Kilpatrick (1987) describes the fundamental tenets of radical constructivism as follows:

1. Knowledge is actively constructed by the cognizing subject, not passively received from the environment.
2. Coming to know is an adaptive process that organizes one's experiential world; it does not discover an independent, pre-existing world outside the mind of the knower.

(p. 211)

While the first of these tenets is now widely accepted amongst the mathematics education community (Lerman, 1989), the second of these has proved to be far more controversial as it leaves radical constructivism open to accusations of solipsism – that is, that knowing anything outside of the mind cannot be justified (Ernest, 1990). The consequences of this position are, firstly, that it appears to exclude any role for social interaction in the process of intellectual development and so brings into question how teaching can take place, and, secondly, that it excludes the possibility of gaining the accumulated knowledge or wisdom of a community (Cobb, 2000a; Lerman, 1989, 1996).

Some radical constructivists have attempted to argue that there is a role for social interaction within their theoretical field (e.g., Steffe & Thompson, 2000) by claiming Piagetian interactionism, and by association von Glasersfeld's (1984) radical constructivism, always assumes some interaction even if it is not mentioned. Thus, it is claimed, radical constructivism is in fact compatible with social theories of intellectual development such as Vygotskian socio-cultural theory. This is an argument difficult to sustain because if the ideas of interaction, intersubjectivity, and social goals are fundamental to the constructivist theory these elements must be explicitly stated.

Other theories that have emerged from the constructivist tradition, however, have attempted to answer criticism of radical constructivist principles related to the limited attention paid to social and cultural processes in intellectual development. In order to integrate social and cultural considerations, socio-constructivists have argued that social interaction can provide the source of the cognitive conflict that is necessary for intellectual growth to take place within individuals. This conflict is viewed as promoting disequlibrium in personal mental structures that in turn leads to their reorganisation in order to accommodate, and finally assimilate, any inconsistencies between an individual's initial world view and the episode that stimulated the conflict. The processes of accommodation and assimilation lead to a more sophisticated organisation of the individual's knowledge and reasoning systems. Research in this area is typified by the work of Cobb and associates (Cobb, 1994, 1998; Cobb & Bauersfeld, 1995; Cobb, Yackel & Wood, 1992), although it is still contended that "the focus is on the individual, autonomous learner as he or she participates in social interaction" (Cobb & Bauersfeld, 1995, p. 7). This still places social interaction as merely an external "catalyst for otherwise autonomous intellectual development" (Cobb, 2000a, p. 279).

Thus, while recognising the importance of interaction in thinking and learning, socio-constructivism still maintains an alliance with the radical constructivist principles in that the development of mind is viewed as an individualistic and personal transformative process. An alternative position, that consciousness is created and intellectual development fostered through interaction between human beings, is central to socio-cultural theories of learning.

2.1.3. Socio-culturalism

While the history of the social perspective on the mind is long (see, for example, Valsiner & van der Veer, 2000), seminal work in this area is generally attributed to Vygotsky (1978) who conducted research into intellectual development in Russia during the 1920s and

1930s. Vygotsky argued that aspects of cognition and learning, such as higher order thinking processes and problem solving, must first be enacted between people interpsychologically before being internalised by an individual. The type of social interaction that leads eventually to this internalisation is typically initiated by a more expert member of a social group who structures events in a way that enables participants to engage in activities that they, by themselves, would not be capable of attempting successfully. Internalisation is the process by which, through repeated involvement and interaction, initially less capable participants are eventually able to function at a higher level of expertise. It is important to understand that this is not merely about the acquisition of new knowledge or mental processes but also describes the transfer of control of the process in which learners are engaged and thus represents part of the individual's developing intellectual maturity (Cole, 1985).

From a Vygotskian perspective, as described by Luria, Cole and Cole (1979), there can be no strict separation of an individual from his or her social environment. In this view, cognitive development is the process of acquiring culture and so the individual and social must be regarded as complementary elements of a single interacting system (Leont'ev, 1981).

Because Vygotsky emphasised the critical role of a student's own activity in learning and thinking within a social context he also shifts attention from individual to social modes of thinking and emphasises the role of language in learning, both as a tool for thinking and as a medium for communication.

The sections that follow describe three key socio-cultural concepts which are relevant to this study. These concepts, zones of proximal development and scaffolding, the use of tools, and the situated nature of learning are either derived from Vygotsky's original work or have been developed by later theorists working within the socio-cultural tradition.

Zones of Proximal Development and Scaffolding

The process of intellectual development comes about in interaction between the learner and a more experienced other working in what Vygotsky termed *zones of proximal development* (ZPD). The ZPD can be conceptualised as a set of possibilities for development that become actualised when students interact with more knowledgeable people, for example teachers, and their learning environment. Vygotsky's development of this idea was driven by a desire to describe not just a child's intrapsychological functioning, which restricted any assessment of a learner's abilities to current achievements, but also their potential for intellectual growth (Wertsch & Stone, 1985). He believed this potential for growth could only be revealed through the study of an individual's interpsychological function, that is, their observable capabilities when working with more expert others.

The support for intellectual growth an individual receives by working with more expert others has become known as scaffolding. The concept of scaffolding was not part of Vygotsky's original writings but it emerged from educational research, based on Vygotskian theory, during the 1970s and 1980s, into how experts assisted novices to develop intellectual capability. The term scaffolding was used by Wood, Bruner and Ross (1976) to describe this process during problem solving activity. While scaffolding has become almost synonymous with Vygotskian theory of intellectual development its meaning and use in practice has often been distorted. Forman (2003) argues that work in this area was limited in the following ways:

> …an over reliance on the tutorial model of teaching; a tendency to discount other forms of learning, such as peer collaboration and play; a focus on task mastery rather than task understanding with little or no attention given to sign mediation; and a relatively decontextualized model of teaching and learning – as an activity that occurs in a historical vacuum apart from particular cultural practices and institutions, and between people whose emotions, values and beliefs are irrelevant to the instructional process.
>
> (Forman, 2003, p. 336)

It is also important to remember that the purpose of scaffolding is only to support the transformative process of an individual in being pulled forward within the ZPD that the scaffolding creates. Once the transformation has taken place, the scaffolding is no longer necessary as the learner now lives and works comfortably within this new ZPD.

The limitations identified by Forman (2003) above, in relation to work on scaffolding, emphasise the importance of other aspects of socio-cultural theory: the role of cultural tools in mediating action, and the situatedness of learning.

Use of Tools

Also central to socio-cultural theory is the idea that human action is mediated by cultural tools, and is fundamentally transformed in the process (Wertsch, 1985). These tools include technical and physical artefacts, and also concepts, reasoning, symbol systems, modes of argumentation and representation. Learning is achieved by appropriating, and using effectively, cultural tools that are recognised and validated by the culture as a whole.

From a socio-cultural perspective, cultural tools can be used to amplify or reorganise cognitive processes through their integration into the social and discursive practices of a knowledge community (Resnick, Pontecorvo & Saljo, 1997). Amplification takes place when a tool provides a more efficient procedure or pathway for engagement in a task, for example, through the use of a microscope to study creatures that cannot be seen readily with the naked eye. Cognitive reorganisation, on the other hand, occurs when the use of tools mediates a qualitative change in an individual's way of thinking about an idea or concept, or their approach to a problem solving task. This type of transformation is evident when a learner's use of a tool allows them to view a concept, situation or problem in a fundamentally different way. The critical deconstruction of Shakespearean text, for example, promotes the possibility of multiple perspectives on the meaning of a selected section of dialogue as opposed to its literal interpretation.

It is important to remember, however, that learning is about more than the changes to mental structures that result from tool use, but also entails the appropriation of methods of reasoning and discourse, within a learner's social context, that incorporate tool use. Thus, the introduction of a tool into a learning environment challenges the learner to go beyond the mastery of that tool towards new modes of reasoning and action.

The Situated Nature of Learning

Just as Vygotsky argued that the formation of higher order mental functions has its origins in social interaction, others have looked at how modes of reasoning and understanding develop within social groups. In *Cognition in Practice* (1988), Jean Lave challenged the notion that mathematical practices outside of schooling were merely the application of school mathematics. In a study of grocery shoppers and dieters, Lave observed that their strategic decision making was heavily influenced by the contexts in which they were working, that is, the knowing and the processes for decision making were situated within a social milieu. Consistent with that view, Bishop (1988) argued that mathematics is a culturally developed way of structuring a learner's experience that represents a unique way of knowing.

Building on Lave's (1988) earlier work, Lave and Wenger (1991) describe learning as a form of apprenticeship where novices are initiated into a learning community, or *community of practice*, through a process they termed legitimate peripheral participation.

> A community of practice is a set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice. A community of practice is an intrinsic condition for the existence of knowledge, not least because it provides the interpretive support necessary for making sense of its heritage. Thus, participation in the cultural practice in which any knowledge exists is an epistemological principle of learning. The social structure of this practice, its power relations, and its conditions for legitimacy define possibilities for learning.
>
> (Lave & Wenger, 1991, p. 98)

In their view, learning is not associated with the individual internalisation of knowledge, but rather can be conceptualised as participation in a particular community of practice. Experts or more knowledgeable peers within the community are responsible for the induction of learners new to the community into the culture of that community including beliefs, values, modes of discourse and means and methods of knowledge creation. Judgments about learning are therefore based on the increased range of participation of the learner within the community.

Participation in the community of practice is seen by Lave (1996) as the mechanism for learning or *becoming*:

> Rather than particular tools and techniques for learning as such, there are ways of becoming a participant, ways of participating, and ways in which participants and practices change. In any event, the learning of specific ways of participating differs in particular situated practices. The term "learning mechanism" diminishes in importance, in fact it may fall out altogether, as "mechanisms" disappear into practice. Mainly, people are becoming kinds of persons.
>
> (Lave, 1996, p. 157)

Learning is therefore located in action and distributed across the social context in which thinking and meaning making take place. The social milieu includes the tools, both cultural and physical, that may be used as mediators of reasoning and knowing. *Becoming*, in this sense, is the degree to which a participant adopts the values, and reasoning and discourse practices of a learning community.

The *becoming* Lave identifies in the above quote is what Wenger (1998) proposes as the formation of an individual's identity within a community of practice. A learner's identity within a community is strongly influenced by their personal affiliation with that community.

> Because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming - to become a certain person or, conversely, to avoid becoming a certain person. Even the learning that we do entirely by ourselves contributes to making us into a specific kind of person. We accumulate skills and information, not in the abstract as ends in themselves, but in the service of an identity.
>
> (Wenger, 1998, p. 215)

How an individual acts and interacts with others, and the role they play as part of a community, is determined by the identity they establish. At the same time, this identity has been influenced by the community itself and the individual's sense of belonging to that community. The relationship between the individual and the community is thus reflexive – one evolving with the other.

2.1.4. Further Insights into the Roles of Discourse and Tools

Out of the social-constructivist and socio-cultural theories, described above, emerge two themes worthy of further elaboration, from the perspective of this thesis: the roles of voice and discourse and of the role of tools in the development of reasoning.

Voice and Discourse

While Vygotsky provided new insight into the social aspects of learning, his description of the process of communication has been criticised for not reflecting the complexity of social discourse (Van Oers, 2002), and in particular, the reciprocal nature of discursive negotiation as new ideas and meanings are explored. It is out of this concern that Bakhtin's theory of voice and, more generally, the field of discourse have emerged.

Bakhtin's theory of voice emphasises the active, situated and functional nature of speech as employed by various groups (Renshaw & Brown, 1998). An act of communication, in this view, must always be constituted by a range of "voices" – the voice of the speaker but also traces of the voices of other members of the learning community who have previously used similar words or methods of argumentation acceptable to the community.

> …we would say that people's utterances in a communication process are not regulated by the processes that occur in direct interaction, but also by the historically developed style of communicating in that particular community of practice.
>
> (Van Oers, 2002, p. 68)

The development of such a voice allows members to recognise themselves as part of what Bakhtin called a sign community, in which a shared identity was manifest. The extent to which a speaker appropriates the style of communicating in the sign community can be used to make judgments about different levels of performance (e.g., explanation, justification, problem solution).

It is important to note the reciprocal nature of the generative process of communication between the individual and the community. Just as the individual communicates in order to receive confirmation of their appropriation of the voice of the community, they may also progress the collective knowledge of the community and so change its voice. This constitutes development in the consciousness of the individual and also the collective.

This shift from a view of learning that is based on the individual acquisition of knowledge to a social interpretation that characterises intellectual development as change in the way one communicates with others is what distinguishes studies concerned with learning discourse – the basis of the field of discursive psychology (Kieran, Forman & Sfard, 2002). In Sfard's view (e.g., Sfard, 2001, 2002; Sfard & Kieran, 2001), learning mathematics is an initiation into a certain well defined discourse: she uses the metaphor of thinking-as-communicating to frame her research. This discourse is reliant on symbolic artefacts as communication-mediating tools and on meta-rules that regulate communication. Tools, and symbolic tools in particular (for example, language, graphs, tables, algebraic formulae), are not viewed as simply the means of, or media for, communicating pre-existing knowledge, but rather they are intertwined with the act of communicating and thus with cognition itself. This represents an expansion of Vygotsky's original concept as tools can be conceptualised as cognitive intermediaries for communication within a community. Meta-discursive rules, on the other hand, guide the course of communicational activities within a community of learners.

While Sfard acknowledges her view of the field of discourse extends from Vygotskian theory, other authors such as Cobb (Cobb, 2000; Cobb & Bauersfeld, 1995; Cobb, Boufi, McClain & Whitenack, 1997; Cobb, Yackel & Wood, 1992) assign their understanding of the role of discourse to a different psychological tradition. According to Cobb and colleagues, children's mathematical development may be strongly influenced by social interaction and their participation in cultural practices but analysis of students' reflective discourse must be tempered with considerations for students' individual activity as they participate in the discourse of a classroom community. Thus mathematical learning is a dualistic process of individual construction that is facilitated via enculturation into the mathematical practices of a classroom learning community. This enculturation is effected by the adoption of socio-mathematical norms (Yackel & Cobb, 1996; Yackel, Rasmussen & King, 2000) "that sustain classroom microcultures characterised by explanation, justification, and argumentation" (Yackel & Cobb, 1996, p. 460).

The position taken by Cobb and his colleagues acknowledges that the act of learning cannot be separated from social contexts which lead to the adoption of modes of reasoning and argumentation that are central to a classroom culture. The separation of socio-mathematical norms from individual knowledge construction, however, appears contradictory or at best internally inconsistent. To claim social interaction merely influences, rather than is integral to learning, means that the role of culture is a peripheral rather than a key concept in

Cobb's social-constructivist theoretical framework. If the role of culture in mathematical learning is not seen as central then the importance of the discourse that frames socio-cultural norms is also diminished. This leaves the role of social aspects of learning as an adjunct to the construction of meaning and understanding rather than a foundational tenet. As a result, this position does not attend to the criticism of Vygotsky's ideas on discourse; which is a lack of acknowledgement of the reciprocal nature of discursive negotiation in knowledge creation.

Distributed Cognition

Because the field of discourse focuses on the importance of language in the development of consciousness, it places greater emphasis on semiotic tools such as language and specialised symbolic systems than it does on physical artefacts as mediators for learning and thinking. Despite studies that consider physical artefacts, such as computers, as having a vital role to play in supporting discourse and, as a result, intellectual development (see, for example, Cobb, 2002; Kirschner & Erkens, 2007; Manouchehri, 2004; McDonald, Huong, Higgins & Podmore, 2005; Pozzi, Noss & Hoyles, 1998), these tools do not appear to be given the same prominence as symbolic tools in theorizing the act of cognition within discursive literature.

An alternative theoretical perspective is that of distributed cognition in which cognition is not merely a social practice but an act distributed across individuals, collectives, symbolic and physical artefacts, and symbolic, virtual and physical environments. Drawing on aspects of Vygotskian socio-cultural theory and recognising the potential of computer technology, Pea (1985, 1993) argues that humans are elements in a reasoning system that includes human minds, social contexts and tools.

Hutchins' (1995) account of the process of navigation on a naval vessel (as described by Cobb, 2007) considers the whole navigation team, including all physical and symbolic tools, as the reasoning system that provides for the safe piloting of the vessel into port. Further, this reasoning system is constituted by elements that exist in the moment of the act, for example, the navigator and the ship's guidance system, but also by elements that preceded the event that led to the process of navigation and the artefacts used to navigate. This is because traces of the intelligence of other minds that developed the procedures that guide navigation and of those that designed the maps or guidance system used by the navigator remain in those procedures and devices.

Pea (1985) argues that tools, and in particular, technological tools can be used to reorganise mental processes which in turn alter the tasks as they were originally conceived.

> Computers are commonly believed to change how effectively we do traditional tasks, amplifying or extending our capabilities, with the assumption that these tasks stay fundamentally the same. The central point I wish to make is quite different, namely, that a primary role for computers is changing the tasks we do by reorganizing our mental functioning, not only by amplifying it.
>
> (Pea, 1985, p. 168)

Pea cites the example of how the spreadsheet has transformed the practice of modern financial advisors; where once financial models not only took much time to develop, they also took at least as long to use when exploring possible fiscal scenarios. The spreadsheet has not only made this process faster, which could be considered as the amplification of the advisor’s reasoning processes, but has completely changed the task because of the range and complexity of the models that can be realistically considered in any available timeframe. As a result, the process of financial planning through economic modelling has been transformed. Further, the action of developing an appropriate model no longer exists within the mind of the planner alone, but is distributed between the planner, the computer and whatever source, human or non-human, is providing input data. In this sense, intelligence is not resident in “minds” but is manifest in activity.

As argued above, reorganisation of mental processes that is evident in activity leads to the transformation of tasks. This transformation, however, must lead to a re-evaluation of the activity which may result in a further reorganisation of mental processes. As explained by Pea:

> ...human nature is not a product of environmental forces, but is of our own making as a society and is continually in the process of "becoming." Humankind is reshaped through a dialectic, or "conversation" of reciprocal influences: *Our productive activities change the world, thereby changing the ways in which the world can change us.* By shaping nature and how our interactions with it are mediated, we change ourselves.
>
> (Pea, 1987, p. 93, original emphasis)

Thus the nature of this intelligence is not always predictable as it emerges from the activity that is shaping intelligence in the process of engaging with the activity.

2.1.5. Summary of Paradigms of Learning/Thinking – Where is the Mind?

The paradigms of learning/thinking, described earlier in this chapter, are discernable from each other by a number of philosophical and epistemological characteristics. Two of the clear differences are the degree of mutuality, or the level of collaboration with others, required for learning or intellectual development to take place, and the primacy of tool use in mediating this intellectual growth. A summary describing the differences between behaviourism, constructivism and socio-culturalism is set out in Figure 2.1 below.

Individualism ⟷ Mutuality

Behaviourism	Constructivism	Socio-culturalism
Learning is through transmission; transferring knowledge from one individual to another. Thus learning is a uni-directional process. Result of learning is knowledge and understanding located in the "head" of an individual.	Learning is through construction of knowledge. Knowledge is either constructed individually or influenced by interaction with the environment or with more expert others. While learning can be influenced by interaction the emphasis remains on the reorganisation of individual mental structures. The result of learning is that knowledge and understanding are still located in the "heads" of individuals.	Learning is a collective process of enculturation into the practices of communities. Knowledge is appropriated through interaction with more expert others but mediated by cultural tools or artefacts. Result of learning is that knowledge and understanding are still located within the community of learners.

Minimal recognition of the role of tools ⟷ Central role for tool use

Figure 2.1. Comparision of theories of learning in relation to degree of mutuality and use of tools

Despite its popularity as a framework for promoting learning during most of the twentieth century, behaviourism fails to provide a mature theory for the role of social interaction and the use of tools in intellectual development, other than the part they might play in the reinforcement of desired behaviours or the discouragement of undesirable activity.

While social interaction was not a key concept in early constructivist thought, more recent developers of social-constructivist theory (see, for example, Cobb, 2000; Cobb & Bauersfeld, 1995; Cobb, Yackel & Wood, 1992; Steffe & Thompson, 2000) have argued that social interaction has an important role to play in constructivist theories of learning. In their view, interaction is fundamental to the process of disequilibrium as it is in social contexts that conflicting ideas between individuals may emerge (Palincsar, 1998). Collaborative discussion also plays a role in the resolution of the conflict and its incorporation into new knowledge and meaning structures. While the role of tools *per se* receives less explicit attention in constructivist literature compared to writings within socio-cultural frameworks, cultural tools receive recognition as facilitators of cognitive conflict (Cobb, 1995, 2002).

By contrast, socio-culturalists view interaction as the means by which individuals are enculturated into the established practices of a community of learners. From a Vygotskian perspective (as described by Luria, Cole & Cole, 1979), there can be no strict separation of an individual from his or her social environment. In this view, cognitive development is the process of acquiring culture and so the individual and social must be regarded as complementary elements of a single interacting system.

The constructivist position, while acknowledging that social interaction has a role to play in intellectual development, still holds as a central tenet that learning is the self organisation or reorganisation of mental structures within an individual, that is, within an individual's own head. By contrast, learning and thinking within a socio-cultural paradigm take place not just in the heads of individuals but also between individuals, are situated in social action and are mediated by both intellectual and physical tools. These two positions have not only led to questions about thinking and learning but also stimulated debate into the location of the mind. Whether the mind is situated in an individual's own head or is distributed across individuals, tools and contexts, as in the distributed cognition position, still remains a point of contention despite decades of debate.

The situated view of learning has been criticised by Anderson, Reder and Simon (1996) for overstating the following four tenets:

1. action is grounded in the concrete situation in which it occurs;
2. knowledge does not transfer between tasks;
3. training by abstraction is of little use;
4. instruction must be done in complex, social environments.

Anderson et al. (1996) argue that the four claims listed above, which are commonly held to be central to the socio-cultural perspective, have all been overstated and that evidence can be found, within the body of literature associated with the cognitive view of learning, that refutes each of these claims and cite evidence, for example, of situations in which learning has transferred across contexts.

Greeno (1997) replies to this criticism by arguing that these claims are not axiomatic foundations of the situated perspective. In countering Anderson et al.'s (1996) second assertion, Greeno argues that the situated view does not challenge the possibility of knowledge transfer between tasks, but rather that the situated perspective provides insight into why this appears to be problematic for learners, as has been documented in many studies (see, for example, Boaler, 1998b, 2000b; Greeno & MMAP, 1998).

Cobb (1994, 1998) also attempts to reconcile the social-constructivist and socio-cultural positions by arguing that each perspective has merits in particular research contexts and that an investigation should be designed within the theoretical framework that best matches the research questions it pursues. He goes on to suggest that the socio-cultural perspective is best suited to studies that focus on conditions for the possibility of learning, while a constructivist theoretical framework is best aligned with research into processes by which student learn. Cobb therefore does not support the hegemony of one perspective over another but, rather, claims that the aim should be the coordination of these perspectives in mathematics education.

Cobb's position is dismissed by Lerman (2000a) in reaffirming the differences between socio-cultualism and constructivism and arguing that constructivists draw on a weak image of the role of social life that cannot accommodate the complexity associated with learning in social contexts. Thus any attempt to describe thinking and learning with theories which are not founded on the primacy of social interaction is doomed to an insipid portrayal of the development of consciousness and intelligence.

2.2. Socio-cultural Theory and School Classrooms

By locating the origin of higher mental functions within social interaction and cultural practice, Vygotsky emphasised the importance of participation within a community to the development of modes of thinking and reasoning. Drawing from Vygotsky's original work, or in some cases developed independently, new socio-cultural perspectives on learning have emerged including those of apprenticeship (Rogoff, 1990) and participation (Lave & Wenger, 1991; Wenger, 1998; Wenger, McDermott & Snyder, 2002), in which the situated nature of reasoning, meaning and knowing are central. These new socio-cultural perspectives recognise that learning takes place in communities of inquiry or communities of practice. The next two sections of this chapter discuss the theoretical underpinnings of learning from a Vygotskian theoretical framework and the implications of socio-cultural theory for school classrooms.

2.2.1. The ZPD and Communities of Practice

From the Vygotskian perspective, participants in a learning community are viewed as having partially overlapping ZPDs that can provide the basis for productive partnerships in which knowledge is created, validated and shared through a process that involves participants pulling each other forward into adjacent ZPDs. While the ZPD is normally applied to individuals working with a more experienced other, recently it has been applied to both small groups and whole classes of learners in a way that seems to be consistent with socio-cultural theory (Goos, 2004). Goos (2004) argues that the ZPD may be viewed as an egalitarian partnership that involves equal status relationships and suggests "there is learning potential in peer groups where students have incomplete but relatively equal expertise, each partner possessing some knowledge and skill but requiring the others' contribution in order to make progress" (p. 263).

2.2.2. School Mathematics Classrooms as Communities of Practice

While authors such as Lave and Wenger (e.g., 1991) have posited theory on the nature of learning, neither has drawn from the context of school mathematics classrooms in the development of ideas about situated learning. Others, however, have further developed the situated perspective on learning to establish theoretical frameworks through which to interpret observations of mathematics classrooms. Brown and Campione (1994), for example, consider increased participation in a community to be the way learners are inducted into more disciplined and scientific modes of thinking that include exploration, conjecture, validation and sometimes proof. They see the orchestration of this activity as taking place through differing frameworks of participation such as peer tutorial, reciprocal teaching, teacher-led lessons and collaborative problem solving lessons. These activities challenge students to move from established modes of knowing and meaning making towards new shared practices of knowledge creation and verification.

The selection of exemplar studies which follow is designed to illustrate how contemporary researchers are working with socio-cultural theories in mathematics education.

Drawing on observations collected during a 2 year project of middle school mathematics classrooms aimed at fostering high level thinking and problem solving skills for students from economically disadvantaged backgrounds, Forman (1996) developed a comparison between the range of activity settings in traditional and reform classrooms. She found that students in classrooms conducted along lines consistent with the U.S. mathematics reform movement participated in a wider range of activity settings than in traditional classrooms.

She observed that students in traditional classrooms had less opportunity to initiate topics, redirect discussion, provide elaborated explanations, or debate issues. Forman contrasts traditional learning environments with classrooms where the following interactional scripts were observed: whole class recitation led by the teacher, whole class presentations led by one or more students, small group work led by one or more students with the teacher's intermittent assistance, individual seat work and unofficial peer group activities. Figure 2.2 compares the characteristics of activity settings in traditional and reform mathematics classrooms identified by Forman.

Activity Settings and Personnel	Values	Task Demands	Scripts	Purposes
Traditional Mathematics Classrooms				
Teacher led recitations	Teacher or text are sources of learning Automaticity and accuracy	Internalize mathematics facts and algorithms	Recitation script	Introduce basic skills
Individual seatwork	Teacher or text are sources of learning Automaticity and accuracy	Practice mathematics facts and algoritms	Work independently	Individual mastery of basic skills
Reform mathematics classrooms				
Student-led presentations	Multiple sources of learning Multiple solutions Effective strategies and explanations	Pedagogical and communication skills	Instructional conversation	Establish community of learners
Small group work	Multiple sources of learning Multiple solutions Effective strategies and explanations	Cooperation and communication skills	Instructional conversation	Establish community of learners Foster collaborative problem solving

Figure 2.2. Charateristics of Official Activity Settings in Classrooms (Forman, 1996, p. 122)

Forman further argued that the increased range of activity associated with reform classrooms brings with it new task demands, values and purposes. She viewed the appropriation of these new demands as the instantiation of a community of practice where students initially participate peripherally. As students appropriate new skills, norms and reasoning practices of a community they move to greater participation in the community and so demonstrate their learning. She also notes, however, that some students can be resistant to this participation even when other students in the classroom are working as a community. These students may demonstrate resistance to participating only within certain modes of interaction, for example, small group work, or may reject the collaborative norms of the classroom community altogether and remain passive and fail to contribute in any way.

In a longitudinal study of approaches to teaching mathematics, Boaler (1999) compared the learning cultures of classrooms in two different schools. In one classroom, teaching and learning was heavily algorithmic, text book focused and oriented toward mastery through repetition. The other classroom focused on open ended problem solving through investigative approaches and connections of mathematics to contexts inside and outside of the classroom. Boaler found that students from the second school out-performed students from the first in the national school leaving examination – the General Certificate of School Education (GCSE) –on tasks that required the use of mathematics in non-classroom contexts. While originally concluding that this was because the forms of knowledge developed by students in the first school were inadequate (Boaler, 1997, 1998a, 1998b, 1999), Boaler recognises this initial view as

> insufficient, because it is based upon the students' individual cognitive attributes, and pays insufficient attention to the ways that their cognitive attributes were mediated by their classroom environment. The students' behaviour as they worked through their textbook exercises seems to indicate the situated nature of their learning and the impact of the norms of their mathematics classroom upon their emergent mathematical knowledge.
>
> (Boaler, 1999, p. 267)

By recognising the situatedness of the knowledge being constructed by the two groups of students, Boaler implies the difference in achievement between the two groups of students is attributable to the individual classroom cultures that had been developed. In the more traditional school, students had become attuned to the particular constraints and affordances of their classroom. Thus, without the explicit, or implicit, prompts available when using a textbook, by following exemplars, or from direct instruction from a teacher, they struggled with making choices about the mathematics required for a particular mathematical task or adapting what they knew to a particular life related context. Their learning in mathematics had been limited by the highly structured situatedness of their classroom experiences. Because the strict adherence to school and mathematical rules at the expense of exploratory thought had become part of these students' mathematical identity, developed in their classroom culture, they had difficulty adapting to authentic activity.

By contrast, the mathematics learning of students from the second school was structured around activities and projects in which the need to learn particular mathematical techniques emerged from the activity itself. Students in this situation were not able to rely on textbook based cues for the selection of mathematics they needed to employ; rather, they were forced to adapt known methods or learn new ones. As a result, the mathematical identities developed by this group of students were more compatible with how mathematics is used and learned in authentic situations outside the classroom. These students did not see a difference between in-school mathematics and mathematics used outside of school because the culture in which they learned mathematics was consistent with how they used it away from school. As their mathematical identities remained relevant in both contexts they were not confronted by any differences between these environments of learning and action.

Brown and Renshaw (2000) also report upon the greater range of communicative spaces available in classrooms constituted as a community of collaborative learners. Through a series of studies based in primary school contexts (see for example Brown & Renshaw, 1995, 1996), they developed a pedagogical strategy they termed Collective Argumentation in which students are introduced to key words – represent, compare, explain, justify, agree and validate – that facilitate students' co-construction of understanding. Students worked in small groups to initially represent the task, compare their representations with other group members, explain and justify competing representations to each other, before presenting their group's finding to the whole class for validation.

In Brown and Renshaw's (1995) study, the teacher also played a key role by initially negotiating acceptable norms for knowledge development and sharing, as well as modelling appropriate methods of reasoning that led to the resolution of disputes. This process is designed to draw students into participating in the conventions of the classroom community of inquiry. Brown and Renshaw present evidence that collective argumentation provides students with a set of resources that they are able to access when working on non-routine problems. Finally, this study, as in the case of Forman's (1996) research, also reported that some students, at least initially, found it difficult to accept this pedagogy as it was different from what they had previously experienced, and they refused to participate in discussions. Others found it difficult to work with specific students in small groups due to personality clashes and having to work so closely and intensively. Brown and Renshaw identified this as a problem that needed to be anticipated and then worked through with students.

Goos, Galbraith and Renshaw (1999) and Goos (2004) carried out a two year study set in the upper secondary classrooms of one teacher - a single Year 12 class in the first year and a single Year 11 class in the second year - to investigate how he established a community of inquiry. While acknowledging that the students' role in establishing a community of inquiry is also of importance, Goos et al. identify five goals the teacher worked towards in attempting to develop such a community of learners.

1. Mathematical thinking is an act of sense-making, and rests on the process of specializing and generalizing, conjecturing and convincing.
2. The processes of mathematical enquiry are accompanied by habits of individual reflection and self-monitoring.
3. Mathematical thinking develops through teacher scaffolding of the process of enquiry.
4. Mathematical thinking can be generated and tested by students through participation in equal-status partnerships.

5. Interweaving of familiar and formal knowledge helps students to adopt the conventions of mathematical communication.

(Goos, 2004, p. 269)

Goos's (2004) analysis included the specific practices used by the teacher to create a variety of ZPDs for students – through scaffolding, peer collaboration and the interweaving of spontaneous and theoretical concepts.

Initially the teacher provided scaffolding by modelling a problem solving process that included sense-making, ownership, self monitoring and justification. As time went by the teacher slowly withdrew this scaffolding as students began to operate more independently. This growth was also characterised by students' adoption of the teacher's scaffolding techniques as they worked with less advanced peers to establish meaning in relation to a particular problem outcome. This included the interpretation of errors or inconsistencies and how these were to be addressed. Thus, for many students, working in the ZPD had become a process whereby learners moved from personal interpretation to validation and acceptance from their broader mathematical community.

The adoption of scaffolding techniques initially modeled by the teacher led, in turn, to the type of peer interaction that formed the foundation for a collaborative ZPD in which students attempted to validate mathematical conjectures through argumentation with peers. The teacher's role in this development extended beyond the initial modelling to an ongoing role that included encouraging students to use their peers as intellectual resources to be brought to bear on a specific problem if they themselves were "stuck", or as a sounding board, or critical friend, against whom new ideas and findings or the reasoning used to derive results could be tested.

2.2.3. Living on the Margins of a Community of Practice

In the process of documenting a teacher's capacity to create a functioning community of inquiry, as described above, Goos (2004) acknowledged that the nature of engagement and extent of participation varied between students. While this is consistent with Lave's idea of *becoming* within a community of practice, Goos noted that a small number of students resisted the adoption of this specific community of inquiry's modes of knowledge creation and validation, a finding consistent with those of Forman (1996) and of Brown and Renshaw (2000). Further, in other studies Goos (1994) makes the observation that peer interaction can sometimes even interfere with decision making, a position supported by Stacey (1992) who notes peer interaction does not necessarily lead to gains in the productive use of metacognitive skills.

Boaler and associates (Boaler, 2000b; Boaler & Greeno, 2000) identify the reason for many students' alienation from mathematics in traditional classrooms as the result of the conflict between the identity they would have to assume to participate in such classrooms and their own identity formed in the real world of experience. In their view, students simply reject the need to assume the "traditional" identity necessary to fully participate in such classrooms as they do not value the modes of knowing and meaning making promoted in these communities.

As a corollary, it seems likely that students who have formed their mathematical identities in traditional classrooms may also have difficulty in reconciling themselves to the development of new identities when challenged to participate in classrooms that function as a community of practice. This means that students may well reject traditional classrooms or those conducted along non-traditional lines as either might not align with a student's mathematical identity. As Lerman observes:

> Practices should be seen, therefore, as discursive formations within which what counts as valid knowledge is produced and within which what constitutes successful participation is also produced. Nonconformity is consequently not just a feature of the way that an individual might react as a consequence of her or his goals in a practice or previous network of experiences. The practice itself produces the insiders and outsiders.
>
> (Lerman, 2000b, p. 27)

This reminds us that inclusion does not necessarily guarantee participation or the appropriation of teachers' aspirations for the way in which the community of inquiry should manage itself.

2.3. Summary and Conclusion

This chapter has reviewed relevant literature on theories of cognitive development, thinking and learning. The implications of relevant theoretical frameworks for mathematical teaching and learning were also examined, particularly in relation to the mathematical behaviours of learners both as groups and as individuals.

While psychological theories of learning have influenced approaches to mathematics instruction since its inception (Boaler, 2000a), it appears that theory is increasingly influenced by disciplinary perspectives outside of mathematics education such as anthropology and sociology. Educational theories that recognised the situated and social nature of learning are becoming increasingly apparent in educational research, school curriculum reform movements and in advice related to improving pedagogical practice (Boaler, 1999; Forman, 2003; Lerman, 2000a, 2000b, 2006).

Constructivist theories of learning now recognise the role of social interaction in mediating learning, however, it remains fundamental to this perspective that the construction of meaning is both personal and individualistic. Issues of intersubjectivity and of the development of common shared meaning therefore remain problematic from this perspective. The work of Cobb and a range of associates, in the development of a constructivist position which incorporates the role of enculturation into situated social worlds via the adoption of socio-mathematical norms, begins to answer this challenge, particularly in acknowledging the importance to the learning process of interaction through discourse. Transfer of knowledge still appears difficult to explain from a view of knowledge, reasoning and meaning that is considered personal and "in one's head". Cobb and his group also acknowledge the role of signs and tools, but there appears little in constructivist literature that describes the role of physical artefacts in learning, other than the role they play as aids in the development of personal internal representations of a mathematical idea or concept.

Socio-cultural theory argues that learning is a situational and quintessentially social activity that takes place within a community of practice (Lave & Wenger, 1991). Classrooms that function as communities of practice value the contributions of all members, both teachers and students, as they work to co-construct mathematical ideas and understandings within a collaborative and critically supportive environment. Here learning is viewed as a collective process of enculturation into practices of mathematical communities. The classroom as a community of mathematical practice supports a culture of sense making, where students learn by immersion in authentic practices of the discipline. In classroom communities of practice, students and teachers work as mathematicians exploring a new idea for the first time. When faced with an unfamiliar mathematical situation students are encouraged to initially conjecture and postulate. Their colleagues then either support or challenge these proposals through the use of patterns of interaction based on forms of speech, representational conventions, and ways of thinking valued by the wider community of mathematicians. Thus their mathematical understanding, and also their justification of ideas, is developed, tested and reconstructed by and with the community in which they are participating.

It must be remembered, however, that not all students feel comfortable, at least initially, with participating and contributing to highly collaborative classroom communities. This might be related to the need to adopt a new mathematical identify, particularly if a student's previous identity was formed in a traditional mathematics classroom. The need to realign an identity with the social norms of a new classroom community may be intimidating and so very challenging for some learners.

The situated perspective on learning also recognises that the mathematical practices that emerge from within communities of learners are influenced not only by relationships between other learners but also other aspects of their environments such as physical artefacts which are used as tools to mediate knowing and understanding. The theoretical perspective of distributed cognition, which developed from within the socio-cultural tradition, views thinking and learning as activities that take place across systems of individuals, tools and environments. While proponents of distributed cognition, such as Pea (1985, 1987, 1993a, 1993b), argue that technological tools have a particularly powerful role to play in problem solving, an understanding of how these tools can promote thinking and learning is still evolving. Current research into how technological tools promote knowing and understanding is examined in the next chapter as well as theoretical frameworks which attempt to describe students' behaviours, both as individuals and in groups, when working with technology to support their mathematical learning.

Chapter 3

Perspectives on Technology Enhanced Learning and Understanding – A Turn to the Social?

Lerman's (2000b) view of a "turn to the social" perspective in mathematics education research, as articulated in the previous chapter, is reflected in calls from educational researchers, policy makers and practitioners to acknowledge the role of social interaction in learning and teaching mathematics. Within the same time frame, incorporating ICTs into teaching and learning has been a major focus for schools. Yet little theoretical attention has been given to the role learning technologies play in promoting, facilitating or mediating collaborative learning and teaching practices.

This chapter examines the question of whether there has been a parallel theoretical turn to a more social perspective in research on teaching and learning processes within technologically rich mathematical learning environments. The discussion which follows:

1. provides a general overview of current research into the field of technology supported mathematics learning;
2. outlines frameworks that describe students' use of technology in mathematical contexts; and
3. examines literature that addresses the influence of technology in promoting collaborative learning activity.

3.1. Overview

The call for including mathematically enabled technologies, such as computers with appropriate software and graphing calculators, as an essential adjunct to the teaching and learning of mathematics has been echoed in the literature and curriculum documents for at least the last two decades (e.g., Churchhouse, 1986; Demana & Waits, 1990; Kaput, 1992; Kaput & Thompson, 1994; Zbiek, Heid, Blume & Dick, 2007). A decade ago experts in the field argued that significant changes in the affordability of these technologies, the development of new hypermedia technologies and the convergence and packaging of mathematically important technologies into "super-calculators" or hand-held "Personal Mathematical Assistants" (Kissane, 1995), as well as the ever increasing dependence of society on technological infrastructure, means that "to expect that schools and teachers can continue to exist apart from serious technological support is hopelessly myopic" (Kaput & Thompson, 1994). Despite long term attention, the use of computers and other technologies in school mathematics classrooms has been restricted to date by economic, social and practical constraints (Guin & Trouche, 1999; Kaput, 1992; Kemp, Kissane & Bradley, 1996; Mariotti, 2002). As Mariotti observes, " the entry of computers into schools has been slow, and their integration in school practice even slower" (Mariotti, 2002, p. 720).

The potential for technology to enrich mathematics curricula, teaching, and learning has been noted in documents such as the NCTM *Standards* (National Council of Teachers of Mathematics, 1989, 1991, 2000), which explicitly advocate the use of technology in mathematics classrooms.

> Electronic technologies – calculators and computers – are essential tools for teaching, learning and doing mathematics……They can support investigation by students in every area of mathematics, including geometry, statistics, algebra, measurement and number. When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving.
>
> (National Council of Teachers of Mathematics, 2000, p. 24)

In Australia it is similarly recommended that all students have ready access to appropriate technology as a means to support and extend their mathematics learning experiences (Australian Association of Mathematics Teachers, 1996, 2000). The communiqué that followed from a national conference on graphing calculators, sponsored by the Australian Association of Mathematics Teachers, for example, concluded:

> The use of graphics calculators enhances student learning and addresses important issues of equity and relevance of school mathematics to the wider world. There is a compelling case for the advantages offered to students who use graphics calculators when learning mathematics. They are empowering learning tools, and their effective use in Australia's classrooms is to be highly recommended.
>
> (Australian Association of Mathematics Teachers, 2000, p. 2)

Parallel to this advocacy, research into the use of digital technologies to enhance the learning and teaching of mathematics has proliferated (Hoyles & Noss, 2003). Studies have included investigations into approaches to concept development (e.g., Vonder Embse, 1992), number (e.g. Kieran & Guzmán, 2005), algebra and calculus (e.g., Ferrara, Pratt & Robutta, 2006; Portafoglio, 1998; Weber, 1998), geometry (e.g., Laborde, Kynigos, Hollebrands & Strasser, 2006.), applications and modelling (e.g., Dance, Nelson, Jeffers &

Reinthaler, 1992) and visualization (e.g., Ruthven, 1990). Robust findings, however, have been mixed due to the accessibility of technological tools within schools and the equally limited availability of appropriate and willing testing sites (Zbiek, Heid, Blume & Dick, 2007). In addition, researchers have found it difficult to define and then investigate the field because of the rate of technological innovation and change and its interface with schooling systems, teachers and students. As Kaput (1992) commented:

> Anyone who presumes to describe the roles of technology in mathematics education faces challenges akin to describing a newly active volcano - the mathematical mountain is changing before our eyes, with myriad forces operating on it and within it simultaneously.
>
> (p. 515)

Also problematic are review findings that reveal conflicting results. Dunham and Dick (1994), for example, in a review of the use of graphing calculators, found mixed results when examining studies that aimed to link student achievement with calculator use. Some investigations indicated the use of calculators increased achievement by a significant margin, while others showed no difference or, in a few cases, favoured control groups who were not using calculators. These studies also compared the achievement of students between study groups where the conditions for the use of calculators varied. Some groups used calculators during regular lessons but were not permitted to use them in tests and examinations, while other groups used calculators in both learning and testing contexts. This sort of problem highlights the difficulty of attempting to discern the impact of technology on learning and teaching through quasi-experiemental designs that make use of treatment and control groups. As Dunham and Dick note, such research, which aims to isolate the effects of the availability of graphics calculators, is likely to impose such tight constraints that findings would be of little practical use in real classrooms.

Zbiek et al. (2007) note that "as a consequence, research on technology-intensive mathematics teaching and learning has only recently begun to mature into a well-articulated area of scholarship" (p. 1170). None-the-less there is now a large corpus of literature available from which some authors have developed meta-studies which aim to discern any emergent trends in the findings of research in this area to date.

In a review of student achievement and attitudes in computer and calculator technology based mathematics courses from 1970 to 1995, Barton (2000) found the effect of introducing technology into mathematics classrooms was generally favourable, although some negative outcomes were also documented. In relation to achievement, she concluded that the literature reviewed in her scan strongly supported the use of technology in the development of conceptual knowledge, with 75% of studies providing results that indicated higher levels of achievement among groups that used technology compared to control groups that did not. In addition, results concerning procedural knowledge found little evidence that algebraic skills were adversely affected. While Barton's analysis is less conclusive with respect to the effect of technology use on student attitude, a negligible number of the studies examined recorded a decline in attitude towards mathematics. In fact, only one study reported a negative attitude to the introduction of technology and this documented students' reaction to a classroom setting in which technology based course materials were viewed as an add-on rather than an integrated part of the unit of study.

However, Barton cautions that while her examination of the literature indicates strong support for the use of technology in learning mathematics, studies that attempted to isolate technology as a variable by controlling other aspects of learning, such as curriculum, text and relevant teacher variables, did not find significant difference in overall achievement between treatment and control groups, thus indicating that access to technology is not the only factor in enhancing learning. This is also a view supported by Ruthven (1990), whose early research was noteworthy for suggesting that the influence of technology is dependent on the ways in which it is used to mediate learning, and by Dunham and Dick (1994), who similarly argue the impact of graphics calculators is as much to do with associated changes to curriculum and instruction as access to technology itself.

More recently, in an examination of 180 published research reports about handheld graphics technology, Burrill, Allison, Breaux, Kastberg, Leatham, and Sanchez (2002) found evidence that students with access to handheld technology were more likely to engage in exploratory or investigational approaches to learning mathematics than those without access. Students with access were more flexible with their solution strategies and moved more freely between different mathematical representations. This finding was qualified by evidence that the role of teachers, and in particular their beliefs about mathematics and mathematics education, was also crucial to students' effective use of this technology. Further, they concluded that teachers should understand the importance of various profiles of student behaviour, in relation to technology, in order to design and implement appropriate learning activities. These activities included exposure to the limitations of a technology in order to interpret displayed results that seem imprecise or inconsistent with what they understand mathematically. For example, the display of a discontinuity in a graph may appear fuzzy or indistinct because of the pixilated nature of the handheld’s display screen. This measure was deemed important to guard against the danger of extending or reinforcing students’ misconceptions of an idea or concept.

Burrill et al. (2002) remain cautious, in concluding that research on the use of handheld technology is not robust and that the literature base is still far from presenting a comprehensive and coherent portrayal of the advantages and shortcoming of using handheld technology in learning mathematics.

Templer, Klug, Gould, Kent, Ramsden and James (1998) have also expressed the conviction that technology should be used to encourage students to explore and investigate mathematical concepts. They argue their case in response to the problems observed to emerge when students worked with technology in a self-monitoring symbolic manipulator environment (*Mathematica*). Specifically they noted that having mastered the rudiments, the majority of students focused on completion of the activity in the shortest time rather than on meaningful learning. This again indicates that enhanced learning outcomes require more that the simple provision of technology and implies that approaches to instruction are also a critical element.

In a comprehensive review of research in the use of graphics calculators in mathematics education, Penglase and Arnold (1996) found that students’ attitudes towards the study of mathematics generally became more favourable as a result of regular use of graphics calculators. A firm belief in the helpfulness of graphics calculators in learning mathematics was also evident in a range of studies examined, the only reservation expressed being related to a fear of "de-skilling" or loss of competency in doing certain forms of mathematics without the support of a graphics calculator. Despite the large number of studies reviewed, however, they noted only a small number addressed learning environments and teaching approaches designed to maximise learning benefits. This position is supported by Asp and McCrae (2000) in their review of Australasian research in the subsequent period 1996-1999. They comment that this particular gap had not been seriously addressed in the body of literature reviewed, although substantial work on other aspects of graphics calculator use was noted.

The major focus of investigations into digital technologies and learning, as outlined above, has been on the impact of technology on learning as an individualistic, cognitive activity. Many studies appear to be based on quasi-experimental designs in which the comparison of treatment and control groups fails to account for the situatedness of the learning and of the role of social interaction. Further, in general these studies focus primarily on the question of whether or not the introduction of technology makes a difference and not on *how* or *why* such differences occur. The next section of this chapter looks at two studies which attempt to outline the ways in which students make use of technology and to explain how technology, through interaction with its user, is developed as a tool for mathematical learning. This process provides insight into why the introduction of technology makes a difference to the type of learning which takes place.

3.2. Frameworks for Technological Tool Use in Mathematics Classrooms

This section outlines two frameworks that seek to describe students' use of technological tools in mathematics classrooms. Both of these frameworks consider technological tools as far more than physical artefacts which possess innate utility in relation to supporting the learning and doing of mathematics, and recognise that the user has at least as important a role to play in the co-construction of a tool's utility. This approach acknowledges a unique interaction or reflexivity between user and tool where one is not wholly formed, in the contexts of learning and doing, without the other.

3.2.1. Doerr and Zangor's Co-construction of Tools

Based on a qualitative, classroom-based investigation which focused on the role of the teacher in supporting students in their use of graphics calculators to learn mathematics, Doerr and Zangor (2000) developed a typography of technological tool use by students. Working from a theoretical perspective in which psychological aspects of learning are coordinated with social aspects through students' interaction with tasks, each other and their teacher, they studied the co-construction, by the students and the teacher, of the graphics calculator as a tool for mathematical learning as "it is through these interactions that the meaning of the graphing calculator as a tool for mathematical learning within the classroom is constructed by both teacher and student" (Doerr & Zangor, 2000, p. 146).

Thus, meaning for the tool does not precede its use in constructing mathematical meaning; meaning is constructed *with* the tool through its interaction with participants working on mathematical tasks. In other words, a tool's meaning, and thus the particular use of a tool, is situated in specific contexts – in this case the context of doing mathematics in a school classroom. The situatedness of a tool's meaning and purpose ties its use to the cultural practices and social norms of a specific context. This intertwining of artefact and social context means that factors such as students' and teachers' beliefs about the nature of mathematics and how it is learned are highly influential in the use of a tool.

As a critical aspect of the social context of the classroom in which this co-construction took place, the emergent norms for tool usage were also examined by Doerr and Zangor (2000), resulting in the identification of five modes of graphics calculator use. Within specific contexts, graphics calculators were observed to be used as:

- computational tools – where the calculator was routinely used by students to evaluate numerical expressions;
- transformational tools – where tedious computational tasks were transformed into interpretative tasks by focusing students' efforts on the interpretation of results rather than on any associated computation;
- data collection and analysis tools – here the calculator was used as a tool for data collection, through the use of peripheral devices such as motion detectors, and the analysis of such data sets;
- visualisation tools – the calculator here was used to: develop visual parameter matching strategies to find equations that fit data sets; find appropriate views of the

graph and determine the nature of the underlying structure of the function; link the visual representation to the physical phenomena; solve equations;

- checking tools – where the calculator was used to check conjectures made by students as they engaged with the problem investigations.

These categories were emergent from data gathered in both problem solving and instructional contexts. It is important to note that in certain situations the graphics calculator can take on more than one role simultaneously – again reinforcing the notion that the role the tool serves is highly dependent on context.

Curiously, despite the collaborative nature of the co-construction of the tool by all classroom participants, Doerr and Zangor (2000) found that the tendency for students to use the graphics calculator as a private device regularly led to the breakdown of small group interactions. When this occurred, it also led to the disruption of the class's collective endeavour to engage with a whole class task. This observation is almost contradictory to the general findings of Doerr and Zangor's (2000) study – that the meaning and purpose of a tool can only be constructed through the interaction of students and teachers with tasks and tools. Their observation suggests that, once fully formed, a tool then facilitates the dissolution of the milieu which gave it meaning

3.2.2. The Instrumental Approach

In a series of papers written from the French curriculum context (e.g., Artigue, 2002; Guin & Trouche, 1999; Trouche, 2003) an *instrumental* approach to viewing students' activity in technology enhanced environments was developed. These French researchers drew on Verillon and Rabardel's (1995) distinction between an artefact, which only becomes a mediating tool, and an instrument, which emerges once a meaningful relationship develops between the artefact and the user when dealing with a specific task. Different tasks will require different relationships between the user and the artefact, and the development of these relationships is referred to as the instrumental genesis of the artefact. This instrumental genesis has two components (as described by Artigue, 2002). Firstly, the transformation of the artefact itself into an instrument is known as *instrumentalisation*. In this process the potentialities of the artefact for performing specific tasks are recognised. Secondly, the process that takes place within the user in order to use the instrument for a particular task is known as *instrumentation*. Here we see the development of schemas of instrumented action, which are developed either personally or through the appropriation of pre-existing social schemas. Thus an *instrument* is composed of the artefact, along with its affordances and constraints, and the user's task specific schemas, which are built around the use of the artefact in specific contexts. In concert these provide direction for the use of the instrument in a given context. Finally, as pointed out by Drijvers and Gravemeijer (2005), the process of instrumental genesis is two dimensional in that the possibilities and constraints shape the conceptual development of the user, while at the same time, the user's conceptualisation of the artefact and thus its instrumentation leads, in some cases, to the user changing the instrument.

A teacher's activity in promoting a student's instrumental genesis is known as *instrumental orchestration* (Trouche, 2003, 2005). Social aspects of learning are recognised within this process and take the form of student activity that makes explicit the schemas that individuals have developed within a small group or whole class. These schemas are thus available for appropriation by other class members. Thus utilisation schemas are essentially individual, even though instrumental genesis may take place through a social process (Drijvers & Gravemeijer, 2005).

3.2.3. Commentary on Frameworks for Use of Technological Tools

Each of the frameworks described above acknowledges that the purpose and use of a tool are inseparable from its user and the activity on which the tool is brought to bear. In doing so, these frameworks acknowledge the importance of context in the process by which a tool acquires meaning and purpose. Both frameworks also assign a role to social interaction in the formation of a tool's utility. However, these frameworks have inherent internal contradictions or incongruities in relation to their theoretical standpoints and the cultural practices and social norms which frame teaching and learning in school mathematics classrooms.

While both frameworks acknowledge the importance of social interaction in the development of an artefact into a tool, they propose fundamentally different types of interaction and roles for this interaction in tool development. Doerr and Zangor's (2000) theorisation of the formation of a tool is founded on a notion of co-construction in which teachers and students make decisions about the constraints and affordances of an artefact within a particular learning context. These constraints and affordances are strongly influenced by a teacher's beliefs about the role of technology in learning and meaning making. For example, if a teacher believes that a calculator should only be used to check the results of pen and paper calculations, then the calculator may only be used in this role, no matter what other facilities it may offer. In turn, the teacher's view of the role of the calculator will limit students to the range of functions that are permitted. This means that the formation of what might be a multi-functioned artefact is restricted to that of a checking tool alone.

Also, as noted above, Doerr and Zangor's (2000) observation of the isolating effect of technology, in terms of social interaction, seems at odds with their principles of how an artefact becomes a tool. The idea that a tool is formed by co-construction, but, once formed, contributes to the break down of the social fabric of a working classroom, appears to be a contradiction. Further, it implies that the formation of a tool is definite and finite within a particular context; that is, once a tool is formed there is no further development to be done via students' continued interaction with the tool. This means that any further development would depend upon further intervention from the teacher, and so a tool's formation is driven by the teacher's construct of the purpose of a tool alone.

Proponents of the process of *instrumental genesis*, on the other hand, take the position that the formation of a tool is an individual act of cognition; that is, the formation of a tool's meaning and purpose takes place at the level of the individual and not through a co-constructive process. This is despite the fact that many of the observations used to substantiate the proposed framework took place in classroom contexts when students were working in pairs or small groups. The role of social interaction within this framework is actualised through the process of *instrumental orchestration* in which a teacher co-ordinates the sharing of students' individual schemas so that those that are deemed most useful are appropriated by other members of a class. This position also gives primacy to the role of the teacher in the process of tool formation, and so the transformation of an artefact to a tool is limited by the teacher's vision of what a particular tool should *be* in a particular context.

The result of the teacher driven perspectives on a tool's formation is that there appears to be no attention given to the possibilities that exist for the development of a tool's meaning and purpose in the different social configurations that are commonplace in school classrooms, such as small group work and whole class discussion. And so, while these frameworks describe processes for the formation of a tool and students' use of technology in classroom environments, neither outlines the role of technology as a mediating tool for social interaction among peers or makes provision for the role of interaction and discourse between learners in the transformation of an artefact into a tool.

Other authors (e.g., Burrill, 1992; Galbraith, Renshaw, Goos & Geiger, 1999; Geiger & Goos, 1996), however, have suggested that the most significant changes related to the introduction of technology into mathematics classrooms will be in the ways students and teachers interact. From this perspective, questions such as the role technology can have in mediating social interaction, or how technology is entwined into the fabric of a learning discourse in collaborative learning environments, receive greater primacy. The next two sections outline studies which focus on the role of technology in mediating collaborative learning practices across a range of instructional contexts.

3.3. A Turn to the Social in Technology Rich Environments

This section begins by examining the proceedings of a series of major international conferences in order to document the growing influence of the social perspective on studies in technologically enhanced mathematical learning.

3.3.1. A Brief History of the Development of a Social Perspective on Research into Technologically Enhanced Teaching and learning in Mathematics

In order to evaluate Lerman's (2000b) appraisal of a turn towards socially orientated theoretical frameworks in relation to the use of digital technologies, it is informative to consider three events sponsored by the International Commission for Mathematics Instruction (ICMI) that have taken place across a span of some two decades. The first of ICMI's seventeen studies to date, *The Influence of Computers and Informatics on Mathematics and its Teaching*, is chosen because it was in the mid-1980s that micro-computers were having their first significant impact in educational contexts. The study commenced with a meeting of educators interested in the potential of computers to improve the teaching and learning of mathematics and the proceedings of this initial meeting (Churchhouse, 1986) are reviewed here in order to establish an early baseline for gauging interest in the role of digital technologies in promoting social aspects of learning. The ninth

International Congress on Mathematical Education (ICME 9) in 2000 was included because it was at this congress that Lerman made the observation that opens this chapter. Finally, because it is the most recent study in the area of technology and mathematics education, the proceedings of the conference associated with ICMI's seventeenth study, *Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain* (Hoyles, Lagrange, Son & Sinclair, 2006), has been included in this review.

Early Accounts

The symposium, *The Influence of Computers and Informatics on Mathematics and its Teaching* (Churchhouse, 1986), was organised under three themes, and the third, *How can the use of computers help the teaching of mathematics?*, is of relevance here.

The report on this theme opens with a discussion of what mathematics and mathematical activity might comprise in a future classroom. It was felt, in particular, that

> the experimental aspects of mathematics assume greater prominence, and there is a corresponding wish to ensure that provision should be made for students to acquire skills in, and experience of, observing, exploring, forming insights and intuitions, making predictions, testing hypothesis, conducting trials, controlling variables, simulating, etc. (pp. 24 – 25)

Curiously, despite a description of what we would consider now to be activities students might engage in as a group, there is no commentary on how students might work with each other, or how such interaction could promote learning.

Later in this section there is acknowledgement that technology has the potential to influence classroom dynamics as "this creates new interactions and relationships between students, knowledge, computer and teacher" (p.25). The use of the singular "student" is a further indication, however, that interactions between students were not a concern at that time. The advantage of the computer was seen as supporting the development of mental images that would assist in the acquisition of mathematical concepts and processes within individuals.

A New Millennium

Despite Lerman's optimism for the uptake of a social perspective in education research articulated at the ninth ICME in 2000, the working group on *The Use of Technology in Mathematics Education* provided only a modicum of support for his position. The reports of each subgroup of this theme reveal only one reference to the contribution of technology to the social aspects of learning. This appears within subgroup 4: *Conceptual and professional development of learners and teachers in technologically rich classrooms* which notes "several informative empirical studies were presented that were rooted in theoretical work in the socio-cultural perspective" (p.277).

One such paper, *Classroom voices: Technology enriched interactions in a community of mathematical practice* (Goos, Galbraith, Renshaw & Geiger, 2000), theorised four roles for technology as a tool for amplifying students' cognitive processes and reorganising interactions between human and technological agencies. This paper demonstrates a clear association with socio-cultural perspectives on teaching and learning but was only one of a very few of its type.

Current Climate

The seventeenth ICMI study, *Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain,* which took place in Hanoi, Vietnam, in 2006, provides evidence of increasing interest in socially orientated aspects of learning within the field of technologically enhanced mathematics instruction. The proceedings of this symposium were examined for indicators of a study's alignment with a social theme. These included references to socio-cultural theory, collaboration, learning communities and classroom discourse. Of the 77 papers included in the proceedings, 14 papers were framed around these ideas or made direct reference to them in their theoretical frameworks. This represents 18% of the studies included in the symposium. Further, an additional 10 papers were framed around, or made reference to, the theoretical position of instrumentalisation. While it is arguable that this is a social perspective, the concept of semiotic mediation through technological tools is often traced to Vygotskian theories of intellectual development and by association socio-culturalism. If these papers are included in the analysis, then 31% could be considered to be aligned with educational theory related to the social aspects of learning and teaching. Considering either figure, and acknowledging the broad brush nature of the analysis, 18 – 31% of papers represents a noteworthy shift in the interest of this branch of mathematics education towards the social and supports the claims of Lerman seven years earlier.

3.3.2. The Use of Different Technologies in Fostering Social Interaction

The scan of the proceedings of ICMI 17 also reveals differing elements of design in the technologies used in studies in which a socially orientated theme was evident. Four models of different types of technology used to support mathematical interaction are identified. These technologies are designed for:

1. learning mathematics but not specifically for collaboration;

4. both learning mathematics and for collaboration;

5. collaboration but not necessarily learning mathematics;

6. neither learning mathematics nor collaboration.

These categories are elaborated upon and exemplified below.

Mathematics but not specifically for collaboration

Technologies in this category include those designed for working with mathematical ideas but not necessarily for the promotion of social interaction. This group of technologies includes devices such as graphics calculators and mathematically enabled software, for example, Maple. A paper that reports on the use of this class of technology is Geiger's (2006) account of a series of episodes in a secondary mathematics classroom in which a learner, who initially rejects the collaborative nature of his mathematics classroom, is eventually drawn into the developing community of practice. His "recapture" is facilitated through his interest in designing mathematics based videos through the programming of his calculator. The need to share his creations with other members of the community catalyse the student's participation in whole class interaction during this incident and then into the future. This episode is reported in greater detail in Chapter 8 of this thesis.

Collaboration but not necessarily learning mathematics

A number of papers from the symposium concerned the use of internet clients, designed for use as a communication tool among groups of learners, as the medium for interaction in on-line mathematics courses. Beatty and Moss (2006), for example, describe research into the use of a web-based collaborative workspace, *Knowledge Forum*, to support Grade 4 students in generalizing with patterns as part of their research in early algebra. The investigation revealed that the opportunity to work on a student-managed database supported students in developing a community practice in which the offering of evidence and justification for their conjectures form part of the discourse of knowledge sharing and validation.

Both mathematics and collaboration

This category includes the use of tools with the facility for learners to work with mathematical concepts in a virtual environment designed for collaboration. Technologies of this type include the *Space Travel Games Construction Kit* developed by Kahn, Hoyles, Noss and Jones (2006). In this simulation of computer game development, participants are provided with a construction kit that includes small program fragments together with tools for customising and composing them. Game development is designed to take place within the context of a metagame where learners are presented with a goal and need to interact with other members of their team in order to share components and acquire the knowledge to proceed. A component of this knowledge is mathematical in nature and so learners acquire mathematical ideas through interaction with peers.

Neither mathematics nor collaboration

Reports from other participants noted the collaborative activity of learners which ensued from interaction with technologies that were not designed specifically for the learning of mathematics or to act as catalysts for social interaction. A study by Fernandes, Fermé and Oliveira (2006) of K-8 level students investigated the potential for the use of robots to act as mediators between students and mathematics. This paper documented the collaborative practice that ensued when students were presented with problems that challenged them to program robots to follow a predetermined path. The researchers reported that the collaborative activity of teams of students led to the co-definition of knowledge and understandings gained from the activity.

The typology presented above provides insight into the range of environments in which social interaction and collaborative practices can take place. They range from mainstream classroom contexts in which teachers and students are present within one physical location through to others when none of the participants are in close proximity to each other and their interactions take place in virtual space. Interactions can take place synchronously or asynchonously. Electronic connectivity might be a feature of the interaction or interaction might happen directly, face-to-face, with technology acting as a mediator. Each of these contexts represents vast areas for potential exploration, although the common theme around which these constructs are based means that there is also great potential for the cross-fertilisation of ideas and approaches to collaborative approaches to learning. While this thesis focuses on the role of technologies identified as *for mathematics but not specifically for collaboration*, literature from the other related areas is drawn upon in developing new theoretical constructs and frameworks that identify the role(s) of technology in mediating collaborative mathematical interaction.

3.3.3. The Role of Technology in Mediating Socially Oriented Mathematical Practice

While there is now a growing interest in social perspectives of learning with technology, this interest is relatively recent and so the corpus of related literature still remains limited. None-the-less, a number of authors have attempted to define the territory. Simonsen and Dick (1997), for example, in a study of teachers' perceptions of students' use of graphics calculators, conclude that this technology has a role to play in shifting the orientation of the classroom towards more student centred, discursive and exploratory approaches. Teacher competence with using technology, however, is an insufficient condition alone for improving mathematics instruction in concert with technology (Thomas & Hong, 2005). Changes in pedagogical approaches are also dependent upon teachers' personal philosophies of mathematics and mathematics education (Tharp, Fitzsimmons & Ayers, 1997; Thomas, Tyrrell & Bullock, 1996). Thus, the availability of technology alone will not ensure the development of collaborative practices in a learning environment and so the classroom teacher, or, in other circumstances, the designer of the virtual learning environment, has a vital role to play in mediating the type of social interaction that is regarded as collaborative within a community of learners.

In order to consider the roles technology can play in mediating collaborative practice it is helpful to begin with Taylor's (1981) description of the three ways in which technology, specifically computers, is used in education:

- as a tutor – in which the computer environment is programmed in such as way as to provide instruction on some topic within a program of study;
- as a tutee – in which the learner takes an active part in the programming of the computer environment and learns something about specific non-computer oriented content domains as a consequence;
- as a tool – in which students make use of the capabilities of a computer to perform mathematical activities they would otherwise have conducted in some other way or to perform tasks that may have been beyond their capabilities without the assistance of computer technology.

While this is a useful starting point to theorizing the role of computers in teaching and learning mathematics, and in particular for identifying the computer as a tool that can enhance the capabilities of humans, there is no attempt to discriminate between how technology might be used by individuals or groups of learners. Building on Taylor's commentary, Willis and Kissane (1989) also added the category of *Computer as a Catalyst*. In this mode the computing environment is used as a means of provoking mathematical explorations and discussion or to invoke the use of problem solving skills. This addition recognises the potential of technology to support learning focused interaction between students and suggests a mediating role for technology in learning.

The metaphor of *Computer as a Catalyst* is further extended, by Goos and Cretchley (2004), in a review of the role of technology in education in the Australasian region. Their development of the metaphor refines the view of the computer as a tool and catalyst for visualization; higher order thinking; and collaboration. While it is important to recognise that the three categories listed here are far from unrelated, this typography is useful to identify the primary focus of the research reviewed by these authors. In particular, Goos and Cretchley note that the role of technology in supporting students' knowledge building in a mathematical community of learners, such as in studies of Computer Supported Collaborative Learning (CSCL), has emerged as a significant theme for research.

Commenting on the design of environments that are supportive of CSCL, Stahl (2006) observes that the use of cognitive tools varies between individual and collaborative settings and outlines some considerations with respect to the differences between the two:

1. The use of cognitive tools by a collaborative community takes place through many-to-many interactions among people, not by individuals acting on their own.
2. The cognition that the tools foster is inseparable from the collaboration that they support.

3. The relevant cognition is the “group cognition” that is shared at the small group unit of analysis; this is a linguistic phenomenon that takes place in discourse, rather than a psychological phenomenon that takes place in an individual’s mind.
4. The tools may be more like communication media than like a handheld calculator—they do not simply amplify individual cognitive abilities, they make possible specific forms of group interaction.
5. Rather than being relatively simple physical artefacts, tools for communities may be complex infrastructures.
6. Infrastructures do not have simple, fixed affordances designed by their creators; they are fluid systems that provide opportunities that must be specified by users and enacted by them.
7. The community must interpret the meanings designed into the tools, learn how to use the tools, share this understanding and form social practices or methods of use.
8. Analyzing the effectiveness of these tools requires a special methodology that can analyze the methods developed by the community for taking advantage of the infrastructure to accomplish its collaborative activities.
9. The community with its tools forms a complex system that cannot be modelled through simple causal relationships, because the whole is both over-determined and open-ended; the community is made possible by its infrastructure, but also interprets the meaning of its tools and adapts their affordances.

(Stahl, 2006, p. 2)

These observations are strikingly redolent of the socio-cultural perspective on learning with respect to the role of interaction in learning and also that of physical artefacts or tools, as outlined in the previous chapter. They also differ from the frameworks, described earlier in this chapter, for tool formation and use in that the role of the community, rather than the individual, in this process is afforded a position of prominent influence. The inseparability of cognitive activity from both the process of learning within the group and from the tools that help mediate the activity are consistent with a Vygotskian view of the social nature of learning and Pea's (1993b) description of the role of cognitive tools in distributed cognition. However, other authors (Geiger, 1998; Geiger & Goos, 1996; Goos, Galbraith, Renshaw & Geiger, 2000a, 2000b; Trouche, 2005) have claimed similar social interactions can take place in learning environments that make use of technological tools which have not necessarily been specifically deigned for collaborative activity. Studies which exemplify the use of technologies designed primarily as mathematical tools to mediate collaborative practice include those of Geiger and Goos (1996), Manouchehri (2004) and Sinclair (2005).

In a case study designed to investigate the social and material mediation of computer-based learning in an upper secondary mathematics classroom, Geiger and Goos (1996) found that interaction was both tool and task dependent. The computer was intended to act as both a tool, in enabling students to generate and manipulate data in a spreadsheet, and as a catalyst, in provoking exploration of the patterns that emerged from the data. However, the extent to which such exploration occurred depended on the type of task the students were given. Differences in the social organisation of students' work, identified in the function of their talk and the structure of their interaction, were associated with differences in task focus, with a focus on process, rather than products or means, producing collaborative discussion. Results implied that computer environments do not automatically facilitate peer interaction and that careful attention needed to be given to the structure of tasks if they are to elicit high level verbal reasoning. Students were most likely to interact if there was a genuine problem to be solved and they felt comfortable with the operation of the technology itself.

In a study involving undergraduate preservice teachers using NuCalc, an interactive algebra application, Manouchehri (2004) observed that students' mathematical discussions displayed greater complexity while using NuCalc than when they used no mathematical application. Manouchehri identified the following four ways that the software supported discourse:

1. by assisting peers in constructing more sophisticated mathematical explanations;
2. by motivating engagement and increased participation in group inquiry;
3. by mediating discourse, resulting in a significant increase in the number of collaborative explanations constructed;
4. by shifting the pattern of interaction from teacher directed to peer driven.

Further, Manouchehri concluded that because of the immediacy of feedback to students, the software also supported a culture of conjecturing, testing and verifying, formalizing mathematics and collaboration and shifted the locus of power from the teacher to the students.

Sinclair's (2005) study of high school students' use of Geometer's Sketchpad/JavaSketchpad to examine geometric concepts found that the role of the "partner" in pair interactions had greater impact on the learning environment than the researcher had expected. This included pointing out details, arguing over interpretation, actively connecting to the task and to one another, and correcting one another in interactions where pupils were not just sharing information but actively collaborating to develop their understanding.

These studies offer qualified support for the premise that technology can play a role in the mediation of collaborative learning processes. The immediacy of the feedback offered by technology can offer enhanced possibilities for classroom cultures where conjecturing, testing and verifying mathematical argumentation in support of findings is part of a classroom's social norms. It must be noted, however, that introduction of technology alone is no guarantee that collaborative practices will ensue. Other factors such as task design and existing classroom culture which is framed by students' and teachers' beliefs about mathematics and how it is learned are also critical.

Authors such as Borba and Villareal (2006) offer a more integrated view and see learning as a collaboration between collectives of human individuals and technology.

> They (computers) interact and are actors in knowing. They form part of a collective that thinks, and are not simply tools which are neutral or have some peripheral role in the production of knowledge.
>
> (p. 5)

Borba and Villarreal propose the existence of an intershaping relationship between learners and technology. In this relationship technology influences the way in which students learn and come to know through mediation, while at the same time students interact with technology in ways unanticipated by the designers. Thus knower and technology shape each other. In this way Borba and Villarreal dismiss the dichotomisation of humans and technology. Knowledge is produced through the efforts of collectives of humans-with-media or humans-with-technologies. Further, this collective also produces different mathematical knowledge so the discipline is influenced to change.

3.4. Summary and Conclusions

This chapter has provided a general overview of current research in the area of technology enhanced mathematical learning. While this field is attracting increasing interest from researchers, and there is evidence of improved student outcomes from achievement and affective perspectives, a mature theory of how and why the introduction of technology makes a difference to what and by which processes students learn is still emerging. Current theories recognise that the formation of technological tools is dependent on both the inherent capacities of the tool and the capabilities and limitations of the tools' users within their socio-cultural contexts. Available theory, however, does not accommodate, in a fundamental sense, the role of social interaction between all members of a functioning classroom community. Specifically, current theory recognises the important role of the teachers in the determination of what is possible and what is allowed in the pursuit of mathematical knowledge, yet does not recognise the potential of student-student interaction in the formation of a tool's capability or of a tool's role in mediating the type of productive social interaction between students and teachers that leads to new insight into mathematical ideas and processes.

Further, how the formation of tools is related to particular social contexts, such as small group discussion in comparison to student directed whole class presentations, appears to be an area which has yet to receive due attention. How a tool is formed and its potential to mediate social discourse and learning after its formation are the foci of this thesis.

The next chapter outlines the specific research questions that frame this study and the methods used to gather and analysis data.

Chapter 4

Research Design and Methodology

4.1. Overview

The method of inquiry for a study in the field of education must be determined by the nature of the research problem and the theoretical framework used to develop the research questions which frame the investigation. As discussed earlier in Chapter 2, the theoretical perspective that best aligns with the intent of this study, that is, the investigation of interactions between learners while working with digital technologies, in authentic classroom settings, is that of socio-culturalism. The focus of the socio-cultural perspective is on the changes that take place in individuals while working with others, and in groups as a collective entity, during activity, rather than on the personal knowledge acquisition of an individual working in isolation (Forman, 2003, p. 390). Changes to an individual's or a group's thinking and learning processes are mediated by cultural tools which include physical artefacts. This study aims to explore the relationships that evolve between human participants, and with technological tools, as learners work in individual, small group and whole class contexts on mathematical ideas and problems.

Research into interactions between multiple participants and between participants and digital tools, in an authentic classroom setting, must employ a methodology with the capacity to accommodate educational phenomena that are situated, temporal and complex. Further, the nature of the classroom environment brings with it, in the case of this study, the prospect of unanticipated or emergent outcomes in terms of both the usage of digital technologies and in the type and quality of the interactions between participants and technologies.

As the focus of this study is on broader communities, or activity systems, rather than on the individual learning processes alone, the research design must provide a structure for analysing *change-in-action*, and so must be holistic and interpretive rather than positivist and deterministic. As a result, this excludes designs based around experimental and control groups which tend to focus on individual outcomes rather than "intact activity systems" (Greeno & MMAP, 1998) and which seek to limit the effects of emergent phenomena rather than embrace them.

The strength of the qualitative methodology known as naturalistic inquiry (Lincoln & Guba, 1985) is the capacity to accommodate, and perhaps even work with, the richness, as well as the "messiness" of a mathematics classroom *in situ* as "these methods could enable the researcher to focus on complexities and qualities in educational action and interaction that might be unattainable through the use of more standardized methods" (Burns, 2000, p. 390). Because this study is situated in an authentic classroom setting, and that it seeks to investigate student-student-technology interaction during mathematical activity, a naturalistic methodology that is based on ethnographic and case study traditions is employed to investigate the following research questions first introduced in Chapter 1.

1. What are the dispositions and preferences of students towards using technology in learning mathematics?

2. What are the perceptions of students with respect to their global facility and confidence with digital technologies as a personal resource?
3. What choices of specific forms of technology use are favoured by students?
4. What choices of general strategic purposes for technology use are favoured by students?
5. What roles can be identified for technology in mediating individual student learning?
6. What roles can be identified for technology in mediating collaborative student learning?

It has also been argued that social interaction can be best understood from the point of view of the participants in the interaction (Cohen, Manion & Morrison, 2000; LeCompte, Preissle & Tesch, 1993). In this view the identification and understanding of patterns of interaction and the meaning participants ascribe to such patterns cannot be accomplished from the vantage point of a detached, objective observer. Rather, insight into these matters can only be gained by researchers who are themselves participants in the phenomena under investigation. Consistent with this view, the documentation and analysis of classroom events was completed by a participant observer, the author of this thesis and the teacher in the focus classroom, with the support of research assistance.

Because realities are multiple, constructed and holistic, studies of this type should take place in their natural settings as context is an essential element in understanding any naturalistic phenomena (Lincoln & Guba, 1985). In order to capture the complexity of events at the centre of this study, multiple research methods were employed (Lincoln & Guba, 1985) including ethnographic techniques such as: participant observation; structured and semi-structured interviewing of both individuals and the whole class; stimulated recall interviews; surveys; and video and audio taping of individuals, small groups and the whole class.

Also consistent with a methodology based on naturalistic inquiry, theory was generated progressively from the data as they were collected and analysed rather than attempting to verify an existing hypothesis (Burns, 2000). As theory developed, techniques and instruments of data collection and analysis were adapted in order to provide additional evidence to confirm theory or to further explore and theorise phenomena as they emerged.

One of the criticisms of research based on participant observation is the difficulty teacher-researchers experience in attaining the necessary level of detachment to record, code and analyse data effectively (Woolcott, 1997). This, and further issues related to sampling, generalisability, reliability and validity are addressed (Section 4.7) in order to establish the trustworthiness of the study's findings.

4.2. Contexts of the Study

In a naturalistic study it is important to describe influential contexts that colour and shape the teaching and learning setting in which the investigation takes place. A description of these contexts follows.

4.2.1. School Context

Hintsay College (a pseudonym) is a coeducational, non-government secondary school that had been established for approximately 15 years at the time of this study. The school is medium sized by Australian standards with a ceiling of 550 students. The decision, by the School Council, to limit student numbers is consistent with the school's declared aspiration to develop and maintain a close knit community with a focus on balanced academic, spiritual and physical development in an environment of respect and care.

The school caters for students in Years 8 to 12 and accepts approximately equal numbers of boys and girls. The fee structure is moderate when compared to other "established" schools as part of a commitment by the School Council to make enrolment at Hintsay College accessible to students from a broad range of socio-economic backgrounds. Despite this practice, students at Hintsay College typically come from white, Anglo-Australian, middle class families.

Although aligned with the Anglican Church, Hintsay College is an independent school managed by a School Council. Membership of the School Council is reflective of other democratic processes that operate in the school as the Council must consist, in accordance with its constitution, of a majority of teachers. The school community could be characterised as happy and stable as evidenced by a low level of student exclusions and expulsions as well as a low level of staff turnover.

4.2.2. Curriculum Context

The study took place within senior (Years 11 and 12) secondary mathematics classrooms in the state of Queensland, Australia. Secondary schools in Queensland work within a school based externally moderated assessment system**.** The Board of Senior Secondary School Studies (BOSSS) offers three mathematics subjects – Mathematics A, Mathematics B and

Mathematics C – to students in Years 11 and 12 (the final two years of secondary schooling). Mathematics A is principally concerned with applicable mathematics that promotes informed citizenship. This subject covers topics such as applied geometry, financial mathematics and applied statistics. Mathematics B consists largely of the study of calculus and advanced statistics. This subject is typically a prerequisite for tertiary courses dependent on the mathematical sciences, for business, and for high demand professional studies such as medicine. Most students choose either Mathematics A or Mathematics B. Mathematics C is a specialist mathematics subject that includes the study of advanced calculus, vectors, matrices and dynamics. This subject is designed for students who are either very interested in mathematics or who have aspirations to study mathematically intensive professional preparation courses such as engineering. Relatively few students across the state study Mathematics C because of its specialised nature and because of the requirement to study Mathematics B as well.

Although the curriculum framework for each school must be based on state-wide subject syllabuses, these syllabuses are only mildly prescriptive and, as a result, schools enjoy a high level of flexibility in the design and implementation of curriculum and assessment programs. There are, however, a number of non-negotiable aspects of syllabuses that were current at the time of this investigation (e.g., Board of Senior Secondary School Studies, 1992). Every school program was required to address three global objectives common to each mathematics syllabus – Mathematical Techniques, Mathematical Applications and Communication. As a consequence, teaching, learning and assessing of mathematics within application based contexts as well as purely mathematical contexts were mandated. Mathematical assessment tasks had to appear in contexts that were both familiar and non-routine to the student. Modes of assessment could not be limited to timed tests, with very limited access to text-based, electronic or human resources, but also had to include non-traditional items such as assignments and reports. Non-traditional items were introduced into the requirements of assessment regimes in an attempt to encourage investigative and collaborative approaches to teaching and learning. This form of assessment was implemented in a wide variety of formats across the state, but within the focus classroom they were typically extended, open-ended tasks that placed few limitations on the type of resources students were permitted to access.

The communication of mathematical reasoning as well as results and conclusions was also emphasised by these syllabus documents. This was an aspect of the syllabuses designed to broaden teachers' and students' perceptions of what it means to work and think mathematically so that elements of reasoning and argumentation were given primacy alongside the capacity to work accurately with familiar mathematical techniques.

The use of technology in the learning and teaching of mathematics was strongly encouraged within syllabus documents but was not prescribed in any syllabuses current at that time. The decision to make use of technology was school based and was dependent on available resources and expertise within individual schools.

4.2.3. Technology Context

There were no restrictions on the form of educational technology employed in learning, teaching or assessment within the mathematics classroom that is the subject of this study. Students and the teacher-researcher were permitted to use whatever technology was available. The school's mathematics department had decided some years previously that the learning of mathematics could be greatly enhanced through the use of technology. As a result it had become mathematics department policy that technology was integrated into student learning experiences whenever it was appropriate and possible. The types of technology included computers with appropriate mathematically enabled applications, including function graphing, symbolic manipulation, and dynamic geometry software, access to the internet, and graphing calculators. The students who participated in this study made use of graphing calculators in particular. There were two types of calculator used. These are listed below with a brief outline of the features available on each (Figure 4.1):

TI-92 Calculators (Texas Instruments)
- Graphical and numerical representations of functions
- Statistical capabilities
- CAS capabilities - including matrix editor and complex number manipulation
- Dynamic geometry module

TI-83 Calculators (Texas Instruments)
- Graphical and numerical representations of functions
- Statistical capabilities

Figure 4.1. Calculators and associated features

Students were encouraged to use these facilities whenever they believed it would assist their learning of mathematical ideas and concepts or to solve problems.

4.3. Participants

Participants in this study included students involved in the pilot and main studies, a teacher-researcher, and a research assistant provided through funding provided by an Australian Research Council grant. Ethical clearance for this study was obtained as part of the ethics requirements for the larger study in which this investigation is embedded. Sample permission forms for both parents and students are included in Appendix 1.

4.3.1. Selection of Cohort

A number of factors influenced the selection of the two cohorts of students who participated in this study. Firstly, the curriculum framework documents that were most supportive of the type of classroom activity this study aimed to investigate were the senior mathematics syllabuses (Years 11 and 12). These included statements supporting collaborative and investigative approaches to teaching and learning, and also the use of technology. Secondly, in order to minimise any disruption to school wide teaching programs is was desirable for the study to be situated within a single self-contained subject cohort. The number of students who studied Mathematics C at Hintsay College filled only one class in Years 11 and 12. Because senior mathematics syllabuses promoted instructional approaches consistent with the nature of the research study, and Mathematics C was the only subject taught by a single teacher, this subject was identified as a preferred option in which to situate the study. An outline of the school's Mathematics C program of topics, which includes relevant technologies, appears in Appendix 2.

Hintsay College has a tradition of involvement with educational research and was keen to support the proposed research project. To this end they were prepared to guarantee that the teacher-researcher would have responsibility for Mathematics C within the school for the duration of the project.

4.3.2. Students

Two cohorts of students were involved across the three years, 1997 (Year 11) – 1998 (Year 12) and 1998 (Year 11) – 1999 (year 12), that bounded the implementation of the pilot and main studies. Students were typically 16 – 17 years old. The pilot study initially involved 14 students (3 females and 11 males) but two withdrew during Year 11, leaving 12 (3 females and 9 males) students who completed the entire course. All students involved in the pilot study attended Hintsay College for their Year 10 education.

In the main study, 16 (6 females and 10 males) students initially enrolled in Mathematics C although two (2 females) withdrew early in Year 11. This loss was offset by the addition of one student (female), who joined Hintsay College from another school in the first term of Year 11. As a result, 15 students (5 females and 10 males) studied Mathematics C continuously through Years 11 and 12. Of these students, 2 had completed their Year 10 studies at other schools.

Both the pilot and main study cohorts of participants were high achievers in earlier years of school mathematics and possessed the backgrounds skills necessary for continued success in Years 11 and 12 Mathematics. This is evidenced by the Year 10 results of Hintsay College students, displayed in Figure 4.2 below, who participated in this study.

All students involved in either the pilot or main studies within this investigation are referred to by pseudonyms within this thesis.

1997/98 Cohort (Pilot)	
Level of Achievement	*Number of Students*
Very High Achievement	5
High Achievement	7
Sound Achievement	0
Limited Achievement	0
Very Limited Achievement	0

1998/99 Cohort (Main)	
*Level of Achievement**	*Number of Students*
Very High Achievement	3
High Achievement	10
Sound Achievement	0
Limited Achievement	0
Very Limited Achievement	0

*Note: Two students from the main study did not attend Hintsay College in Year 10

Figure 4.2. Students' Year 10 results: Pilot study (1997/98) and main study (1998/99)

4.3.3. The Teacher-Researcher

In this section I will speak in the first person as it would be difficult to write about myself in any other voice. Talking about myself is a process of unmasking – of revealing those things about myself that must necessarily impact upon my neutrality as a participant observer.

At the beginning of this investigation, I was a secondary teacher of some fifteen years experience, the last seven of which had been spent at Hintsay College as the mathematics coordinator. In this role I was responsible for the development of mathematics teaching/learning and assessment programs within the school and had designed the structure and sequence of learning experiences for Mathematics C. Before joining Hintsay College I had worked in both government and non-government schools in Australia and in

Britain. My involvement in the broader education community had extended to leadership roles within teacher professional associations at both state and national levels. I had also served on syllabus development committees and district and state assessment review panels.

Since completing my initial teaching qualifications I had pursued further professional studies including a Master of Educational Studies degree in which I completed a small scale thesis, which meant I had previous, though limited, experience as a researcher. As a teacher-researcher I was responsible for the design and implementation of this study, although my own observations were complemented by those of a university based research assistant with whom I had worked over a number of years.

4.3.4. Researcher's Consul

The capacity of a research approach based on ethnographic, participant observation to represent the lived experience of a community from the perspective of its members is tempered by the danger that the prejudices and dispositions of the researcher could bias the data (Burns, 2000). Woolcott (1997) identifies the threat of "the researcher who already knows", that is, the researcher who imposes predetermined interpretations of classroom events on data rather than eliciting meaning through interaction with other participants, as the greatest problem to realizing the potential of ethnography in educational settings. In addition to this concern, Janesick (2003) cautions that is possible to become too comfortable in a research setting and, as a result, fail to recognise and then document important or significant events. In this study the technique of *investigator triangulation*

(Denzin, 1978) was used to enhance the trustworthiness of the teacher-researcher's perception of individual student's behaviours as well as students' interactions with other members of their classroom community.

As indicated in Chapter 1, the investigation which resulted in this thesis was part of a larger study supported by an Australian Research Council grant, so funding was available for research assistant support. The research assistant managed a range of resources associated with day to day data collection as well acting as a *Consul* with respect to the observation and interpretation of classroom events. The *Consul* provided another set of eyes in the classroom and was able to observe classroom activity and interaction closely while the teacher-researcher was involved in active teaching.

The *Consul*:

- directed the focus of recording instruments to the activity and interaction of potential interest;
- recorded field notes during each session for which they were present;
- reported observations to the teacher-researcher or ratified observations made by the teacher-researcher himself;
- conducted semi-structured interviews with students as requested by the teacher-researcher;

- acted as a foil for the interpretation of classroom events by the teacher-researcher which in turn led to the creation of theory which gave meaning to observed action and interaction between all classroom participants;
- conducted a semi-structured interview with the teacher-researcher at the end of the study.

Further details of these activities are provided in section 4.6 of this chapter.

4.4. Research Design and Chronology

This is an intensive longitudinal study of a classroom *in situ* with the teacher-researcher living within, contributing to, and so helping to create the culture of the learning community at the centre of this investigation. The data collection methods were selected for their suitability to a research design in which participants maintained their sense of agency rather than merely being treated as subjects of an experiment. These methods, both qualitative and quantitative, were selected, trialled and then further developed in response to developing theory. Three main approaches to data collection were used:

1. surveys of students' dispositions and preferences in relation to the use of technology when learning and using mathematics (the *Technology Questionnaires*);
2. classroom observations of student-student and student-technology interaction in individual, small group and whole class settings; and
3. interviews with individual students, small groups and the whole class.

The development of the specific data collection instruments evolved through the course of the pilot study and the two years of the main study (details can be found, where relevant in other sections of this thesis). A chronology of data collection activity is provided in Tables 4.1 (a), 4.1 (b) and 4.1 (c).

4.4.1. Pilot Study (1997/98)

The pilot study began in Term 3 of 1997 (Year 11) and concluded in Term 2 of 1998 (Year 12). Its purpose was to determine the effectiveness of the chosen data gathering instruments and to trial methods for the analysis of students' responses. A *Technology Questionnaire* was developed and completed by students towards the beginning of the pilot study, mid-way through Year 11, and again at the beginning of Year 12, at the conclusion of the pilot phase. The questionnaires were designed to obtain information about students in relation to their attitudes towards technology and their perspectives on the role of technology in learning mathematics. The questionnaires were distributed in class and completed under supervised conditions. All 14 students completed the initial questionnaire while 12 responded to the second questionnaire. Analysis of these data appears in Chapters 5 and 7.

Methods for video and audio recording classroom events were trialled during the pilot phase in order to refine procedures for the effective collection of data in individual, small group and whole class settings. Recordings were transcribed and analysed. Video and audio recording data were used in conjunction with questionnaire data to progressively develop theory through the pilot study.

The whole class interview, in particular, provided valuable feedback. During the whole class interview, students were presented with an analysis of their responses to the two *Technology Questionnaires*. This interview, conducted in the final week of the pilot study, provided deeper insight into patterns of student responses that emerged from the *Technology Questionnaires*. A detailed analysis of this episode is reported in Chapter 5.

A data collection calendar for the pilot study appears below in Table 4.1 (a).

Table 4.1 (a). Data collection calendar: Pilot study (1997-1998)

Data Collection	Term 3, 1997	Term 2, 1998
Technology Questionnaire	14 students	12 students
Video Observation (lessons – 45 minutes)	6	
Whole Class Audio Interview (lessons – 45 minutes)		1
Whole Class Video Interview (lessons – 45 minutes)		1

The pilot study led to the modification of the questionnaire (outlined in section 4.5) and highlighted the value of documenting students' interpretations of their own responses to questionnaire data, as occurred during the whole class interview.

4.4.2. Main Study (1998)

The main study began with a new Year 11 cohort at the start of 1998. The *Technology Questionnaire* was distributed in February, at the beginning of Year 11 and again, after further modification (see section 4.5), towards the end of Year 11 in November. The questionnaires were completed in class time, under supervised conditions in the presence of the teacher-researcher. The first questionnaire was completed by 16 students with 12 students responding to the second questionnaire.

At least one lesson was videotaped, and associated field notes recorded, in most weeks of the school year. There were periods of time, however, where it was not possible to complete such a schedule due to school holidays or assessment requirements. The purpose of these observations was to gain insight into patterns of student-technology and student-student interactions. Most videotaped lessons also included a segment that focused on at least one smaller group of students as they worked with mathematics and technology. The selection of these focus groups was based on the degree of audible engagement exhibited while working on a new mathematical idea or on a problem. Over the course of the year, however, all students were videotaped while working within a small group setting.

Two sequences of intensive observation were also employed – in April and May (Term 2). During these sequences, every lesson over a period of approximately one week was recorded. The timing of these sequences corresponded with topics that the teacher-researcher believed had the greatest potential for student-student-technology interaction. This provided opportunity to follow the development of students' thinking and understanding of a new mathematical idea or process over an extended period of time.

Video and field note data were complemented by audiotaped interviews of individual students. Interviews were conducted by the teacher-researcher's *Consul*, after withdrawing interviewees from the classroom, for the purpose of gaining greater insight and understanding into classroom events including students' individual actions and their interactions with others. A calendar for data collection procedures appears below in Table 4.1 (b).

Table 4.1 (b). Data collection calendar: Main study (1998)

Data Collection	Term 1	Term 2	Term 3	Term 4
Technology Questionnaire	16 students			12 students
Video Observation (lessons - 45 minutes)	5	18	6	6
Audio Interview (individual students)			1 session (4 students)	2 sessions (4 students and 2 students)

Students were kept informed of developments in the research project, the analysis of data, and any theoretical constructs that were being developed. For example, students were invited to read and provide informal comment on a research paper based on data gathered during the first year of the main study.

4.4.3. Main Study (1999)

The main study continued in 1999 with the same cohort of students, now in Year 12. While the principles of the research design of this study remained unchanged in its second year, a number of modifications were made to data gathering processes.

In response to developing theory, the *Technology Questionnaire* was altered and expanded (see section 4.5 for a detailed description) to allow students to respond to the metaphors that were being developed to describe their modes of interaction with technology. The questionnaire was again administered in class by the teacher-researcher and completed under supervised conditions by 15 students.

Videotaping of lessons continued at a rate of about one lesson per week when the requirements of school life (e.g., camps, vacations, examination periods, etc.) permitted. Taping again focused on the activity of smaller groups within the class. Intensive observation sequences were conducted in May and October. Audiotaped interviews with individual students, as a follow-up to activity observed in previous lessons, were again used in an attempt to capture students' perceptions of classroom events.

These procedures were supplemented by a number of modified data gathering processes. Firstly, audio interviews were complemented by video interviews with individuals, about a range of technology related issues that had arisen so far in the study. Secondly, four whole class interviews were conducted in relation to students' use of particular technologies or their choice of technology, from those available, when working on a mathematical problem or in specific settings, for example, in examinations. These interviews were used to gain greater insight into observed student behaviours by documenting their own perspectives on events. The data collection calendar appears in Table 4.1 (c) below.

Table 4.1 (c). Data collection calendar: Main study (1999)

Data Collection	Term 1	Term 2	Term 3	Term 4
Technology Questionnaire				15 students
Video Observation (lessons – 45 minutes)	7	10	10	9
Audio Interview (individual students)				1 session (2 students)
Video Interview			2 sessions (2 students and 8 students)	
Whole Class Interview (lessons – 45 minutes)	2	2		

4.5. Data Collection Instruments and Procedures

The data collection instruments associated with this study are described below. Blank copies of these instruments are available in Appendix 3. It should be noted, that whenever administering data collection instruments care was taken to remain sensitive to the risk of influencing student responses through a desire to satisfy their teacher's expectations. In each administration of an instrument, students were encouraged to be "honest" in relation to their opinions and views.

4.5.1. Technology Questionnaire

A questionnaire was developed to elicit students' responses to questions related to:

1. disposition of students towards using technology in learning mathematics;
2. development of collaborative preferences (or not) by students as they work with technology in mathematics learning;
3. choices of specific forms of calculator use favoured by students;
4. choices of general strategic purposes for calculator use favoured by students;
5. perceptions of students with respect to their global facility and confidence with graphing calculators as a personal resource.

The questionnaire was structured around six sections consisting of a combination of Likert items (adapted from Galbraith & Haines, 1998), alternative response items and open-ended questions. Structured Likert items were grouped in Sections 1 to 3 and 5. Section 4 consisted of a combination of alternative response items and an open ended response item, while Section 6 invited open-ended responses. Figure 4.3 summarises the purpose of each section of the Technology Questionnaire along with exemplar items.

The *Technology Questionnaire* was administered on five occasions in successively modified forms (although it remained unaltered on the second occasion). The stability of the majority of the question set allowed for valid comparison of responses across time, while the process of theory building, based on the continuous analysis of data, informed the enhancement of the *Questionnaire* before each cycle of administration and analysis. Thus, while the pilot study provided initial feedback on the viability of a larger study and the effectiveness of the instrument, it also helped provide substance for initial theory building. This, in turn, informed decisions about the further development of theory and how questionnaires could be further enhanced.

The cycle of survey analysis and adaptation was repeated through the duration of the main study, thus providing direction on how to view gathered evidence, both classroom observation and student interview, in relation to developing theory.

A summary of the structure of each *Technology Questionnaire,* with an indication of the changes implemented in each cycle, appears in Figure 4.4. Sections that were modified are asterisked. The details of these modifications are discussed in the relevant data chapters (Chapters 5 to 8). A clean copy of each versions of the Technology Questionnaire is included in Appendix 3.

Section	Purpose and Exemplar Items
1	Purpose Confidence with using technology in the context of a mathematics classroom.
	Stimulus I feel confident I can use technology when faced with a new problem in maths class.
2	Purpose Preference in relation to working with technology individually or in collaboration with others.
	Stimulus I prefer to work with others when using technology because I often get good ideas from them.
3	Purpose Perspective on the role of technology when learning mathematics.
	Stimulus I prefer to learn the mathematics first, without technology, and then learn the technology to do the mathematics more quickly.
4	Purpose Access to and type of technology available in the home environment.
	Stimulus If you use a computer at home, name the kind of software you use, ie spreadsheets, games, graphing packages etc.
5	Purpose "Mathematical" use of technology within home and school environments.
	Stimulus I use technology at home for exploring ideas about mathematics begun in class
6	Purpose Students' views on learning and teaching mathematics within technologically enhanced settings
	Stimulus Are there any advantages in using technology instead of pencil and paper? If so, explain how technology helps you learn better. (Give specific examples.)

Figure 4.3. Purpose for each section of the *Technology Questionnaire*

Section	**Data**			
	Pilot	*TQ1*	*TQ2*	*TQ3*
1	Confidence with using technology in the context of a mathematics classroom. (5 statements)	Confidence with using technology in the context of a mathematics classroom. (5 statements)	Confidence with using technology in the context of a mathematics classroom. (5 statements)	Confidence with using technology in the context of a mathematics classroom. (5 statements)
2*	Preference in relation to working with technology individually or in collaboration with others. (5 statements)	Preference in relation to working with technology individually or in collaboration with others. (5 statements)	Preference in relation to working with technology individually or in collaboration with others. (5 statements)	Preference in relation to working with technology individually or in collaboration with others. (6 statements)
3	Perspective on the role of technology when learning mathematics. (12 statements)	Perspective on the role of technology when learning mathematics. (12 statements)	Perspective on the role of technology when learning mathematics. (12 statements)	Perspective on the role of technology when learning mathematics. (12 statements)
4	Access to and type of technology available in the home environment. (3 questions)	Access to and type of technology available in the home environment. (3 questions)	Access to and type of technology available in the home environment. (3 questions)	Access to and type of technology available in the home environment. (3 questions)
5	Mathematical use of technology within home and school environments. (11 statements)	Mathematical use of technology within home and school environments. (11 statements)	Mathematical use of technology within home and school environments. (11 statements)	Mathematical use of technology within home and school environments. (11 statements)
6*	Advantages/disadvantages of technology in relation to learning. (3 questions)	Advantages/disadvantages of technology in relation to learning. (3 questions)	Advantages/disadvantages of technology in relation to learning. Preferences for different types of digital technologies. Perspectives on teacher's role in technology rich classrooms. (5 questions)	Advantages/disadvantages of technology to learning. Preferences for different types of digital technologies. Perspectives on teacher's role in technology rich classrooms. Modes of calculator use as a personal resource. (8 questions)

Figure 4.4. Structure and Purpose of Pilot and Technology Questionnaires 1 – 3

4.5.2. Video-recording of Lessons

As described in Section 4.4 one lesson per week was videorecorded whenever circumstances permitted. In addition, there were intensive periods of observation where every lesson over a set period of time was recorded during learning sequences where it was felt interaction of the type of interest to the study might take place. Lessons were typically of 45 minutes duration although there were a number of occasions when 90 minute double lessons were recorded.

Each lesson was recorded using a palm sized VHS-C video recorder which was operated by the teacher-researcher's *Consul*. The video camera was mounted on a tripod and generally positioned to the side or the back of the room. This allowed the *Consul* to video record interactions between students and technology in individual, small group or whole class contexts regardless of their chosen position in the room. Further, the camera was equipped with a high fidelity plug-in microphone that could be placed centrally within groups of students. The range of the microphone permitted the clear recording of discussion in student groupings of up to four or five in number. Students readily adapted to the presence of both the video camera and the microphone. In the main, after an initial period of time, students appeared to pay little attention, if any, to either recording instrument.

The focus of recordings was determined by the teacher-researcher. Initially, different groups of students were targeted systematically, so that each group of students experienced being videotaped within the first few weeks of the study. After this initial period, the camera's focus was determined by the history of students' interaction with technology either as individuals or in a group setting while still remaining responsive to unanticipated classroom events. Consistent with this sensitivity to emergent phenomena, there were occasions when the focus of the observation was redirected during a lesson if a noteworthy interaction took place within a group different from that initially selected.

Field notes were hand recorded by the *Consul* during the lessons. These notes included the time and date of the lesson, the topic being studied, the general structure of the lesson, any feature that might be of potential interest and any additional resources such as handouts or worksheets. As the study progressed these notes were then summarised in a Lesson Observation Record with additional information such as the mode of recording (audio or video) and the suitability of the lesson for transcription. An example of a set of field notes from one lesson and a sample Lesson Observation Record for the duration of one semester have been included in Appendix 4.

While the use of a small video camera and high fidelity microphone provided a relatively unobtrusive method for documenting the interactions between students and technology, there were a number of limitations associated with this approach to data collection. Firstly, the camera could only be directed at the whole class or a smaller group with the whole class in the background. This meant that although the judgments made about where to point the camera were based on well considered criteria there were conversations, discussions and other interactions that were not recorded. As a result, interpretations and inferences about the nature of a conversation, discussion or use of technology are restricted to what can be gleaned from the video data viewed at some time after the actual event.

4.5.3. Individual Student Interviews

In order to develop understanding of observed classroom events, and so inform developing theory, individual student interviews were conducted. The aims of these interviews were to:

1. clarify an account of an observed event or discussion;

2. probe students' recollections of their thinking during an observation or sequence of observations;
3. test the teacher-researcher's interpretation of student-technology or student-student-technology interaction.

Students were selected for interview on the basis of their participation in an event of interest. The interviews were generally conducted by the *Consul* either by withdrawing interviewees from a class while in session, or arranging to meet interviewees after class or between class periods. The duration of interviews was typically 10 minutes and they were conducted in an area outside the students' classroom. Because of the number of students interviewed, usually four or more, interviews were sometimes conducted up to two days after an event of interest. As a result, this procedure is limited by a student's capacity to recall events accurately due to the time between the event and the interview. Further, in cases where interviews were conducted over an extended period of time the possibility of collusion between students when reporting interpretations of events could not be excluded.

Interviews were semi-structured and based around events, activities or behaviours that required further illumination or elaboration. Two examples of protocols for semi-structured interviews appear in Appendix 5. Interviews were recorded on either video or audio tape and selected passages transcribed. The selection of events for transcription was based on the clarity and insight an interview contributed to understanding of thinking and development of meaning that took place during an activity or student-technology or student-student-technology interaction.

4.5.4. Whole Class Interviews

Whole class interviews were conducted near the beginning (Term 1, 1998) and half way through the second year of the main study (Term 2, 1999). These interviews were used to:

1. elicit students' recollections of earlier classroom events;
2. inform students of the progress of the study;
3. test the validity of the theory that was being built through the study against students' perspectives on events.

These interviews were semi-structured and conducted by the teacher-researcher with support from the researcher's *Consul*. The interviews were carried out as close as was possible to an event of interest. Three of the four whole class interviews in the main study relied on students' unassisted memories of previous classroom events while the remaining interview used the technique of video-stimulated recall.

4.6. Method for Analysis and Interpretation of Data

Ramsden (1997) argues that while we should not seek to use a technology for a purpose that it is patently unsuited to, emergent uses should be productively sought, including uses that no one (including the designers) could have predicted. This study seeks to document and interpret emergent patterns of behaviour evident in interactions between humans and technological tools. Because of the emergent nature of this study a naturalistic research design was chosen in order to accommodate and build theory around classroom events as they occurred *in situ*. Consistent with a naturalistic methodology, data collection and analysis were conducted simultaneously with theory building. Patterns of emergent behaviour were documented and categorised. Theoretical insight was then gained through an iterative process similar to that described by Goos (2004). This process included the phases of category creation, category confirmation, category refinement and theory development.

Categories were initially created through the analysis of both qualitative and quantitative data, including field notes, video and audio recordings and discussion with participants in student-technology or student-student-technology interactions. Because the creation and confirmation of categories demanded the use of different sets of data analysis techniques, detailed descriptions of analysis procedures are reported in conjunction with the data to which they are applied. As a result, the data chapters which follow in this thesis outline the specifics of analysis techniques, findings and the limitations of the chosen methodological approach. A general outline of analysis procedures, as they relate to specific research questions is presented below.

Research Question 1: What are the dispositions and preferences of students towards using technology in learning mathematics?

Data for this question were provided by the *Technology Questionnaires*, including both Likert and open-ended response items, and whole class interview transcripts. Totals for aggregated responses to Likert items were used to determine students' dispositions and preferences in relation to technology usage in learning mathematics and how these developed over time. Follow-up whole class interviews were employed to confirm or illuminate emergent student views.

Research Question 2: What are the perceptions of students with respect to their global facility and confidence with digital technologies as a personal resource?

This question was considered from the perspectives of students using digital technologies as individuals and during presentations to the whole class. Scores from Likert items from Section 1 of the *Technology Questionnaire* were aggregated and used to benchmark students' confidence levels, in relation to use of technology, through the duration of the study. A fine grained analysis of a series of classrooms episodes, captured on video tape and transcribed, provided insight into the support students felt was made available via technology for presenting mathematical ideas and conjectured solutions to problems presented to their peers as a whole class group.

Research Question 3: What choices of specific forms of technology use are favoured by students?

Open-ended response items and whole class interview transcripts were used to address this question. Students were asked to state their preferred form of technology (calculators or computer) directly and to provide reasons for their preference. A whole class interview employed a stimulated recall approach where students were shown a video recording of classroom activity and asked to explain observable behaviours related to their choice of digital technologies in the video. This question was considered from both individual and collaborative perspectives.

Research Question 4: What choices of general strategic purposes for technology use are favoured by students?

Students' preferences in relation to the strategic use of digital technologies and how these changed over time were determined through examination of totals of aggregated Likert scores on items that specifically targeted this issue and from responses to open-ended items.

Research Question 5: What roles can be identified for technology in mediating individual student learning?

Metaphors for students' technological use were created by examining responses to the open-ended items from Section 6 of the *Technology Questionnaire*. These categories were tested and verified by providing students with the opportunity to self-identify their current mode of technology use against these metaphors via an additional question in Section 6 of the final *Technology Questionnaire*. A re-examination of open-ended response data against the identified metaphors allowed for the construction of a finer grained categorisation of responses. The frequency of students' responses within categories against time was also studied in order to gauge if some types of technology use took longer to present than others.

Research Question 6: What roles can be identified for technology in mediating collaborative student learning?

Preferences for and dispositions toward learning with technology in collaborative contexts were established via reference to the Likert measures used in Section 2 of the *Technology Questionnaire*. Open-ended questions in Section 6 of the Questionnaire were used to elaborate further on these responses, especially in the last version of the *Technology*

Questionnaire as questions in this section were modified to increase the focus on collaborative aspects of learning with technology. Excerpts from transcripts of videotaped classroom episodes are used to illustrate students' collaborative behaviour in small groups and whole class settings with reference to metaphors for students' technology use. Student interviews were also used to probe for individuals' perspectives on classroom events.

4.7. Trustworthiness

While frameworks for judging the validity of qualitative research are emerging, there is no single procedure that has yet won universal acceptance. This is related to the variety of approaches that exist under the umbrella of qualitative methodology and the practice within tradition of not privileging one methodological practice over another (Denzin & Lincoln, 2003). Thus, while general criteria have been developed to assess the methodological worth of research practices and findings these criteria need to be contextualised to each individual study.

4.7.1. Establishing Standards of Trustworthiness

Lincoln and Guba (1985) argue that naturalistic inquiry sits apart from conventional modes of scientific inquiry for two major reasons. Firstly, they claim qualitative research is not about determining and isolating causes and effects. Thus the only standard applicable to truth value (or internal validity) is how credible are findings to those who provide the data. Secondly, external validity (and so applicability and consistency) does not apply to naturalistic inquiry. In their view researchers are only responsible for describing clearly and comprehensively the contextual conditions of their studies. It is the responsibility of the audience, or readers of the research study, to determine whether the transfer of findings is potentially possible. Thus it is the researcher's responsibility to provide the "thick descriptions" so that judgments about the similarity of conditions and contexts can be made in order to ascertain the likelihood of similar findings from a different research site.

In a review of criteria for establishing the quality and validity of qualitative research Eisenhart and Howe (1992) acknowledge the contribution of Lincoln and Guba (1985), among others, but also argue that their position is too remote from the conventional view of scientific research to gain wide acceptance. They instead propose the following general criteria.

Standard 1: Fit between research questions, Data Collection Procedures, and Analysis Techniques.

Standard 2: The Effective Application of Specific Data Collection and Analysis Techniques – Data Collection and Analysis Techniques should be competently applied.

Standard 3: Alertness to the Coherence of Prior Knowledge – Arguments must be built on some theoretical position, or contribute to some substantive area or practical arena.

Standard 4: Value Constraints – The conduct of research is subject to both external and internal value constraints that relate to the worth in importance or usefulness of the study and its risks.

Standard 5: Comprehensiveness – Involves judgment about technical aspects such as the overall clarity, coherence and competence of the study as well as the balance between overall technical quality and the value and importance of the study.

While these standards are comprehensive they have been designed to accommodate all of the manifestations of the family of research methodologies that identify as qualitative. These standards require further interpretation in order to provide the specificity necessary for their operationalisation within the context of a refined tradition of qualitative investigation.

Of relevance to this study are criteria that are appropriate to longitudinal investigations that are largely dependent on the analysis of video and transcript data. Cobb and Whitenack (1996) present criteria relevant to this context starting from a position that “the trustworthiness of the findings depends on the extent to which they are reasonable and justifiable given the researcher's interests and concerns” (p. 224). They argue that the trustworthiness of an ethnographic study, of the type described above, can be established through attention to three practices:

1. the systematic analysis of the data set by continually testing provisional conjectures;
2. the prolonged engagement of the researcher with the participants of the study;
3. the extent to which the analysis has been critiqued by other researchers.

Schoenfeld (1992) describes the development of nonstandard methods for the analysis of videotapes of problem-solving sessions and the associated standards for judgments about validity, reliability, and the communication of methods and data. He outlines five processes for establishing the quality and validity of a study as:

1. Establish the context, describing the issues to be addressed.
2. Describe the rationale for the method.
3. Describe the method in sufficient detail that readers who wish to can apply the method.
4. Provide a body of data that is large enough to allow readers to (a) analyze it on their own terms, to see if their sense of what happened in it agrees with the author's, and (b) employ the author's method and see if it produces the author's analyses.
5. Offer a methodological discussion that specifies the scope and limitations of the method, as well as the circumstances in which it can profitably be used, and that treats issues of reliability and validity.

(Schoenfeld, 1992, p. 181)

When viewed as complementary practices the criteria set out by Cobb and Whitenack (1996) and Schoenfeld (1992) provide a framework for evaluating trustworthiness that is specific to the methodology employed in this study and at the same time provide coverage of the general standards outlined by Eisenhart and Howe (1992). Overlap exists between the method specific and general standards, and the major alignments between these criteria are set out in Figure 4.5.

Eisenhart and Howe	**Cobb and Whitenack**	**Schoenfeld**
Standard 1	Criterion 1	Criteria 1 and 2
Standard 2	Criterion 2	Criteria 2 and 3
Standard 3	Criterion 3	Criterion 5
Standard 4		Criterion 4
Standard 5	Criterion 3	Criterion 5

Figure 4.5. Alignments between criteria for trustworthiness

4.7.2. Standards of Trustworthiness Applied to this Study

Judgments about the trustworthiness of a study must necessarily remain with the readers of education research and can only be made against established criteria of a thesis as a whole. The intent of this chapter has been to provide a description of the methodology employed in data collection and analysis. Further, issues related to the contexts and conditions of the education site and the participants have also been discussed. These descriptions address Cobb and Whitenack's (1996) second criterion and Schoenfeld's (1992) criteria 1, 2, 3 and 5. Judgments about issues related to the quality of the implementation of the methodology and the analysis and interpretation of data can only be made after the reader has examined the chapters related to the data collected during the course of this study and the data's associated interpretation (Cobb and Whitenack, criterion 1; Schoenfeld, criterion 4).

Finally, Cobb and Whitenack's (1996) third criterion has been addressed through the course of this study via exposure of ideas, interpretations, initial inferences, developing theory to participants. The researcher's *Consul* also provided feedback on the researcher's interpretations of classroom events and comments related to categories and constructs as they were developed by the teacher-researcher. In addition, the university based supervisors of this thesis followed the study closely and, because of their relative distance from its "coal face", were able to provide independent advice. Scrutiny of research practice and developing theory was also invited from a wider audience of researchers through conference presentations and publication (e.g., Galbraith, Goos, Renshaw & Geiger, 2001; Galbraith, Renshaw, Goos & Geiger, 1999; Geiger, 2005, 2006; Geiger, Galbraith, Goos & Renshaw, 2002; Goos, Galbraith, Renshaw & Geiger, 2000, 2003)

4.8. Framework for the Presentation of Findings

The metaphor of a zoom-lens (Lerman, 2001) has been adopted as a means of presenting data and findings in a logical format. The act of "zooming-in" or zooming-out" described the shifting focus employed by an investigator when considering student-technology or student-student-technology interactions in individual, small group or whole class settings. Individual student-technology interactions are observed or documented by "zooming in" to behaviours that take place while working on mathematical ideas or problems (Chapters 5 and 6). By "zooming-out" to the middle ground, student-student-technology interactions can be studied while students work in small group settings (Chapter 7). By again "zooming-out" to the landscape view of student-student-technology interaction in openly public whole class settings, such as when students present findings of a mathematical enquiry to the whole class using technological tools, can be examined (Chapter 8).

Chapter 5

Orientations, Dispositions and Preferences

This chapter reports on students' dispositions in relation to the use of technology to learn mathematics and their preferred choices from available types of technology for solving problems or expressing new mathematical ideas. Data were drawn from: two *Technology Questionnaires* administered in the pilot study; a whole class interview conducted during the pilot study; three *Technology Questionnaires* administered during the main study; one video taped stimulated recall interview with the whole class as part of the main study; and one further follow-up whole class interview conducted after the completion of an assessment item.

Details of the administration of the *Technology Questionnaires* are explained in the previous chapter (Chapter 4), but it should be noted that the substance, number and timing of questionnaires varied between the two studies in response to the process of continuous theory building and reflection. These changes are identified as appropriate through this chapter.

The analysis which follows examines the results of the two phases of data gathering and then compares and contrasts results in order to address the following research questions:

1. What are the dispositions and preference of students towards using technology in learning mathematics?

2. What are the perceptions of students with respect to their global facility and confidence with digital technologies as a personal resource?

3. What choices of specific forms of technology use are favoured by students?

4. What choices of general strategic purposes for technology use are favoured by students?

5.1. Orientations and Dispositions

Section 5.1, below, focuses on research questions 1 and 2.

5.1.1. Preliminary Findings - The Pilot Study

Classroom Context

The pilot study was situated in a Mathematics C class, over a six month period, between July 1997 (Year 11) and May 1998 (Year 12) in a co-educational independent secondary school. The use of graphing calculators occupied an important and integrated role throughout the course in both teaching and assessment. Graphing calculator technology was augmented by computer based learning experiences. Students had free access to both of these technologies during class time and were permitted to take home calculators at any time.

Data Sources

In order to capture a snapshot of students' attitudes to the role of technology when learning mathematics the first *Technology Questionnaire* was administered at the start of the pilot study, during the third term of Year 11 after the *Introduction to the Theory of Chaos and*

Fractal Geometry unit (see Appendix 2). The questionnaire was completed at this time, as a judgment was made that students had been engaged in a wide enough range of technology based experiences to respond in a meaningful way to the survey instrument. These experiences included exposure to both graphing calculator and computer technologies.

A second *Technology Questionnaire* was administered in the first term of Year 12 after the students had gained further experience with these technologies and their modes of use. Trends, patterns and differences, noted through the analysis of the two *Technology Questionnaires*, were then used as stimuli to structure discussion in a whole class interview. Structured items designed to address the areas represented by research questions identified above are grouped respectively in sections 1, 3 and 5 in Table 5.1.

The item format used strong agreement (SA) (5) to strong disagreement (SD) (1) in Sections 1 to 3, and Always (5) to Never (1) in Section 5. In order that high scores reflect more of the property of interest, asterisked items were reverse coded. For example, in Section 3, question 1 is an item where agreement represents a negative predisposition toward *using technology*. Thus after reverse coding, a high score represents a positive predisposition.

The following analysis is based on the responses of the 12 students who completed both *Technology Questionnaires,* and the whole class interview.

Results

Score totals are included with the questions in Table 5.1. The elements in the ordered pairs denote the Year 11 and Year 12 responses respectively. Since 12 students completed both questionnaires the total score on any item can vary between 12 and 60. Totals, rather than means, have been displayed because they are more informative. For example, a shift of 6 can be thought of as equivalent to the net effect of half the class changing their score by 1 in the same direction, or more radical moves by fewer students and so on. On this basis, shifts of 6 or more are considered noteworthy changes (indicated by shading in table). The rating magnitudes are also important. Totals of 48 (75th percentile) and above are considered as indicative of strong group support for the relevant construct, and conversely for very low scores of 24 (25th percentile) or below (both indicated by bolding in table). Moderate support is indicated by scores between 36 (50th percentile) and 47.

Table 5.1. *Technology Questionnaire* outcomes Sections 1, 3 and 5: Pilot Study

Section 1		
1	I enjoy using technology during mathematics classes	(**56**, **54**)
2	I will work with technology for long periods of time if I think it will help me solve a problem	(**57**, **54**)
3	I feel confident I can use technology when faced with a new problem in maths class	(**53**, **50**)
4	If I make a mistake when using technology I am usually able to work it out for myself	(**50**, 47)
5	Using technology makes me feel more confident about learning mathematics because I can check answers and ideas as I go	(**52**, **53**)
Section 3		
1*	I prefer to just learn the mathematics and find the need to learn technology as well a burden	(**54**, **52**)
2*	Good students don't need the assistance of technology to understand mathematics	(**51**, **48**)

3*	Technology is only there to check what you do with pen and paper	(**56**, **52**)
4	Technology allows me to explore my own ideas about mathematics as well as those discussed in class	(**51**, **49**)
5	I am sometimes forced to use new mathematics when exploring the use of technological tools	(47, 45)
6	By looking after messy calculations technology makes it easier to learn essential ideas	(**55**, **51**)
7*	I prefer to learn the mathematics first, without technology, and then learn the technology to do the mathematics more quickly	(40, 33)
8*	I tend to use technology to do calculating basic tasks but not much else	(**51**, **48**)
9	I find technology particularly useful when exploring unfamiliar problems	(**57**, 45)
10	Technology allows me to learn mathematics more easily because I can work through a greater number of examples more quickly	(**53**, 45)
11	Technology helps me to link knowledge, eg the shapes of graphs and their equations	(**54**, **51**)
12*	I can often solve problems using technology in the classroom but when thinking about the same mathematics latter I feel I don't really understand it	(47, 40)
Section 5		
1	I use technology at home for:	
a	entertainment (eg games, surfing the internet)	(**46**, 47)
b	writing up school assignments	(**56**, **56**)
c	doing mathematics homework	(35, 39)
d	exploring ideas about mathematics begun in class	(30, 31)
e	exploring ideas about mathematics of my own	(26, 27)
2	I use technology at school:	
a	when the teacher tells me	(**56**, **53**)
b	when I get stuck on a problem	(**48**, **48**)
c	to look at a problem in a different way, eg a picture or a table	(**48**, 47)
d	as a way of discussing a problem with others	(**48**, 41)
e	when I feel pen and paper isn't helping	(**52**, **49**)
f	as a first resort when looking at a mathematical problem	(40, 35)

Less Extreme Views

Student responses indicated that they had become less extreme in their opinions with fewer scores of 5 (105 vs 60) and 1 (37 vs 20) recorded on the second response to the questionnaire in relation to the first response. Exploration of this observation during the whole group interview indicated that students had greater experience with the technologies under discussion and, in general, were more confident with their use. This meant that students were less likely to be intimidated, or impressed, by the novelty of the technology alone. They were also more aware of the strengths and weaknesses of these technologies and so less emphatic about their views on related issues. This stance is reflected in the following student comment from the whole class interview.

> Fran: Since the first time we've had more experiences and so are less definite about our answers.

Dispositions When Using Technology

Responses from Section 1 of the questionnaire indicate stable and overwhelming support for the use of technology when studying mathematics (see Figure 5.1).

The total scores on items from Section 1 ranged between 50 and 57 on the first response to the questionnaire and between 47 and 54 on the second. Students enjoyed the use of technology, were confident when using technology to explore new problems and in fixing mistakes, and were prepared to persist with its use even when the solution to a problem was not immediately apparent.

These attitudes were attributed, during the whole class interview, to the options provided by technology to do problems in different ways in a shorter time span. Further, the facility to verify hunches and conjectures "on the fly" without needing to stop and lose a train of thought, and the opportunity to study a visual representation of a problem, especially in relation to work on functions, were also commented upon. In addition, the use of technology was viewed in a positive light because it provided an alternative to the classroom practice of using pad and pen.

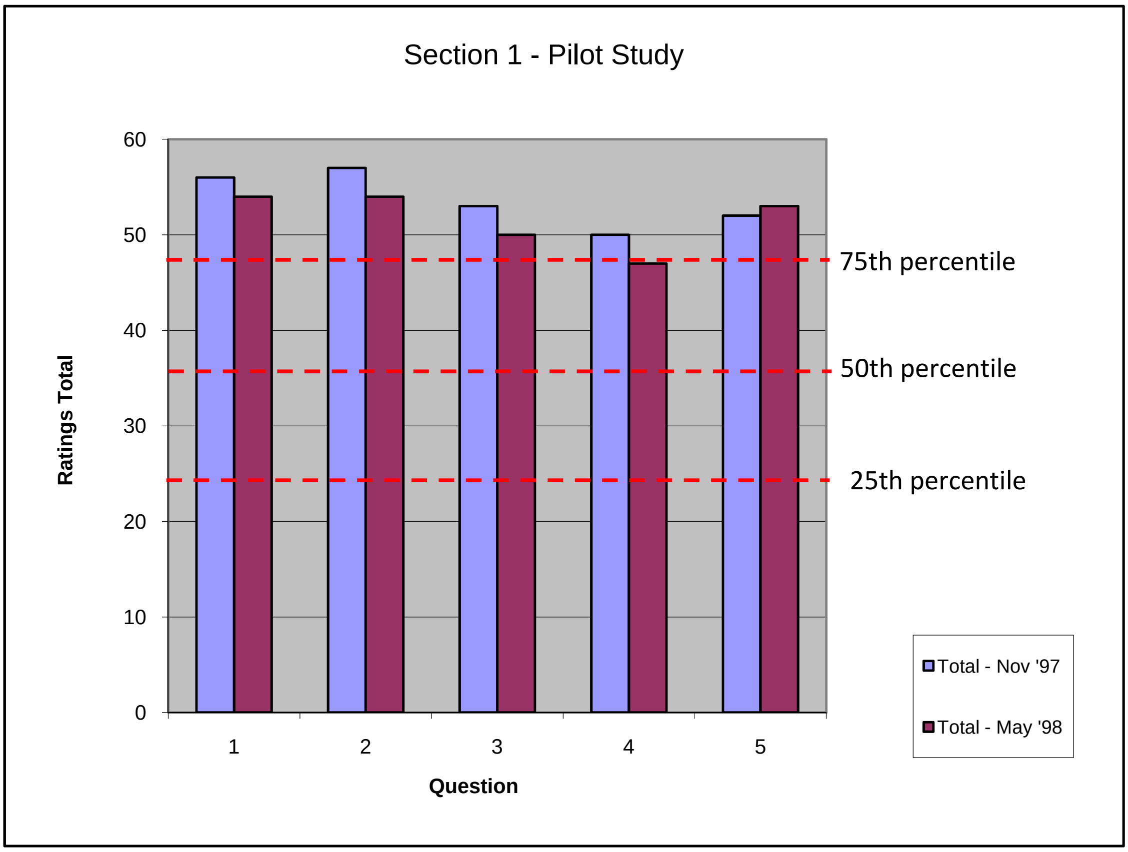

Figure 5.1. Students' responses to Section 1 of *Technology Questionnaires* 1 and 2: Pilot Study

Technology as Mathematical Assistant

Students revealed a sophisticated understanding of how they used technology to learn and do mathematics. The Likert items in Section 3 (see Figure 5.2) indicate students were comfortable with using technology with 9 out of 12 items recording a total equal to or above the 75th percentile on the first questionnaire and 7 out of 12 with a total equal to or above the 75th percentile on the second questionnaire.

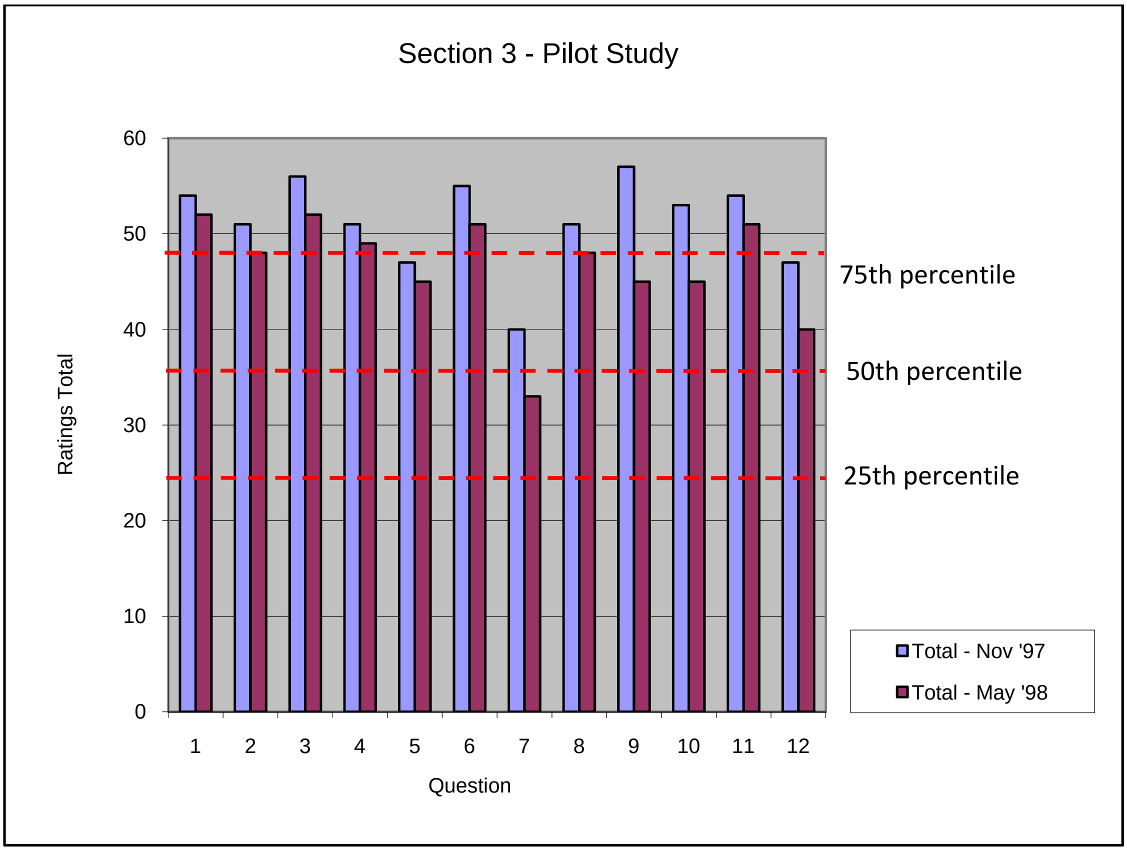

Figure 5.2. Students' responses to Section 3 of *Technology Questionnaires* 1 and 2: Pilot Study

While the proportion of responses above the 75th percentile in the second survey is lower this is consistent with the observation, discussed earlier in this chapter, that students' views became less extreme over time. In addition, it should be noted that 2 scores on the first survey (each scores of 47) and 3 scores on the second survey (each scores of 45) were only marginally below the 75th percentile.

The whole class interview supported questionnaire data in relation to students' enthusiasm for working with the assistance of technology. Students expressed the view that the advantages offered by technology were principally related to the efficiency and power available for exploring a situation.

> Fran: So you don't have to use pen and paper (alone); it's easy to change what you have typed in, it helps with investigating.

Question 7 referred to the order in which technology and a mathematical idea or concept should be presented for learning. Questionnaire data indicated an increase in support for learning how to use technology after students had studied a mathematical topic. As interpreted by Jack:

> Jack: Better to understand first before using a calculator.

Students also related this change to the technology based activities used to introduce some units of work. The practical nature of these activities, as to be expected, was subject to experimental variation and so assumed an acceptance of some level of tolerance in results. Some students found this conflicted with their view of mathematics as an exact science.

> Jessie: Theory is always right!
>
> Fran: I get confused with "explore" activities using technology. It just doesn't work for me.

Thus some students placed little value in practical activities and preferred being taught theory from the beginning.

A large drop in support (-12) was recorded in Question 9 for the use of technology. This item asked for an indication of students' preferences when using technology to investigate unfamiliar problems. When brought to their attention, during the whole class interview, students also related this change to the experiential approach used by the teacher to introduce new mathematical ideas and concepts. The change in students' responses was put down to unease about how well an activity "worked":

> Jack: I've seen more failures.

Exposure to activities that were based on the measurement of real time data gathered from natural phenomena were naturally subject to experimental error, which seemed to marginally erode students' confidence in using technology to investigate an unfamiliar situation. It should be noted, however, that despite this trend, students' reseponses in question 9 were still positive in relation to the use of technology in unfamiliar contexts (totals of 57 and 45).

Other noteworthy downshifts were recorded for Questions 10 (-8) and 12 (-7). These questions canvassed student opinion on the advantages offered by technology in relation to its computational power, and the longevity of their technology assisted learning in mathematics. This drop in support is consistent with the general downward trend students identified as the passing of the "gee wow" phase, as experience has taught them that the advantages offered by technology, while helpful, are not panacean. This indicates that students were moving to a more balanced realisation of the strengths and weaknesses of technology rather than viewing it as a cure all, that is, as a helpful if fallible collaborator or assistant.

Home and School

Section 5 of the questionnaire was designed to compare and contrast commonalities and differences between school and home use of technology. Analysis revealed that there was very little home use related to school activities other than writing up assignments (Figure 5.3). This section, combined with the whole class interview, did provide, however, insights into a developing capacity among the students to make discriminating use of technology.

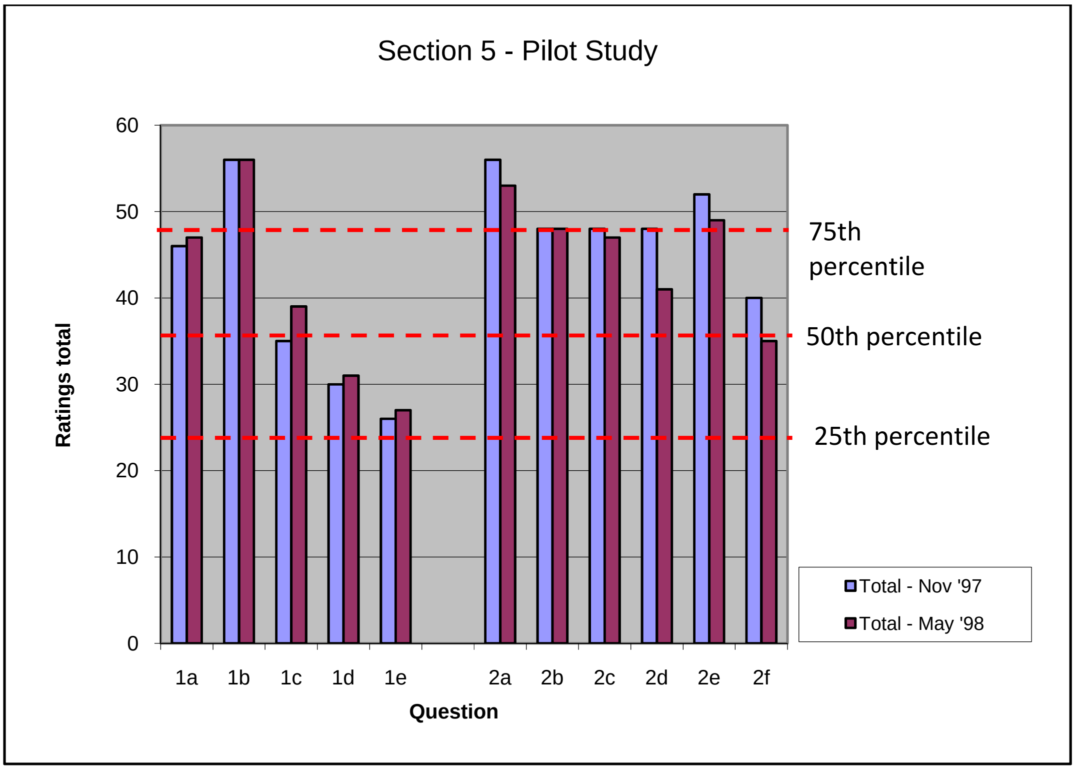

Figure 5.3. Students' responses to Section 5 of *Technology Questionnaires* 1 and 2: Pilot Study

The trend in Question 2 of Section 5 is one of strong support for the use of technology at school, in both sets of responses to the questionnaire, except for 2(f) where the feeling appears to be neutral. When students were asked, during the whole class interview, for an explanation of the difference of views in 2(f) compared to the rest of the items in Question 2, students made it clear they believed it is often best to think about a problem first rather than reach for a calculator as a matter of course.

> Fran: Only use technology if you think it will help, look at the problem first.
>
> Jessie: You need to decide what's more effective (i.e., to use technology or not).

Students had thus shown they were developing a mature and flexible understanding of, and approach to, the use of technology. They were aware that technology could be used in a variety of ways and were developing the capacity to make decisions about when and how it could or should be used.

5.1.2. Significant Observations

The Questionnaire

It appeared the questionnaire had targeted identified issues well and that the follow up interview had corroborated, or elaborated upon, observations emergent from the data. It was decided these data might be enriched by making use of a third “tracking” of student attitudes and opinions by making use of a *Technology Questionnaire* earlier in the school year – that is, before students had any great experience of the particular teaching and learning culture that existed in this teacher’s classroom.

Observations

Students' responses to the questionnaires revealed strong support for the use of mathematically enabled technologies in the mathematics classroom. Further, there was a strong belief that the use of these technologies provided valuable support for the learning of mathematics and the investigation and exploration of mathematical situations set in unfamiliar contexts.

The longitudinal nature of this pilot study provided evidence that tangible benefits to the learning and doing of mathematics may be accessible and actualised through the careful implementation of technologically supported classroom environments. It demonstrated that students feel positively about the use of technology and documented their belief that digital tools can assist them to learn and understand mathematics, and to solve problems. These findings should be tempered, however, with a caution that enthusiasm for the use of technology to learn mathematics appears to moderate over time. The pilot study provided a foundation for a more detailed and longer term investigation of the issues raised.

5.1.3. Further Investigation –The Main Study

Classroom Context

The main study was carried out with a new cohort of Mathematics C students; also over a two-year (Years 11 & 12, 1998-1999) period in the same co-educational independent secondary school. The type of technology and its availability was identical to that of the pilot study. Course structure and content were also very similar to that of the pilot study; the only variations being related to timing due to changes in the school calendar year.

Data Sources

At the beginning of Year 11 and at the end of Year 11 and 12, students completed a *Technology Questionnaire*, as described earlier, on their attitudes towards technology, its role in learning mathematics, and its perceived impact on the life of the classroom. The introduction of a third questionnaire was in response to the analysis of data from the pilot study that suggested an extension to the process of tracking students' views was necessary.

Unlike the pilot study, it was not possible to arrange a final whole class interview with this group of students in order to record their reaction to the results of these questionnaires because of school organisational and assessment requirements towards the end of the year. Thus, the analysis that follows is based on the structured Likert item component of the *Technology Questionnaires* alone. Of the 17 students who began the course in Year 11 in 1998, 15 continued until the completion of Year 12. Of these 15, 11 completed the questionnaire instrument on all three occasions. It is from these 11 students that the Likert data, analysed below, is based.

Results

Responses were coded using the same procedure as the pilot study and are recorded in Table 5.2 below. Scores at the end of each item represent totals for that item recorded in February 1998, November 1998 and November 1999 respectively. Since, in this case, 11 students completed all three questionnaires the total score on any item can vary between 11 and 55. Thus totals of 44 (75th percentile) and above are taken as indicators of solid support or conversely for 22 (25th percentile) and below (both bolded). Moderate support is indicated by scores between 33 (50th percentile) and 43. A shift of 5 or more is taken as a noteworthy shift (shaded) for the same reasons explained under Section 5.1.1 of the pilot study.

Table 5.2. ***Technology Questionnaire*** **outcomes Sections 1, 3 and 5: Main Study**

Section 1		
1	I enjoy using technology during mathematics classes	(43,**46**,**46**)
2	I will work with technology for long periods of time if I think it will help me solve a problem	(43,**44**,**47**)
3	I feel confident I can use technology when faced with a new problem in maths class	(43,42,43)
4	If I make a mistake when using technology I am usually able to work it out for myself	(41,39,37)
5	Using technology makes me feel more confident about learning mathematics because I can check answers and ideas as I go	(41,**46**,43)
Section 3		
1*	I prefer to just learn the mathematics and find the need to learn technology as well a burden	(42,43,42)
2*	Good students don’t need the assistance of technology to understand mathematics	(41.5,40,40)
3*	Technology is only there to check what you do with pen and paper	(37,**47**,**47**)
4	Technology allows me to explore my own ideas about mathematics as well as those discussed in class	(42,38,39)
5	I am sometimes forced to use new mathematics when exploring the use of technological tools	(38,39,39)
6	By looking after messy calculations technology makes it easier to learn essential ideas	(40,**49**,**47**)
7*	I prefer to learn the mathematics first, without technology, and then learn the technology to do the mathematics more quickly	(27,26,31)
8*	I tend to use technology to do calculating basic tasks but not much else	(39,40,41)
9	I find technology particularly useful when exploring unfamiliar problems	(43,40,**44**)

10	Technology allows me to learn mathematics more easily because I can work through a greater number of examples more quickly	(39,37,**46**)
11	Technology helps me to link knowledge, eg the shapes of graphs and their equations	(**48**,**46**,**47**)
12*	I can often solve problems using technology in the classroom but when thinking about the same mathematics latter I feel I don't really understand it	(40,37.5,33)
Section 5		
1	I use technology at home for:	
a	entertainment eg games, surfing the internet	(38,39,43)
b	writing up school assignments	(43,**44**,**49**)
c	doing mathematics homework	(29,27,31)
d	exploring ideas about mathematics begun in class	(23,21,24)
e	exploring ideas about mathematics of my own	(18,17.5,23)
2	I use technology at school:	
a	when the teacher tells me	(**53**,**49**,**50**)
b	when I get stuck on a problem	(41,41,43)
c	to look at a problem in a different way, eg a picture or a table	(41,38,42)
d	as a way of discussing a problem with others	(34,33,42)
e	when I feel pen and paper isn't helping	(37,40,43)
f	as a first resort when looking at a mathematical problem	(30,37,38)

The following observations are noted in relation to this data.

Dispositions When Using Technology

Responses to Section 1 (illustrated in Figure 5.4 below) were consistent with those of the pilot study, indicating support for the use of technology in tandem with studying mathematics as Questions 1, 2, 3 and 5 all registered scores of 41 or higher on each iteration of the questionnaire - 5 scores out of 15 with high ratings equal to or greater than 44 (75th percentile) and 8 scores between 41 and 43 indicating moderate to high ratings. The total scores on items from Section 1 ranged from 41 to 43, 39 to 46 and 37 to 47 on each version of the questionnaire respectively. One noteworthy positive shift was recorded between the first two surveys in question 5 indicating students recognized the value of technology as a means of checking results when working through a mathematical problem. These results are consistent with the results returned in the pilot study and demonstrated a high level of acceptance for the use of technology within this mathematics classroom. All ratings maintained high values over the course of the study, except for item 4 where scores decrease slightly while remaining in the moderate range. This is consistent with a general wariness that any student might be expected to express when considering the challenge of finding and correcting errors.

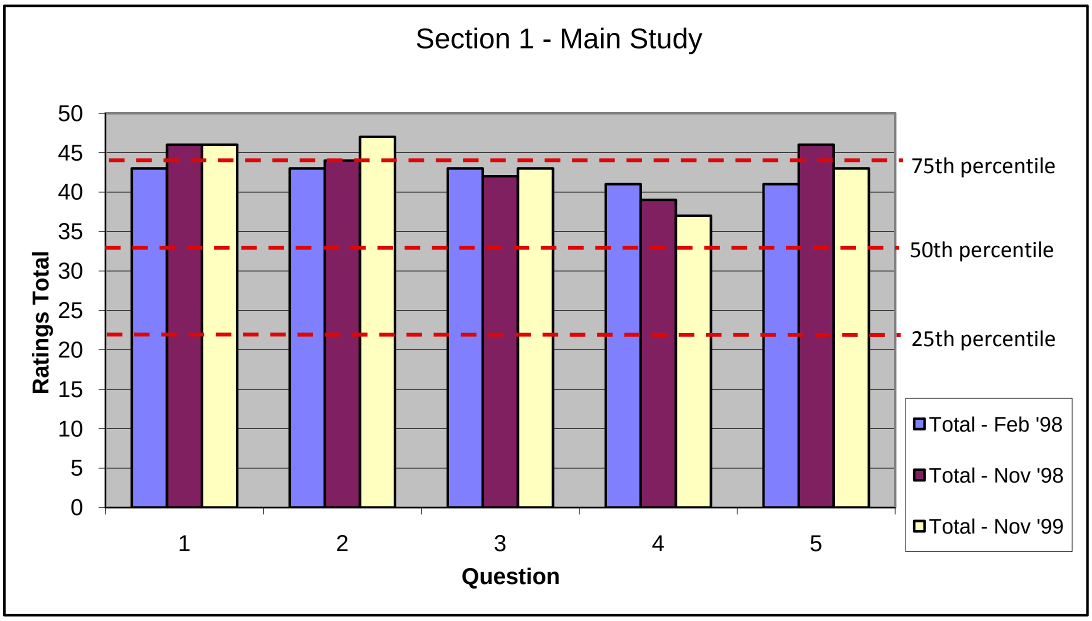

Figure 5.4. Students' responses to Section 1 of *Technology Questionnaires* 1, 2 and 3: Main Study

Technology as Mathematical Assistant

Section 3 canvassed specific preferences of students in relation to the use of technology for different mathematical purposes (illustrated in figure 5.5 below). The maintenance of high ratings across items 3, 6, 9, 10, 11 (5 out of 12 items) or moderate to high ratings in items 1, 2, 4, 5, 8 (5 out of 12 items) by the end of Year 12, indicates that the class viewed technology as a beneficial adjunct while learning mathematics. In particular, technology was valued for its capacity to assist students in exploratory approaches to problem solving and to link different types of mathematics; which represents a more sophisticated and powerful use of technology than merely checking results.

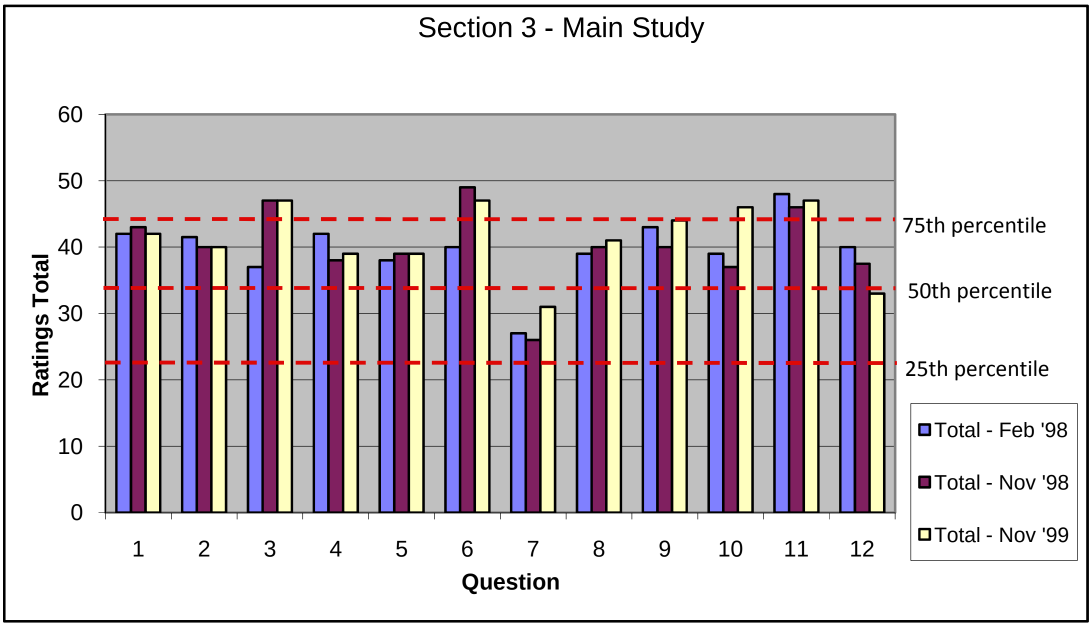

Figure 5.5. Students' responses to Section 3 of *Technology Questionnaires* 1, 2 and 3: Main Study

Further, noteworthy positive shifts occurred for items 3, 6 and 10 (positive shifts of 10, 7 and 7 respectively across the entire study); strong indication that students developed an appreciation of the advantages offered by technology in terms of efficiency when handling large calculations and developing competency with new ideas and concepts.

All items received a high or moderate to high rating with the exceptions of Question 7 and Question 12, with the latter also subject to a negative shift of 7. These ratings are consistent with the pilot study data. The decrease in support for Question 12 occurred steadily over the two years (ratings of 40, 37.5, 33) and so does not appear to be related to a "one-off" negative experience. The opinion expressed here strikes at the heart of what it means to know and do mathematics and is directly related to the issues of deep, genuine understanding as well as the retention of skills. These are issues for mathematics classrooms in general and so it would be unwise to infer too strongly that this is specifically related to the use of technology alone. None-the-less, the students' views expressed on both surveys indicated that the issue of the potency of their learning, and especially the retention of mathematical understanding and skill, proves of greater concern as they progress through the course. This increased concern is perhaps triggered when performance on assessment takes on greater importance as students move into the summative assessment regime that exists in Year 12.

Question 7 addresses students' preferred approaches to the introduction of technology into mathematics learning. While there is some movement in favour of the integration of technology throughout the learning process by the end of Year 12 (a positive shift of 4) there remain clear and distinct preferences that are disguised by the raw totals. In fact very few ratings of 3 were given. On the first occasion 9 out of 11 students recorded a clear positive or negative preference (seven students providing a rating of 1 or 2 and two students a rating of 4 or 5) followed, on the second questionnaire, by 7 out of 11 (six students with 1 or 2 and 1 student with 4), and finally 8 out of 11 (five students with 1 or 2 and three students with 4 or 5). The totals indicate that the majority retained a preference for the introduction of mathematical theory first, followed by instruction in the use of technology. It should be noted, however, that by the end of Year 12, neutral to strong positive ratings were in the majority (6 out of 11). This indicates that students maintained a strong preference for studying mathematical theory first and then technology throughout Year 11 but that this preference was moderated by the end of Year 12. Here a more even distribution of preferences is evident, indicating that while the approach taken by the teacher had been more generally accepted over time, students' initial preferences, which are most likely rooted in their previous experience of learning mathematics, are strong and persistent.

Home and School

The results for Question 1 (multiple parts) of Section 5 (see Figure 5.6) are consistent with those of the pilot survey in indicating that little use of technology is made at home except for writing up school assignments and playing games. Interestingly, this contrasted with the results of Question 2 in this section, which targeted school use, where high or moderate to high ratings are recorded in all 6 items by the end of Year 12. Striking positive shifts in preferences were evident in items d, e and f of this question indicating a developing preference for working collaboratively with peers while using technology and a growing appreciation for the potential of technology to enhance the investigation and exploration of mathematical problems.

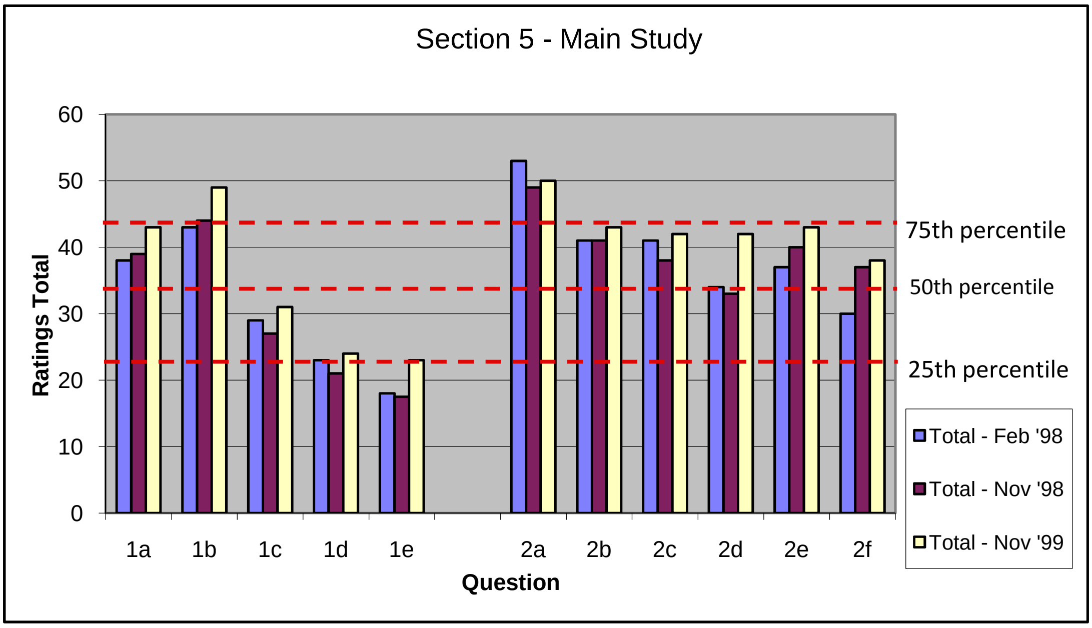

Figure 5.6. Students' responses to Section 5 of *Technology Questionnaires* 1, 2 and 3: Main Study

5.1.4. Significant Observations

The results of the three questionnaires, in general, indicate positive support from students for the integration of technology into the process of learning/teaching mathematics. This is evident in both pilot and main studies. Students acknowledge advantages lie with using technology in terms of efficiency when attempting to solve problems that carried a high computational load and when working through a sufficient number of examples in order to acquire competence in the use of a mathematical skill. Further, students saw the technology as a powerful tool in the investigation and exploration of problems including those set in unfamiliar contexts. This positive position needs to be tempered, however, with a strong expression of concern, by some students, about the quality of their learning, particularly in relation to their retention of mathematical procedures, ideas and concepts.

Students made little use of technology at home unless playing games or working on school assignments yet positively embraced its use over a range of different aspects of learning mathematics while at school. This was most relevant to strategic aspects of mathematical exploration, such as looking at a problem in a different way, although it was not rated highly as a first resort when stuck on a problem.

A larger number of items in the main study received noteworthy positive shifts than was evident in the pilot study (10 vs 0). This is most likely due to the recording of students' opinions upon entry into the course as well as at the end, as was the case in the main study, versus the limited number of months over which the pilot study data was sampled. Positive changes in student views on learning/teaching approaches that incorporate the use of technology appear to take time to develop. Further, this observation might also be taken as evidence of the persistent nature of attitudes and opinions, developed from earlier experience but then carried by students into a new learning/teaching environment. This was no more evident than in the persistent line taken by students about their preferred order for the introduction of new mathematical ideas versus associated technology skills. Additionally, this highlights the great variability that can exist, in terms of learning styles and preferences, in any one classroom.

5.2. Choice of Technology

This section examines students' declared preferences for the type of technology they use when engaged in mathematical activity (research questions 3 and 4). Details of the types of technology available to students are presented earlier in the Methodology section of this thesis (Chapter 4). Students' preferences are examined at two levels. Firstly, their preferences for graphing calculators versus computers is examined through the analysis of responses to a question, included on the three *Technology Questionnaires* administered during the main study, that targets this issue. Secondly, a stimulated recall interview, conducted early in Year 12, is used to provide insight into the model of calculator preferred by students.

5.2.1. Graphing Calculators Versus Computers

Data Sources

The data examined here are based on students' responses to the open-ended questions included in Section 6 of each of the *Technology Questionnaires*. In order to determine students' preferred technology in relation to graphing calculators and computers the following question was included as Question 2 on the first *Technology Questionnaire* (TQ1) and the second *Technology Questionnaire* (TQ2).

6. Do you have a preference for using computer or graphing calculator technology? Write down how strongly you feel about this and why, i.e., you might discuss what you see as the advantages and disadvantages of both devices as a way of justifying which one you prefer.

While the question supplied useful data, students' replies tended to be of a general nature and lacked the detail necessary to form specific conclusions about the reasons for their declared choices. In an attempt to capture this missing detail the question was modified to the form below and included as Question 5 in Section 6 of the third *Technology Questionnaire* (TQ3).

5. Which do you prefer to use—computer or graphing calculator technology? Write down the advantages and disadvantages of both, how strongly you feel about this and why.
    - Advantages of a computer:
    - Disadvantages of a computer:
    - Advantages of a graphing calculator:
    - Disadvantages of a graphing calculator:
    - State how strong your preference is for one or the other:
    - What are the major reasons for your choice?

The responses to these questions are now analysed below.

Results

The distribution of students' preferred choice of graphing calculators or computers are recorded in table 5.3.

Table 5.3. Students' preferences graphing calculator versus computer: Main Study

	Computers	Graphing Calculators	Both	Indeterminate
TQ1	4	4	6	1
TQ2	1	8	2	1
TQ3	4	6	4	0

Both TQ1 and TQ3 exhibit a close to even spread of preferences for computers, calculators or the use of both technologies. While the results for TQ2 appear to show a strong preference for graphing calculators the detail of these responses indicated caution should be applied to any conclusion drawn here. It should be noted there was variation in the number of students who returned each survey. Thus students who recorded a preference in one survey may not have been present for a follow up questionnaire. None-the-less it does appear that students who completed the second survey showed a strong preference for the use of graphing calculators. However, this trend does not continue through to the final survey where students again record a relatively even spread of preferences. A detailed review of students' preferences across time includes only the 11 students who were present for at least TQ1 and TQ3. Of these 11, a steadfast core of students (55%) did not change preferences over the course of the study. Of those remaining, changes of preference are evenly spread across the possibilities as follows: Both to Computers (1); Computers to Both (1); Both to Graphing Calculators (2); and Computers to Graphing Calculators (1). These results suggest that although approximately half of the students were able to decide which technology was more suitable to their needs from early in the study and did not change their position, almost the same number changed their preferred choice of technology.

Students who did not change preference sometimes expressed a very clear and unwavering position as does Russel in the following sequence of responses:

> Graphics calculators are easier to use, because they have dedicated keys, easy to lug around, and have relevant functions for maths. Unfortunately you cannot print graphs. (TQ1)
>
> We can carry and use calculators anywhere. Computers are less than mobile – sure they have more functions – but calculators make more sense. (TQ2)
>
> Calculator unless I NEED excel. I hate using elaborate drawn out methods on a spreadsheet to work out something I can work out on a calc in 2-3 seconds. (TQ3)

Comments from students who changed preferences through the study indicated their choice of technology often depended on the task.

> Susie: I don't have a preference – I use them for different things. The computer for spreadsheets and the graphics calcs for graphs. (TQ1)
>
> Sean: Not really strong, it would depend on particular circumstances. (TQ3)
>
> Sylvester: My preference only depends on the task the technology is needed for. Calculators and computers are good for different tasks. (TQ3)

Even students who expressed a preference for one technology over another often conceded that both technologies offered advantages.

> Gloria**:** Computer is much more useful because you can print your results, however you can use your graphics calculator to do more mathematical things. (TQ1)
>
> Keira: I prefer to use graphing calculator probably because I have greater access to one. I think as computer is better because of the bigger screen & clearer symbols, writing etc, however calcs are more compact, work on battery power & are generally easier to

understand & can do much the same things (apart from typing assignments etc).

(TQ2)

Francis: For maths use they both have there (sic) merits so there (sic) kinda the same. Computer maybe just be ahead.

(TQ3)

Sometimes students simply determined their preferences on the basis of familiarity with a technology.

Francis: I have access to them (computers) everywhere. I've been using computers longer. (TQ3)

Keira: I've had more exposure through school so I'm more familiar with them (calculators).

(TQ3)

Demi: Familiar with them (calculators) and how they work. Very quick. (TQ3)

This familiarity was not static, though, and could vary according to the activities of a classroom at the time. For example, Nicole made the following comments on each of the 3 *Technology Questionnaires*.

Computer – it is more straight forward and easy to understand. (TQ1)

Graphing calc. better due to it being able to be taken around with you and is easier to understand due to not being so complex. The computer is intimidating, in that I feel I am not able to use school computers for fear of breaking them but no fear at home for this. (TQ2)

I've been around computers a lot more, and am $\therefore$ more comfortable with them. (TQ3)

Students who preferred computers commented on the advantage of the larger screen offered by computers and the capacity to perform a greater range of functions.

Johnny: Well I prefer computers because you can get programs that can do it all on computers and screens are better. (TQ1)

Alternatively, students who preferred graphing calculators noted their portability; ease of operation and suitability to mathematically orientated tasks.

Adam: I prefer graphing calculator because it is compact, easy to learn and can do many mathematical functions out of the box without extra software. (TQ2)

5.2.2. Choice of Calculators

Data Sources

Data here are drawn from three 45 minute lessons spaced over a three week period. These lessons were video taped and then transcribed. In the first lesson students worked through a combinatorics problem that was unfamiliar in nature. It was noted that students made use of the available technology in a number of different ways and so a follow-up whole class interview was conducted two weeks later using the parts of the video from the previous lesson for stimulated recall. Approximately one week after this session students underwent assessment for their course work. During this assessment it was again observed, by the classroom teacher, that students were using different calculators and using them in different ways. A second whole class interview was conducted during the next available lesson in relation to the students' use of calculators during assessment. Students' comments from both interview sessions, as indicated, are used in the analysis below.

Technology Context

As was outlined in the Methodology Chapter of this thesis (Chapter 4) students had access to two types of calculator – a TI-92 and a TI-83. A brief outline of the features available on each calculator is repeated in Figure 5.7.

TI-92 Calculators (Texas Instruments)

- Graphical, numeric representations of functions
- Statistical capabilities
- CAS capabilities - including matrix editor and complex number manipulation
- Dynamic geometry module

TI-83 Calculators (Texas Instruments)

- Graphical, numeric representations of functions
- Statistical capabilities
- Matrix editor and complex number manipulation

Figure 5.7. Features of TI-92 and TI-83 calculators

A Problem in Combinatorics

In general, new ideas in mathematics were introduced to students through a practical context. The fundamental ideas of combinatorics, for example, were introduced to students through discussions on the number of possibilities for the choice of a committee of set size from a larger number of nominations. These discussions, however, eventually led to a more formal and abstract treatment of the topic. In the first of the series of lessons that are the focus of this section, students' addressed the problem below:

$$ {}^{n}C_3 = \frac{5}{18}\,{}^{n}C_5 \qquad \textit{Find } n $$

Students were expected to solve this problem through an understanding of the relationship between permutations and combinations as illustrated below.

$$nC_3 = \frac{5}{18} nC_5$$

$$\frac{nP_3}{3!} = \frac{5}{18}\frac{nP_5}{5!}$$

$$\frac{nP_3}{6} = 5\frac{nP_5}{2160}$$

$$\frac{2160}{30} = \frac{nP_3}{nP_5}$$

$$72 = \frac{n(n-1)(n-2)(n-3)(n-4)}{n(n-1)(n-2)}$$

$$72 = (n-3)(n-4)$$

$$n^2 - 7n - 60 = 0$$

$$(n-12)(n+5) = 0$$

$$n = 12 \; or \; n = -5$$

It was observed that the class used both TI-92 and TI-83 calculators to assist them in solving the problem. Some students made use of the permutation and combination facilities on their calculators in an attempt to solve the problem via trial and error. Others used the CAS capabilities of the TI-92 graphing calculator in order to manage some of the algebraic aspects of the problem. While some students made use of one type of calculator or the other a number also made use of both in a complementary fashion.

Student Revelations

Two weeks after this lesson, students were shown video segments of the whole class working on the problem. The teacher asked why the students chose one calculator type over another (TI-83 vs TI-92). One student argued that you should only use technology as powerful as is needed for the task at hand.

> Teacher: But Russel, why would you have picked the 83 rather than the 92?
>
> Russel: Because I don't think there is anything more the 92 could have done that the 83 couldn't have…
>
> Teacher: So Russel, the idea, what you are telling me is that unless you really need the facilities of a 92 you'd prefer to use the 83?
>
> Russel: Yeah.
>
> Teacher: Is there any reason why…that you prefer to use it?
>
> Russel: Well it types faster.

Other students joined in and commented that they preferred to use one calculator over another simply because they used it more often (e.g., in other courses) and were more familiar with its operation.

> Keira: 'cause we use it (TI-83) more often, it's easier to know where all the functions are.
>
> Demi: We're used to it.

Another student commented that the TI-92 calculator offered more direct access to the commands than the TI-83 which was more menu driven.

Teacher: Whoa, whoa…let's slow down…. So you think the nice thing about the 92 …'cause you (talking to Matthew) tend to use it as a first preference, don't you, is that because you know the command you can just type in and do things straight away, whereas on the 83, you have to go through the menus to get the commands?

Matthew: Yes.

This indicates that the student had developed a level of familiarity and expertise with the device and that his focus was now on working with the mathematical task and the calculator was a useful tool to be used as efficiently as possible.

Other students, in the first interview session, indicated that the choice of calculator depended on the task and the particular device's suitability to that task.

Adam: Well it depends on what kind of work you're doing….with permutations and combinations, I find it easier on this (signalling TI-83) but with complex numbers and things (another student adds matrices) yeah, matrices - it's so much easier on the 92.

Some students were even prepared to use two calculators at once to satisfy a desire to use, in their view, the technology that was most suited to a task, or sub-task, as described in these comments recorded during the second interview session.

Keira: I was doing normal calculations, I'd use the little one because I could look at my matrix and do the calculators at the same time, that sort of thing.

Teacher: You're using both at once because…

Keira: ... you could see the matrix really clearly on the big calculator.

Teacher: You use that as just a way of looking at it, but you do all the manipulation on the other one.

Keira: I like using the big one for complicated stuff like "i" and stuff like that, but the little one is just easier to use when you are doing all that little basic stuff.

Students also commented on differences between the way the calculators displayed certain inputs and outputs. They indicated a preference for a visual display that is closest to how mathematics is written with a pad and pen, or when properly type set. Students indicated they found this more useful for checking their inputs and looking for mistakes.

(First interview)

Heath: It (the TI-92) gives you what you typed in. It actually writes out the matrices you are multiplying together instead of giving just "a" or"b".

Teacher: Okay, so you can see it...you like seeing the matrix, not just a symbol for it?

Demi: Visually this (the TI-92) is a lot better.

(Second interview)

Demi: Um, I usually pick the big one (TI-92) when we've had matrix problems, where we had big numbers…you could see the whole thing.

Tom: Because it's easier to see and you can find the mistakes.

Students were also asked to comment on their choice of methods when solving the combinatorics problem that had been the focus of the initial lesson. A number claimed they had simply made use of the calculator to implement whatever method of solution had occurred to them at the time. For some this meant a "trial and error" approach, for others, perhaps more familiar with the "equation solver" capacities to the calculator, this meant the use of a numeric approach to solution. The teacher also inquired of the class, what a student, who was absent on the day of the interview, was doing on the video. Students who had worked with the missing student replied as below:

Teacher: You know what she (Gena) was doing Heath?

Heath: She was expanding.

Teacher: So she (Gena) wasn't solving like Tom, she was expanding using it (the TI-92) like an "Algebra Assistant". She was using it as a way of checking her algebra.

Keira: She used it to *do* her algebra.

This student appears to have used the calculator to compensate for a lack of capacity to perform algebraic operations. Her understanding was at a level of sophistication where she knew an algebraic approach was appropriate but did not have the capacity or lacked the confidence to follow this up without assistance. The CAS facility of the calculator had provided the scaffolding she needed to continue with her chosen approach. Given her need for this support it seems unlikely she could have completed this problem using this method if she did not have the "assistance" offered by the calculator available. Interestingly, Gena's usual preference was the TI-83 calculator, but on this occasion she chose to use the TI-92 because of a particular facility she required to assist her progress to a solution.

Significant Observations

A number of observations emerge from both the questionnaire data and from student interviews. Familiarity with a technology and confidence in using technology appear to be important determinants in a student's choice of digital tool – in the case of a choice between computer and calculator technology and between two different graphing calculators. It should be noted, however, that this is not the only influence.

It was clear that students had the capacity to make a decision on the choice of technology based on a device's suitability to a task. So, for example, a student might decide to use a TI-83 for arithmetic work but change to a TI-92 in order to work with matrices, or use a computer if spreadsheeting was required. This awareness of choosing the right technological tool to fit the mathematical job was commented upon directly by students when responding to the *Technology Questionnaires* and in interviews but was also evident in the way students changed their preferred technology over the period of the three questionnaires. A specific example of this choice was observed when at least one student made use of the CAS capabilities on one the calculators to support her use of an algebraic approach to solving the combinatorics problem. She chose a calculator with a CAS capability even though it was not her preferred technology.

Students also reflected on the differences between visual displays or screen size when making a choice between computers and different types of calculators. A number of students commented on the importance of seeing the whole of their input or their outputs while working with a calculator. They felt less comfortable in working with a problem or detecting errors when some part of the input or output was off screen because of its length. Further, students also expressed a preference for displays that represented mathematics in a similar fashion to how they wrote it with a pen on paper or how it appeared in a book. Clearly they found unfamiliar styles of symbolic representation at best distracting and at worst unintelligible. Thus the “visual” aspect of students’ interaction with mathematics and technology should not be underestimated.

Finally it was noted that students did not distinguish between the use of technology in class or during assessment when discussing use and preferences in relation to technology - all discussions were confined to their familiarity with a technology or to the availability of a facility.

5.3. Summary and Conclusions

Students in both the main and pilot studies exhibited positive dispositions to the use of technology in the learning of mathematics. This stands in agreement with other studies which have investigated attitudes about learning in computer (e.g., Lehtinen & Repo, 1996) and graphing calculator (e.g., Penglase & Arnold, 1996) enhanced environments.

The advantages perceived by students to be available through technology included: the power to accommodate both complex and/or tedious calculations; the capacity to provide visual representation as support for learning a new concept or for solving a problem; the enhanced capacity to take exploratory approaches to solving problems because of the computational power and the wider range of strategies available; and the potential to act as the focus for discussion of a problem with peers.

A drop in support for technology was evident in questionnaire data during the pilot study. In a follow up interview, students were at pains to point out that this was merely a consequence of their experience and familiarity with the available technologies and that they required less assistance as they progressed. Students made it clear they believed technology to be of benefit whenever they were faced with a challenging problem or new learning experience. It was also evident students had begun to understand the limitations of technology and made choices about its appropriate use.

This moderation of enthusiasm for the use of technology was not evident in the main study where support generally remained high throughout the duration of the 2 year program. Some caution was recorded, however, as students reported a degree of unease in relation to the longevity of the learning that took place in technology rich contexts. This concern is mirrored by Templer, Klug, and Gould (1998) who, although convinced that technology should be used to encourage students to explore and investigate mathematical concepts, expressed concern about the quality of learning that takes place when students work with technology in a completely self-directed environment. This serves to highlight the importance of teacher-student interaction in promoting students' deep understanding of mathematical ideas.

Little support was found in either study for the use of mathematically enabled technologies at home, other than to complete homework tasks.

Questionnaire and interview data indicate that while students developed preferences for the use of one technology over another, as a default, some also made strategic choices about the technological tools that were appropriate for a particular mathematical task. Preferences were based on a student's familiarity with a particular technology as well as attributes such as screen size versus portability and power compared to suitability to a specific task. Most students, however, were able to identify the advantages and disadvantages of different types of technology with a number of students commenting that they chose a technology on the basis of the task at hand. The flexibility of students in relation to their choice of available technologies is an issue worthy of further investigation as those who are reluctant to make use of technologies with which they are less comfortable restrict the repertoire of tools they have available when exploring a mathematical idea, concept or problem.

The above analysis provides a backdrop of students' orientations, dispositions, preferences and opinions in relation to the integration of technology into the learning/teaching experiences of a mathematics classroom. These preferences, opinions and views are in many ways about the relationship between students' learning, both individually and collectively, and technology. The answers to the research questions that drive this study lie somewhere in and around these relationships. The next chapter will begin to identify and describe these relationships by building on the observations recorded in this chapter through insights provided by students in the open ended response sections of the questionnaires.

Chapter 6

Technology as a Personal Assistant – Master, Servant, Partner and Extension-of-self

In the previous chapter students' general orientations, dispositions and preferences in relation to technology were established through the analysis of questionnaire and interview data. This chapter is one of three that considers the specific role technology plays in students' learning by examining the different relationships that exist between individual students, their peers and technology. The role of technology in mediating the learning of individual students is considered first, in this chapter, followed by the role it plays in small group and then whole class settings in the succeeding chapters of this thesis. Evidence, provided by students' written responses to open-ended questions included in the three *Technology Questionnaires*, administered during the main study, is used to develop categories of use to address the following research question:

5. What roles can be identified for technology in mediating individual student learning?

6.1. A Framework for Analysing Students' Use of Technology

While a substantial number of studies have investigated how electronic technologies are used by students or what impact these technologies have on instruction, fewer have sought to develop typographies of student behaviour in relation to the use of technology to learn mathematics. An example of such a typography is proposed by Doerr and Zangor (2000). In a case study of pre-calculus classrooms they identify five modes of graphing calculator use: computational tool; transformational tool; data collection and analysis tool; visualisation tool; and checking tool. Alternatively, Guin and Trouche (1999) developed *profiles of behaviour* in relation to students' use of graphing calculator technologies. The modalities outlined in these profiles are characterised by random, mechanical, rational, resourceful, or theoretical behaviours in terms of students' abilities to interpret and coordinate calculator results.

Both of these typographies provide useful descriptions of students' use of calculators when working on mathematical tasks, however, neither of these frameworks appears to acknowledge the social milieu in which mathematics learning and practice is situated. This thesis develops a framework based on the mathematical behaviours, practices and reflections of students working within a classroom that has been modelled on mathematical community of inquiry principles. The resulting framework, therefore, is underpinned by assumptions strongly tied to a socio-cultural view of learning and so attempts to accommodate those practices that are related to student interaction as well as the acquisition of personal mathematical knowledge. Analysis of students' responses to three questionnaires provides evidence for the development of the first part of this framework which describes the interactive relationship between individual students and technology.

6.2. An Evolving Theory

Consistent with the methodological approach outlined in Chapter 4, data collection and analysis were conducted simultaneously with theory building. In this case questionnaires, field notes, video and audio recordings provided data that supported the processes of category creation, category confirmation, category refinement and theory development.

6.2.1. Doing the Groundwork

The open response questions, under consideration here, were distributed with the Likert items described in the previous chapter as part of the *Technology Questionnaires* students completed at: the beginning of Year 11 (TQ1); the end of Year 11 (TQ2); and the end of Year 12 (TQ 3).

The following question appeared in Section 6 of *TQ1* by way of eliciting reflective comments from students at an early point in the study:

1. Write down in your own words what you think about using technology to learn mathematics.

Due to the emergent nature of the research design, this question was further developed, as the study progressed, to allow students the opportunity of providing responses that offered greater depth and insight into their reactions to learning mathematics in a technology rich environment. After viewing responses from the first survey, and in response to observations made in the classroom, Question 1 from TQ1 Section 6 was further expanded, and included on TQ2, in order to probe further what students saw as the advantages/disadvantages of learning mathematics with the assistance of technology (Questions 1 and 2).

1. Are there any advantages in using technology instead of pencil and paper? If so, explain how technology helps you learn better. (Give specific examples.)

   Are there any disadvantages in using technology instead of pencil and paper? If so, explain how technology gets in the way of your learning. (Give specific examples.)

2. If you had not been able to use technology to learn maths this year, what difference would this have made to your understanding? (Give specific examples.)

A final adjustment to Question 2, above, was made for TQ3 to elicit responses from students in relation to individual understanding versus collaborative classroom practices.

2. If you had *not* been able to use technology to learn maths this year, what difference (do you feel) would this have made to:

   your understanding? (Give specific examples.)

   the way you work with others in class?

The first part of this question is relevant to the analysis below.

Students' responses to all questions described above were collected and analysed in conjunction with the growing body of observational and interview data.

6.2.2. Finding the Words

Based on responses to questionnaire data, associated student interviews and observational data, and through the process of category generation described earlier, four general metaphors for the way in which technology can mediate learning were developed (see Galbraith, Renshaw, Goos & Geiger, 1999 for further detail). These metaphors, *Technology as Master*, *Technology as Servant*, *Technology as Partner*, and *Technology as Extension-*

of-self, describe the varying degrees of sophistication with which students work with technology. While these metaphors are hierarchical in the sense that they represent increasing levels of complexity, they do not comprise a developmental framework that describes levels of attainment through which students would be expected to progress. Rather, the demonstration of more sophisticated usage indicates the expansion of a technological repertoire where an individual has a wider range of modes of operation available to engage with a specific task. This means, for example, that a very capable individual may well use technology as a servant if the task at hand is mundane and there is no reason to invoke higher levels of operation.

A description of these metaphors is outlined below:

Technology as Master – The student is subservient to the technology; a relationship induced by technological or mathematical dependence. If the complexity of usage is high, student activity will be confined to those limited operations over which they have competence. If mathematical understanding is absent, the student is reduced to blind consumption of whatever output is generated, irrespective of its accuracy or worth.

Technology as Servant – Here technology is used as a reliable timesaving replacement for mental, or pen and paper computations. The tasks of the mathematics classroom remain essentially the same but now they are facilitated by a fast mechanical aid. The user ‘instructs’ the technology as an obedient but ‘dumb’ assistant in which s/he has confidence.

Technology as Partner – In this case rapport has developed between the user and the technology, which is used creatively to increase the power that students have over their learning. Students often appear to interact directly with the technology (e.g., a graphing calculator), treating it almost as a human partner that responds to their commands, for example, with error messages that demand investigation. The calculator acts as a surrogate partner as students verbalise their thinking in the process of locating and correcting such errors. Calculator or computer output also provides a stimulus for peer discussion as students cluster together to compare their screens, often holding up graphing calculators side by side or passing them back and forth to neighbours to emphasise a point or compare their working.

Technology as Extension-of-self – This is the highest level of functioning, where users incorporate technological expertise as an integral part of their mathematical repertoire. The partnership between student and technology merges to a single identity, so that rather than existing as a third party technology is used to support mathematical argumentation as naturally as intellectual resources. Students working together may initiate and incorporate a variety of technological resources in the pursuit of the solution to a mathematical problem.

Survey and additional observational data were then used to further confirm, refine and extend the initial categories. A description of this process follows.

6.3. Confirmation and Extension

A final Technology Questionnaire was administered at the end of Year 12 (TQ3). The purpose of this final questionnaire was firstly to confirm the viability of the theorised metaphors, and secondly, to develop a finer grained framework of technology use. Consequently, questionnaire items were adapted in order to focus more closely on emerging aspects of developing theory. The final version of the relevant questions appears below.

1. Are there any *advantages* in using technology instead of pencil and paper? If so, explain *how* technology helps you learn better. *(Give specific examples.)*

   Are there any *disadvantages* in using technology instead of pencil and paper? If so, explain *how* technology gets in the way of your learning. *(Give specific examples.)*

2. If you had *not* been able to use technology to learn maths this year, what difference would this have made to:

   your understanding? (Give specific examples.)

   the way you work with others in class?

3. Are there any ways in which you believe technology helps you to *think* differently, for example, the approaches you might use when solving unfamiliar problems or an investigation? *(Give specific examples.)*

4. Are there tasks (in mathematics) you would never use technology for?

   What kinds of tasks?

   Why?

8. The use of technology by students to learn mathematics has been described in the following ways

   i. Technology as master

      Here the student is subservient to the technology – this happens when the student does not feel confident with the use of technology and so can only use a limited number of functions, or when their knowledge of the mathematics being studied is *limited so they are prepared to simply accept whatever is on the computer/calculator screen.*

ii. *Technology as servant*

In this role technology is basically used as a reliable timesaving replacement for mental, or pen and paper computations. The tasks of the mathematics classroom remain the same -- but now they are facilitated by a fast mechanical aid. Unlike the previous category the user is in control, and 'instructs' the technology as an obedient but 'dumb' assistant.

iii. *Technology as partner*

Here a 'rapport' has developed between the user and the technological device -- which may be addressed in human terms. A graphing calculator, for example, becomes a friend to go exploring with, rather than merely a producer of results. Explorations, for example in graphical work, lead to situations where the output needs to be checked against the known mathematical properties of related graphical forms. It is possible for the calculator to be 'wrong', as the student knows enough mathematics to be able to have a sense for what the answer should be.

iv. *Technology as an extension of self*

This is the highest level of functioning, and involves users incorporating technological expertise as an integral part of their mathematical toolkit, so that the partnership between student and technology merges to a single identity. Here powerful use of calculators and computers forms an extension of the user's mathematical powers. Rather than existing as an external tool, a calculator may be used to share and support mathematical arguments to support a conjecture, as when students share and compare computer output as part of their *own* contribution to a solution process.

Which of the above best describes the way you use technology in the classroom? Explain why. (Give specific examples.)

Categories were confirmed by matching responses from the third and final *Technology Questionnaire* (TQ3) to the existing broad categories and by providing participants with the opportunity to comment on the appropriateness of the categories, identified by the four metaphors described previously in this chapter, and to self identify their respective behaviours in relation to technology and these metaphors. A finer grained description of the categories defined by these metaphors was developed by firstly matching student responses to the descriptions of the role of technology as *Master*, *Servant*, *Partner* and *Extension-of-self* and then applying the same process of category creation used to develop the original metaphors.

6.3.1. Data Collection and Results

Sixteen students completed the questionnaire (TQ1) at the beginning of Year 11. Of these twelve also provided corresponding data at the end of Year 11 (TQ2) and fifteen at the end of Year 12 (TQ3). Students enter Year 11 with a variety of backgrounds, and during this year the culture of the classroom is established (see Goos, Galbraith, Renshaw & Geiger, 2003); this included experience with the various technologies and a variety of teaching and learning approaches. During this time the expectations of the teacher were also made clear. Thus, by the end of Year 11, students were able to respond in a more informed way to the questionnaire items than would be expected at the beginning of that year. Through Year 12 students gain further experience with learning mathematics in a technology rich context as well as developing further confidence in their participatory role within a classroom conducted according to the principles of a community of inquiry. During this year students also establish personal preferences for how they make use of technology to learn and do mathematics as reported in the previous chapter (Chapter 5).

Responses were initially categorised into one of the four overarching clusters by matching a response to the descriptor of a relevant metaphor. The partitioned responses were then re-examined, leading to the emergence of the sub-categories described below. In addition, a sub-set of the student responses was provided to the Consul for independent checking of the categorization of students' responses. It should be noted that some student responses provided data such that more than one frequency was reported from a single question. Thus, it is possible for the sum of frequencies for a category to exceed the total number of students.

6.3.2. Response Categories

The categories of responses and their associated sub-categories appear below along with illustrative examples of student responses.

Technology as Master

*S*tudents' responses indicated that their relationship with technology was one of subservience in some way for the following reasons. Firstly, a lack of competence with using a technology could restrict a student's capacity to make progress with a task that required a specific facility. A lack of competence with the matrix module of a graphing calculator, for example, would restrict a student's progress on a problem that required the manipulation of large matrices as this would prove very difficult using pen and paper methods alone. Secondly, students' comments indicate that there is a danger of developing a dependence on technology that supplanted the need to understand underlying mathematical processes. This is consistent with findings from the *Technology Questionnaire* data reported in Chapter 5 and reflects the concern that the use of technology can be simply a "black box" approach (e.g., Buchberger, 1989) to the study of mathematics. Thirdly, the input and output conventions (syntax) used by different technologies is identified as a negative influence on students' confident use of calculators and computers. Each of these sub-categories appears below together with an example of a representative student comment.

Lack of technology skill

> Technology can also cause confusion if you are not competent enough with the machine to understand why it may make mistakes.

Mathematical dependence

> Some times you can rely on it too much. And then not understand the full process.

Unfamiliar conventions

> Technology can often confuse the issue because it uses different conventions and symbols than normal.

Technology as Servant

In this category students identified a range of ways in which technology could be used as a fast reliable replacement for mental computation or pen and paper algorithms. It should be noted that these are not approaches to technology that transform the task or significantly change how a student attempts to solve a problem. Rather, technology is used to complete tasks more quickly, more neatly or more efficiently rather than more creatively. The only possible exception to this description is the sub-category *Reduces errors in calculation and checking answers* as students were often observed working interactively with the calculator over a series of checks; adjusting their initial solutions on the basis of the output they received from the technology. While students are essentially using technology as a *Servant* in this case there is a sense of partnership in the way they progress toward a solution. Operating with technology in this way may well be an indicator that the student is in transition towards using technology at a more sophisticated level. Sub-categories with representative examples appear below.

Accommodating large calculation and tedious repetitive methods

> It gives you something to blame when things go wrong. It does all the small calculations you can't be bothered to do.

Performs calculation more quickly and efficiently

> I much prefer technology because of its efficiency. The work can be done much quicker.

Reduces errors in calculation and checking answers

> Less chance of error in calculations.
>
> When graphs or functions are needed or to check answers of $\int$ or derivative.

Presentation

> Displays everything in a neater and more succinct manner. You can illustrate eqns graphs etc.

Technology as Partner

Responses in this category indicate that students believed there were two different ways that technology assisted them in approaching mathematical tasks. These sub-categories describe the capacity technology provides to take an exploratory approach to looking at a problem, and so gain a different perspective, or to facilitate understanding by providing scaffolding such as the reduction of cognitive demand or the provision of a visual representation of a mathematical task. The first sub-category represents the level of operation described by Templer et al. (1998) who advocate that the genuine promise of working with technology lies in the potential for students to explore and investigate new mathematical ideas and concepts. The second sub-category can be illustrated by way of an example of scaffolding that was observed when students were challenged by a problem in which algebraic facility was required but was not the focus of the task. Students who were not strong users of algebra, for example, were sometimes able to achieve success through the use of the symbolic manipulation system available on their calculators for the part of the task that required such facility.

An interesting counterpoint to the generally positive responses to the deployment of technology in this category was the comment that computers inhibit visualisation. This is perhaps a reference to the previously mentioned concern that acquisition of skills can be retarded if technology is used exclusively to perform a task, that is, if technology is used to replace a skill instead of being used to amplify a skill already developed by an individual.

Exploration and different perspectives

> With the learning of integration and differentiation, the seeing of the examples graphically helps understand the whole concept, and thus makes you think on a wide scale (graphically and manually) when doing a problem.

Facilitating understanding (e.g., looking after cognitive demand, scaffolding, via visualisation)

> Yes – it quite often helps to simplify steps in a complex problem.

> Can do problems that I usually cannot do myself because of lack of basic skills.
>
> The study of chaos theory would have been virtually impossible as the graphs enable us to visualise the functions more clearly.
>
> The computers inhibit visualisation.

Technology as Extension-of-self

While the sub-categories within this metaphor received relatively low frequencies of response (2 each), this is consistent with the conceptualisation of this category as the highest level of function within the framework. At this level students have a complete repertoire of technological skills. Such mastery permits students to seamlessly transform a task via technology in order to explore conjectures that are the product of a student's intuition. There is again, however, an interesting counterpoint by way of the third comment under the sub-category of *Mind expander*. The student in this case reminds us that, ultimately, potential uses of technology are only limited by the creativity of the human mind and that this cannot be replaced by technology alone.

Mind expander

> Technology allows you to expand ideas and to do the work your own way.
>
> Think differently? No – act differently yes. To work out unfamiliar problems you must first figure out what type of process you need to solve it, then execute the process. What you use is irrelevant.

Freedom

> You have much more freedom.

The expanded *Master*, *Servant*, *Partner* and *Extension-of-Self* (MSPE) typography is represented below in Figure 6.1.

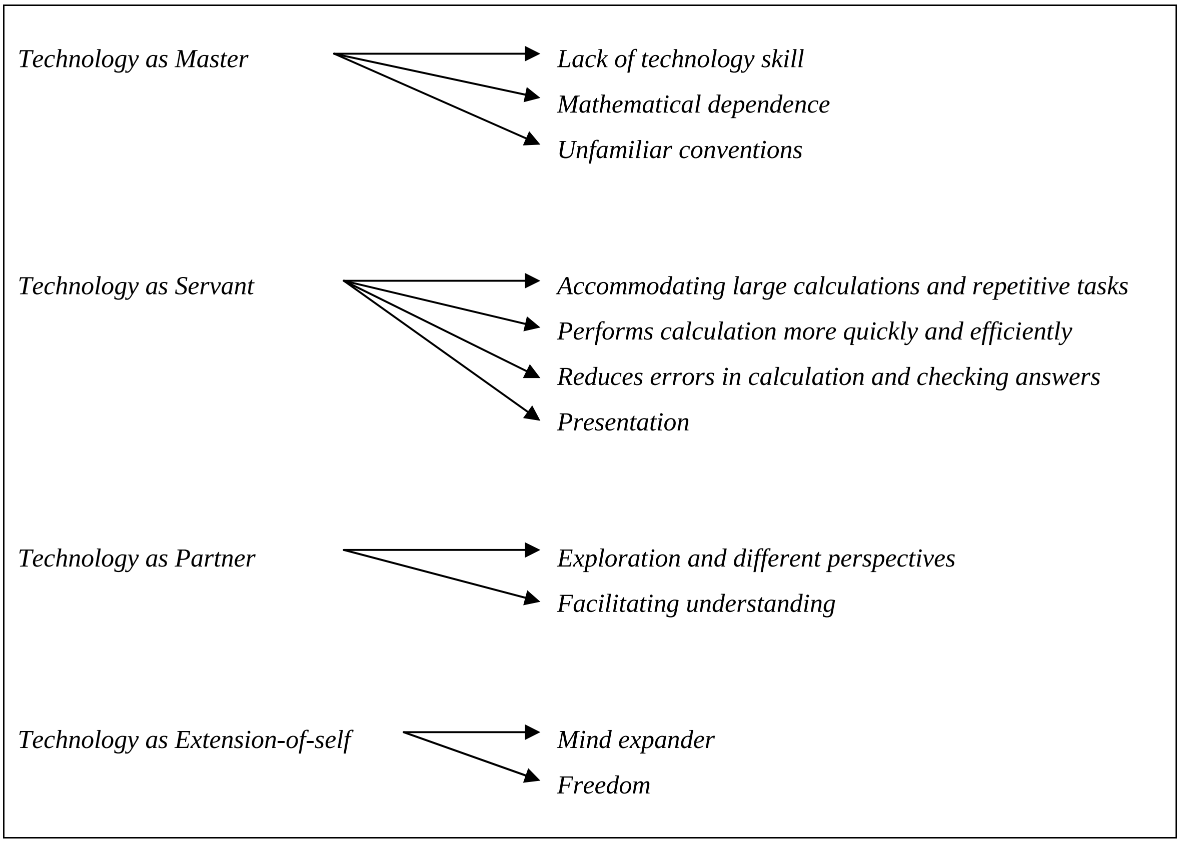

Figure 6.1. Typography of students' individual usage of technology

6.3.3. Distribution of Responses

How students perceived their learning was influenced by technology changed over time. This is analysed through attention to the frequency of categorised responses (Table 6.1).

While it is important not to generalise inferences based on these data because of the small number of participants, there are a number of features that suggest noteworthy changes to how students view their use of technology while learning and doing mathematics.

Table 6.1. Distribution of student responses against MSPE categories

		TQ1	TQ2	TQ3
Technology as Master	*Lack of technology skill*	5	4	3
	Mathematical dependence	4	8	12
	Unfamiliar conventions	0	0	2
Technology as Servant	*Accommodating large calculations and repetitive tasks*	0	5	4
	Performs calculation more quickly and efficiently	4	6	9
	Reduces errors in calculation and checking answers	2	4	4
	Presentation	1	5	2
Technology as Partner	*Exploration and different perspectives*	1	2	10
	Facilitating understanding	0	8	8
Technology as Extension-of-self	*Mind expander*	0	0	2
	Freedom	0	0	2

Although the frequencies within the sub-categories of *Technology as Master* are relatively stable, there seems to be a marked increase in the concern about the potential to become mathematically dependent (*Mathematical dependence*) on technology as this category of response increases by 100% and then 50% for each subsequent survey (TQ1, 4; TQ2, 8; TQ3, 12). This has been commented upon earlier as an indicator that students understand the danger of not understanding the underlying mathematical ideas and concepts they interact with while using technology. It would appear that students become more attuned to this danger as they become more experienced with technology and as a consequence of their general development as mature learners.

Performs calculation more quickly and efficiently, a sub-category from within *Technology as Servant,* also displays a marked increase in frequency of response (TQ1, 4; TQ2, 6; TQ3, 9). This is most likely attributable to students developing skill and confidence with the use of technology through the course of study.

Both sub-categories within *Technology as Partner* record noteworthy changes in their rates of response. *Exploration and different perspectives* (TQ1, 1; TQ2, 2; TQ3, 10) demonstrates a marked increase in frequency, especially between the end of Year 11 and the end of Year 12. This is evidence that making use of technology to explore a problem at a strategic level requires significant experience over time. The responses that fall into the category of *Facilitating understanding* (TQ1, 0; TQ2, 8; TQ3, 8) also support a view that the use of technology to aid understanding takes an extended period of time to develop. This is also evidence that *Technology as Partner* is a higher level of function in relation to using technology to learn and do mathematics. It should be noted, however, that the change only takes place between the beginning and end of Year 11, which indicates that some students were ready to make the transition to this high level of function during the first 12 months, while others were unable to do so even after participating in this class for two years. The failure of the second group of students to progress in this area is worthy of further investigation.

The two aspects of *Technology as Extension-of-self*, *Mind expander* and *Freedom*, record identical frequency profiles (TQ1, 0; TQ2, 0; TQ3, 2). While again, necessary caution must be observed in any inference drawn, these numbers indicate that this is a level of operation with technology that is difficult to attain and that takes time and experience to do so.

A further general caution must be considered in relation to these finding. Consistent with the emergent nature of the research design the *Technology Questionnaire* was continuously updated, increasing the explicitness of the questions from TQ1 through to TQ3. This may have resulted in eliciting more thoughtful and elaborate responses from students across time which in itself may have produced the change in frequencies reported above.

6.4. Students' Perspectives on Their Own Place within the Taxonomy

6.4.1. Students' Self Assessments

Students were asked, in Question 8 of Section 6 of TQ3, how they saw their own adequacy with technology in relation to the major descriptors that define the taxonomy. No student appeared to have difficulty identifying a metaphor which best described the way they made use of technology in the classroom and in doing so displayed a high degree of self-awareness of the learning modality in which they operated when using technology. Examples of typical students' responses are presented below.

Technology as Master

> Sean: (i) this is because I often don't understand how to use every specific function of the technology. Thereby limiting the use of such technology. I often don't know if I've used it correctly and as a consequence I can't be sure if my answer is correct or not.

Technology as Servant

Nicole: (ii) It is not my friend or companion, it simply helps me do Q, which would have taken me some time to do.

Demi: (ii) – it is a lesser being – it is a machine. I am above such things. No, really – I am not always 100% comfortable using new things on a calculator – and I choose not to waste time exploring what they can do

Gena: (ii) Because I do not have enough knowledge of technology to be able to investigate concepts. However I do regularly use it for familiar tasks purely as a time saver and to verify and check my answers.

Technology as Partner

Geoffery: I am 3 because my calculator has become my best friend. His name is Wilbur. Me and Wilbur go on fantastical adventures together through Maths land. I don't know what I'd do without him. I love you Wilbur.

Francis: 3 (iii) I have found it good to understand the process and then work out how to use a program or write a program for it.

Technology as Extension-of-self

Russel: (iv) My calc is practically a part of myself. Its like my 3rd brain. I use it whenever it can help me do anything faster.

Tom: (iv) it allows you to explore and go off in your own direction.

Students clearly see these metaphors as adequate descriptions of their capacities in relation to the use of technology in mathematics classes. Sean, for example, feels that he is, to an extent, subservient to the technology because of a perceived lack of facility with his graphing calculator. Nicole and Demi, however, see technology as a tool designed simply as a labour saving device and do not seem to acknowledge the potential of the technology to explore and investigate mathematical ideas. Gena, on the other hand, recognises this potential, but does not feel confident enough with her technological skills to take advantage of technology's potential in this area.

Geoffery and Francis, also recognise this potential. Geoffery, somewhat tongue in cheek, describes a relationship where he uses technology as a genuine partner in the investigation of mathematical ideas and concepts.

Finally, Russel and Tom describe their relationship with technology as an emancipating experience where the use of technology has become a seamless adjunct to their own thinking processes.

One student, however, indicated that she was in transition between categories of the framework.

Keira declares:

> I thing [sic] I'm between i and ii. I tell the calc what to do sometimes but only stick to what I know usually. I don't know exactly what it allows me to do and if I haven't been taught I won't look for it.

Keira has begun to view the calculator as a tool but feels restricted by her knowledge of the calculator's facilities. Not only does she lack confidence in her own ability to explore mathematics with technology, but she also lacks confidence in exploring the facilities of the calculator itself. While students were observed operating in different modes, depending on the demands of the task, Keira's comment suggests that one of the limitations of moving between modes is familiarity and experience with the range of facilities available through a technology. Thus students' expertise with a technology is a factor in the development of a technological repertoire. This indicates that the transition between the various modalities of operation is neither discrete nor final.

The framework acts as a description of the potential modes of operation students may assume when working with mathematics and technology. Clearly, students were able to identify the level at which they perceive they operate, or in the case of Keira, a level at which she has begun to operate. It is a matter for further investigation if self-knowledge with respect to an individual's current mode of technology use is a factor that can assist in the transition from one level to the next.

6.4.2. Cluster Frequencies

The number of responses that fell into each category appears below. The response of the student who indicated she was in transition has been coded as 0.5 within each of the two categories between which she was transitioning.

Technology as Master (1.5)

Technology as Servant (7.5)

Technology as Partner (2)

Technology as Extension-of-self (4)

These results indicate that, by the end of Year 12, half of the students (50%) make use of technology as a labour saving tool but they do not yet feel they can use technology as a means of enhancing their understanding or to explore new ideas or concepts in mathematics. Still, 40% of students believed they could use technology as a *Partner* or as an *Extension-of-self* to investigate and/or explore the mathematics they encountered in course work. In addition, the proportions reported above confirm earlier observations that the last two levels of operation, *Technology as partner* and *Technology as an extension-of-self*, are more difficult to attain.

6.5. Summary and Discussion

Goos and Cretchley (2004), in a review of the research on the use of computers and other non-calculator technologies in mathematics education, argue:

> If technology is assumed to change the nature of classroom learning environments, then awareness of students' attitudes towards technology becomes a central concern in evaluating the impact of computers on learning.
>
> (Goos & Cretchley, 2004, p. 157)

The metaphors of *Master*, *Servant*, *Partner*, and *Extension-of-self*, and the sub-categories proposed here, are intended to capture some of the diversity of students' attitudes toward, and perceptions of, their personal interactions with digital technology within the context of a mathematics classroom.

Students' responses to the open-ended questions in the survey were readily grouped into clusters defined by the metaphors developed by Galbraith, Renshaw, Goos, and Geiger (1999). Sub-categories emerged from within these clusters forming the Master, Servant, Partner, Extension-of-self framework (MSPE framework) of personal technology use (for further detail, see Geiger, 2005).

The frequency pattern of responses within categories and sub-categories indicated that, by the end of Year 12, students generally felt confident using technology as a labour saving device to complete large calculations or to reduce the tedium of using pen and paper techniques repetitiously to solve problems that were procedural in nature. At the same time, fewer than half of these students indicated confidence in their ability to use technology to explore and investigate mathematical ideas or problems. This minority, however, expressed a strong belief that technology was of great assistance in the way they worked with mathematics, not only for its computational advantages but also because of the capacity of technology to support a range of approaches to solving problems.

The validity of the technology metaphors from the perspective of the student participants was supported by evidence in the form of students' responses to an open end section on the third *Technology Questionnaire* that invited them to comment on which metaphor best described their use of technology when working with mathematics.

The frequency profile of students' responses within categories and sub-categories also indicates that it took an extended period of time for students to develop the capacity to use technology at a level greater than procedural tasks, if at all.

Finally, there were a small number of students who still viewed themselves as operating at the lowest level of function within the MSPE framework, even after two years of working within a classroom environment that was supportive of the use of technology in exploratory and investigative modes. Despite the level of support available to all students in this class, these students still lacked confidence in their use of technology to the extent that they believed that, at times, it restricted their ability to solve problems considered by other students as elementary. These results act as a reminder that there will always be a range of performances within any domain of learning, no matter what assistance is available or how supportive the classroom environment.

The development of these categories and sub-categories support the theorised extension of aspects of socio-cultural theories of learning including the Vygotskyian definition of Zones of Proximal Development. The *Master*, *Servant*, *Partner* and *Extension-of-self* framework, described in this chapter, portrays the role of technology in mathematical activity as a series of relationships between individual students and technology. Students' reflective comments provide evidence that, at least for students operating at the two highest levels of the taxonomy, technology plays an interactive role in the mediation of their learning. This means that artefacts, in this case technological tools, play a more expansive role than the mere amplification of ideas, as the tools, here, appear to be adopted by students as a means of transforming their knowing of a mathematical idea or construct. This introduces the notion of engagement with a non-human partner, into the construct of the ZPD, as a legitimate social interaction.

It is argued that the MSPE framework, developed in this chapter, describes the various roles technology can play when students engage in mathematical activity. These roles have been represented as a series of personal relationships between students and technology as they attempt to develop an understanding of a new mathematical idea or when solving a problem. The next chapter explores further the notion of technological relationships by examining the role of technology in social contexts that include students' interactions with peers as well as technology.

Chapter 7

Technology as a Team Member – Working with Technology in a Small Group

In the previous chapter, categories for the ways in which individual students worked with technology were theorised and refined according to principles described in Chapter 4. This chapter represents another stage in the process of theory development as the prospect of broadening these categories to categories that accommodate students working in small group contexts is considered. This necessitates the analysis of the intricacies of interaction that take place between members of a group and their chosen technology while they investigate a mathematical idea or solve a problem. Evidence that supports the findings of this chapter is based on classroom observation notes, video and audio recording of groups of students working *in situ* and *Technology Questionnaire* data which targeted the questions:

1. What are the dispositions and preference of students towards using technology in learning mathematics?
3. What choices of specific forms of technology use are favoured by students?
6. What roles can be identified for technology in mediating collaborative student learning?

7.1. Students' Views on Working Collaboratively with Technology

Students' responses to Section 2 of the *Technology Questionnaire*, from both the pilot and main studies, are examined here. Section 2 of the *Technology Questionnaire* was designed to elicit responses that indicated students' dispositions toward working with others, or by themselves, while using technology. As described earlier, in Chapter 4, *Technology Questionnaires* were administered during the pilot and main studies according to the following schedule (Figure 7.1).

Year	**Pilot**	**Main**
1997	November (Year 11)	
1998	May (Year 12)	February (Year 11)
1998		November (Year 11)
1999		November (Year 12)

Figure 7.1. Schedule for administration of Technology Questionnaire

The structured Likert items in Section 2 of the *Technology Questionnaire* were identical for both the pilot and main studies except for the inclusion of an additional question, Question 6, on the final questionnaire (TQ3) of the main study. This additional question was included in response to observational data that suggested students made use of technology in collaborative, problem solving ventures, or during episodes of concept development, to appropriate the ideas of others.

The set of questions that make up Section 2 appears in Figure 7.2.

Section 2	
1*	I prefer to work with technology on my own when studying mathematics
2	I prefer to work with others when using technology because I feel I need help if something goes wrong
3	I prefer to work with others when using technology because I like to discuss what I see on the screen
4*	I don't like others to see the work I do with technology in case they criticise what I've done
5	When I use technology to study mathematics I really feel I need to share with others what I find
6	I prefer to work with others when using technology because I often get good ideas from them

Figure 7.2. *Technology Questionnaire* Section 2 (Question 6 added for TQ3 only)
Note: Asterisked items were reverse coded so a high score represents a positive predisposition

An analysis of the responses to these questions follows which reports on the pilot study and then on the main study respectively.

7.1.1. Pilot Study

As in Chapter 5, totals rather than means have been displayed and, as before, a shift that has the net effect of more than half the class changing their score by 1, in the same direction, will be considered noteworthy (see Table 7.1). Here, totals of 48 (75^{th} percentile) and above are considered indicative of solid to strong group support for the relevant construct, and conversely for very low scores, that is, 24 (25^{th} percentile) or below (both bolded). Moderate support is indicated by scores between 36 (50^{th} percentile) and 47. Twelve students completed both questionnaires and so the total score on any item can vary between 12 and 60. As a result a shift of 6 or more in successive totals indicates an important change in students' perspectives in relation to an item (shaded). The elements in the ordered pairs denote successive questionnaire results. Score totals have been included with the questions in Table 7.1. Consistent with the approach taken in Chapter 5, some items were reverse coded in order that high scores reflect more of the property of interest. These items have been indicated by an asterisk.

Table 7.1. *Technology Questionnaire* totals Section 2: Pilot Study

Section 2		
1*	I prefer to work with technology on my own when studying mathematics	(36, 31)
2	I prefer to work with others when using technology because I feel I need help if something goes wrong	(44, 35)
3	I prefer to work with others when using technology because I like to discuss what I see on the screen	(**51**, 45)
4*	I don't like others to see the work I do with technology in case they criticise what I've done	(**54**, **50**)
5	When I use technology to study mathematics I really feel I need to share with others what I find	(41, 39)

As indicated by the totals reported in Table 7.1, and illustrated in Figure 7.3, students generally displayed a moderate stance towards working together with technology in three out of the five items (Q. 1, 2, 5); the other two items indicating strong positive support. There was a downward shift of 6 or more on two of the five items (Q. 2, 3). This indicates that enthusiasm for working together had been somewhat moderated during the period between the two *Technology Questionnaires*.

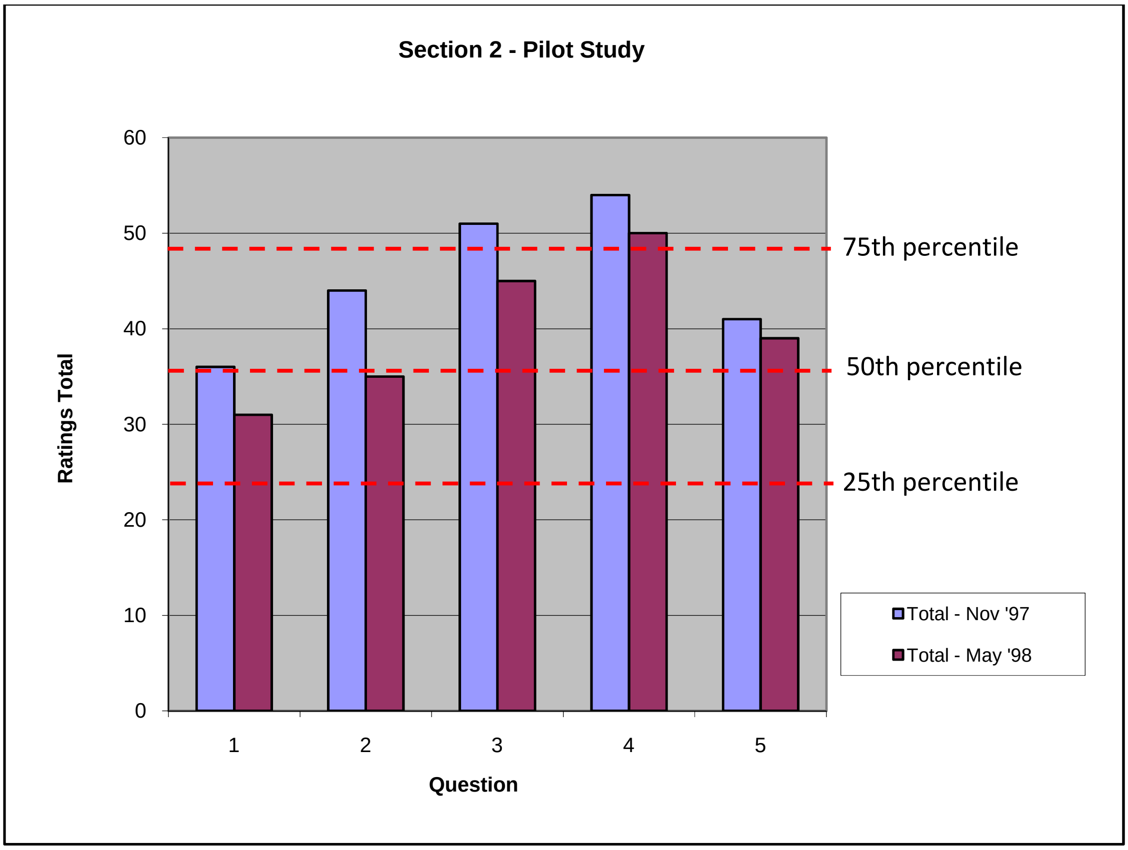

Figure 7.3. Students' responses to Section 2 of *Technology Questionnaires* 1 and 2: Pilot Study

When presented with this analysis, during a follow-up whole class interview, students firmly emphasised they favoured a collaborative working environment. For example:

Colin: I prefer working with others.

The contrast between this position, and the trend in the data, was explained by students in two ways. First, the role of experience was cited as an influence. Students felt they needed less assistance as they gained experience in the use of a technology as indicated by the noteworthy downward shift recorded in Question 2. Since all participants had gained in both experience and confidence, due to further exposure to digital technologies between administrations of the questionnaire, they were less likely to require assistance from others.

Collaborative activity, therefore, was more likely to be associated with the mathematics of a problem rather than any technology related issues.

The second issue is related to screen size, students indicating that it was far more difficult to share ideas, confer and focus a discussion around an image on a graphing calculator screen in comparison to a computer screen.

> Fran: With calculators it's hard to confer, but it's different with computers. I like to work together.
>
> Sean: Calculators are more personal.
>
> Fran: With calculators it's too hard to show each other; the computer screen is easier for more than one person to see. "Technology" is too broad a term (referring to the use of the word in the questionnaire which doesn't differentiate between calculators or computers in this section), it (referring to a student's response) depends on the type of technology.

Despite these comments, students were observed to collaborate around a graphing calculator, often passing the calculator around small groups so individuals could study an output. Sometimes an individual would make a change to a calculation or display, as the calculator was being passed around, as a way of advancing a mathematical argument or challenging a conjecture by another group member. Thus, graphing calculators were regularly used by students as part of a collaborative approach to engaging with mathematical ideas. The comments above, therefore, would appear to be an expression of preference of one technology over another only and not an indication that one digital device was supportive of collaborative practices while another was not.

Students also commented that they did not always feel a great need to share discoveries and so tended to interact with peers primarily when "stuck" on a problem. This emphasises the important role of the task in promoting collaborative interaction and supports earlier findings by Geiger and Goos (1996) who contrasted the higher level of productive interaction observed when students worked on tasks that required exploratory approaches compared to procedural tasks in a study of secondary students working with mathematics in a technologically rich classroom.

Significant Observations

The questionnaires and follow up interview indicated students felt very positively about using technological tools while collaborating with each other. This was more likely to occur while using a computer, rather than a graphing calculator, because the size of the screen made it easier to make public the ideas being explored by members of a group.

Students' comments during the follow-up interview also indicated that the nature of the task was a crucial influence in the mediation of collaborative interaction as they were most likely to feel the need to interact if there was a genuine problem to be solved – given they felt comfortable with the operation of the technology itself. This finding also serves to emphasise the importance of sensitive and sensible interpretation of the questionnaire data as the Technology Questionnaire data alone did not communicate the subtlety of the students' positions in relation to their enthusiasm for using technology in collaborative contexts. Insight into the perceived changes to students' attitudes towards working collaboratively with technology was sought during the whole class interview session in order to gain an interpretation of events consistent with the students' overall positive view of this type of interaction.

7.1.2. Main Study

Students’ responses to the relevant questionnaire items in the main study are presented in Table 7.2 and illustrated in Figure 7.4. Data from only the 11 students who responded to all three questionnaires have been considered and so the total score on any item can vary between 11 and 55. Thus totals of 44 and above are taken as indicators of solid support or conversely for 22 and below (both bolded). Moderate support is indicated by scores between 33 (50th percentile) and 43. A shift of 5 or more is taken as a noteworthy shift (shaded) for the same reasons explained earlier in the section of this chapter devoted to the pilot study.

Table 7.2. *Technology Questionnaire* totals Section 2: Main Study († Q6 did not appear on TQ1 and TQ2 and totals for these questionnaires are recorded as 0)

Section 2		
1*	I prefer to work with technology on my own when studying mathematics	(28, 32, 37)
2	I prefer to work with others when using technology because I feel I need help if something goes wrong	(35, 36, 40)
3	I prefer to work with others when using technology because I like to discuss what I see on the screen	(38, 36,**44**)
4*	I don’t like others to see the work I do with technology in case they criticise what I’ve done	(**46**, **46**, 38)
5	When I use technology to study mathematics I really feel I need to share with others what I find	(35, 40, 41)
6†	I prefer to work with others when using technology because I often get good ideas from them	(0, 0, **45**)

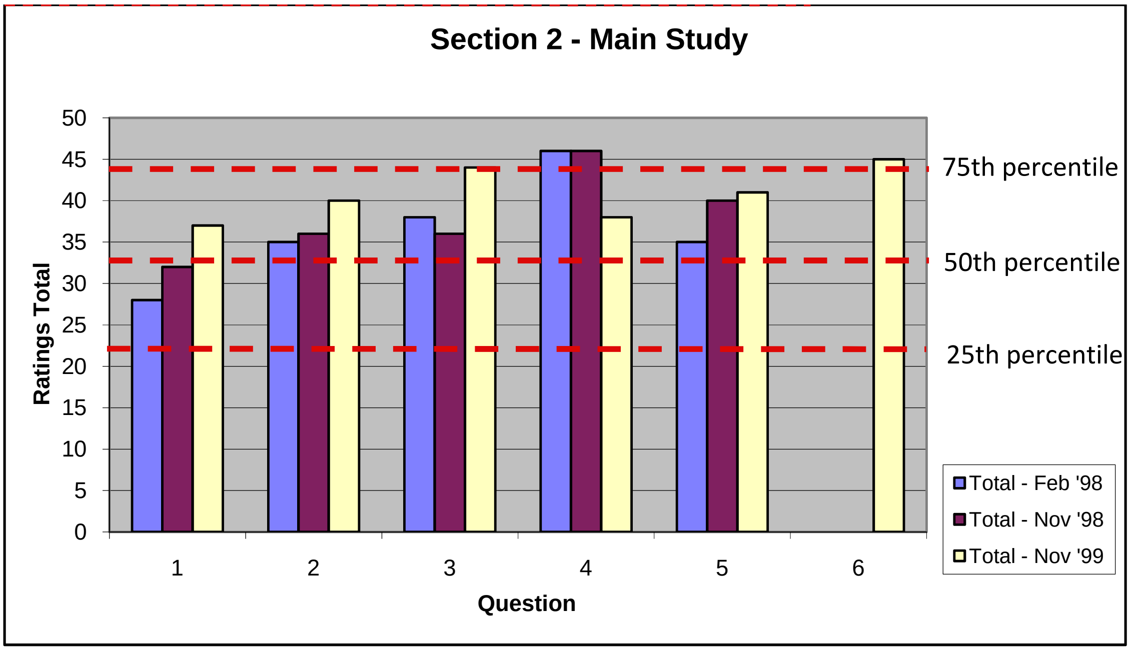

Figure 7.4. Students' responses to Section 2 of *Technology Questionnaires* 1 and 2: Main Study

Totals indicate that students' attitudes towards the use of technology in collaborative contexts was initially moderate to strongly positive (between 33 and 55 in four out of five items) and changes toward being slightly more strongly positive (44 and above on two out of six items compared to one out of six items) by TQ3.

Of greatest interest, though, are the noteworthy positive shifts (Questions 1, 2, 3, 5) that take place in four out of the five items included on each of the questionnaires. A particularly large shift was evident on Q1 (+9) which provided students with an opportunity to express displeasure with collaborative activity. While the final rating (37) was still within a zone of moderation there was a clear positive change in attitude toward working within a culture of collaboration.

The remaining items, in this section, aimed to probe reasons for support (or otherwise) of collaborative activity. Items 2, 3 and 5 all return strong positive or near positive ratings by the end of Year 12 and each has been subject to a noteworthy shift of positive support (+5, +6, +6 respectively). This result is a greater confirmation, by students, of the culture of teaching/learning favoured by the teacher-researcher than was the case in the pilot study. It should be noted, however, that the magnitude of this change would not be as strong if judgments did not include data recorded across the two years of the main study. This implies the positive changes noted in the data from the main study may be related to the longer period of time available during this phase of the project when compared to the pilot study.

The noteworthy drop in the score on item 4, between Year 11 and Year 12, is in fact substantially due to two students who clearly felt vulnerable to peer comment – they each recorded a negative shift of 3 points in their Year 12 response. This is a timely reminder that collaborative classrooms involve more risk taking, and associated sensitivities need to be recognised.

Due to the real school life considerations of preparation for final assessment, the administration of this assessment and the celebratory events associated with a Year 12 class leaving the school, there was no opportunity to conduct a final follow up whole-class interview. However, an open-ended response item, included on the final *Technology Questionnaire*, provides further insight into students' perspectives on the use of technology in collaborative contexts. As discussed earlier in Chapter 4, the *Technology Questionnaire* evolved through the course of this study and items were added or modified in response to observations or as an attempt to assist in clarifying developing theory. This was particularly true of Section 6 of the *Technology Questionnaire* which consisted of open-ended items. Of relevance to the current analysis is a question which aimed to elicit students' views on the use of technology to learn mathematics. This question was first included on TQ1 in the following form.

1. Write down in your own words what you think about using technology to learn mathematics.

This is phrased in a broad sense with the aim of capturing any views students wished to elaborate upon given their current state of experience. In response to the need to gain more specific insights this question was modified to the form below for the second questionnaire (TQ2).

1. Are there any advantages in using technology instead of pencil and paper? If so, explain how technology helps you learn better. (Give specific examples.)

   Are there any disadvantages in using technology instead of pencil and paper? If so, explain how technology gets in the way of your learning. (Give specific examples.)

2. If you had not been able to use technology to learn maths this year, what difference would this have made to your understanding? (Give specific examples.)

Finally, the following modification was made to Question 2, above, and included on TQ3 in order to gain student responses specific to the issue of working collaboratively with technology.

2. If you had *not* been able to use technology to learn maths this year, what difference (do you feel) would this have made to:

your understanding? (Give specific examples.)

the way you work with others in class?

The second part of this question is of relevance to this current analysis.

Of the 15 responses to this question on TQ3, 5 indicated that they believed that the use of technology had little impact on the way they worked as collaborators in the classroom and 8 recorded responses that indicated that technology promoted collaborative interaction. Of the remaining responses one indicated that technology had the potential to inhibit student interaction while the final response was irrelevant to the question and so could not be categorised.

Students who indicated technology offered no advantages to collaborative interaction seemed to view the development of collaborative relationships as independent from the technological tools available.

Sean: Not much at all. I work with others when I don't know how to do something. This happens both when using technology and when not using technology.

Keira: I don't think it really would have made a difference. I would not have been as close to the others in my class as I probably wouldn't have needed their help as much.

There was a range of reasons offered by students for why they believed the availability of technology made a positive impact on collaborative interaction when they were working with mathematics. Firstly students commented on how technology provided a medium for the facilitation of discussion.

Nicole: But I think the work we were working on would not be talked about as much without the aid of technology.

This was because of the efficiencies offered by communicating through technology based forms as opposed to pen and paper.

> Gena: ...it has helped us communicate together and help each other and if we simply worked with pen and paper we wouldn't communicate as efficiently.

Or technology could provide the visual focus for discussion:

> Francis: It makes it easy to interact in the class. It gives us something to look at and discuss.

This visual aspect of technology use can provide a common medium for the illustration and clarification of ideas that are under discussion:

> Matthew: Technology helps to discuss problems on common grounds. It lets us compare our efforts easily.

Students also identified the potential of technology for the sharing of ideas:

> Tom: Tech allows us to share new ideas and give people my point of view.

One student offered a contrasting view, reminding us of the legitimate concern that the mastery of technology, for some students, can be as big a challenge as the learning of new ideas and concepts in mathematics.

> Sylvester: Technology tends to make the class struggle on using technology and therefore less communicable (????) whereas more competent class members skip ahead.

Significant Observations

Evidence from the main study indicates that students generally displayed a positive disposition to the use of technology during collaborative endeavours. It is important to note, however, that the strength of this view only emerged over the duration of the main study, and so, it appears that an appreciation of the advantages offered by technology in mediating collaborative practice may take more than a few months to develop. This position contrasts, in part, to the view offered by the students in the pilot study who argued that a parallel existed between their increasing expertise and the decreasing frequency of their collaborative activity. The only issue identified via the Likert item set that could be perceived in any way to be negative was the concern of two students over their potential loss of self-esteem if they offered incorrect, incomplete or inappropriate contributions within a group setting. This highlights the importance of affective issues in any consideration of the activity that takes place in collaborative contexts.

Responses to the open-ended item provided additional insight into the view expressed by a minority of students that collaborative activity was independent of the presence of technological tools and that this type of interaction would have occurred even without the availability of digital technologies. The majority of students, however, assigned technology an important role in the mediation of collaborative interaction. These students felt that technology provided the medium for productive discourse to take place, and in certain circumstances represented the tool that provoked it. When viewed in this way, technology can be considered a provocateur of discussion through its capacity to offer a visual representation of a problem or as a medium through which students could make use of a common representational language. That is, it provided students with a means to share ideas they may have had difficulty expressing if they were reliant on verbal descriptions or pen and paper based symbolic approaches alone. A note of caution, however, was provided by one student, who commented that there remains the ever present danger that technology can inhibit the progress of students' mathematical understanding if they become distracted by a desire to simply master the technology itself.

7.2. The Role of Technology as *Master, Servant, Partner* and *Extension-of-self* in Small Group Contexts

In the previous section, students' perspectives on the role of technology when working in small groups were reported. Now, a series of episodes, taken from two different lessons, are presented to illustrate students' use of technology within small group contexts with reference to the *Master*, *Servant*, *Partner* and *Extension-of-self* (MSPE) framework, first described in Chapter 6.

7.2.1. Episode 1

Students (Year 11, April, 1998) were asked to use the geometric facility on their TI-92 calculators (a version of Cabri Geométrè) to draw a line $\sqrt{45}$ units long. The teacher's aim was to encourage students to think about the geometric representation of irrational numbers, the topic being studied at that time. It was anticipated that students would solve the problem by making use of the geometric facilities of their calculators to explore possible Pythagorean relationships that would provide a solution to the problem. It was hoped students would eventually realise the relationship $6^2 + 3^2 = (\sqrt{45})^2$ was a basis for the construction of a right-angled triangle with a hypotenuse equal to $\sqrt{45}$. This meant the other two sides must be 3 and 6 units respectively. This is illustrated via the calculator screen-shot in Figure 7.5.

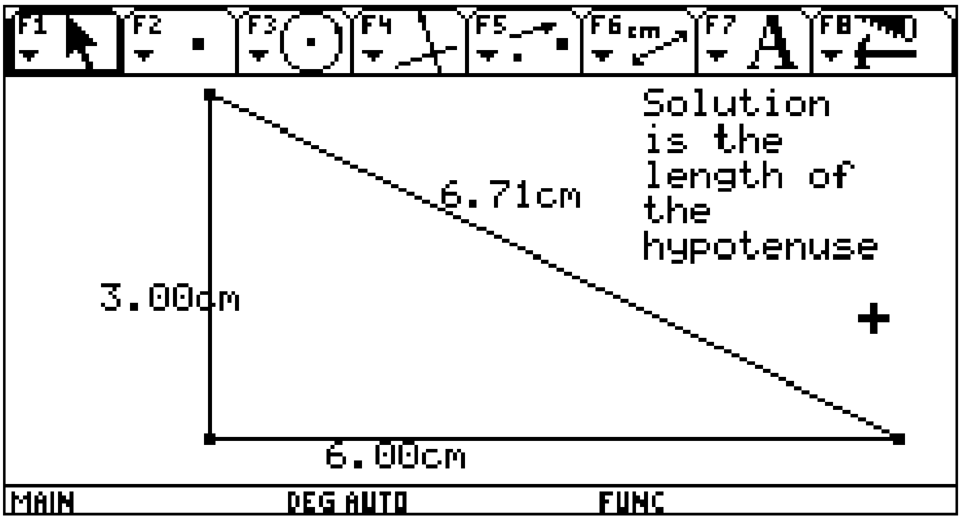

Figure 7.5. Model solution to the length of √45 problem

Students had been previously provided with experience in working with this facility through assignment work earlier in the year, although not all had become confident users as a result. The class was set to work on the task with little further instruction in the use of this application.

The teacher allowed some time for students to explore the problem. The first excerpt, which follows, concerns three students, Susie, Keira and Gena. Initially Keira and Gena worked together, while Susie worked independently. Later, Susie joined the conversation as Keira and Gena raised issues that are also troublesome to Susie. During discussion of the problem, Keira and Gena made use of a TI-92 calculator in a variety of ways. Each used the calculator to perform procedural calculations, such as taking the square root of numbers they wish to evaluate as part of their exploration of the problem. After each calculation, however, they passed their calculators to each other as a way of sharing what they had found. Thus the calculator was used as means of communicating findings between students. The openness of this process of sharing was evident when Gena passed her calculator to Susie who was not immediately next to Gena but on the other side of Keira. The passing of a calculator from one student to another was not always just about the simple transmission of results. On a number of occasions, each student in this cluster accepted the calculator of another and modified or added to what was displayed. In this way the calculator is acting as a medium for the progress of the students' thoughts and ideas towards the solution of the problem and plays the role of *Partner* in its own right. Within the working cluster, each student's calculator display appears to be public property on which ideas are offered up for comment and critique and are then transformed through the modification of existing ideas or the addition of new ones.

After allowing some time for investigation of the problem, the teacher called for volunteers to present preliminary results of the investigation. Gena offered the solution, developed by her group, to the class. She moved to the front of the room, plugged her calculator into the viewscreen, and entered $\sqrt{45}$ followed by the ENTER key. This produced a result with 10 decimal places which Gena assumed was a terminating decimal because of the calculator's known capacity to display up to 12 decimal places. Other students in class, however, pointed out that $\sqrt{45}$ is irrational and so could not terminate. They then offered counterexamples for Gena to work "live" in order to illustrate the misconception. The problem was identified, with the assistance of the class, as a setting on the calculator which fixed the results of calculations to 10 decimal places. At this stage, Gena acknowledged the error although she was unable to suggest any improvements to her approach to the task.

Gena's group initially used their calculators as *Servants*, firstly, to perform procedural operations such as finding the square root of 45 and secondly, to communicate their individual findings to other members of the group. At the same time the passing of calculators from one to another provided the opportunity and the medium to progress ideas or try new directions by altering the display and return it to an original group member for further consideration, in the same way that a scratch pad might be used by a group of people working on a common problem. Here technology was used as a *Partner* to make public individual contributions to the problem solving endeavour and also as the canvas on which all members of the group worked toward a solution.

There is another aspect to this episode that is worthy of comment. Gena's initial, faulty solution is a result of a misconception, clearly shared by all her group members, related to the way irrational numbers are represented on a calculator. Because of this misunderstanding, the group's potential to find an appropriate solution was limited by their knowledge of the technology and in this way it acted as their *Master*.

The episode, described above, also illustrates the capacity of technology to act as a *Partner* in whole class settings as the combination of calculator and viewscreen permitted Gena to present findings which, in turn, allowed other members of the class to identify a misconception and to correct the source of the problem. While noted here, the role of technology to act as a *Partner* in whole class settings will be more thoroughly examined in the next chapter.

7.2.2. Episode 2

This section continues from the previous episode and documents attempts to solve the same problem by other groups of students.

Rather than directly indicating Gena's group had not produced the result he had expected, that is, Gena had not produced a line $\sqrt{45}$ units long, the teacher noted her contribution as a conjecture and asked for other solutions. At this invitation, Adam volunteered to present his group's contribution.

> Adam: Okay, to factorise things? Two! Factor! There! Now root 45 and we've got to close brackets twice.

Adam used the "factor" facility of the calculator to produce the result $3\sqrt{5}$. This was certainly a different approach to the task but was not consistent with the original problem – to produce a line exactly $\sqrt{45}$ units long. At this point the teacher made the decision to try and direct students back to the original question – to provide a geometric representation of $\sqrt{45}$. After a period of rhetorical questioning by the teacher, for example, "But what would this *look* like?", Sylvester offered a suggestion that was potentially useful.

> Sylvester: Um, well you could have a triangle, and if you have one….
>
> Teacher: (Interrupts) Um, hang on! Before you tell everybody…….(to the class) I think Sylvester's given you a hint! ……. Sylvester's given you a big hint. The word is triangle.

Students: Cool!

The teacher then asked students to return to work in their small groups to explore the problem further. After 5 to 10 minutes, Nicole, who was working with Adam and Susie, asked a question of the teacher.

Nicole: Has it got something to do with Pythagoras?

Teacher: Way to go!

Satisfied with this reinforcement the students return to work.

Susie: So, we're trying to relate it to 45! The hypotenuse.

Nicole: So the side of "a" could be $\sqrt{5}$ and the side of "b" could be $\sqrt{40}$, so $\sqrt{40}^2$ and $\sqrt{5}^2$ is 45.

Susie: If you had square root of 5, both of the sides would be …….what would be that?

Nicole: I thought it would be …. I'm stupid!

Susie: Pardon?

Nicole: I don't know I think it is….

Susie: If you had maybe 5^2

Nicole: What's half of 45?

Susie: 22.5

Nicole: So we can do that, like that square root of that, plus the square root of that.

Through the preceding dialogue students continued to explore the problem from a numerical perspective and remained resistant to the teacher's coaxing towards a geometric approach. There was an understanding that the use of Pythagoras' rule was an important key to solving the problem but their focus was to simply investigate values that would work for Pythogoras' rule to produce $\sqrt{45}$ as a result. Their effort proceeded almost independently of any geometric considerations, such as the one raised briefly by Susie when responding to Nicole's suggested triple of $\sqrt{5}$, $\sqrt{40}$ and $\sqrt{45}$.

Susie: But the length of the sides will be an irrational number.

Here, Susie recognised that this problem involves more than simply finding any set of Pythagorean triples as the actual drawing of a simple right triangle, with hypotenuse $\sqrt{45}$, will not be possible if the other two sides of the triangle have irrational lengths. This realization, however, does not provoke a change in direction by the group and is soon passed over to return to "number plugging" activity.

Eventually the teacher realised most students were still attempting approaches that were not consistent with the demands of the task and interrupted the whole class, again, to try and redirect students.

Teacher: There seems to be a lot positively related to the work we were doing yesterday, but walking around, there were five of you doing the geometry and the rest of you were on your calculators working only with numbers. So, some of us are going to have to take a little risk and get out of our "comfort zones". We like working with numbers because it's comfortable, but just because you're busy, doesn't mean it's productive. Other people have given you big hints. You need to try to work with that.

The students stopped and listened attentively but resumed work exactly where they left off – effectively ignoring the teacher's advice.

Nicole: $-\sqrt{5}^{2}$, which can give you 45. (While the girls are arguing, Adam is trying to get Nicole's attention)

Adam: I got it (Nicole turns to him). You can't add two roots together.

Nicole: Yes it does! It works! You square it. You square it so it gets rid of the square root (Adam is sceptical), so it ends up with 40 and 5; but if you end up with 20 and 25, then you can do $\sqrt{20} = 2\sqrt{5}$, and $\sqrt{25} = 5^{2}$, so that's squared.

Nicole performed these calculations on her TI-92 and showed Adam as a way of supporting her argument.

Adam: Is that it? Is that $\sqrt{5}$? I'll tell you whether I'm right or not….

Nicole: Ahhh! I just feel so relieved! It just works! It really does work!

Adam: It does. Oh it's true! It really does work.

Nicole: I'm so excited about it!

Adam: We worked it out on that! (Adam shows the calculator to the teacher)

Teacher: What are you doing with that? (expressing dismay at Adam's persistence with a numeric approach)

Adam: Because we didn't have the calculator.

Teacher: But I said stop playing with that! You should be in Geometry!

Nicole: What are we supposed to be doing? Geometry?

This last comment by Nicole emphasises her misdirected focus, and to an extent, that of her group. The students in this group appear to have been lulled into a false sense of security through the process of searching for a solution through a numeric approach alone, despite the fact that the problem is strongly geometric in nature and that an attempt to represent the problem geometrically may have provided a critical perspective on the nature of the problem. It is almost as if they are transfixed, much as a rabbit in a set of headlights, by the numeric approach they have chosen and are unable to change direction despite the insightfulness of one group member and the direct advice of the teacher who was aiming to create an appropriate ZPD for the students. The teacher’s comment seems most appropriate as students seem to have settled into a numeric “zone of comfort” that restricts what they are able to achieve in relation to this problem. The technology in this case has acted as a *Master* because of the students’ reluctance to consider a geometric approach, either because they cannot break away from a numeric "zone of comfort", or because they lack the expertise or confidence with using the geometric facility of the calculator. Either scenario resulted in students producing solutions that were inconsistent with the problem requirements.

At this point in time, a huddle of students, including Susie, Keira, Gena, Francis and Demi, formed around Heath.

Demi: We’ve got two that are the same. So 45 equals 9 plus 36.

Heath: Yeah-

Gena: Yeah, so that equals 9. Yeah, so write that down! Write that down!

Heath: Square root of 9 is 3.

Keira: Tom wants your attention.

Gena: What are you trying to say Tom? What are you trying to say?

Tom: No, because that squared plus that squared...Whatever the number what...6 point something...So, to get that number, you need a right-angled triangle with a side of 6 and 3 and that-

Francis: I get it now.

Tom: And that one there!

Heath: How do you know it's going to be a rational number?

Tom: That! That's the root of 45. If you want to know how to draw it...

Heath: That line is going to be...

Tom: By drawing that and that and the right-angle, you get that. (In response to this the girls cheer "Yay! Cool!" and return to their seats.

Demi: So Tom, this has to be 3 and that has to be 6, so it ends up equaling the 6 point something number.

Tom: Yep! (Turns to Heath) You had the $\sqrt{20}$ and $\sqrt{25}$.

Heath: It was easier that way!

Tom: $\sqrt{36}^2 + \sqrt{9}^2 = 45$ and $6^2 + 3^2 = 45$, and that, the square root of that is 45...

In this excerpt, Tom was working primarily by himself while still monitoring the progress of the group of students close by. He injected himself into their discussion to show them what he believed to be a viable solution. Tom had, in fact, taken into account the factor that was missing from the approach taken by Nicole's group, that is, that the sides of the right angle triangle used to draw a line $\sqrt{45}$ long must be rational, and ideally integers. After some discussion he convinces others that his solution is appropriate and supports his argument by passing around the calculator to show the audit trail of his exploration. Tom came to this solution with very little input from others but used the calculator as a *Partner* to make public his findings and to provide support to his argument within his group. It is worth noting, that in this instance, Tom has been able to choose the moment of his contribution; a moment that was potentially less threatening than the type of whole class presentation contributed earlier by Gena. Tom waited until he was sure he had something worth sharing and used his calculator to check results before intervening in the work of a nearby group. Here, he used the calculator as a symbolic *Partner* that provided him with the confidence he needed to introduce his conjectured solution into the hurly burly of a vigorous debate.

After Tom's intervention, and eventual acceptance of his solution, the group turned their attention to the original context of the problem, the actual drawing of a line $\sqrt{45}$ units long.

> Francis: How do you draw a certain length of line? (The girls are discussing how to draw a line)
>
> Tom: Oh yeah, you could do that, eh?
>
> Francis: Where did the line go?
>
> Tom: You're trying to use the line? I'll just do a line.
>
> Demi: No! Don't!

Gena: Do you do that, now?

Keira: Yeah!

Tom: Through this point, how do you mark a right angle triangle? Six, angle…90 degrees! No, don't want to do that!

The excerpt above shows students struggling with the geometry facility of the TI-92 calculator. Their lack of competence restricts their capacity to fulfill the task requirement of drawing a line $\sqrt{45}$ long. Technology, here, acts in the role of a *Master* as Tom, Gena, Keira and Francis are unable to complete the final phase of the problem. This is despite Tom's competence and confidence when using the calculator in a different episode earlier and illustrates the task specific and technology specific nature of the MSPE framework.

7.3. Confirmation and Extension

The above lesson transcript describes small group activity in which technology plays the roles of *Master*, *Servant* and *Partner*. What follows, here, is another description of small group activity, from a lesson a month later, where there are variations in how these technological roles were enacted.

7.3.1. Episode 3

In this excerpt (Year 11, May 1998) students were introduced to the Leontief Input-Output Model of an economy as a real life application of matrices and as a context for developing understanding of the inverse of a matrix. While working on this topic students were asked to use their graphing calculators to help them solve the problem shown in Figure 7.6.

An economy with the four sectors manufacturing, petroleum, transportation, and hydroelectric power has the following technology matrix:

$$T = \begin{pmatrix} 0.15 & 0.18 & 0.3 & 0.1 \\ 0.22 & 0.12 & 0.37 & 0 \\ 0.09 & 0.3 & 0.11 & 0 \\ 0.27 & 0.05 & 0.07 & 0.1 \end{pmatrix}$$

Find the production matrix if all the entries in the demand matrix are 200.

Figure 7.6. Leontief matrix problem

To solve this problem, students needed to develop a demand matrix and a 4 × 4 identity matrix. These were then combined with the transition matrix using the operations of subtraction, multiplication and inverse, as shown below in figure 7.7, to produce the desired production matrix.

$$D = \begin{pmatrix} 200 \\ 200 \\ 200 \\ 200 \end{pmatrix}$$

$$\begin{aligned} P &= (I - T)^{-1} D \\ &= \begin{pmatrix} 579.25 \\ 572.31 \\ 476.21 \\ 464.83 \end{pmatrix} \end{aligned}$$

Figure 7.7. Leontief matrix solution

Three students, Gena, Keira, and Heath, compared their workings and results, as illustrated in the following transcript.

Gena: Yeah, it's the identity of ... the inverse of the *I* take away *T* is supposed to be a four by four, and the four by four times the one column one ... the answer's got to be four rows, one column.

Keira: I got four columns, one row! (holds up her calculator for Gena to see) Look, I know I got that. Is that right?

Gena: (inspects Keira's calculator screen) I haven't done it like that.

Keira: What did you get, Heath?

Heath: (still pressing buttons) Ah, just give me a minute.

Gena falters for a moment when describing the inverse of the identity matrix, but recovers to reason that the answer must be a matrix with four rows and one column. In this instance she has used the calculator as a *Servant*, to rapidly work through a series of calculations as she attempts to find the source of her error as part of her contribution to the group's efforts.

Gena: (to herself) Row, column. (balances calculator on her head as she thinks)

Keira: (to Gena, looking at her calculator screen) How did you get that? (no reply, issues general plea to whole class) Has anyone done it?

Gena: Yeah, in about two seconds. (to herself) Give it a name. What was the other one called? Three by one is 200, 200, 200 (entering elements of demand matrix) Okay!

Heath: (to Gena) All righty, what have you got?

Gena: Hang on, I got it! (verbalises keystrokes) D times 'Kan' (label for her matrix) (groans and lowers head to desk)

Keira: What happened?

Gena: (reading dejectedly from calculator screen) Dimension problem!

Keira: Did you go 200 down that way or across?

Gena: Down.

Keira: I've got to check that. Heath ... (he is not listening, talking to another student) Heath! (Keira taps his arm) What did you get?

Heath: (turns back to Keira) This! (indicating calculator screen) I wrote all that (indicating pen and paper notes) to get that!

Keira: (inspects Heath's screen carefully, compares with her own) Oh my God, oh my God ! I got it right!

Heath: (grinning at Keira's excitement) Happy now?

Keira: Yes, very happy!

Heath: Good!

In this exchange Gena treated the calculator as a *Partner*, verbalising the menu choices and keystrokes and responding with despair when the calculator returns an error message – *"Dimension problem!"* – indicating the dimensions of the matrices the group is attempting to multiply together are mismatched. This message represents another aspect of technology as *Partner* as the calculator has provided direct feedback that indicated there was an error, and also indicated the error type. Gena also shared the display with Keira and so involved her in the partnership with her calculator. While the error is noted, it does not appear as if any of the group members consider the specific advice on offer.

Intense emotional involvement was also apparent in Keira's surprise and delight as she *"got it right!"* which also places her in a position to provide knowledgeable assistance to Gena.

Gena: (still trying to identify the source of her error) Maybe my inverse is wrong.

Heath: (to Gena, wanting to help) So what did you get? What did you get?

Keira: (to Gena) What did you get for your inverse?

Gena: (dejected) It tells me there's a dimension error, and I don't know why.

Heath: Did you get that? (passing his calculator to Gena so she can look at his working)

Keira: (also showing her calculator working to Gena) It should be that.

Gena: (comparing her working with the other two screens) That's what I had!

Keira: So then you ...

Gena: (puzzled, comparing screens with Heath) Is that what you have? It's exactly the same as mine!

Keira: Yeah, and you times that by 200, 200, 200 *down* (referring to demand matrix)

Gena: (sudden insight) Oh hang on ... That should be *four* ... Oh God!

Keira: What did you do?

Gena: I didn't do four 200s!

Keira: Oh you big dork! (Gena and Keira laugh) You've only got three 200s! (referring to number of entries in demand matrix – there should be four rows, not three)

Gena: (chastened) God I'm a moron! (talking to her calculator as she presses buttons) Second, quit. Now ... (re-does the calculation. Asks Keira) Did you get that?

Keira: (inspects Gena's screen) Yeah! (Gena jumps up from her seat in delight)

Heath: (to Gena) Look at mine.

Gena: (goes over to Heath) Did you get this? (Heath holds both calculators up side by side, compares his screen with Gena's. Gena pulls his hair when he hesitates in replying.)

Heath: (with cheeky grin) Yes!

Gena: Thank you!

The students clustered around their calculators, holding them up side by side to compare the working on the screens, sometimes passing them back and forth to emphasise points as they spoke. Gena even imagined her calculator spoke to her, *"It tells me there's a dimension error"*. Once the error is located through the help and encouragement of other members of her group, who have had access to her calculator display, she and her technological *Partner* quickly re-calculate and the results are verified by again referring her display to peers.

7.4. Learning Mathematics in Small Groups

The public perception of technology is often that of a personal device used for the purpose of introspective approaches to completing a task and with little potential for mediating interaction between individuals. Doerr and Zangor (2000), for example, found that the use of the graphing calculator as a private device led to the breakdown of small group interaction. The excerpts, from the lessons presented above, provide evidence that technology can act as the mediator of interaction between small groups of students in ways consistent with the MSPE framework first described in Chapter 6. The role of technology in these interactions extends the framework in ways described below.

Technology as Master

Firstly, the use of technology may be problematic because of a group's limited cumulative knowledge of the available facilities of a digital device. Such a limitation may mean a technological resource that might have provided insight into a problem, or a more direct approach to solving a problem, is left unexploited. Secondly, a disposition towards using technology in a particular way may inhibit a group using an alternative and potentially productive technological approach. For example, the ease with which a problem can be investigated numerically is an exploratory vortex into which students can find themselves drawn and from which they have difficulty extracting themselves. Thirdly, a limited knowledge of the direct help available from a technology, as in the case of the “dimension error” discussed earlier, may limit a student’s response to such a prompt. Finally, technology can act as a *Master* if students are unaware of the nature of the constraints on mathematical representation imposed by software design (Strasser, 2006), for example defaults such as those related to the floating decimal point display as was the case in Gena's solution to the "length of a line $\sqrt{45}$ units long" problem.

Technology as Servant

The role of technology in this category is in supporting group argumentation by providing evidence for conjectures and refutations through representations of a problem that can be shared or by handling large or tedious calculations. This role is closely aligned to but is different from, that of technology as a *Partner,* as the role of *Servant* does not imply a strategic use of a digital tool. The calculator as *Servant* also has a physical dimension when the sharing information is facilitated by passing a calculator from one group member to the next, or when shared by all members of a group at the same time.

Technology as Partner

The capacity to mediate discussion is one of the potential benefits of the use of technology. In the examples presented earlier, students passed their calculators between members of a group as a way of confirming or challenging the conjectured solutions or suggested approaches to a task. Calculators were used as a digital canvas on which two or more students progressed their work on one display; the ownership of the work in progress being shared between students. This action led to further interaction and discourse in the form of argumentation.

Technology was also observed to provide support for a student, initially working as an individual, to join a discussion in order to share findings with a group. In this case technology provided the physical facility necessary to present findings in a more public forum and also seemed to offer a form of moral support, as he waited until, through the assistance of his calculator, he was sure he was correct before offering a contribution.

A technological partnership can be more direct if a technology has the capacity to provide direct advice in relation to perceived errors. The capacity to interpret and act on this advice, however, is a precondition to progress further as a result of such a prompt.

Technology as Extension-of-Self

No evidence was found, in the available data, for technology in this category for small group settings. However, it is possible to hypothesise the existence of such a role on the basis of its identification by students working as individuals (see Chapter 5) as well as the evidence presented for the existence of this mode in the next chapter on the role of technology in *Public* contexts. Given the identification of this category of use in both private and public contexts, it seems likely that technology will also have a role in mediating collaborative practice when students work in small group settings. This is a topic of discussion in the Conclusion chapter of this thesis.

7.5. Summary and Conclusions

This chapter has been concerned with the role of technology in mediating collaborative interaction within the context of small group settings. To this end, a series of episodes from two separate lessons have been considered in which students have been working on problem solving tasks. Observational, video, audio and questionnaire data were analysed, firstly, to establish students' dispositions towards the use of technology in collaborative settings, and secondly, to extend the *Master*, *Servant*, *Partner* and *Extension-of-self* framework first developed in Chapter 6.

Questionnaire and follow-up interview data revealed that students, in general, held a positive disposition to the use of technology in small group contexts. Although data from the pilot study indicated a disenchantment with technology developed through the period of the study, students indicated, in the follow-up whole class interview, that while they felt discussion that was inspired by technical aspects of using technology had waned over time, they were just as enthusiastic about the use of technology when placed in problem solving contexts where they were unsure of how to proceed. This is consistent with the findings of Geiger and Goos (1996) who concluded that tasks that were problem orientated were more supportive of collaborative interaction than procedural activities.

Students from the pilot study also made the point that the size of the screen was a critical factor in the mediation of collaborative activity between the students (reported in greater detail in Geiger, 1998). This is a finding consistent with Lehtinen and Repo's (1996) assessment of the reasons for apparent facilitation of positive peer to peer interaction during sessions with computer assisted algebra programs.

Findings from the main study varied from those of the pilot study in a number of ways. Students in the main study generally remained positive, or expressed enhanced enthusiasm, about the use of technology in collaborative endeavours, throughout the duration of the project. A minority of students, however, believed a high level of collaboration would have been evident even without the availability of technological tools. The majority of students, though, expressed the view that technology had an important role in facilitating productive discourse between members of a group. Students identified a range of ways in which technology facilitated collaboration between peers. This included the use of visual representations that could be viewed by all group members to develop common conceptual ground and shared meaning. This view partially mirrors the perspective of the students in the pilot study who identified the size of a technology's screen as a factor in the effective use of technology to mediate collaborative activity – the larger the screen, as in the case of a computer, the more opportunity existed for focused student discussion. While students in the main study did not place such importance on the size of a screen, they may have obviated the need for a large common screen by developing techniques, over a two year period, to share calculator displays.

In both studies, however, the views expressed by students are consistent with the socio-cultural notion of a ZPD in which the acts of conjecture, clarification, justification, and interrogation of anomalies are mediated by cultural tools, in this case the mathematically enabled technologies available within students' classrooms.

Observational data, represented by the lesson excerpts analysed in this chapter, support the student view of technology as a mediator of collaborative interaction. As noted earlier, collaboration in these instances was characterised by the sharing of ideas through technological tools, either via a large common screen or by passing calculators between students. Computer and calculator displays became common spaces where complete, or nearly complete ideas, were proposed by individuals and shared with the group and then progressed to maturity through the contributions of multiple authors on one screen. This form of action illustrates the role of technology as a *Partner*, as students make public their thinking in the process of developing ideas and solutions or in the location and correction of errors. In all cases reported here, dialogue was driven by both the desire to solve a problem as well as a need to understand and to share understanding. Technology played a number of different roles in assisting students to realise these needs including: the moral support offered to a usually hesitant Tom to make public, and so share, a well developed solution with colleagues who were struggling; the mediation of mathematical discourse through the provision of a common "space" for the expression and debate of conjectures and their further development; and the direct assistance through error messages and other in-built facilities that provided direct advice about progress.

The role of technology as a *Partner* is often supported through its initial use as a *Servant*. Computer screens and calculator displays are used as the physical platform through which information is passed between members of a group. Technology is also used as a *Servant* to perform tedious or time consuming calculations that are then brought back to the collaborative team as support for, or evidence against, a conjecture or as substance for maintaining the process of argumentation.

The use of technology is not without its pitfalls, however, and a number were identified by students. Sylvester expressed the view that technology can inhibit collaborative interaction if a student's lack of experience with technology causes him or her to focus on the technology rather than on a group's attempt to solve a problem. This observation is symptomatic of a context in which technology is the *Master*. While a group's ability to work productively can be limited by a lack of expertise in the use of a technology's facilities, knowledge of what Strasser (2006) describes as the constraints of software in representing mathematical entities is also a factor. Gena displayed such a lack of understanding when interpreting her calculator's use of a floating point decimal to represent an irrational number. Students' responses to an error message may be limited if their communal expertise is insufficient to allow them to interpret the specifics of such a prompt. Another form of servitude was observed when students submitted to the ease of exploring a problem numerically to the exclusion of other approaches. In the example reported in this chapter, the temptation to use this mode of investigation exclusively inhibited progress towards a solution when an alternative approach might have provided more beneficial insight.

The evidence needed to verify the existence of the category *Extension-of-self*, within small group settings, was not available and is acknowledged in this chapter. None-the-less the existence of this category seems likely given its identification as a role for technology in other settings. A more thorough discussion of this matter will be included in the Conclusion chapter of this thesis.

Finally, the transcripts presented above provide clear evidence that the metaphors that form the basis of the framework, *Master*, *Servant*, *Partner* and *Extension-of-self* are descriptive of a student's technology based activity "in the moment" and do not prescribe how a student will operate universally within technology rich contexts. Thus, the MSPE framework should not be viewed as a developmental framework with each metaphor representing a level of operation that, once attained, carries the expectation of constant and consistant operation at that level. Rather, the framework represents different modes of operating with technology which, except for the *Master* phase, all have their role in productive mathematical activity. The aim, therefore, should be to assist students in developing a complete technological repertoire from which they can select the appropriate option given the mathematical and technological context.

Chapter 8

Going Public – Working with Technology in Whole Class Settings

In the three preceding chapters students' orientations and preferences for working with digital technology were examined (Chapter 5) as well as the role of technology in promoting student-technology and student-student-technology interaction in individual (Chapter 6) and in small group (Chapter 7) settings. Interactions in both individual and small group settings were categorised and then theorized, leading to the development of the *Master*, *Servant*, *Partner* and *Extension-of-self* (MSPE) framework. This chapter extends this work by examining student-student-technology interactions within public or whole class settings. Firstly, the findings of Chapter 5 related to students' orientations towards and preferences for working with technology in collaborative contexts are explored in greater detail. Secondly, the MSPE framework is further extended by theorizing the student-student-technology relationships which develop when interactions are played out in public forums, for example, whole class student presentations. Evidence in this chapter is drawn from data sources which include: versions 2 and 3 of the *Technology Questionnaire* used in the main study (TQ2 and TQ3), classroom observation notes, and video and audio recording of whole class collaborative activity. Analysis of these data uses the same techniques exploited in Chapters 5, 6 and 7 where categories that emerge from the data are theorized and refined according to processes outlined in Chapter 4. Findings based on these data address the following research question.

6. What roles can be identified for technology in mediating collaborative student learning?

8.1. Preferences for, and Orientations to, Learning with Technology During Whole Class Presentations

8.1.1. Classroom Context in Relation to Technology Supported Whole Class Presentation

Within the classroom at the centre of this study, all students were encouraged to take the responsibility of leading the class via a whole class presentation. Most students volunteered, on a regular basis, to present their work through the use of a graphing calculator and a projection device known as a viewscreen although there were a small number of class members who were reluctant to participate in this way.

After an initial stage where the use of technology to enhance a presentation was modelled by the teacher, students were regularly invited to present their findings after they had worked in groups on a new idea or on a problem. The teacher constantly reinforced the collegial nature of the practice of presentation by reminding students that the audience must be both supportive of presenters and rigorous in terms of their critique of the ideas presented. Students' competence and confidence in relation to this practice improved over time to the point where over the last six months of the main study this process had evolved to a point where students were able to conduct themselves almost autonomously when in this mode. By this stage the teacher's role was often that of another class member with most of the direction for learning being shared by the class as a whole.

8.1.2. Data Sources

The data examined in this section were collected as part of the *Technology Questionnaire* described in Chapter 4. These data consist of students' responses to questions aimed at gaining an insight into their perspectives on the benefits or disadvantages of presenting to the whole class, with the assistance of a viewscreen or computer datashow. These responses were collected as part of *Technology Questionnaires* 2 and 3 which were administered in November 1998 (near the end of Year 11) and November 1999 (near the end of Year 12). The timing of these surveys, towards the end of each school year of the main study, meant that most students had presented to the class and all students had been part of an audience in which a student had made a presentation prior to the respective survey.

In response to classroom observations, during the first year of the main study, which indicated there were benefits to learning for all participants when students publicly presented the findings of an investigation or solution to a problem, the question below was included in Section 6 of the second *Technology Questionnaire* (TQ2) (Year 11, November 1998)

3. Are there any benefits in having students present their calculator work to the class via a viewscreen and overhead projector? If so, what are they? (Are there benefits for the presenter? Benefits for those watching and listening?)

After examining students' responses to this question, it was further refined for the third and final *Technology Questionnaire* (TQ3) (Year 12, November 1999) to provide greater focus on the benefits or disadvantages to specific classroom participants – the presenter, the audience and the teacher. The revised question appears below.

6. Are there any benefits in having students present their calculator work to the class via a viewscreen and overhead projector? If so, what are they?

   Benefits for the presenter:

   Benefits for the audience:

   Benefits to the teacher:

   Any other benefits:

An analysis of students' responses to these questions (on both TQ2 and TQ3) follows with the exception of those responses related to "benefits to the teacher". This section was excluded as it was not a focus of this study. As a result, any reference to benefits to classroom participants, in the analysis which follows, is directed at student participants alone.

8.1.3. Analysis of Survey Data

Students' responses were examined and classified into categories that represented perceived benefits to the presenter and the audience. Once partitioned, these responses were further sectioned into sub-categories that emerged from the data. Frequencies related to these sub-categories, which are associated with the public sharing of mathematical activity, are recorded in Table 8.1. It should be noted that some student responses provided data such that more than one frequency was reported from a single question which meant that it was possible for the total frequency in a category to exceed the total number of students in the study. Students identified a range of benefits to classroom participants. For the purpose of this analysis, a category is judged as having noteworthy support from students if it is nominated by at least twenty-five percent of class members.

This means a category is deemed noteworthy if it records a frequency of 3 or more on TQ2 (3 out of 12) or 4 or more on TQ3 (4 out of 15) (both bolded).

Table 8.1. Students' perceptions of the benefits of presenting publicly: Main Study

	Questionnaire	
Presenter	TQ2	TQ3
Clearer medium for presentation	**3**	1
Supportive environment	**5**	**8**
Improved understanding due to the responsibility of presenting to others	1	**4**
Enhanced feeling of participation in classroom community	0	2
Audience		
Improved clarity and variation of presentation	**3**	2
Improved explanations because of the structured nature of presentations due to technology and immediate feedback	**7**	**8**
Improved understanding by viewing other students' approaches to viewing a problem and the capacity to check results and processes	**6**	**7**
Enhanced feeling of participation in a supportive classroom community	1	1

Benefits to the Presenter

Initially students identified the improved clarity of presentations by class members when using technology as noteworthy.

> Johnny: The presenter can work faster than writing on the board and everyone watching can read the

computer writing which is not always so when trying to read peoples scribble. (TQ2)

Adam: Visual aide helps him explain to class. (TQ3)

Support for the categories in this class of response, however, dropped away by the time of the second survey, perhaps indicating that as students' presentations became more commonplace they were seen as less extraordinary.

Far more frequently, though, students commented on how the experience of presenting publicly with technology enhanced classroom involvement. Students' responses indicated that they believed talents were shared in a supportive environment where members' (including the teacher's) constructive critique either affirmed the presenter's understanding of a mathematical idea or provided a forum in which all participants could improve their knowledge and understanding.

Geoffery: Presenter gets a boost of confidence when their work is admired by peers. (TQ2)

Nicole: When you go up and demonstrate what you've done, you're able to receive constructive criticism and advice on how you could improve it or otherwise. (TQ2)

Johnny: Yes they get showered in praise or helped with problem. (TQ3)

Tom: Share their talents. Help others. (TQ3)

This sub-category registered as noteworthy on both TQ2 and TQ3.

Students appeared to develop a greater appreciation for the role presentation can play in promoting understanding of a topic by the presenters themselves.

Keira: The presenter is also able to understand the material better by explaining to a group of people. (TQ2)

Gena: The presenter is forced to confront the problem head on and make sure they fully understand it. (TQ3)

Sean: They have to understand everything in order to present it to the class. (TQ3)

Class members also recognized the responsibility of presenting new ideas clearly to peers. It should be noted that this category attracts a noteworthy level of response on TQ3 while it was only identified by one student as an important benefit on TQ2. This may indicate a greater appreciation of the benefits of presentation as students gained further experience and competence during the period of time between the two questionnaires.

Benefits to the Audience

Students noted the improved clarity of presented information when technology was used – this time for the audience. Consistent with responses in the *Presenter* category the importance of this observation waned over time indicating that benefits of this approach has less impact from the perspective of the audience.

There was strong recognition of the value of technology enhanced presentations to an audience because of the structured nature of the presentations (TQ2 – frequency 7; TQ3 – frequency 8). Students' comments indicated they believed that this was due to a discipline imposed on the presenter by the technology.

Demi: …it displays things well and people can be shown how to do things step by step very quickly. (TQ2)

Gena: The audience gets shown how to do an example in a neatly packaged way. (TQ3)

Students also recognized the opportunities for learning made available through technology mediated interactions with the whole class either through viewing other's approaches to solving problems or to check results and processes while on public display (TQ2 – frequency 6; TQ3 – frequency 7). Responses in this section mirrored those recorded under *benefits to presenters* indicating that students recognized the opportunities available for deeper learning through whole class interaction between technology and a community of learners.

> Francis: Yes, if you don't get it you can see how others have done it. (TQ2)
>
> Sylvester: Allows them time to think about the work more and understand it. (TQ3)
>
> Adam: More in depth understanding can be gained. (TQ3)
>
> Gena: If they are unable to do it then they can learn. You can see other approaches of solving the same problem. (TQ3)

Here, students recognised that the benefits of working with a peer presenting their ideas on a mathematical activity are beyond that of merely checking answers. A noteworthy number of students indicated they were interested in *alternative* methods for solving the problem – not just the one *correct* way. The desire to understand alternative pathways to solving problems is indicative of students who wish to develop a more connected understanding of mathematics and who wish to expand their repertoire of procedures and processes.

8.1.4. Reflection upon Students' Views of Presenting Publicly

The discussion above documents students' perceptions of the value of public presentation of their work through the use of technology. In their role as presenters students valued the opportunity to work in a supportive environment where they could receive assistance when needed and praise and encouragement when they had done well. As members of the audience they placed great value on the chance to deepen their knowledge through well structured presentations and by viewing the different approaches others took to solving problems.

Each of these points is redolent of a milieu in which learning is viewed as a participatory activity in which the responsibility is owned by the whole of the community of inquiry. The roles of the members of this community, however, vary as conditions and contexts change. These roles are now further explored and theorized from the perspective of the MSPE framework developed earlier in this thesis.

8.2. The Role of Technology as *Master*, *Servant*, *Partner* and *Extension-of-self* in Whole Class Settings

This section presents a series of episodes, based on observation and transcript data taken from a number of different lessons, as well as follow-up interviews with individual students, to illustrate the role technology can play as a *Master*, *Servant*, *Partner* and *Extension-of-Self* in whole class settings.

8.2.1. Episode 1

Students (Year 12, September 1999) were asked to develop programs for their TI-83 or TI-92 calculators that found the angle between two vectors, in both 2 or 3 dimensions, as an application of the scalar product of vectors, and also as a means for validating results found via pen and paper techniques. In effect, this task required students to develop programs capable of taking inputs for either two dimensional vectors, for example, $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$, or three dimensional vectors, for example, $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$, substituting these inputs into the equations $\theta = \frac{\cos^{-1}(a \times c + b \times d)}{\sqrt{a^2 + b^2} \times \sqrt{c^2 + d^2}}$ and $\theta = \frac{\cos^{-1}(a \times d + b \times e + c \times f)}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{d^2 + e^2 + f^2}}$ respectively to produce the desired outcome.

The teacher provided only minimal instruction in basic programming techniques, and expected individual students to consult peers, who had varying degrees of knowledge, for assistance. Students were given one lesson to work on their programs and advised that they would be expected to present their results at the next meeting of the class. Three volunteers offered to demonstrate their programs, via a calculator viewscreen, the next day. The presentations of the three volunteers are reported in the following excerpts.

Excerpt 1

Demi was the first of the volunteers to demonstrate her program. She had been a regular presenter and usually performed competently and with confidence. On this occasion she responded to the call for presenters without hesitation. She moved to the front and after plugging her calculator into the viewscreen proceeded to demonstrate her program.

Demi: What do you want me to tell you about it?

Teacher: Just what each of those bits do (pointing to the command lines of the program displayed through the OHT).

Demi: OK. The Prompt feature … like when you want to do a vector in 3D it just goes A, B, C, etc. so that A, B and C are the first vector and D, E and F are the second one … and then it just...

Francis: (shouting out assistance) Does it!

Demi: Yeah! Does it!

Teacher: Do you want to show us an example?

Demi: Sure.

(Demi begins the program and enters the components of two vectors.)

Francis: (reading out what Demi is entering) -1, 2, 3, 3, -1, 2.

(Demi activates the program but recognizes immediately her output is in radians, not in the form she wants, which is degrees.)

Demi: It's not in the right mode.

(Demi opens up the mode menu and makes the appropriate adjustment.)

Teacher: Will it work in degrees?

(Demi completes the adjustments.)

Demi: Yeah.

(Demi re-enters her example and calculator displays the desired output.)

Demi: Is it right now? (to audience)

Francis: Yeeaaaahhh!

Demi produced a program that functioned without error. She made minor adjustments "on the fly" in order to produce an output of the form required. This was a confident display in which she received few prompts or advice from the class. Information was presented in a clear and well structured manner and the class seemed satisfied as they did not ask questions for clarification or any other purpose. Demi has used the calculator and viewscreen as a *Servant* to demonstrate her approach to this task. The nature of the task is essentially procedural and her mastery of the technology is such that the creation of a calculator program that makes use of a simple algorithm is routine. As the task itself does not require any exploration of the mathematics at the core of the activity the calculator and viewscreen are used purely for presentation purposes and not as a medium for the stimulation of discussion and debate.

Excerpt 2

The second volunteer was Tom. Tom was a co-operative and capable student of mathematics who was always prepared to contribute to small groups but had rarely offered to lead a whole class presentation. On this particular occasion, though, Tom was one of the first to offer to present his program.

At the beginning of Tom's presentation the class disputed the answer he obtained by executing the program shown in the first frame of Figure 8.1.

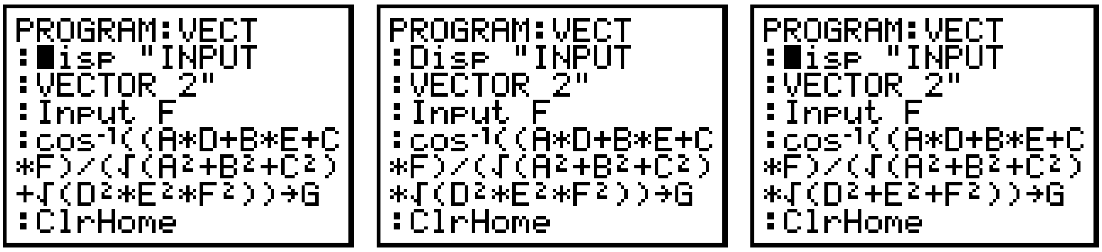

Figure 8.1. Tom's program for solving a 3 dimensional vector problem

Demi: That is not the answer!

Russel: Maybe he has it in radians?

Kearnu: No it's just the wrong answer!

On the advice of fellow students, the presenter scrolled down through the program and replaced the plus sign in the second last line with a multiplication sign (Figure 8.1, second screen).

Johnny: There it is – that's the wrong formula…… That plus there (getting up and pointing) should be a times.

Matthew: Yeah the plus should be a multiplication sign.

The amended program produced another incorrect answer, and the further correction of changing the multiplication signs to addition signs within the brackets of the second last line (Figure 8.1, third screen) was suggested by students before the correct output was obtained.

Whole class: Yeeaah! (followed by loud applause).

The public inspection of Tom's work revealed programming errors that were subsequently corrected by other members of the class. Tom's inability to find these errors himself meant that technology was his *Master* as his limited facility with programming affected his capacity to find a correct solution and present this to the class. At the same time, however, technology supported interaction with his peers resulted in the repair of a faulty calculator program. Technology in this circumstance provided the medium to make public a particular student's work; holding it up for scrutiny and providing the opportunity for supportive critique. The use of technology in a whole class setting assisted the class to progress the development of an individual who was struggling by himself. The contribution of technology in this episode is more than that of a simple tool; it is an instance where the boundary between the physical tools which mediate learning and the human learners themselves is blurred. In this case, technology assumed the role of *Extension-of-self* in that it offered a skill or expertise that the group required to complete the task.

Excerpt 3

The third volunteer was a student who consistently rejected the teacher's invitations to participate in whole class discussion and to contribute to thinking with his peers. It was quite surprising when Geoffery offered to present in relation to this task, as he had often been resistant to working in public forums or to contributing to any endeavour that required his active contribution. For example, prior to an earlier assessment item, the teacher asked class members to participate in the development of a revision sheet for the upcoming exam. This involved each student contributing a question and model solution to the revision sheet that the teacher offered to organise, edit, and then print for each class member. Geoffery attempted to avoid participation in the activity.

Teacher: So..., what's your contribution going to be?

Geoffery: Not much?

In response, the teacher decided to assign a section of work to Geoffery in order that he make a contribution to the class effort. Geoffery expressed his opposition to this approach.

Geoffery: But I wouldn't have a bloody clue.

Teacher: But that's part of the point ...this is probably a really good way of revising.

Geoffery: Yes but me revising one thing isn't going to help me much!

After considering what was being asked of him further, Geffory decided to make one more attempt to avoid the activity.

Geoffery: Does that mean if I choose not to take a revision sheet I don't have to write a question?

Several other students: Aw just grow up!

There appears to be at least two reasons for this student's reluctance to contribute to the class revision sheet. Firstly, Geoffery did not believe that learning can be a collaborative activity and, as a result, he views preparation for assessment as an individual responsibility. Secondly, Geoffery seems to operate from a system of beliefs about learning that ascribes the role of a student to that of a passive receiver of knowledge from an expert source – in this case the teacher. From Geoffery's perspective, his only responsibilities as a learner are to attend classes, to listen to the instruction provided by the teacher and to consolidate knowledge through exemplars provided by the teacher. In this instance, Geoffery believed the creation of a revision sheet was the teacher's responsibility and finds it difficult to accept that this role should be shared by the students.

Geoffery's participation in classroom events began to change when he was drawn into the activity described above. He offered to present his program which included the initial screen illustrated in Figure 8.2 (Screen 1) and also two screens which contained a question that appeared after vector components were entered but before the final calculations was displayed (Screens 2 & 3). If option 1: YES was chosen (from Screen 3), the user received an answer to the problem (Screen 4). The selection of option 2: No resulted in being displayed – a taunt (Screen 5).

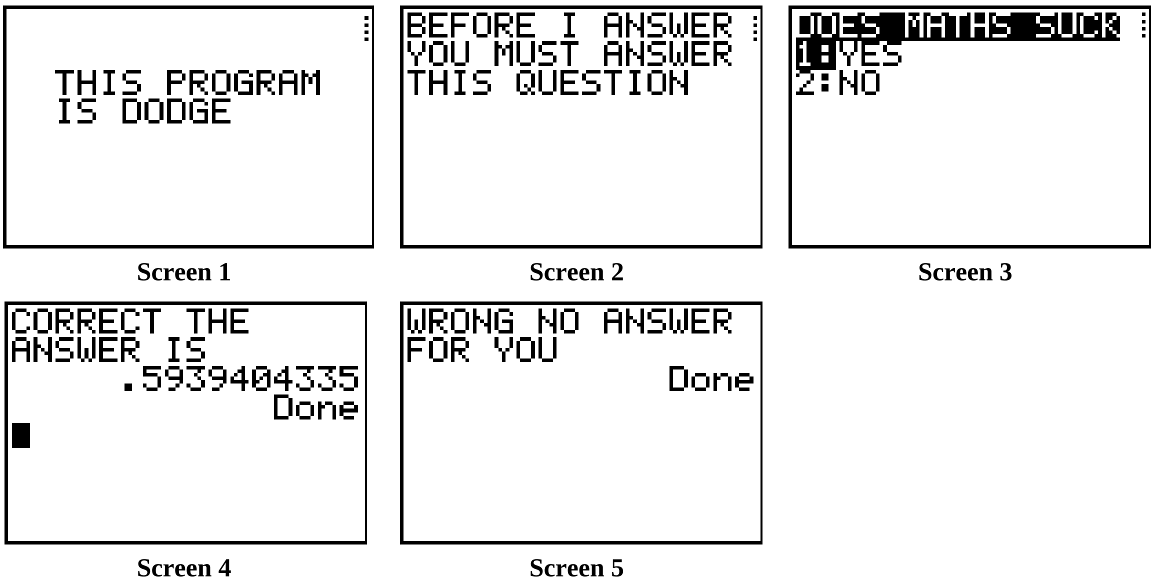

Figure 8.2. Geoffery's program for solving a 3 dimensional vector problem

Geoffery used the task to demonstrate dissent in relation to the culture the teacher had established in this mathematics classroom. This was a clever use, by Geoffery, of the very method of discourse the teacher was using to encourage students' participation in a classroom community of inquiry, in order to record a protest. Despite such an open challenge, the teacher did not issue a reprimand of any type, as he recognized that this would only be counter-productive. Instead, Geoffery was complemented on his ingenuity and praised for the quality of his program.

Geoffery responded, over subsequent lessons, by increasing his involvement in classroom presentations whenever technology was used to mediate discussion. This included presentations to the whole class of improved, and increasingly sophisticated, versions of his initial program.

After some weeks, Geoffery asked if he could present an animated program he created which depicted the adventures of mathematical objects (various irrational numbers) as human-like characters – *Dodge: The Movie*. The enthusiastic and admiring response to his "movie" (and the sequel – *Dodge II: The Revenge of Dodge*) was significant in drawing this student into the kind of mathematical discussion he had previously resisted, and he became a willing participant in subsequent discussions both technology-focused and otherwise.

Geoffery had initially used a method of discourse he had previously resisted to register dissent in relation to the way his mathematics classes were conducted. However, after receiving positive reinforcement from his peers (and no negative feedback from the teacher) for his initial and subsequent presentations, he was slowly drawn into the ways of interacting with his learning community that he has previously shunned; initially when technology was involved and then, eventually, at other times. In this episode, technology has again fulfilled the role of more than a simple tool. Firstly, technology was used as a *Partner* that assisted Geoffery to express a personal frustration that resided in a conflict between his view of how to learn and do mathematics and the social and cultural norms for doing this in his particular classroom. Technology had acted as a *Partner* "in crime" in this instance. Secondly, technology was used as a supportive *Partner*, a "go-between", that encouraged him to move from the fringes of his learning community into the mainstream.

8.2.2. Episode 2

This episode spans two consecutive lessons in Year 11 in August 1999 (see Goos, Galbraith, Renshaw & Geiger, 2003 for a more extended analysis). As the option existed within the Mathematics C syllabus for schools to design and teach a topic of their choice, the teacher had chosen to introduce students to iteration as one of the central ideas of chaos theory. This topic was presented as a teacher-prepared booklet containing a series of spreadsheet examples and tasks for students to work through at their own pace. One particularly challenging task involved using iterative methods to find approximate roots of equations such as $x^3 - 8x - 8 = 0$. The equation may be expressed in the form $x = F(x)$, and a first approximation to the solution is obtained by estimating the point of intersection of the curves $y = x$ and $y = F(x)$. This approximate solution is used as the initial value in a two column spreadsheet, where the first column provides input *x*-values for $F(x)$ in the second column, and the output of $F(x)$ becomes the input of subsequent iterations. Figure 8.3 shows the calculation when $F(x) = \frac{x^3}{8} - 1$. Cell B4 contains the formula =(1/8)*((A4)^3)-1 and cell A5 contains =B4. Both of these formulae were then copied down into the other cells in these columns.

Depending on the way in which the original equation is rearranged and the initial value chosen, the iteration may converge on a solution (as in Figure 8.3), or generate increasingly divergent outputs and hence no solution (for example, see Figure 8.4).

	A	B	C
1	x=(x^3/8)-1		
2			
3	x	F(x)	
4	1	-0.875	
5	-0.875	-1.08374	
6	-1.08374	-1.15911	
7	-1.15911	-1.19466	
8	-1.19466	-1.21313	
9	-1.21313	-1.22317	
10	-1.22317	-1.22875	
11	-1.22875	-1.2319	
12	-1.2319	-1.23369	
13	-1.23369	-1.23471	
14	-1.23471	-1.23529	
15	-1.23529	-1.23562	
16	-1.23562	-1.23581	
17	-1.23581	-1.23592	
18	-1.23592	-1.23598	
19	-1.23598	-1.23602	
20	-1.23602	-1.23604	
21	-1.23604	-1.23605	
22	-1.23605	-1.23606	
23	-1.23606	-1.23606	
24	-1.23606	-1.23607	
25	-1.23607	-1.23607	
26			

Figure 8.3. Spreadsheet method for solving $x^3 - 8x - 8 = 0$ by iteration, rearranged as $F(x) = \frac{x^3}{8} - 1$, with initial value x= 1.

	A	B	C
1	x=(x^3/8)-1		
2			
3	x	F(x)	
4	4	7	
5	7	41.875	
6	41.875	9177.558	
7	9177.558	9.66E+10	
8	9.66E+10	1.13E+32	
9	1.13E+32	1.79E+95	
10	1.79E+95	7.2E+284	
11	7.2E+284	#NUM!	
12	#NUM!	#NUM!	
13	#NUM!	#NUM!	
14	#NUM!	#NUM!	
15	#NUM!	#NUM!	
16	#NUM!	#NUM!	
17	#NUM!	#NUM!	
18	#NUM!	#NUM!	
19	#NUM!	#NUM!	
20	#NUM!	#NUM!	
21	#NUM!	#NUM!	
22	#NUM!	#NUM!	
23	#NUM!	#NUM!	
24	#NUM!	#NUM!	
25	#NUM!	#NUM!	
26			

Figure 8.4. Spreadsheet method for solving $x^3 - 8x - 8 = 0$ by iteration, rearranged as $F(x) = \frac{x^3}{8} - 1$, with initial value x= 4.

Rearranging $x^3 - 8x - 8 = 0$ as $x = \frac{x^3}{8} - 1$ yields only one of the three roots (–1.236). To find the other roots of this cubic equation (–2 and 3.236), students must investigate other rearrangements and a range of initial values. Thus the task afforded the use of technology as a *partner* in the sense that the spreadsheet approach provided a new way for students to tackle the task of solving cubic equations.

Some students chose to use their graphing calculators by plotting the graphs of $y = x$ and $y = F(x)$ to make a realistic first approximation to the roots of the equation (see Figure 8.5). In addition to spreadsheets, function graphing software and well as TI-83 graphing calculators were available to tackle this task.

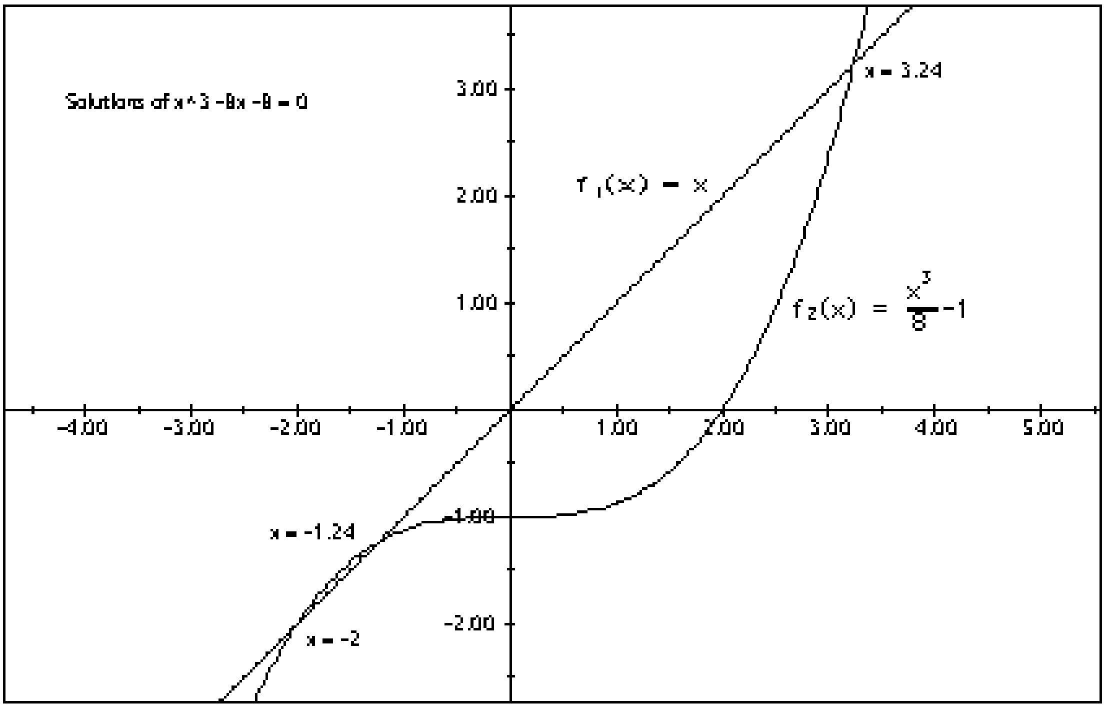

Figure 8.5. Graphical approach to solving the equation $x^3 - 8x - 8 = 0$

Excerpt 1

Four students (Keira, Gena, Demi, Adam) clustered around a laptop computer, sharing the responsibilities of pencil-and-paper and keyboard work. (Other similar groups were working on the same task in the classroom.) They ignored the written instructions, on how to use the spreadsheet method, which accompanied the task, and instead launched the graphing software installed on the laptop computer:

> Keira: Should we be using the spreadsheet?
>
> Gena: I don't think so ... the spreadsheet's just a way of checking.

The students rearranged $x^3 - 8x - 8 = 0$ as $x = \frac{x^3}{8} - 1$ and plotted it on the same axes as $y = x$. Three intersection points were clearly visible (see Figure 8.5), much to their dismay:

> Demi: Oh no no! It's gone through it too many times!

They zoomed in on only one intersection point to find the *x*-coordinate, and obtained an approximate value of 3.24. Ignoring the other solutions, they used the TI-83's Equation Solver with this value entered as an initial guess. The group accepted this as "the" solution – there was no attempt to explore the other two intersections. They then moved on to the next problem.

After a few minutes Gena reminded the others that they zoomed in on only one intersection point for the cubic equation.

> Gena: We ignored the other two. Why did the Solver only pick up one?

The students seemed unaware of the limitations of the calculator's Equation Solver, which yields one solution that is closest to an initial guess within specified bounds. The lesson ended before this anomaly could be explored further.

In this segment, the students deferred to the graphing calculator and accepted the initial output without questions. In doing so, technology has acted as a *Master* as this group of students blindly accepted the output produced by the Equation Solver without monitoring its reasonableness in the light of the graphical evidence before them.

Excerpt 2

At the start of the lesson the *Consul* (the research assistant whose role is described in the methodology chapter of this thesis – Chapter 4) mentioned to the teacher that this group of students had not used spreadsheets at all. The teacher repeated the task instructions to the whole class, emphasising the importance of the spreadsheet approach.

Consul: You accepted this (i.e. $x = 3.24$) as the only solution ... Did it occur to you to explore the possibility of other solutions at all?

Demi: We didn't realise! We only did when [the teacher] told us to.

Here the teacher simply wanted the students to follow the task instructions and begin to apply the spreadsheet as a tool to carry out the repetitive calculations involved in the iteration process. His intervention at this point moved the students away from their uncritical acceptance of the Equation Solver answer from the previous lesson, towards using technology as a *Servant* in order to demonstrate the utility of a spreadsheet in performing time consuming calculations.

The students started on the cubic problem again, this time using a spreadsheet. They entered a formula equivalent to their original rearrangement of the equation ($x = \frac{x^3}{8} - 1$) and "filled down" the columns until the values converged. However, their answer, –1.23 (see Figure 8.3), did not match the graphical result obtained earlier:

Demi: But we got 3.24!

Keira reminded the group that there were three intersection points visible on the graph, and suggested they might find the other two solutions if they continued scrolling down their spreadsheet. When this was not successful they called the teacher over and requested clarification as to how the spreadsheet worked. He re-focused the group on the important elements of the task, and issued a challenge:

Teacher: Is it possible to use the spreadsheet to get all three solutions?

By juxtaposing the spreadsheet, showing only one solution, with the graph, which displayed all three, the teacher attempted to have the students use technology as a *Partner* to re-organise their thinking and engage with the task in the way he had originally intended.

The students found that trying different initial values made no difference to their position: the spreadsheet values either converged on –1.23 or became increasingly large. David reproduced the graph previously plotted on the computer with the aid of the TI-83, thus enabling the graph and spreadsheet to be viewed simultaneously. The *Consul* questioned the students about this action during a follow-up interview.

Consul: I noticed you used the TI-83 to draw graphs.

Adam: It's quicker than multi-tasking!

Gena: Otherwise we'd have to swap around (i.e. between spreadsheet and graphing program) using the computer and it takes ages.

Adam's words seem to imply he viewed the TI-83 as a technological *Servant* that provided a more efficient way of viewing both representations at the same time. However, the very act of coordinating different types of technology in this way also resonates with the metaphor of technology as a *Partner* that transforms the nature of mathematical tasks and hence the reasoning processes students need to employ in solving them.

The students continued trying different initial values, to no avail. After conferring once more, they called on the teacher again:

Adam: Are you going to tell us what to do now?

Teacher: No ... I'm going to tell you to take a walk around the class and see how other people have done it.

Gena: Have they done it?

Teacher: Other people are trying it. It might interest you to see how.

Through his intervention at this point the teacher reinforced the role of technology as a *Partner* in mediating mathematical discussion between students. He was aware that other groups of students had rearranged the cubic equation in different ways and thus obtained different solutions, and, realising that the focus group of students had exhausted their own intellectual resources, he wished to prompt further discussion focused on other groups' computer screens.

The four students dispersed to consult with other groups, and discovered two other ways of rearranging the equation: $x = \sqrt[3]{8x+8}$ and $x = \dfrac{8x+8}{x^2}$. These gave the "missing" spreadsheet solutions of 3.24 and –2 respectively. During the follow-up interview with students, the *Consul* explored this idea further:

Consul: Would you have thought of doing that (i.e. visiting other groups) on your own?

Adam & Demi: [in unison] No – We're too self-centred!

On reconvening the group, the students pieced together the information they had obtained, set up the relevant spreadsheets and confirmed they had found all three solutions. This resulted in some excitement as no other group had managed to do so.

Making a spur of the moment decision, the teacher asked the group to connect their laptop computer to the data projector and present their findings to the class. The students quickly decided who would operate the computer keyboard, data projector remote control (which permits scrolling and zooming independently of the computer), and laser pen. Although they had no time to prepare explanations, a communally constructed argument emerged through questioning by the teacher and other members of the class. The teacher's comments and queries had the effect of drawing attention to salient aspects of the task and ensuring that other students saw how different technologies created different representations of the task:

Adam: (showing spreadsheet) Basically the very first equation we - that we used we reorganised from the basic equation was eight minus *x* cubed over negative eight, and that was just using all terms and stuff. We rearranged it –

Teacher: OK slow down. So what we are establishing here are that there are different ways of arranging the equation, which is a very important thing. Most people don't recognise that for a start.

Gena: We found that there are three different ways ...

Teacher: There are at least three different ways?

Adam: Yes. To start with our group actually used the graph to find the three intersections.

Teacher: Have you got the graphs there?

Adam: (shows graph) And that shows the three intersection points.

Mathematical and communications technologies were thus seamlessly integrated to share and support argumentation on behalf of the group of students, suggesting that technology became an *Extension of self* for the members of this group. The excitement was obvious in the follow-up interview conducted by the *Consul*.

Consul: What made this task exciting compared with other things you'd been doing?

Students: [overlapping] It was new! Like a prac, very hands on. You didn't have to sit there and listen. And we got involved because we were working with friends. We were doing it ourselves, not just listening to the teacher. And seeing something visual helped our understanding.

Keira: You feel you've achieved something when you did it all by yourself!

Consul: So you created something that was yours, very uniquely yours.

Adam: We'll call it Demi's conjecture! (referring to the teacher's practice of naming conjectures after the students who propose them)

The students' recollections of this experience hint at the sense of autonomy and power associated with appropriating technology into one's personal repertoire of mathematical practice, that is, as an extension of self.

8.3. Learning Mathematics as a Whole Class Activity

Technology is often viewed as a neutral tool used for the illustration of mathematical ideas and concepts but with little potential for mediating interaction (Doerr and Zangor, 2000) The episodes described above demonstrate the potential of technology, including associated presentation tools, for mediating whole class collaborative activity. This included the drawing in of students who are initially reluctant to engage in, or in some cases resist, the social and cultural norms of this community of learners. This suggests the MSPE framework can be extended to encompass uses of technology which promote student-student-technology interaction in whole class settings. This further elaboration is described below.

Technology as Master

The use of technology in public forums is problematic. This might be due to an inability to make effective use of the available technology or an inability to make corrections "on the fly" when errors are identified during a presentation. Technology also acts as *Master* if an audience blindly accepts a faulty idea or solution to a problem by deferring to the authority of the technology without question.

Technology as Servant

In this mode technology is used for the public delivery of pre-worked solutions to tasks. Because of the non-contentious nature of the presentation, little exploration or debate results as a consequence. Technology is essentially used as an electronic blackboard.

Technology as Partner

Technology is used to explore and investigate a problem or idea “live” in a public forum. Here, technology assists in focusing the intellectual resources of the community in order to explore ideas, offer critique of existing work, or suggest improvements to work where faults are identified. Technology is also used to provide support for the engagement of members of the community.

Technology as Extension of Self

This expression of the framework is characterised by the seamless use of technology for public investigation of problems or presentation of proposed solutions. Technology may be used to orchestrate and sustain community wide enquiry into a task or problem or to invite the critique of a proposed solution to a novel task.

8.4. Summary and Conclusion

This chapter has drawn on students' responses to questionnaires, classroom observations and transcripts of students’ in-class conversations to document their perspectives on the use of technology in whole class settings and to extend the MSPE framework to include roles of technology in public forums such as whole class discussion.

The analysis of questionnaire data indicated that students recognised the existence of a range of benefits associated with using technology to present mathematical ideas and problem solutions to the whole class. Most notable among the advantages were: the clarity of explanations that appeared associated with technology enhanced presentations; the high level of understanding required by a presenter of a problem and solution in order that they are clearly communicated to an audience; and the benefits to the audience of viewing and discussing other students' alternative solutions. Further, students recognised that the nature of presenting with technology changed the role of all class members to that of participants in a community who were responsible for the learning of the group as a whole.

The interpretation of observation and transcript data provided evidence that technology has the potential to mediate whole class interaction in a productive and inclusive fashion. These data provided examples of students helping each other to repair faulty solutions "on the fly" in a public setting and of students orchestrating the exploration of a problem by utilizing the talents of the whole class. In addition, an instance is described where technology appears to mediate the interaction of a student, who usually participated at the margins of the classroom community, with the whole class.

The range of whole class interactions described in this chapter implies the MSPE framework should be extended to include student-student-technology interaction in whole class settings. In public settings, technology was viewed as a *Master* if a student was unable to correct a fault or act on advice from class members while presenting to the class. Technology was used as a *Servant* to present pre-worked solutions to problems which did not require further interaction – much as writing the worked solution to a problem on a traditional blackboard. The episodes described above revealed the use of technology as a *Partner* in a variety of formats. Firstly, technology was used as a vehicle for exploring a problem by tapping into the intellectual resources of whole class, as in the case of the "Chaos" problem where students gathered information from other groups by viewing their computer or calculator screens and then assembling these different perspectives into a complete solution. Secondly, technology was used by Geoffery as a *Partner* in expressing his dissatisfaction with the modes of knowledge creation and confirmation that were part of the culture of his mathematics classroom by creating an amusing twist to a mathematical program. Thirdly, technology acted as a *Partner* in supporting Geoffery to participate more fully in a class culture he had all but rejected initially through presentation of his calculator movies and eventually at all other times. Evidence for the use of technology as *Extension-of-self* is found in the final presentation of the group engaged in the "Chaos" problem where they presented their finding with little notice. In this instance, students constructed a communal argument "on the fly" and integrated the use of technology to support their solution in a seamless fashion.

These episodes suggest that technology might be reconceptualised as a quasi-peer in some circumstances. The notion that digital technologies can be regarded as quasi-peer within a community of practice extends Vygotsky's notion of a ZPD to include technology as contributing member to a group of learners rather than only as a cultural tool.

Chapter 9

Conclusion

In the final chapter of this thesis the various themes pursued in each of the preceding chapters and associated findings are synthesised. The theoretical significance of these findings as well as their practical implications for school mathematics classrooms are considered. Finally, opportunities for further research, suggested by this study, are discussed.

9.1. Summary and Synthesis

This study has investigated aspects of students' use of digital technologies in learning mathematics. Students' dispositions and preferences for using technology have been explored in personal, collaborative, private and public contexts. While there have been previous attempts to theorise students' usage of technology in the process of mathematical learning (e.g., Doerr & Zangor, 2000; Guin, Ruthven & Trouche, 2005), these appear to be founded on individualistic notions of knowledge development and so fail to incorporate the role of collaboration between classroom participants, in concert with technology, in learning, reasoning and understanding. This study has taken a socio-cultural perspective which places interaction and activity, between both human and non-human participants, at the centre of theory development. The situation of this study in a working mathematics classroom grounds theory, including the associated practical implications, in a context which is more readily identified by practitioners as relevant to their needs.

The direction of this thesis has been guided by the following research questions:

1. What are the dispositions and preference of students towards using technology in learning mathematics?
2. What are the perceptions of students with respect to their global facility and confidence with digital technologies as a personal resource?
3. What choices of specific forms of technology use are favoured by students?
4. What choices of general strategic purposes for technology use are favoured by students?
5. What roles can be identified for technology in mediating individual student learning?
6. What roles can be identified for technology in mediating collaborative student learning?

These questions dictated the direction of the study's strands of inquiry of this study. The questions were shaped by the theoretical framework and operationalised via the research design, both of which are revisited below.

9.1.1. Theoretical Framework

The theoretical framework for the study was formed from two bodies of research literature. Firstly, the corpus of literature related to intellectual development and cognition was considered. Secondly, research into the role of digital technologies in enhancing student learning in mathematics was discussed.

The roles of digital technologies in enhancing learning, reasoning and understanding in mathematics, in both individual and collaborative contexts, are central issues pursued in this study. The perspective offered by socio-cultural theory was chosen to frame this investigation as it emphasises both the role of students' own activity and interaction in intellectual development, and the importance of tools in mediating learning. The related field of discourse emphasises the critical importance of language in the appropriation of conventions of reasoning and knowledge generation within learning communities. These are vital processes in classrooms where learning is viewed as the co-construction of knowledge by all participants and so research into communities of inquiry also brings a complementary perspective on events described in this thesis.

How tools, such as the digital technologies examined here, mediate learning, particularly in collaborative contexts, is an area of limited attention in current research literature. Some studies have concluded that while the formation of technology as a tool for learning requires interaction and negotiation between students and teachers, the use of technology after this formation can inhibit productive, collaborative interaction in the mathematics classroom (e.g., Doerr & Zangor, 2000). Other studies have attempted to incorporate a social dimension to how students learn in concert with technology by theorising the role of the teacher as an "orchestrator" of social interaction (e.g., Guin, Ruthven & Trouche, 2005). Neither of these positions, however, place social interaction at the centre of the process of thinking, reasoning and learning, nor do they support theory that suggests technology can be seamlessly integrated into ongoing collaborative processes.

The theory of distributed cognition and research into collaboration with digital technologies, such as that being conducted into interaction and discourse in CSCL (e.g., Stahl, 2006), appear to have greater potential to inform researchers of the nature of social interaction and collaboration in technology rich environments. Borba and Villarreal's (2006) call to consider a single unit of analysis, humans-with-media, in studies that focus on learning mathematics through the use of technological tools also points to a direction in research that attempts to conceptualise the role of technology in social contexts, such as school mathematics classrooms, in a more holistic manner. Many such studies, however, have been based around technologies designed for collaboration within virtual communities rather than those in which participants are physically proximate – as in most school mathematics classrooms. Thus, while both of the bodies of literature that form the theoretical framework of this thesis provide a critical foreground to the investigation framed by the research questions, the gap in knowledge that currently exists lies in how these two bodies of knowledge connect in school classroom settings, as illustrated in figure 9.1 below.

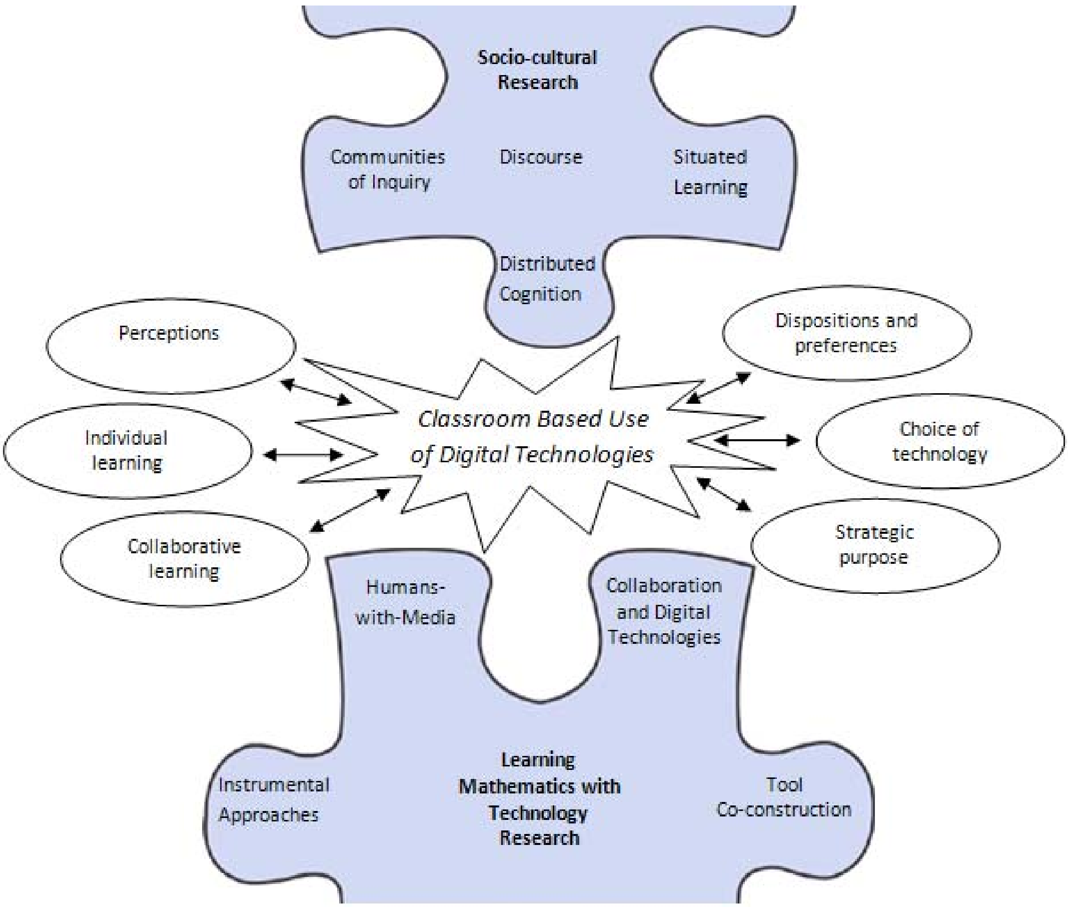

Figure 9.1. Overview of theoretical framework

This study sought to investigate interactions which take place between learners working in concert with technological tools in the complex social setting of a school classroom. The research questions which guided this investigation arose from gaps in research knowledge outlined in section 9.1 above. These questions considered interactions between students and technology in individual, small group and whole class contexts and focused on the role of technology in mediating *individual* and *collaborative* learning and students' *dispositions* and *preferences* in relation to their *modes* of working with, and *choices* of, available digital technologies. Further, students' *strategic* use of technological tools to enhance both learning and problem solving capabilities was also explored.

Because data were gathered from within a complex learning environment in which it was difficult, if not impossible, to control or partition out the effects of a wide range of variables, as would be the case in experimental studies, a naturalistic methodology was employed. The investigation was longitudinal over a period of three years and utilised participant observation (video and audio taping), student interviews and student surveys (employing both multiple choice and open response items).

The metaphor of a zoom-lens was adopted for the different foci applied to student activity and interaction within the classroom. By "zooming in" the focus was fixed on how individual students worked and interacted with digital tools. By "zooming out" to the middle ground, the investigation examined technology mediated interactions between students and technology in small groups. Finally, by "zooming out" again, the broader landscape of how students worked with technology in public, whole class settings was brought into view.

While the notion of a lens can imply a tightly focused search for knowledge, excluding any peripheral events that occur during an investigation, this study sought to embrace emergent uses of technology. As emergent uses of technology can sometimes provide the most exciting outcomes and point the way to more innovative and creative uses of a technology than for which it was designed (Ramsden, 1997), emergent uses have been actively sought after as part of the data gathering processes for this thesis.

9.1.2. Main Findings

Findings can be viewed as interpretations of the technologically rich relationships that exist between the student participants of the classroom at the centre of this study, as illustrated in figure 9.2 below, and the choices they make, preferences they exhibit and dispositions they display while working within these relationships.

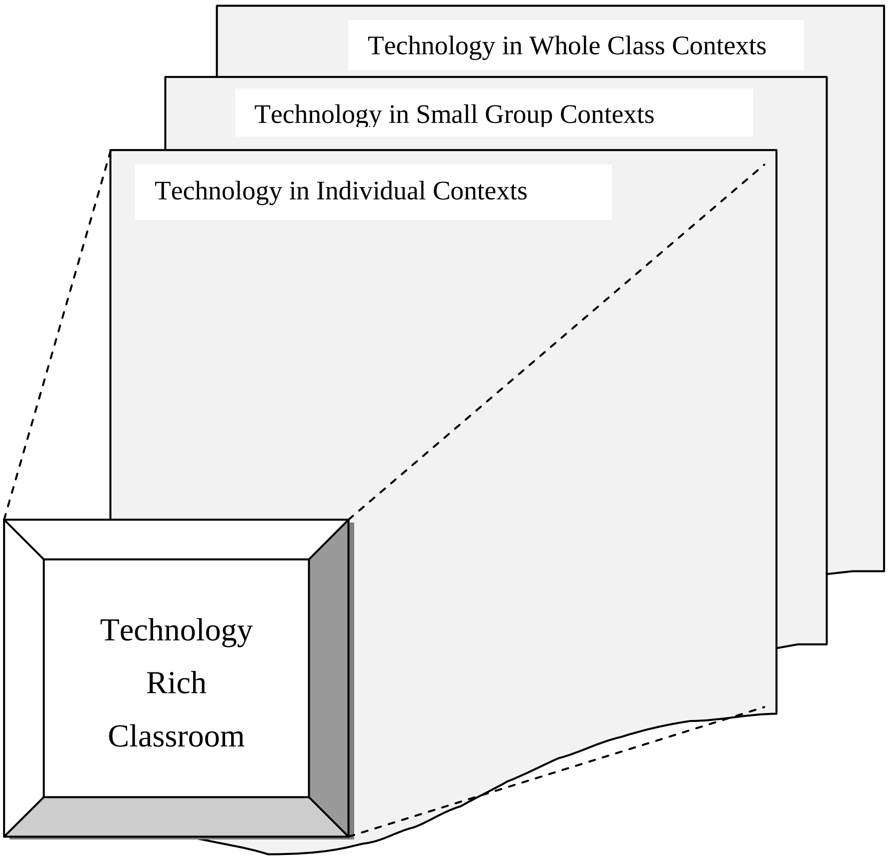

Figure 9.2. Contexts for technological usage

From the detail available by zooming in to the level of the individual, students in the pilot study generally favoured using technology to explore mathematical problems or for fixing mistakes. They were also prepared to persevere with the use of digital tools even when solutions were not immediately apparent. Students indicated that technology provided them with options for looking at mathematical problems in different ways as it extended the repertoire of representations available. These representations were accessible in an efficient manner in terms of time and effort. This allowed students to explore and seek solutions to problems by testing and verifying hunches and conjectures "on the fly". Students thus appreciated technology for the capabilities, power and efficiency it made available to them.

While this group of students was generally supportive of the use of technology in learning mathematics they expressed concern over how long they would retain mathematical knowledge aquired with the assistance of technology. Students also indicated a preference for learning new mathematical concepts first, without the support of digital technologies, before they attempted to solve problems where they were happy to make use of calculators or computers. These dispositions were interpreted as a maturing attitude toward the use of technology as students had become more aware of the advantages and disadvantages offered by digital tools through the duration of the pilot study. This more considered position was further evidenced by comments that stressed the need to think carefully about a problem first before simply rushing in making use of technology in a strategically superficial way.

The main study supported most of these preliminary findings with students recording a high level of acceptance and support for the use of technology to assist in learning mathematics. In particular, students expressed an appreciation for the advantages offered by technology in dealing with problems that were computationally demanding or tedious and for developing competency with new ideas because of the immediacy of the feedback provided by digital tools. Students again expressed concern about the longevity of acquired mathematical competencies when mathematics was learnt with the support of technology and also reported a preference for learning mathematical ideas and concepts before the introduction of technology. While recording these concerns students indicated an increasing appreciation, through the duration of the study, for the use of technology in exploring and investigating mathematical problems. This appears to stand in contrast to their preferred approach to learning mathematics initially but may indicate that positive changes to ways of learning take time to develop and that previous approaches to learning mathematics have a powerful and lasting effect on students' preferences in relation to and dispositions toward instruction.

A range of views was apparent in relation to students' choices of technology. Some students displayed a strong preference for a particular technological tool – a computer in preference to a graphing calculator or one type of calculator over another. Students who indicated an affinity with computers over calculators suggested the larger screen size and greater range of functions as reasons for their preference. Those students who preferred calculators argued that the portability, ease of operation and specialisation toward mathematical activity provided advantages over computer based technologies. Some students indicated, however, that their preference was based on familiarity alone.

Another group of students' preferences were task dependant. These students claimed their preferences were dictated by the technological power required for a particular task. So, for example, they tended to use the simplest available calculator available if the task at hand required only straight forward numerical calculation, but would choose a technology with the relevant power and screen size necessary to accommodate operations with large matrices. This disposition indicates a high level of technological maturity as the capacity to make choices between technologies demonstrates an understanding of the strengths and weaknesses of available digital tools. The emergence of this disposition from the data related to the main study, when it was not prevalent in the pilot, may be related to the greater period of time over which the main study was conducted – 2 years as compared to six months. Students in the main study, therefore, were provided with greater opportunity to develop an understanding of the suitability of different technologies for different types of tasks. This provides further evidence that the capacity to make strategic use of technology takes time and experience even among students who have a strong disposition towards the use of technological tools.

A framework of student behaviour in relation to technology use was developed. The categories that form the framework emerged from observational data of students' use of technology while learning new mathematics or while working on mathematical problems. The existence of these categories was validated by testing the capacity of the framework to align with students' individual responses to open ended questions about aspects of their use of technological tools. These categories, technology as *Master*, *Servant*, *Partner* and *Extension-of-self*, are described in chapter 6 and summariesd in Figure 9.3.

It must be noted that while these categories are generally hierarchical they are neither discrete nor final in relation to a student's technological behaviour but represent classes of students' potential in relation to technological usage. A student, for example, may make use of technology as a *Servant* or as a *Partner*, depending on the nature of a task, but they must have developed the capacity to do both in order to work at these levels.

Some student behaviours indicated that they were in transition between levels, a process confirmed by a number of responses to the self-identification item on the last student technology questionnaire in which they identified their patterns of usage as typical of two different but adjacent levels. The identification of students in transition between levels and the small number who demonstrated they could operate at the highest level indicated that the capacity to operate at different levels within the framework is dependent on time and experience.

Technology in Small Group Contexts

By adjusting the metaphorical zoom lens to the middle ground, student-student-technology interactions can be examined within small group contexts. In general, students expressed views that were firmly in favour of collaborative learning environments. Initially this collaboration supported investigational mathematical activity with technology used to support exploration as well as assisting with technical issues associated with the use of a technology. With experience, however, less support was needed with technical issues and students tended to collaborate only when they reached a mathematical impasse. Thus there was a greater prospect of students making collaborative use of technology if the demands of a task exhibited a complexity beyond that of the procedural. As students indicated that they would have looked to collaborate with others even if no technology was available, then collaborative uses of technology can only be expected when students perceive that digital tools will offer advantages in responding to the challenge offered by the task.

Advantages offered by technology in small group collaborative work include the provision of a common medium for mathematical discourse to take place. A graph, for example, could be used to focus discussion about an aspect of a mathematical problem as it provides a common representation of an individual's interpretation of the problem that can be discussed and manipulated by all members of a group. In this way technology can introduce a common representational language as well as fostering the development of a common reasoning discourse.

It was also observed that not all technology mediated collaborative activity was productive. The problem in which students were required to provide a geometric representation of the square root of 45, described in Chapter 7, is an example where the facilities offered by technology allowed a group to be side tracked as they explored the problem in a manner unrelated to the set task.

The metaphors of *Master*, *Servant*, *Partner* and *Extension-of-self* were also relevant to the small group context. Student behaviours within the MSPE framework in small group contexts are described in Chapter 7 and summarised in Figure 9.3.

Technology in Whole Group Contexts

The potential of technology in this context lies within the capacity of digital tools to mediate whole class presentations and discussions. These tools may also facilitate the inclusion of students who may be reluctant or resistant to participating in public discourse.

Students identified advantages of technology to the presenter and the audience in this context. Public presentation of a mathematical idea or of a problem solution provided presenters with immediate technical support, if a difficulty was related to the technology, and mathematical support if errors or blockages to a solution pathway were identified. In general, students did not register concern about the potential for over enthusiastic criticism of presentations and viewed the opportunity to present as a supportive experience where they could obtain feedback on work in progress.

From the perspective of the audience, students commented on the quality of student presentations and how work was generally presented in a clear and organised fashion. While an organised structure might have been imposed on a presenter by the nature of the technology in use, it is also possible that presenters felt a responsibility to carefully prepare their work because of the public mode of their interaction with the class. Students also commented that the audience would benefit from exposure to alternative approaches to the same problem offered by different presenters.

The MSPE framework is also relevant within public contexts as described in Chapter 8 and is summarised in Figure 9.3.

	Master	Servant	Partner	Extension-of-self
Individual	The student is subservient to the technology – a relationship induced by technological or mathematical dependence. If the complexity of usage is high, student activity will be confined to those limited operations over which they have competence. If mathematical understanding is absent, the student is reduced to blind consumption of whatever output is generated, irrespective of its accuracy or worth.	Here technology is used as a reliable timesaving replacement for mental, or pen and paper computations. The tasks of the mathematics classroom remain essentially the same – but now they are facilitated by a fast mechanical aid. The user 'instructs' the technology as an obedient but 'dumb' assistant in which s/he has confidence.	Here rapport has developed between the user and the technology, which is used creatively to increase the power that students have over their learning. Students often appear to interact directly with the technology (e.g. graphical calculator), treating it almost as a human partner that responds to their commands – for example, with error messages that demand investigation. The calculator acts as a surrogate partner as students verbalise their thinking in the process of locating and correcting such errors.	Users incorporate technological expertise as an integral part of their mathematical repertoire. The partnership between student and technology merges to a single identity, so that rather than existing as a third party technology is used to support mathematical argumentation as naturally as intellectual resources. The relationship between student and digital tool facilitates the transformation of tasks.
Small Group	All group members are limited by their facility with technology. Interaction (in terms of mathematical progress) is also limited as a result. The group might be drawn into a familiar but inappropriate technology based approach to solving a problem. Insufficient knowledge of technical issues related to the function of a digital tool may limit the group's interpretation of displayed results.	Small groups of students use technology to respond to routine tasks. Interaction is limited to comments on technical use of technology (e.g. which buttons to push). The digital tool is used as a physical artefact used to communicate information.	Technology is used to explore and investigate a problem but there is also evidence of the technology playing a part in the facilitation of collaborative processes. Calculator or computer output also provides a stimulus for peer discussion as students cluster together to compare their screens, often holding up graphical calculators side by side or passing them back and forth to neighbours to emphasise a point or compare their working. Work can be progressed "live" on any group members' display. Technology may provide support that facilitates students' engagement in group interaction.	*Students working together may initiate and incorporate a variety of technological resources that help transform the approach in the pursuit of the solution to a mathematical problem.*
Whole Group	The use of technology in public forums is problematic. This might be due to an inability to make effective use of the available technology or an inability to make correction "on the fly" when faults are identified in the idea presented or solution to a task.	Public delivery of pre-worked solutions to tasks. Little need for exploration or debate. Technology is used as an electronic blackboard.	Technology is used to explore and investigate a problem or idea in a public forum. Technology is used to focus the intellectual resources of the community to help explore ideas, offer critique of existing work or suggest improvements to work where faults are identified. Technology is used to provide support for engagement in this community of inquiry.	Seamless use of technology for public investigation of problems or presentation of proposed solutions. Technology is used to orchestrate and sustain community wide inquiry which results in the transformation of a task or problem.

Figure 9.3. The *Master, Servant, Partner* and *Extension-of-self* framework

Note: The descriptor in *Extension-of-self* for small group settings is recorded in italics to indicate it is inferred from behaviours described in individual and whole group settings

9.2. Contribution to Theory and Limitations of the Study

This study investigated various aspects of student-student-technology relationships within an authentic classroom setting. The focus on the interactive and dynamic relationship between students and technology is a significant point of difference from current theory where technology has generally been considered a tool that merely amplifies human capabilities. The detail of the contribution of this thesis to knowledge within the theoretical framework, outlined in Figure 9.3, is presented below along with a discussion of the limitations of this study.

9.2.1. Student-Student-Technology Relationships in Classroom Settings

While there have been attempts to theorise the co-construction or the instrumentalisation of technological artefacts into intellectual tools (e.g., Doerr & Zangor, 2000; Guin, Ruthven & Trouche, 2005), as outlined in Chapter 3, there appears to be little in the literature related to the difference between students' individual and collaborative uses of technology. The MPSE framework developed in this thesis identifies technology use in individual, small group and whole class settings. The major characteristics of the framework are summarised in Table 9.3.

The framework portrays the use of technology as a series of relationships between technology and individual students and between technology and different sized collectives of students. As the notion of relationship implies interaction, the ways in which students engage with technology includes the potential for dynamic, two-way working relationships between human and non-human partners. Indeed, comments from students in Chapter 6 imply that some confer a status on digital devices beyond that of inanimate tool. This portrayal of mathematical learning as an activity that is distributed across individuals, collectives, physical and symbolic artefacts, as well as environments, is consistent with Pea's (1993) concept of distributed cognition, and Borba and Villarreal's (2006) notion of humans-with-media, but extends related theory by identifying the different types of interaction that take place between human participants and digital tools when working as individuals, or in small groups or whole class settings. Further, the ways these modes of interaction are enacted within different settings, as defined by the number of participants engaged in an in-class episode, represents a differentiation in technology influenced behaviour that has not been previously addressed in research literature. While there have been studies that addressed the characteristics of collaborative behaviour in CSCL environments (e.g., Stahl, 2006) and typographies of technology use based the co-construction of tools (Doerr & Zangor, 2000) neither of these investigations recognises the influence of different types of collectives on the use of digital technologies.

Other studies, such as those by Manouchehri (2004) and Sinclair (2005), recognise that technology has a role to play in fostering a collaborative culture where conjecturing, testing and verifying results are essential parts of classroom practice, but do not distinguish between the different ways this role is manifest when students work as individuals, or in small groups or as a whole class.

Evidence for the existence of different categories in the extended MSPE framework was drawn from observational data and confirmed by testing categories against student responses to open ended items and by asking students to self identify their relationship with technology based on descriptions of the framework. A limitation of this framework, however, is the absence of evidence for the *Extension-of-self* category within small group settings. While no data were available to substantiate the position of this category within the framework it seems reasonable to hypothesise its existence given that the *Extension-of-self* mode was observed in both individual and whole class settings. A suggested instantiation of this category is recorded in italics in Figure 9.3. This description is based on the category of *Partner* in small group contexts but extends the collaborative behaviour outlined here to include the transformation of tasks.

9.2.2. New Perspectives on Communities of Inquiry

The use of technology changes and extends the nature of communities of inquiry. Aspects of these changes were documented in this study and are summarised below.

Equal Expertise Peer Scaffolding

Literature within socio-cultural theory emphasises the central role of social interaction in intellectual development and in thinking and understanding (e.g., Boaler, 1999; Boaler & Greeno, 2000; Leont'ev, 1981; Vygotsky, 1978) but has tended to assign primacy to the role of a more expert participant, typically a teacher, in the process of scaffolding students to move forward within their ZPDs or in orchestrating social interaction between students.

Chapters 7 and 8 of this study, which consider small group and whole class settings, provide evidence that technology can mediate interactions between peers of equal expertise who provide each other with the scaffolding necessary for all within a group to move forward.

Within small groups, initial ideas or conjectures were shared via the display on a computer or calculator. These initial thoughts or part solutions served as stimulus for discussion and debate. Conjectures were often tested, and confirmed or refuted, on the basis of evidence obtained via the manipulation of a representation. This could be effected by members of a group sitting in front of a computer, taking turns to introduce ideas or alter an existing image, or on a calculator by passing the device between group members, or explored on a number of calculators simultaneously and comparing outputs. These activities mediate the negotiation of collective meanings and assist in converging interpretations of phenomena that structure a joint problem space.

In whole class settings technology can facilitate the distribution of the processes of knowledge creation and testing among all members of a class. Whole class technology enhanced lessons provided a forum for harnessing the intellectual resources of the classroom community and provided a channel for wider discussion and debate. Class members also assumed responsibility for coordinating the development of shared meanings in relation to mathematical ideas, procedures or processes through discursive negotiation. As noted in Chapter 8, power relationships also change as the teacher assumes the role of a participant *in* rather than director *of* activity.

New forms of Voice and Discourse

Technology engenders different forms of discourse and facilitates the development of new "voices" within a classroom community. Researchers from the field of discursive psychology (e.g., Sfard, 2002; Sfard & Kieran, 2001; Sfard & McClain, 2002) have argued that symbolic tools (e.g., language, graphs, tables) become intertwined with the act of communicating and so with the norms of discourse which frame modes of reasoning, meaning making and understanding. This study provides evidence, through Chapters 6, 7 and 8, that physical artefacts, such as computers and graphing calculators, can also act as tools which mediate language and modes of reasoning and so broadens the notion of tool within the construct of discourse. The technology generated mathematical representations that students used in the episodes documented in this study were central, in many cases, to the reasoning processes employed by students in confirming or refuting hunches and conjectures. Students also used technology generated representations and objects to provide final justification for solutions to problems which means the use of technology has a role to play in the negotiation of a community's acceptance of "truth".

As has been argued earlier, technology can catalyse more active roles for students in processes that lead to new mathematical knowledge and understandings. As these processes require students to adopt new roles within a collective, for example, self-regulatory and negotiative positions, students must also assume appropriate voices in alignment with these roles. These voices can be those traditionally owned by a teacher or new voices that are tied specifically to interactions with technology. Technological voices include those used by an individual with particular expertise in the technical aspects of using a digital tool who can explain its use to others, or a strategic technological voice possessed by a student who is adept at instructing others how to use technology to provide alternate views of a problem.

While other researchers have concluded that knowledge development is bound to the medium of expression when participants communicate online, altering modes of communication and participation (Borba & Villarreal, 2006; Stahl, 2006), the current study demonstrates the relevance of this argument to mainstream classroom settings.

Marginalised Members of a Community

The description of two students who were reluctant participants in the classroom culture, documented in Chapter 8 of this study, is a salient reminder that not all students will readily accept approaches to learning with which they are uncomfortable. While it has been argued in this thesis that technology can provide the medium for connecting students in ways that foster collaborative activity, the provision of technology itself, or the establishment of a supportive classroom culture, does not always promote productive social interaction between all students.

One of the students described in chapter 8, Geoffery, was eventually drawn into the community of inquiry through experiences in which he was encouraged to pursue his interest in designing mathematics-based animations through programming his calculator. The technology in this instance catalysed his participation in collaborative practice – during this incident and then into the future. Consistent with this finding, Goos (2004) also acknowledges that the nature of engagement and extent of participation varies between students and concedes that a small number of students remained resistant to the adoption of a community of inquiry's modes of reasoning and action. This reminds us that even with careful planning there is never any guarantee that students will adopt the forms of reasoning and methods of discourse that characterise a technology enhanced community of inquiry.

Mathematical Identity is Not Fixed

The episode involving Geoffery, referred to in the section above on *Marginalised Members of a Community*, provides evidence that the rejection of a classroom culture by a student and his or her resulting self-marginalisation is not necessarily final. In Geoffery's case technology provided a pathway for his eventual inclusion in a classroom of inquiry culture. This suggests that mathematical identity, also, is not necessarily fixed but malleable. While Lave (1988) has described the process of *becoming* within a community of practice and Boaler (2000) argues that a student's mathematical identity shapes and is shaped by a student's attitudes and predispositions to learning, neither author has documented how a classroom identity can be altered once seemingly established. While recognising that the assertion that technology may have a role to play in engaging students in interactive approaches to learning is based on only one particular case, and so cannot be generalised, this example does provide a new perspective on how to foster inclusiveness in school mathematics classrooms.

9.2.3. Approaches to Research in Technology Rich Classrooms in Collaborative Settings

The findings of this study have implications for the methodologies used in future investigations into collaborative practices in technology rich environments. The rapid development of new digital technologies means that educational researchers will often be working in classroom environments that are being created as they are observed. The unfamiliarity of such environments will produce unanticipated results which researchers must be open to and willing to document. The focus on collaborative classroom interaction in this study also demonstrates that the unit of analysis for similar studies must accommodate student-student-technology interaction.

Emergent uses of technology

Much previous research into collaborative mathematical practice has made use of methodologies that are based on predetermined tasks and on assumptions about how students are likely to interact (e.g., Beaty & Moss, 2006; Kahn, Hoyles, Noss & Jones, 2006). This study, however, has embraced emergent uses of technology as advocated by Ramsden (1997) and so differs from current research orthodoxy.

Studies, such as Sinclair's (2005) investigation of high school students' use of Geometer's Sketchpad/JavaSketchpad, which found that interactions between pairs of students had greater impact on the collaborative learning environment than the researcher had expected, indicate the viability of this approach. It must be conceded, however, that the generalisability of findings will be necessarily limited in studies based around emergent phenomena, as it is not possible to integrate emergent and experimental methodologies. The adoption of such approaches, however, allows for more naturalistic and so less intrusive study designs which may be less intimidating to potential participants – both students and teachers. Such approaches to research may mean teachers are less reluctant to make their classrooms available to investigators and as a result assist in alleviating the problem of finding school based sites, as identified by Zbiek, Heid, Blume & Dick (2007) for future research studies.

Unit of Analysis

Conceptualising technology as an agent which mediates collaborative activity brings into question the unit of analysis appropriate for studies which seek to investigate collaborative learning practices within technology rich environments. Borba and Villarreal (2006) suggest humans-with-media as a construct that captures the intent of research in this area. This construct, however, does not accommodate the diversity of student-student-technology relationships identified in this thesis. Others, such as Lavy and Leon (2004), describe the human-technology construct as a "super entity" in which the capabilities and resources of the computer and individual learners act in synergy to contribute to a shared learning process. Again, however, this notion does not differentiate between different types of collectives working in concert with technology. The unit of analysis in this thesis has ranged from individual-with-technology, to small group-with-technology, and whole-class-with-technology and offers a finer grained perspective on how human-technology collaborations can be viewed.

9.3. Implications for Practice

This study attempted to marry two different demands made of pedagogy as part of the reform movement in mathematics education – to incorporate collaborative approaches to teaching practice and to integrate digital technologies in mathematics instruction (e.g., Australian Association of Mathematics Teachers, 2000; Australian Education Council, 1991; National Council of Teachers of Mathematics, 2000). While the naturalistic approach taken in this thesis limits the generalisability of the study, the events documented here provide evidence that technology can enhance community of inquiry based approaches to learning and that these can be integrated into the life of mainstream classrooms.

Students in this study collaborated in a variety of different ways – within small groups, across small groups and as a whole class. Within each of these settings technology played a variety of roles which suggests that there are a number of options open to practitioners who wish to take advantage of collaborative approaches to instruction that are made available by digital tools. Through this study students have demonstrated the capacity to direct themselves in making use of technology during exploratory mathematical activity. This means the use of technology need not be restricted to teacher directed presentations that focus on mastering technical aspects of a particular technology or as a presentation tool for teacher directed mathematical instruction alone.

The different categories of technology mediated collaborative practice identified in this study indicate consideration should be given to which practice best resonates with a teacher's learning intentions. Small group work, for example, was enhanced by the ready availability of different representations of a mathematical idea through technology and these images provided the stimulus for collaborative discussion within a supportive environment. In larger group settings, ideas and solutions to problems were publicly debated using a student's presentation as a starting point and focus. Presented work was adapted and improved "live" and subjected to further debate until consensus, including that of the teacher's, was achieved. The different settings, from individual to small group, to whole class, also provided a structure for students to grow through until they feel confident they can contribute in all learning formats and forums.

While the events reported in this study generally portray a very positive picture of the potential for using technology to enhance mathematical learning, it must be acknowledged that there are impediments to the adoption of the approaches to teaching and learning outlined here.

The pedagogical approach described in this thesis shares significant responsibility for learning with students. In this case the teacher had a prominent initial role while modelling and structuring practices that acted as the foundation for a community of inquiry but, over time, this role was increasingly shared with students. Such a transition might prove problematic for some teachers as it would also mean the renegotiation of power structures within a classroom. Further, the introduction of technological tools into a classroom appears to mediate independent learning opportunities which in itself can be viewed as another type of power shift. The prospect of sharing the responsibility of teaching and learning with students, and with technology, may be too unpalatable for some practitioners to contemplate.

This study was conducted in a classroom where a range of technologies was freely available, a vital factor in some of the student behaviours documented. This is not the case in most secondary school mathematics classrooms as the availability of one type of digital tool is likely to be the norm. As the facilities available to mathematics teachers are generally limited, the findings of this thesis may be regarded as largely unimplementable or irrelevant by many teachers. None-the-less, the rapid development of educational technologies combined with a decrease in their relative cost means that the range and availability of technologies to schools in the relatively near future will surprise even the most optimistic of observers. New ideas on how to make use of these technologies in school classroom settings will be needed. The findings of this study make a contribution to advancing this cause.

Finally, as the students in this study were generally very able and interested in mathematics, practitioners may perceive that the type of teaching and learning practices described here are only replicable with similar groups of students. The story of Geoffery, however, provides a salient example of a student who was initially marginalised but who was drawn into the mainstream of classroom practice through the mediation of technology.

9.4. Future Directions in Research

Opportunities for further research into a range of issues arise from this study from both methodological and theoretical perspectives.

Given the case study nature of this investigation, broadening the range of schools, classroom settings and participants is warranted in order to investigate whether similar findings are obtained in different contexts. As discussed in 9.3 above, one of the limitations of this study is that it involved generally able and highly motivated students who understood the benefits of high achievement in school subjects in relation to future career aspirations. Further, these students had almost unrestricted access to a variety of technological tools. Future studies should include a broader range of ability groupings, age groupings, classroom contexts and technological availabilities. This is particularly relevant to the finding in this study that mathematical identity is not fixed but rather malleable. The story of the marginalised student, Geoffery, has significance for those students who feel alienated within mathematics classrooms and isolated by what they may perceive to be a learning environment that is incompatible with their beliefs about learning and knowing. Extending this study into settings which include students disaffected with mathematics is of particular importance.

The MSPE framework developed through this study needs to be validated in different settings. The theorised category of *Extension-of-self* in small group settings also requires validation as evidence for the existence of this category could not be produced in this study. While the existence of this category has been conjectured on the basis of evidence that supports surrounding constructs within the framework, additional research is required to confirm that students can operate at the highest level of technological usage within small groups. Perhaps the broadening of methodological aspects of this study in future research, as outlined above, will increase likelihood of finding such evidence.

Finally, if benefits to teaching and learning are dependent on how technology is used rather than what technology is available (Burrill et al., 2002), the role of the teacher is critical in the development of effective learning and teaching practices in mathematics classrooms (National Council of Teachers of Mathematics, 2000). Research is needed into how teachers' technology enhanced teaching identities are developed and maintained. This is especially important in a world where technological change is so rapid. As illustrated through this thesis, new technological developments provide opportunity for new and enhanced pedagogies. These new pedagogies may hold the key not just to enhancing the learning experiences and outcomes for students already engaged in mathematics learning but also for those who, in their present circumstances, are not.

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Appendix 1

Letters of Permission

· GRADUATE SCHOOL OF EDUCATION

THE UNIVERSITY OF QUEENSLAND

Brisbane Qld 4072 Australia
Telephone (07) 3365 6550
International +61 733656550
Facsimile (07) 3365 7199
Telex UNIVQLD AA 40315

Dear Parent/s,

MATHEMATICS LEARNING STUDY

It is our pleasure to inform you that the Mathematics Department at Hillbrook Anglican School has agreed to participate in a study, funded by the Australian Research Council, that will involve your child's mathematics class.

We are university lecturers in the Graduate School of Education at The University of Queensland and members of our research team will be working with your child's teacher, Mr Vince Geiger, to identify more effective ways of teaching the upper secondary mathematics syllabus. In the coming year we will be observing the mathematics classroom of your child, collecting information from class members regarding their work in mathematics and on several occasions video-taping groups of students working together. Confidentiality of the identity of individual students will be maintained at all times in discussing any outcomes of the research.

Our involvement has the full support of your child's teacher, the Mathematics Department head and the Principal of your school. We will be working closely with the school to find better ways of implementing the mathematics syllabus, so that the abilitv of students to understand concepts and solve problems will be enhanced.

If you are willing to allow your child to participate in the study, please sign the consent form below and return it to the school at your earliest possible convenience. Consent can be withdrawn at any time.

If you would like further information or would like to talk to us on any aspect of the study, please do not hesitate to call either Associate Professor Peter Galbraith or Associate Professor Peter Renshaw on 33656376 or 33656497 respectively.

We look forw~rd to a rewarding association with your child in his or her mathematics classroom.

Yours sincerely

Peter Galbraith

Peter Galbraith
Associate Professor

Peter Renshaw

Peter Renshaw
Associate Professor

Please return to your child's mathematics teacher as soon as possible.

I consent to my child ...... (NAME IN FULL) participating in the study on mathematics learning being undertaken at Hillbrook Anglican School in conjunction with The University of Queensland.

Signature of parent/guardian .... Date .....

GRADUATE SCHOOL OF EDUCATION

THE UNIVERSITY OF QUEENSLAND

Brisbane Qld 4072 Australia
Telephone (07) 3365 6550
International +61 7 3365 6550
Facsimile (07) 3365 7199
Telex UNIVQLD AA 40315

Dear Student,

MATHEMATICS LEARNING STUDY

The Mathematics Department at Hillbrook Anglican School has agreed to participate in a study, funded by the Australian Research Council, that will involve you and your mathematics class.

We are university lecturers in the Graduate School of Education, The University of Queensland, who are studying ways to improve mathematics learning in the senior years.

The purpose of our study is to identify more effective ways of teaching the mathematics syllabus to senior students. In the coming year we will be working with your teacher to improve the understanding and success that students have in solving problems. We will be observing the classroom on a number of days each term, collecting information from you regarding your work in mathematics, and on several occasions video-taping you working in a group with your classmates.

Our study will not be used for assessment purposes and so will not influence your grade in mathematics. We hope that it will make mathematics more interesting and increase your understanding and use of mathematical ideas, but the normal assessment procedures employed by your teacher at this school will be followed. Your answers to our interview questions and the video-tape of you working with fellow students on the problem-solving tasks will remain confidential. We hope that the information gained will help us to understand better how different people learn and hence be able to develop better ways of teaching.

If you are willing to participate in the study please sign the consent form below.

Yours sincerely

Peter Galbraith

Peter Galbraith
Associate Professor

Peter Renshaw

Peter Renshaw
Associate Professor

I agree to participate in the mathematics learning project being undertaken at Hillbrook Anglican School in conjunction with The University of Queensland.

Name

Signature...

Date ....

Appendix 2

Topic Sequences – Years 11 and 12 Mathematics C

1997-1999

Year 11 C Outline Semester 1 1997

Term 1

Topic	**Time**	**Technology Resource**
Structures and Patterns ◊ recognition of patterns in well known structures including Pascal's Triangle and Fibonacci sequence ◊ applications of patterns ◊ life-related applications of arithmetic and geometric progressions	3.5 week	TI-83 TI-92 TI-Interactive
Introduction to Groups ◊ concepts of: - closure - associativity - identity - inverse ◊ definition of a group	2.5 weeks	
Real and Complex Number Systems ◊ structure of the real number system including: - rational numbers - irrational numbers ◊ simple manipulation of surds	2 weeks	TI-83 TI-92 TI-Interactive

Term 2

Topic	Time	Technology Resource
Matrices and Applications ◊ definition of a matrix ◊ definition and properties of the identity matrix ◊ matrix operations - addition - transpose - multiplication by a scalar - multiplication by a matrix ◊ determinant of a matrix ◊ singular and non-singular matrices ◊ group properties of matrices ◊ applications of matrices in both life-related and purely mathematical situations ◊ relationship between matrices and vectors	4 weeks	TI-83 TI-92 TI-Interactive
Vectors and Applications ◊ for vectors as structures which are used for the storage of data • definition of a vector • relationship between vectors and matrices • operations on vectors including: - addition - multiplication by a scalar • scalar product of two storage vectors • simple life-related applications of vectors ◊ for vectors describing situations involving magnitude and direction • definition of a vector • relationship between vectors and matrices • two dimensional vectors and their algebraic and geometric representation • operations on vectors including: - addition - multiplication by a scalar • scalar product of two vectors • unit vectors • calculation of the angle between two vectors • resolution of vectors into components acting at right angles to each other • applications of vectors in both life-related and purely mathematical situations	4 weeks	TI-83 TI-92 Cabri Geometry

Year 11 C Outline Semester 2 1997

Term 3

Topic	Time	Technology Resource
Real and Complex Number Systems ◊ structure and representation of complex numbers including: - algebraic definitions and interpretations - geometric definitions (Argand diagram) and interpretations - polar form - conjugates	2 weeks	TI-83 TI-92 TI-Interactive
Structures and Patterns ◊ sum to infinity of a geometric progression ◊ life-related applications of arithmetic and geometric progressions ◊ permutations and combinations and their use in life-related situations	2 weeks	TI-83 TI-92 TI-Interactive
Introduction to the Theory of Chaos and Fractal Geometry (I) ◊ basic notions of recursive functions and iterative processes ◊ exponential growth and decay	2 weeks	TI-83 TI-92 TI-Interactive Spreadsheets On-line resources
Introduction to the Theory of Chaos and Fractal Geometry (II) ◊ basic notions Fractal Geometry ◊ Koch curves	2 weeks	TI-83 TI-92 TI-Interactive Spreadsheets On-line resources

Term 4

Topic	Time	Technology Resource
Dynamics ◊ Newton's Laws of Motion in vector form - straight line motion in a horizontal plane with constant force - vertical motion under gravity without air resistance - projectile motion without air resistance	4 weeks	TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software
Dynamics Cricket Assignment		TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software

Year 12 C Outline Semester 1 1996

Term 1

Topic	**Time**	**Technology Resource**
Real and Complex Number Systems ◊ de Moivre's Theorem ◊ simple, purely mathematical applications of complex numbers	2 week	TI-83 TI-92 TI-Interactive
Structures and Patterns ◊ sequences and series other than arithmetic and geometric ◊ applications of patterns ◊ use of finite differences	3 weeks	TI-83 TI-92 TI-Interactive
Dynamics ◊ derivatives of vectors and their application to straight line motion in a horizontal plane with variable force	2 weeks	TI-83 TI-92 Cabri Geometry

Term 2

Topic	**Time**	**Technology Resource**
Introduction to the Theory of Chaos and Fractal Geometry ◊ the growth equation ◊ stability of systems modelled by differential equations	2 weeks	TI-83 TI-92 TI-Interactive On-line resources
Matrices and Applications ◊ applications of matrices in both life-related and purely mathematical situations	2 weeks	TI-83 TI-92 TI-Interactive
Calculus ◊ approximating small changes in functions using derivatives ◊ solution of simple, linear, first order differential equations with constant coefficients ◊ life-related applications of simple, linear, first order differential equations with constant coefficients	3 weeks	TI-83 TI-92 TI-Interactive

12 Maths C 1997 Semester 2

Term 3

Topic	Time	Technology Resource
Introduction to the Theory of Chaos and Fractal Geometry ◊ strange attractors ◊ the Verhulst equation ◊ stability of systems modelled by differential equations ◊ fractal dimensions ◊ potential real world applications of Chaos Theory and Fractal Geometry	2 weeks	TI-83 TI-92 TI-Interactive On-line resources Spreadsheets
Dynamics ◊ derivatives of vectors and their application to straight line motion in a horizontal plane with variable force ◊ derivatives of vectors and their application to: - vertical motion under gravity with air resistance - projectile motion with air resistance - simple harmonic motion ◊ - circular motion with uniform angular velocity	4 weeks	TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software Motion Detector
Vectors and Applications (3D) ◊ three dimensional vectors and their algebraic and geometric representation ◊ operations on vectors including: - addition - multiplication by a scalar ◊ scalar product of two vectors ◊ vector product of two vectors ◊ unit vectors ◊ calculation of the angle between two vectors applications of vectors in both life-related and purely mathematical situations	2 weeks	TI-83 TI-92 On-line resources Cabri Geometry

Term 4

Topic	Time	Technology Resource
Calculus ◊ integrals of the form $\int \frac{f'(x)}{f(x)}\, dx$ ◊ $\int f[g(x)].g'(x)\, dx$ ◊ simple integration by parts ◊ Simpson's rule	3 weeks	TI-83 TI-92 TI-Interactive

Year 11 C Outline Semester 1 1998

Term 1

Topic	**Time**	**Technology Resource**
Structures and Patterns ◊ recognition of patterns in well known structures including Pascal's Triangle and Fibonacci sequence ◊ applications of patterns ◊ life-related applications of arithmetic and geometric progressions	3.5 week	TI-83 TI-92 TI-Interactive
Introduction to Groups ◊ concepts of: - closure - associativity - identity - inverse ◊ definition of a group	2.5 weeks	
Real and Complex Number Systems ◊ structure of the real number system including: - rational numbers - irrational numbers ◊ simple manipulation of surds	2 weeks	TI-83 TI-92 TI-Interactive

Year 11 C Outline Semester 1 1998

Term 2

Topic	Time	Technology Resource
Matrices and Applications ◊ definition of a matrix ◊ definition and properties of the identity matrix ◊ matrix operations - addition - transpose - multiplication by a scalar - multiplication by a matrix ◊ determinant of a matrix ◊ singular and non-singular matrices ◊ group properties of matrices ◊ applications of matrices in both life-related and purely mathematical situations ◊ relationship between matrices and vectors	3 weeks	TI-83 TI-92 TI-Interactive
Vectors and Applications ◊ for vectors as structures which are used for the storage of data • definition of a vector • relationship between vectors and matrices • operations on vectors including: - addition - multiplication by a scalar • scalar product of two storage vectors • simple life-related applications of vectors ◊ for vectors describing situations involving magnitude and direction • definition of a vector • relationship between vectors and matrices • two dimensional vectors and their algebraic and geometric representation • operations on vectors including: - addition - multiplication by a scalar • scalar product of two vectors • unit vectors • calculation of the angle between two vectors • resolution of vectors into components acting at right angles to each other • applications of vectors in both life-related and purely mathematical situations	4 weeks	TI-83 TI-92 Cabri Geometry

Year 11 C Outline Semester 2 1998

Term 3

Topic	Time	Technology Resource
Vectors and Applications • scalar product of two vectors • calculation of the angle between two vectors • applications of vectors in both life-related and purely mathematical situations	1 week	TI-83 TI-92 TI-Interactive
Real and Complex Number Systems ◊ structure and representation of complex numbers including: - algebraic definitions and interpretations - geometric definitions (Argand diagram) and interpretations - polar form - conjugates	2 weeks	TI-83 TI-92 TI-Interactive
Structures and Patterns ◊ sum to infinity of a geometric progression ◊ life-related applications of arithmetic and geometric progressions ◊ permutations and combinations and their use in life-related situations	2 weeks	TI-83 TI-92 TI-Interactive
Introduction to the Theory of Chaos and Fractal Geometry (I) ◊ basic notions of recursive functions and iterative processes ◊ exponential growth and decay	2 weeks	TI-83 TI-92 TI-Interactive Spreadsheets On-line resources
Introduction to the Theory of Chaos and Fractal Geometry (II) ◊ basic notions Fractal Geometry ◊ Koch curves	2 weeks	TI-83 TI-92 TI-Interactive Spreadsheets On-line resources

Term 4

Topic	Time	Resource
Dynamics ◊ Newton's Laws of Motion in vector form - straight line motion in a horizontal plane with constant force - vertical motion under gravity without air resistance - projectile motion without air resistance	4 weeks	TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software
Dynamics Cricket Assignment		TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software

Year 12 C Outline Semester 1 1998

Term 1

Topic	Time	Resource
Real and Complex Number Systems ◊ de Moivre's Theorem ◊ simple, purely mathematical applications of complex numbers	3 week	TI-83 TI-92 TI-Interactive
Structures and Patterns ◊ sequences and series other than arithmetic and geometric ◊ applications of patterns ◊ use of finite differences	3 weeks	TI-83 TI-92 TI-Interactive
Matrices and Applications ◊ applications of matrices in both life-related and purely mathematical situations	2 weeks	TI-83 TI-92 TI-Interactive

Term 2

Topic	Time	Resource
Matrices and Applications (continued) ◊ applications of matrices in both life-related and purely mathematical situations	1 week	TI-83 TI-92 TI-Interactive
Introduction to the Theory of Chaos and Fractal Geometry ◊ the growth equation ◊ stability of systems modelled by the growth equation	2 weeks	TI-83 TI-92 TI-Interactive On-line resources Spreadsheets

12 Maths C 1998 Semester 2

Term 3

Topic	**Time**	**Resource**
Calculus ◊ solution of simple, linear, first order differential equations with constant coefficients ◊ life-related applications of simple, linear, first order differential equations with constant coefficients	2 weeks	TI-83 TI-92 TI-Interactive
Introduction to the Theory of Chaos and Fractal Geometry ◊ strange attractors ◊ the Verhulst equation ◊ stability of systems modelled by differential equations ◊ fractal dimensions ◊ potential real world applications of Chaos Theory and Fractal Geometry	2 weeks	TI-83 TI-92 TI-Interactive On-line resources Spreadsheets
Dynamics ◊ derivatives of vectors and their application to straight line motion in a horizontal plane with variable force ◊ derivatives of vectors and their application to: - vertical motion under gravity with air resistance - projectile motion with air resistance - simple harmonic motion ◊ - circular motion with uniform angular velocity	4 weeks	TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software Motion Detector
Vectors and Applications (3D) ◊ three dimensional vectors and their algebraic and geometric representation ◊ operations on vectors including: - addition - multiplication by a scalar ◊ scalar product of two vectors ◊ vector product of two vectors ◊ unit vectors ◊ calculation of the angle between two vectors applications of vectors in both life-related and purely mathematical situations	2 weeks	TI-83 TI-92 On-line resources Cabri Geometry

Term 4

Topic	**Time**	**Resource**
Calculus ◊ integrals of the form $\int \frac{f'(x)}{f(x)}\, dx$ ◊ $\int f[g(x)].g'(x)\, dx$ ◊ simple integration by parts ◊ Simpson's rule	3 weeks	TI-83 TI-92 TI-Interactive

Year 11 C Outline Semester 1 1999

Term 1

Topic	Time	Resource
Structures and Patterns ◊ recognition of patterns in well known structures including Pascal's Triangle and Fibonacci sequence ◊ applications of patterns ◊ life-related applications of arithmetic and geometric progressions	3.5 week	TI-83 TI-92 TI-Interactive
Introduction to Groups ◊ concepts of: - closure - associativity - identity - inverse ◊ definition of a group	2.5 weeks	Prepared Handout
Real and Complex Number Systems ◊ structure of the real number system including: - rational numbers - irrational numbers ◊ simple manipulation of surds	2 weeks	TI-83 TI-92 TI-Interactive

Year 11 C Outline Semester 1 1999

Term 2

Topic	Time	Resource
Matrices and Applications ◊ definition of a matrix ◊ definition and properties of the identity matrix ◊ matrix operations - addition - transpose - multiplication by a scalar - multiplication by a matrix ◊ determinant of a matrix ◊ singular and non-singular matrices ◊ group properties of matrices ◊ applications of matrices in both life-related and purely mathematical situations ◊ relationship between matrices and vectors	3 weeks	TI-83 TI-92 TI-Interactive
Vectors and Applications ◊ for vectors as structures which are used for the storage of data • definition of a vector • relationship between vectors and matrices • operations on vectors including: - addition - multiplication by a scalar • scalar product of two storage vectors • simple life-related applications of vectors ◊ for vectors describing situations involving magnitude and direction • definition of a vector • relationship between vectors and matrices • two dimensional vectors and their algebraic and geometric representation • operations on vectors including: - addition - multiplication by a scalar • scalar product of two vectors • unit vectors • calculation of the angle between two vectors • resolution of vectors into components acting at right angles to each other • applications of vectors in both life-related and purely mathematical situations	4 weeks	TI-83 TI-92 Cabri Geometry

Year 11 Maths C Outline Semester 2 1999

Term 3

Topic	Time	Resource
Vectors and Applications • scalar product of two vectors • calculation of the angle between two vectors • applications of vectors in both life-related and purely mathematical situations	1 week	TI-83 TI-92 TI-Interactive
Real and Complex Number Systems ◊ structure and representation of complex numbers including: - algebraic definitions and interpretations - geometric definitions (Argand diagram) and interpretations - polar form - conjugates	2 weeks	TI-83 TI-92 TI-Interactive
Structures and Patterns ◊ sum to infinity of a geometric progression ◊ life-related applications of arithmetic and geometric progressions ◊ permutations and combinations and their use in life-related situations	2 weeks	TI-83 TI-92 TI-Interactive
Introduction to the Theory of Chaos and Fractal Geometry (I) ◊ basic notions of recursive functions and iterative processes ◊ exponential growth and decay	2 weeks	TI-83 TI-92 TI-Interactive Spreadsheets On-line resources
Introduction to the Theory of Chaos and Fractal Geometry (II) ◊ basic notions Fractal Geometry ◊ Koch curves	2 weeks	TI-83 TI-92 TI-Interactive Spreadsheets On-line resources

Term 4

Topic	Time	Resource
Dynamics ◊ Newton's Laws of Motion in vector form - straight line motion in a horizontal plane with constant force - vertical motion under gravity without air resistance - projectile motion without air resistance	4 weeks	TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software
Dynamics Cricket Assignment		TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software

Year 12 C Outline Semester 1 1999

Term 1

Topic	Time	Resource
Real and Complex Number Systems ◊ de Moivre's Theorem ◊ simple, purely mathematical applications of complex numbers	3 week	TI-83 TI-92 TI-Interactive
Structures and Patterns ◊ sequences and series other than arithmetic and geometric ◊ applications of patterns ◊ use of finite differences	3 weeks	TI-83 TI-92 TI-Interactive
Matrices and Applications ◊ applications of matrices in both life-related and purely mathematical situations	2 weeks	TI-83 TI-92 TI-Interactive

Term 2

Topic	Time	Resource
Matrices and Applications (continued) ◊ applications of matrices in both life-related and purely mathematical situations	1 week	TI-83 TI-92 TI-Interactive
Introduction to the Theory of Chaos and Fractal Geometry ◊ the growth equation ◊ stability of systems modelled by the growth equation	2 weeks	TI-83 TI-92 TI-Interactive On-line resources Spreadsheets
Calculus ◊ solution of simple, linear, first order differential equations with constant coefficients ◊ life-related applications of simple, linear, first order differential equations with constant coefficients	2 weeks	TI-83 TI-92 TI-Interactive

12 Maths C 1999 Semester 2

Term 3

Topic	Time	Resource
Introduction to the Theory of Chaos and Fractal Geometry ◊ strange attractors ◊ the Verhulst equation ◊ stability of systems modelled by differential equations ◊ fractal dimensions ◊ potential real world applications of Chaos Theory and Fractal Geometry	2 weeks	TI-83 TI-92 TI-Interactive On-line resources Spreadsheets
Dynamics ◊ derivatives of vectors and their application to straight line motion in a horizontal plane with variable force ◊ derivatives of vectors and their application to: - vertical motion under gravity with air resistance - projectile motion with air resistance - simple harmonic motion ◊ - circular motion with uniform angular velocity	4 weeks	TI-83 TI-92 TI-Interactive On-line resources Measurement in Motion Software Motion Detector
Vectors and Applications (3D) ◊ three dimensional vectors and their algebraic and geometric representation ◊ operations on vectors including: - addition - multiplication by a scalar ◊ scalar product of two vectors ◊ vector product of two vectors ◊ unit vectors ◊ calculation of the angle between two vectors applications of vectors in both life-related and purely mathematical situations	2 weeks	TI-83 TI-92 On-line resources Cabri Geometry

Term 4

Topic	Time	Resource
Calculus ◊ integrals of the form $\int \frac{f'(x)}{f(x)}\,dx$ ◊ $\int f[g(x)].g'(x)\,dx$ ◊ simple integration by parts ◊ Simpson's rule	3 weeks	TI-83 TI-92 TI-Interactive

Appendix 3

Technology Questionnaires

Technology Questionnaire 1

Name:

Sex: Male / Female

Year level: 11 / 12

Subject: Maths B / C

Date:

We are interested in your ideas about learning mathematics with technology. Technology in this case means computers or graphing calculators. If you have any doubts about what is meant here please don't hesitate to ask.

Please answer the questions that follow honestly – we need to know what you really think. If you are unsure what a question is asking please be sure to ask as this is not part of your assessment program.

	Section 1	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I enjoy using technology during mathematics classes.	5	4	3	2	1
2	I will work with technology for long periods of time if I think it will help me solve a problem.	5	4	3	2	1
3	I feel confident I can use technology when faced with a new problem in maths class.	5	4	3	2	1
4	If I make a mistake when using technology I am usually able to work it out for myself.	5	4	3	2	1
5	Using technology makes me feel more confident about learning mathematics because I can check answers and ideas as I go.	5	4	3	2	1

	Section 2	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I prefer to work with technology on my own when studying mathematics.	5	4	3	2	1
2	I prefer to work with others when using technology because I feel I need help if something goes wrong.	5	4	3	2	1
3	I prefer to work with others when using technology because I like to discuss what I see on the screen.	5	4	3	2	1
4	I don’t like others to see the work I do with technology in case they criticise what I have done.	5	4	3	2	1
5	When I use technology to study mathematics I really feel I need to share with others what I find.	5	4	3	2	1

	Section 3	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I prefer to just learn the mathematics and find the need to learn technology as well a burden.	5	4	3	2	1
2	Good students don't need the assistance of technology to understand mathematics.	5	4	3	2	1
3	Technology is only there to check what you do with pen and paper.	5	4	3	2	1
4	Technology allows me to explore my own ideas about mathematics as well as those discussed in class.	5	4	3	2	1
5	I am sometimes forced to use new mathematics when exploring the use of technological tools.	5	4	3	2	1
6	By looking after messy calculations technology makes it easier to learn essential ideas.	5	4	3	2	1
7	I prefer to learn the mathematics first, without technology, and then learn the technology to do the mathematics more quickly.	5	4	3	2	1
8	I tend to use technology to do calculating basic tasks but not much else.	5	4	3	2	1
9	I find technology particularly useful when exploring unfamiliar problems.	5	4	3	2	1
10	Technology allows me to learn mathematics more easily because I can work through a greater number of examples more quickly.	5	4	3	2	1
11	Technology helps me to link knowledge, eg. the shapes of graphs and their equations.	5	4	3	2	1
12	I can often solve problems using technology in the classroom by when thinking about the same mathematics later I feel I don't really understand it.	5	4	3	2	1

Section 4

1 Do you have access to technology at home? Yes / No

2 What type? Computer / Graphing Calculator

3 If you use a computer at home, name the kind of software you use, ie. Spreadseets, games, graphing packages etc.

	Section 5	**Strongly Agree**	**Agree**	**Neutral**	**Disagree**	**Strongly Disagree**
1	I use technology at home for:					
a)	Entertainment eg. games, surfing the internet.	5	4	3	2	1
b)	Writing up school assignments.	5	4	3	2	1
c)	Doing mathematics homework.	5	4	3	2	1
d)	Exploring ideas about mathematics begun in class.	5	4	3	2	1
e)	Exploring ideas about mathematics of my own.	5	4	3	2	1
2	I use technology at school:					
a)	When the teacher tells me.	5	4	3	2	1
b)	When I get stuck on a problem.	5	4	3	2	1
c)	To look at a problem in a different way, eg. a picture or a table.	5	4	3	2	1
d)	As a way of discussing a problem with others.	5	4	3	2	1
e)	When I feel pen and paper isn't helping.	5	4	3	2	1
f)	As a first resort when looking at a mathematical problem.	5	4	3	2	1

Section 6

1. Write down in your own words what you think about using technology to learn mathematics.

2. Do you have a preference for using computer or graphing calculator technology? Wrtie down how strongly you feel about this and why, ie. you might discuss what you see as the advantages and disadvantages of both devices as a way of justifying which one you prefer.

3. Were there any questions on this questionnaire that you found ambiguous, confusing or difficult to answer? Write down which ones and why you found them so.

Technology Questionnaire 2

Name:

Sex: Female / Male

Year level: 11 / 12

Date:

We are interested in your ideas about learning mathematics with technology. Technology in this case means computers or graphing calculators. If you have any doubts about what is meant here please don't hesitate to ask.

Please answer the questions that follow honestly – we need to know what you really think. If you are unsure what a question is asking please be sure to ask Ms Goos or myself as this is not part of your assessment program.

	Section 1	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I enjoy using technology during mathematics classes.	5	4	3	2	1
2	I will work with technology for long periods of time if I think it will help me solve a problem.	5	4	3	2	1
3	I feel confident I can use technology when faced with a new problem in maths class.	5	4	3	2	1
4	If I make a mistake when using technology I am usually able to work it out for myself.	5	4	3	2	1
5	Using technology makes me feel more confident about learning mathematics because I can check answers and ideas as I go.	5	4	3	2	1

	Section 2	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I prefer to work with technology on my own when studying mathematics.	5	4	3	2	1
2	I prefer to work with others when using technology because I feel I need help if something goes wrong.	5	4	3	2	1
3	I prefer to work with others when using technology because I like to discuss what I see on the screen.	5	4	3	2	1
4	I don’t like others to see the work I do with technology in case they criticise what I have done.	5	4	3	2	1
5	When I use technology to study mathematics I really feel I need to share with others what I find.	5	4	3	2	1

	Section 3	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I prefer to just learn the mathematics and find the need to learn technology as well a burden.	5	4	3	2	1
2	Good students don't need the assistance of technology to understand mathematics.	5	4	3	2	1
3	Technology is only there to check what you do with pen and paper.	5	4	3	2	1
4	Technology allows me to explore my own ideas about mathematics as well as those discussed in class.	5	4	3	2	1
5	I am sometimes forced to use new mathematics when exploring the use of technological tools.	5	4	3	2	1
6	By looking after messy calculations technology makes it easier to learn essential ideas.	5	4	3	2	1
7	I prefer to learn the mathematics first, without technology, and then learn the technology to do the mathematics more quickly.	5	4	3	2	1
8	I tend to use technology to do calculating basic tasks but not much else.	5	4	3	2	1
9	I find technology particularly useful when exploring unfamiliar problems.	5	4	3	2	1
10	Technology allows me to learn mathematics more easily because I can work through a greater number of examples more quickly.	5	4	3	2	1
11	Technology helps me to link knowledge, eg. the shapes of graphs and their equations.	5	4	3	2	1
12	I can often solve problems using technology in the classroom by when thinking about the same mathematics later I feel I don't really understand it.	5	4	3	2	1

Section 4

1 Do you have access to technology at home? Yes / No

2 What type? Computer / Graphing Calculator

3 If you use a computer at home, name the kind of software you use, ie. Spreadsheets, games, graphing packages etc.

____

	Section 5	**Strongly Agree**	**Agree**	**Neutral**	**Disagree**	**Strongly Disagree**
1	I use technology at home for:					
a)	Entertainment eg. games, surfing the internet.	5	4	3	2	1
b)	Writing up school assignments.	5	4	3	2	1
c)	Doing mathematics homework.	5	4	3	2	1
d)	Exploring ideas about mathematics begun in class.	5	4	3	2	1
e)	Exploring ideas about mathematics of my own.	5	4	3	2	1
2	I use technology at school:					
a)	When the teacher tells me.	5	4	3	2	1
b)	When I get stuck on a problem.	5	4	3	2	1
c)	To look at a problem in a different way, eg. a picture or a table.	5	4	3	2	1
d)	As a way of discussing a problem with others.	5	4	3	2	1
e)	When I feel pen and paper isn’t helping.	5	4	3	2	1
f)	As a first resort when looking at a mathematical problem.	5	4	3	2	1

Section 6

1. Are there any *advantages* in using technology instead of pencil and paper? If so explain *how* technology helps you learn better. (*Give specific examples*).

   Are there any disadvantages in using technology instead of pencil and paper? If so, explain *how* technology gets in the way of your learning. (*Give specific examples*).

2. If you had not been able to use technology to learn maths this year, what difference would this have made to your understating? (*Give specific examples*).

3. Are there any benefits in having students present their calculator work to the class via a view-screen and overhead projector? If so, what are they? *(Are there benefits for the presenter? Benefits for those watching and listening?)*

4. Do you have a preference for using computer or graphing calculator technology? Write down how strongly you feel about this and why, ie. you might discuss what you see as the advantages and disadvantages of both devices as a way of justifying which one you prefer.

5. Do you think that using technology changes the teacher’s role in the classroom? i.e. Do they teach differently? If so, what are the differences?

Technology Questionnaire 3

Name:

Sex: Female / Male

Year level: 11 / 12

Subject: Maths B Maths C

Date:

We are interested in your ideas about learning mathematics with technology. Technology in this case means computers or graphing calculators. If you have any doubts about what is meant here please don't hesitate to ask.

Please answer the questions that follow honestly – we need to know what you really think. If you are unsure what a question is asking please be sure to ask as this is not part of your assessment program.

	Section 1	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I enjoy using technology during mathematics classes.	5	4	3	2	1
2	I will work with technology for long periods of time if I think it will help me solve a problem.	5	4	3	2	1
3	I feel confident I can use technology when faced with a new problem in maths class.	5	4	3	2	1
4	If I make a mistake when using technology I am usually able to work it out for myself.	5	4	3	2	1
5	Using technology makes me feel more confident about learning mathematics because I can check answers and ideas as I go.	5	4	3	2	1

	Section 2	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I prefer to work with technology on my own when studying mathematics.	5	4	3	2	1
2	I prefer to work with others when using technology because I feel I need help if something goes wrong.	5	4	3	2	1
3	I prefer to work with others when using technology because I like to discuss what I see on the screen.	5	4	3	2	1
4	I prefer to work with others when using technology because I often get good ideas from them.	5	4	3	2	1
5	I don’t like others to see the work I do with technology in case they criticise what I have done.	5	4	3	2	1
6	When I use technology to study mathematics I really feel I need to share with others what I find.	5	4	3	2	1

	Section 3	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I prefer to just learn the mathematics and find the need to learn technology as well a burden.	5	4	3	2	1
2	Good students don't need the assistance of technology to do mathematics.	5	4	3	2	1
3	Technology is only there to check what you do with pen and paper.	5	4	3	2	1
4	Technology allows me to explore my own ideas about mathematics as well as those discussed in class.	5	4	3	2	1
5	I am sometimes forced to use new mathematics when exploring the use of technological tools.	5	4	3	2	1
6	By looking after messy calculations technology makes it easier to learn essential ideas.	5	4	3	2	1
7	I prefer to learn the mathematics first, without technology, and then learn the technology to do the mathematics more quickly.	5	4	3	2	1
8	I tend to use technology to do calculating basic tasks but not much else.	5	4	3	2	1
9	I find technology particularly useful when exploring unfamiliar problems.	5	4	3	2	1
10	Technology allows me to learn mathematics more easily because I can work through a greater number of examples more quickly.	5	4	3	2	1
11	Technology helps me to link knowledge, eg. the shapes of graphs and their equations.	5	4	3	2	1
12	I can often solve problems using technology in the classroom, but wonder afterwards if I really understand it.	5	4	3	2	1

Section 4

1 Do you have access to technology at home? Yes / No

2 What type? Computer / Graphing Calculator

3 If you use a computer at home, name the kind of software you use, ie. Spreadsheets, games, graphing packages etc.

	Section 5	**Strongly Agree**	**Agree**	**Neutral**	**Disagree**	**Strongly Disagree**
1	I use technology at home for:					
a)	Entertainment eg. games, surfing the internet.	5	4	3	2	1
b)	Writing up school assignments.	5	4	3	2	1
c)	Doing mathematics homework.	5	4	3	2	1
d)	Exploring ideas about mathematics begun in class.	5	4	3	2	1
e)	Exploring ideas about mathematics of my own.	5	4	3	2	1
2	I use technology at school:					
a)	When the teacher tells me.	5	4	3	2	1
b)	When I get stuck on a problem.	5	4	3	2	1
c)	To look at a problem in a different way, eg. a picture or a table.	5	4	3	2	1
d)	As a way of discussing a problem with others.	5	4	3	2	1
e)	When I feel pen and paper isn’t helping.	5	4	3	2	1
f)	As a first resort when looking at a mathematical problem.	5	4	3	2	1

Section 6

1. Are there any *advantages* in using technology instead of pencil and paper? If so explain *how* technology helps you learn better. (*Give specific examples*).

   Are there any disadvantages in using technology instead of pencil and paper? If so, explain *how* technology gets in the way of your learning. (*Give specific examples*).

2. If you had *not* been able to use technology to learn maths this year, what difference (***do you feel***) would this have made to:

   your understanding? (*Give specific examples*).

   the way you work with others in the class?

3. Are t**h**ere any ways in which you believe technology helps you to think differently, for example, the approaches you might use when solving unfamiliar problems or an investigation? (*Give specific examples*).

4. Are there tasks (in mathematics) you would never use technology for?

   What kind of tasks?

   Why?

5. Which do you prefer to use – computer or graphing calculator technology? Write down the advantages and disadvantages of both, how strongly you feel about this and why?

   Advantages of a computer:

Disadvantages of a computer:

Advantages of a graphing calculator:

Disadvantages of a graphing calculator:

State how strong you preference is for one or the other:

What are the major reasons for your choice?

6. Are there any benefits in having students present their calculator work to the class via a view-screen and overhead projector? If so, what are they?

   Benefits for the presenter:

   Benefits for the audience:

Benefits for the teacher:

Any other benefits:

7. Do you think that using technology changes the teacher's role in the classroom? i.e.

Do they teach differently?
If so, what are the differences?

8. The use of technology by students to learn mathematics has been described in the following ways.

   i. *Technology as master ("What is it allowing me to do?")*
   Here the student is subservient to the technology – this happens when the student does not feel confident with the sue of technology and so can only use a limited number of functions, or when their knowledge of the mathematics being studied is limited so they are prepared to simply accept whatever is on the computer/calculator screen.

   ii. *Technology as servant ("Tell the ting what to do!")*
   In the role of technology is basically used as a reliable time saving replacement for mental, or pen and paper computations. The tasks of the mathematics classroom remain the same – but now they are facilitated by a fast mechanical aid. Unlike the previous category the user is in control, and 'instructs' the technology as an obedient by 'dumb' assistant.

   iii. *Technology as a partner ("How can we do this together?")*
   Here a 'rapport' has developed between the user and the technological device – which may be addressed in human terms. A graphics calculator

for example, becomes a friend to go exploring with, rather than merely a producer of results. Explorations, for example in graphical work, lead to situations where what appears on the calculator screen needs to be checked against what is known about the mathematical properties of the graph being investigated. It is possible for the calculator to be 'wrong', as the student knows enough mathematics to be able to have a sense for what the answer should be.

iv. *Technology as an extension of self ("Come fly with me!")*
This is the highest level of functioning, and involves users incorporating technological expertise as an integral part of their mathematical toolkit, so that the partnership between student and technology merges to a single identity. Here powerful use of calculators and computers forms an extension of the user's mathematical powers. Rather than existing as an external tool, a calculator may be used as part of a mathematical argument to support a conjecture, as when students share and compare computer output as part of their own contribution to a solution process.

Which of the above best describes the way you se technology in the classroom? Explain why. (*Give specific examples*).

Technology Questionnaire Sample

Name: Keira

Sex: Female

Year level: 11

Date: 23/11/98

We are interested in your ideas about learning mathematics with technology. Technology in this case means computers or graphing calculators. If you have any doubts about what is meant here please don't hesitate to ask.

Please answer the questions that follow honestly – we need to know what you really think. If you are unsure what a question is asking please be sure to ask Ms Goos or myself as this is not part of your assessment program.

	Section 1	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I enjoy using technology during mathematics classes.	5	**4**	3	2	1
2	I will work with technology for long periods of time if I think it will help me solve a problem.	5	4	**3**	2	1
3	I feel confident I can use technology when faced with a new problem in maths class.	5	4	**3**	2	1
4	If I make a mistake when using technology I am usually able to work it out for myself.	5	4	**3**	2	1
5	Using technology makes me feel more confident about learning mathematics because I can check answers and ideas as I go.	5	**4**	3	2	1

	Section 2	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I prefer to work with technology on my own when studying mathematics.	5	4	3	**2**	1
2	I prefer to work with others when using technology because I feel I need help if something goes wrong.	5	**4**	3	2	1
3	I prefer to work with others when using technology because I like to discuss what I see on the screen.	5	**4**	3	2	1
4	I don’t like others to see the work I do with technology in case they criticise what I have done.	5	4	3	**2**	1
5	When I use technology to study mathematics I really feel I need to share with others what I find.	5	4	**3**	2	1

	Section 3	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I prefer to just learn the mathematics and find the need to learn technology as well a burden.	5	4	3	**2**	1
2	Good students don't need the assistance of technology to understand mathematics.	5	4	3	**2**	1
3	Technology is only there to check what you do with pen and paper.	5	4	3	**2**	1
4	Technology allows me to explore my own ideas about mathematics as well as those discussed in class.	5	4	**3**	2	1
5	I am sometimes forced to use new mathematics when exploring the use of technological tools.	5	4	**3**	2	1
6	By looking after messy calculations technology makes it easier to learn essential ideas.	5	**4**	3	2	1
7	I prefer to learn the mathematics first, without technology, and then learn the technology to do the mathematics more quickly.	5	4	**3**	2	1
8	I tend to use technology to do calculating basic tasks but not much else.	5	4	3	**2**	1
9	I find technology particularly useful when exploring unfamiliar problems.	5	**4**	3	2	1
10	Technology allows me to learn mathematics more easily because I can work through a greater number of examples more quickly.	5	4	**3**	2	1
11	Technology helps me to link knowledge, eg. the shapes of graphs and their equations.	5	**4**	3	2	1
12	I can often solve problems using technology in the classroom by when thinking about the same mathematics later I feel I don't really understand it.	5	4	**3**	2	1

Section 4

1 Do you have access to technology at home? **Yes**

2 What type? **Graphing Calculator**

3 If you use a computer at home, name the kind of software you use, ie. Spreadsheets, games, graphing packages etc. **N/A**

	Section 5	**Strongly Agree**	**Agree**	**Neutral**	**Disagree**	**Strongly Disagree**
1	I use technology at home for:					
a)	Entertainment eg. games, surfing the internet.	5	4	3	2	**1**
b)	Writing up school assignments.	5	4	3	**2**	1
c)	Doing mathematics homework.	5	4	3	**2**	1
d)	Exploring ideas about mathematics begun in class.	5	4	3	**2**	1
e)	Exploring ideas about mathematics of my own.	5	4	3	**2**	1
2	I use technology at school:					
a)	When the teacher tells me.	**5**	4	3	2	1
b)	When I get stuck on a problem.	5	**4**	3	2	1
c)	To look at a problem in a different way, eg. a picture or a table.	5	**4**	3	2	1
d)	As a way of discussing a problem with others.	5	4	**3**	2	1
e)	When I feel pen and paper isn't helping.	5	4	**3**	2	1
f)	As a first resort when looking at a mathematical problem.	5	4	**3**	2	1

Section 6

1. Are there any *advantages* in using technology instead of pencil and paper? If so explain *how* technology helps you learn better. (*Give specific examples*).

When using technology there isn't as much room for error and the answers are displayed neatly on a screen and not scribbled on a piece of paper. If using pen and paper and you've made a mistake you could learn something wrong however if you use technology to do it or at least check it there is a better chance of you getting it right.

Are there any disadvantages in using technology instead of pencil and paper? If so, explain *how* technology gets in the way of your learning. (*Give specific examples*).

Sometimes you get to dependant on technology and don't learn pen and paper to the degree you're supposed to. You take the easy option and in the long run I think this can disadvantage you. You don't learn what you're supposed to.

2. If you had not been able to use technology to learn maths this year, what difference would this have made to your understating? (*Give specific examples*).

It would have made matrices very difficult because we would have to be able to solve very large matrix problems. However in other areas I don't think it would have made a huge difference. With simple areas like graphs etc. it would have been more difficult (slightly) but if we didn't know this type of technology was available we wouldn't have missed it.

3. Are there any benefits in having students present their calculator work to the class via a view-screen and overhead projector? If so, what are they? *(Are there benefits for the presenter? Benefits for those watching and listening?)*

It is very helpful to see what were suppose to do on a big screen so we can be walked through it slowly and we can get a similar screen ourselves while 15-20 people are also copying the screen. (It is same reason why teachers write on a big black/white board instead of in a book to teach stuff). The presenter is also able to understand the material better by explaining to a group of people.

4. Do you have a preference for using computer or graphing calculator technology? Write down how strongly you feel about this and why, ie. you might discuss what you see as the advantages and disadvantages of both devices as a way of justifying which one you prefer.

I prefer to use a graphing calculator probably because I have greater access to one. I think a computer is better because of the bigger screen and clearer symbols, writing etx. However, calculators are more compact, work on battery power and are generally easier to understand and can do much the same things (apart from typing assignments etc.)

5. Do you think that using technology changes the teacher's role in the classroom? i.e. Do they teach differently? If so, what are the differences?

I don't think it does change their role although rather than teaching so much they are more supervising and handing out advice. Their focus is more on the technology than the theory which is sometimes a bad thing because more focus needs to be on theory but there should be a good balance, like with everything.

Appendix 4

Observation Record Samples

Cover Sheet – Technology Project

School: **Date:** **Class:** **Teacher:**

CONTENT

Description:	
Purpose:	
Background knowledge:	

GENERAL STRUCTURE

TECHNOLOGY FEATURES

INTERACTION FEATURES

ARC PROJECT – TECHNOLOGY IN SENIOR MATHS - 1999																
LESSON OBSERVATION RECORD																
No.	**DATE**	**SCHOOL**	**SUBJECT**	**TEACHER**	**LESSON TOPIC**	**DETAILS OF RECORD**										**COMMENTS**
						Video Tapes (N	**Audio Tapes (No.)**	**Lesson Report**	**Text Reference**	**Hand-outs**	**Student Working**	**Interesting Maths**	**Interesting Technology**	**Interesting Students**		**General**
68	06/10/99	Hindsay	12 Maths C	Vince Geiger	Revision of Integration	2	0	*	1	0	0	0	1	1		A year 10 maths student visited to find out about Maths C.
69	07/10/99	Hindsay	12 Maths B	Michael Barra	Area under curve	2	0	*	0	1	0	1	1	1		Students used three methods to find the area under a curve. 1. Average of Upper and Lower Riemann Sums (areas of rectangles for different increments using XCEL). 2. TI83 graphing calculation. 3. Definite integral (manual calc).
70	13/10/99	Hindsay	12 Maths C	Vince Geiger	Reversal of the Chain Rule for Differentiation; also "DODGE 3 – THE MOVIE" at intermission! (produced and directed by Graeme).	2	0	*	1	0	0	1	1	1		Students were asked to look for characteristics of a function that might indicate that it could be the end product of a Chain Rule application. They then had to devise a rule for reversing the Chain Rule to find the integral of a function.

ARC PROJECT – TECHNOLOGY IN SENIOR MATHS - 1999															
LESSON OBSERVATION RECORD															
No.	**DATE**	**SCHOOL**	**SUBJECT**	**TEACHER**	**LESSON TOPIC**	**DETAILS OF RECORD**									**COMMENTS**
						Video Tapes (No.)	**Audio Tapes (No.)**	**Lesson Report**	**Text Reference**	**Hand-outs**	**Student Working**	**Interesting Maths**	**Interesting Technology**	**Interesting Students**	**General**
71	20/10/99	Hindsay	12 Maths C	Vince Geiger	Integration by Partial Fractions – Focus students: Daniel R., Stephen, Ben, (occasionally Drew).	2	0	*	1			1	1	1	Vince commented how 'enabling' technology can be when introducing new ideas. Students were obviously out of practice with their factorising and their calculations were valuable assistants in overcoming what could have been a major source of frustration.
72	22/10/99	Hindsay	12 Maths C	Vince Geiger	Simpson's Rule	1	0	*	0	1	1	1	1	1	Students – introduced to Simpson's Rule (numeric technique for approximation of area under a curve when integral too difficult to calculate). Students asked how many intervals (slices) needed for approx. to agree in 4th decimal place with manual method.

ARC PROJECT – TECHNOLOGY IN SENIOR MATHS - 1999														
LESSON OBSERVATION RECORD														
No.	**DATE**	**SCHOOL**	**SUBJECT**	**TEACHER**	**LESSON TOPIC**	**DETAILS OF RECORD**								**COMMENTS**
						Video Tapes (No.)	**Audio Tapes (No.)**	**Lesson Report**	**Text Reference**	**Hand-outs**	**Student Working**	**Interesting Maths**	**Interesting Technology**	**Interesting Students**	**General**
70	13/10/99	Hindsay	12 Maths C	Teacher	Reversal of the Chain Rule for Differentiation; also “DODGE 3 – THE MOVIE” at intermission! (produced and directed by Graeme).	2	0	*	1	0	0	1	1	1	Students were asked to look for characteristics of a function that might indicate that it could be the end product of a Chain Rule application. They then had to devise a rule for reversing the Chain Rule to find the integral of a function.

Comments:

- Last Wednesday 13/10/99 I visited Teacher’s Maths C class and they were doing some ‘serious’ Maths. Students were asked to examine functions and to look for characteristics that might indicate that they could possibly have been the result of the application of the Chain Rule. They then had to devise a rule for reversing the Chain Rule to find the integral of a function. Students wanted to use their graphing calculators to store the eventual proof which the teacher said they may have to develop in an examination. Their intention was to represent the proof in alpha numeric on the calculator screen – sort of ‘cheat’ notes. Students protested about not being allowed to use their calculators in this way. Teacher ‘threatened’ to take the calculators from the students if they were going to use them for this purpose. (Note: This would be the first time this year that I have seen Teacher try to limit students’ use of their calculators in any way). I think he indicated that other students may not have access to this kind of technology, prompting Adam’s interesting analogy, “You don’t take out drinking water way just because the Ethiopians don’t have it!”.

- DODGE 3 – THE MOVIE produced and directed by Geoffery was shown between the two lessons. The class was truly impressed by what Geoffery had produced and downloaded copies of his program using connecting cables. Teacher was also most impressed and asked Geoffery if he could show the ‘movie’ to the TI people who are visiting Australia from USA at present. Geoffery was agreeable and, I feel, very flattered to be asked. He also responded very positively to the questions that were fired at him after some other programming enthusiasts delved into his program. Geoffery took on a very different role in the classroom during this lesson.

- Teacher noted that Russel is much more prepared to help other students than he used to be. In this lesson he was mainly helping Sylvester and he helped Matthew too on occasions. He was quite animated and seemed very keen to assists. The assistance was not offered in a condescending way but rather in a spirit of sharing understanding.

Handwritten notes:

DODGE 3 – THE MOVIE

Geoffery's presentation 11:41 – 11:46

(@ 11:41 frightening watching the speed at which Geoffery deletes programs from Teachers's calculator).

ARC PROJECT – TECHNOLOGY IN SENIOR MATHS - 1999															
LESSON OBSERVATION RECORD															
No.	**DATE**	**SCHOOL**	**SUBJECT**	**TEACHER**	**LESSON TOPIC**	**DETAILS OF RECORD**									**COMMENTS**
						Video Tapes (No.)	**Audio Tapes (No.)**	**Lesson Report**	**Text Reference**	**Hand-outs**	**Student Working**	**Interesting Maths**	**Interesting Technology**	**Interesting Students**	**General**
71	20/10/99	Hindsay	12 Maths C	Vince Geiger	Integration by Partial Fractions – Focus students: Matthew, Russel, Sylvester, (occasionally Tom).	2	0	*	1			1	1	1	Teacher commented how 'enabling' technology can be when introducing new ideas. Students were obviously out of practice with their factorising and their calculations were valuable assistants in overcoming what could have been a major source of frustration.

Comments:

- On Wednesday last, 20 October 1999, I visited a good Maths C lesson at Hindsay. The class was introduced to a new method of Integration using partial fractions. They were using the 'green book' again (Radcliffe and Dan, Geometry and Calculus III, 2nd Edition, Brooks Waterloo 1990) starting with example 22 p.175.

- The teacher sat back watching his students for a while and commented how "enabling" technology ca be when introducing new ideas. Because students were out of practice at factorising, they headed for their calculators and were able to overcome what would have been a major source of frustration in the past. They were able to move on and really get into the application of this new method. They were able to do many more problems in the lesson because of their calculator ‘assistant’ and thus could gain confidence in recognising the types of problems to which this method could be applied.

- The teacher commented, as he sat back (yes, it’s becoming a habit!) and observed how competently the students organised their exam revision, that if he had these students for an 3 months he would be out of a job (he could then do lots of ‘sitting back’!).

  a) Early in the lesson students asked if they could organise a revision sheet this lesson.

  b) The teacher suggested that last 5 minutes could be set aside.

  c) Students negotiated a 10 minute time slot saying that five minutes wasn’t enough.

  d) One student, Francis, headed for the whiteboard and with a lot of help (from students consulting their Semester program) wrote up all the major areas covered. They consulted Vince to see if they had forgotten anything.

  e) Individuals then volunteered to prepare revision questions on the various topics.

  f) Those who weren’t quick to volunteer soon found their name added to the list (and not one protest was heard!).

  g) Susanna then suggested they needed a date by which to prepare their questions for collation.

  h) They then furthered their discussion (stared earlier in the lesson) of a plan for a revision evening to go through the problems at someone’s house.
  (When trying to establish a venue, the teacher said something like “What do you say to your parents? I want to have some Maths nerds over?”).

- Another amazing development in this classroom is that Tom is suddenly finding he’s enjoying Maths. He has decided that he wants to pass. He seems to be understanding everything and explaining everything to everyone and everyone’s commenting on the ‘new Tom in such a positive light. He is quite enjoying the attention. He and Matthew had reached the same answer to a homework problem and were sure the book was wrong. They watched with great confidence as their solution matched the one the teacher was unfolding on the whiteboard until it took an unexpected turn towards the end. I have never seen Tom so animated except maybe when he’s attached to a computer (but solid algebra – unheard of!).

Handwritten notes:

Focus students:

Matthew, Russel, Sylvester, Tom

Appendix 5

Individual Student Interview Protocol Exemplars

Technology Project: Individual Student Interview……..

Hintsay Year 11 Maths C

Individual interviews with…………………

1 I have been watching this class for a long time now and it seems to me that in most lessons there are some people who prefer to work with other people, like they talk a lot, and other people prefer to work pretty much on their own. Do you have a preferred way of working yourself?

2 Because I've noticed that most of the time you tend to work pretty much on your own.

But you know last week up in the computer room when you were all working on that cricket problem, it was really interesting because everyone was talking. Like I noticed that you for example were talking a lot to Marieka and it was different. Like it wasn't people so much on their own and I'm curious about why that was. Do you have any ideas?

3 So it was just a matter of you didn't know what to do, so you needed some help. Was that because it was a hard problem?

4 OK so there s were two things there. It was the problem plus that new software. So were you asking Marieka for help then? Is that what it was about?

5 Oh, OK so you were asking for help and giving help.

6 Oh good, alright, so did it take long to work out how to use that software?

7 And did that work out all right?

8 Did you talk to other peop0le about how to understand the problem itself?

9 What sort of things did you need to ask them about?

10 And did you or did you work it out sort of your own way after getting some help?

11 So yes that's interesting. You didn't just sort of do what other people told you. But that helped your thinking, did it?

12 OK that is all I wanted to talk to you about. Alright.

Technology Project: Individual Student Interview……

Hintsay Year 11 Maths C

Individual interviews with…………………

1 You've made much use of graphing calculators in Maths C. Has this technology helped you to learn the mathematics? How/why? (or why not?)

Can you give me any specific examples of how it has helped/not helped?

If it has helped, was this:

a) In the initial stages of learning new work?

b) In doing calculations/exercises?

c) In exploring problems?

d) In solving problems?

e) In presenting your solution to the class (via OHP)?

f) In listening to/watching other students present their solutions?

g)

2 Are there any advantages or disadvantages of using technology verses pencil and paper?

Explain.

3 How do you feel about using graphics calculators versus computers?

4 Have your feelings about using technology changed since the start of the year? How/why/why not?

5 Do you prefer to work alone or with others when using technology? Why?

6 Now think about the two topics most recently studied: matrices and vectors. Were there any differences in ways you used the technology to learn these two topics? Explain.

Was one type of calculator (TI92 vs TI83) more helpful than the other for either of these two topics?

What if you couldn't use technology to study matrices – what difference would that make to your understanding?

(Repeat question for vectors).

You've been taught to use three solution methods for vector addition: column vectors, i & j unit vectors, and a geometrical method. Do you have a preference for any one of these? Why/why not? Which helps you understand best? Why?

Can you see a place for the different representations:

a) As a way of solving problems?

b) As a way of learning about vectors?

7 How do you find technology most helpful personally, i.e. how do you use it?

Do you tend to use technology at home e.g. to work on maths problems at home, doing homework, any other way? Do you use technology in this way to study other subjects?

8 Does it make a difference if the teacher is good at / enthusiastic about using technology?

Do you think it changes the teacher's role in the classroom (i.e. the way they teach)?

Is there any difference between teacher and peer presentation (an OHP) as far as your understanding or learning is concerned?