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An Inverse Model to Determine the Heat Transfer Coefficient and its Evolution with Time during Solidification of Light Alloys

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Abstract

Infra-red probes linked to pyrometric chains and thermocouple arrays have been used to accurately determine both casting and die surface temperatures during the solidification of an aluminium A380 alloy and the magnesium alloy AZ91D. An inverse model was then used to accurately determine the heat flux densities and interfacial heat transfer coefficients and the rapid evolution of these values with time during high pressure die casting of these alloys.

Keywords Inverse model, Heat transfer coefficient, Solidification, Light Alloys

1. Introduction

The interfacial heat transfer coefficient is a critical parameter in understanding and modelling the development of microstructure during the solidification of light alloys[1]. Numerical simulation in the form of solidification modelling has become an increasingly important tool for pressure die casting. The effectiveness of this simulation is however highly dependent on the accuracy of the heat transfer data utilized by the model. There is very little heat transfer data available that is suitable for numerical simulation or the Modelling of microstructural development during solidification of light alloys [1-7]. A small number of temperature measurements related to the determination of heat flux have been attempted in high pressure die casting [8-11]. Generally these investigations have been limited in scope with no calculation of heat transfer coefficients or determination of the change of HTC and heat flux with time [8,9]. More detailed knowledge of the variation of these heat transfer values with time is therefore critical for the development of solidification models.

A detailed review of the literature and description of the correct procedures to be followed when conducting heat transfer measurements in rapid forming processes like high pressure die casting has been previously discussed in detail by the present authors [12]. The accurate heat transfer coefficient data obtained through these investigations has broad application to a wide range of materials for scientists and engineers concerned with the formation of microstructure during high pressure die casting and particularly in the development of accurate interfacial heat transfer models and numerical simulation techniques.

In order to obtain accurate heat transfer data at high temperatures, a new non-intrusive sensor was designed that uses an infrared probe incorporated into a Pyrometric chain (light pipe + optical fiber + pyrometer) [2]. The pyrometers used in the present investigations were chosen for their rapid response time of 2 ms. In addition, the sensor was manufactured to incorporate six fine thermocouples (configured in pairs for redundancy) located at various depths below the die surface. These thermocouples were laid in grooves along the cylinder and exited through holes in the shoulder. The InfraRed(IR) lightpipe/pyrometric sensor system has a precision of ±0.1°C and the thermocouple system is designed to record at up to 1 kHz. The theoretical considerations critical to the design of the sensors have been discussed in detail by Dour et al. 2006[12-13].

The derivation of heat transfer parameters during solidification and die casting fall into the class of problems called inverse heat conduction problems. Inverse Heat Conduction Problems (IHCP) involve the determination of the density of the heat flux as a function of time of a body from interior temperature measurements in the body [17,18,19]. It is quite different from a direct heat conduction problem that implies the determination of the temperature distribution inside a body from the known initial and boundary conditions such as interfacial heat flux and temperature. IHCP generally concerns the configurations which have complex conditions at the surface where the boundary conditions can not be experimentally measured with contacting devices such as thermocouples, because of the physical and technical limitations imposed by the configuration. Die casting is a good example of such a configuration. Solidification of a casting involves a change of phase and hence interfacial temperatures. Therefore, the IHCP becomes nonlinear. For this reason, numerical techniques generally known as methods of solving the IHCP are required in order to determine the heat flux density history at the interface.

In the most of the investigations discussed in the literature in which an inverse method has been employed in order to analyse the measured temperature data, the heat conduction system was assumed to be on-dimensional as illustrated in Fig. 1 (20).

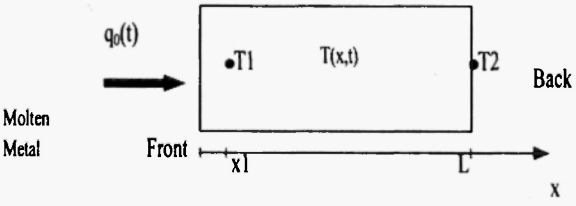


Fig. 1 : One dimensional heat conduction system (20).

The transient heat conduction problem is expressed by the following equations:

$$\frac{\partial^2 T}{\partial x^2}(x,t) - \frac{1}{a}\frac{\partial T}{\partial t}(x,t) = 0 \quad \text{for}$$
$$0 \prec x \prec L \quad \text{and} \quad t \geq 0 \qquad (1)$$

$$-\lambda \frac{\partial T}{\partial x}(0,t) = q_0(t) \qquad (2)$$
$$\text{at } x=0 \quad \text{and for} \quad t > 0$$

The boundary conditions for the configuration presented in Fig are:

$$T(L,t) = T_2 \quad \text{at } x=L \text{ and for } t \geq 0 \qquad (3)$$
$$T(x,0) = T_0 \quad \text{at } t=0 \text{ and for } 0 \leq x < L \qquad (4)$$

Where a is the diffusivity of the body and λ is the thermal conductivity of the body q_0 is the heat flux density crossing the interface.

These equations have to be solved numerically in order to determine the heat flux density at the interface as a function of time. For this purpose, several methods have been proposed in the literature such as: proposed by D'Souza [21], Raynaud and Bransier (22), the function specification method (FSM) or Beck's specification method (least square optimisation) (17), regularization method and iterative regularization method, etc.

Moreover, several comparison studies have been carried out for the main proposed numerical solutions (23, 19). From these studies one can conclude that these solutions in general yield very similar results. However, the FSM is conceptually simpler, can be extended more readily to other parabolic problems, computationally more efficient, suitable for smaller time set ups as compared to other methods. Furthermore, Krishnan and Sharma (24) compared two basic procedures available for the solution of IHCP: Specification and regularization methods. These investigators reported that Beck's method (least square optimisation, future instants) has an advantage over the other numerical procedures because it takes into account inaccuracies in measuring the locations of the thermocouples, statistical errors in temperature measurements and inaccuracies in measuring the locations of the thermocouples by application of the statically principles and use of the future temperature methods.

For this reason, Beck's principle has been employed in most experimental investigations of the HTC or heat flux density at the casting-die interface. The difference among all the investigations is only in the procedures with which the heat transfer equations are solved (finite different method (FDM) (25,26,27), derivative method (28), sequential method (29), semi-analytical method based on Laplace transform (20), singular value decomposition (SVD) (30), enthalpy method (31,32,33,34,35), and implicit or explicit resolution methods. In any case, the solution yield determines the temperature at any positions from the surface.

Furthermore, Beck's FSM is focussed on minimising the following relation.

$$\Delta q = \frac{\sum_{k=1}^{k=m} S_{i+k}\left[T_{i+k}^{\exp} - T_{i+k}(q_{0,i})\right]}{\sum_{k=1}^{k=m} S_{i=k}^{2}} \qquad (5)$$

$$q_{n+1} = q_{n} + \Delta q \qquad (6)$$

Further information on the inverse method and its application to Inverse Heat Conduction Problems is described in [14]. The temperature data obtained from the thermocouple arrays was analyzed with an inverse model as described by Dour et al., [12-14] in order to determine the die surface temperature and the heat flux density. The heat transfer coefficient was then calculated using the heat flux density data and the measurements of the casting surface temperature data obtained from the Pyrometric chain.[12-13] The middle thermocouple (located at 9.5 mm from the die surface) is used to verify the soundness of the inverse method results.

2. Experimental Design

A series of experiments were conducted to concurrently measure heat flow and in-cavity pressure during the high pressure die casting of an A380 aluminium alloy and AZ91 magnesium alloy into an experimental high pressure die casting die (Fig. 2). The experiments were designed to measure both in-cavity pressure and heat flow in the die concurrently. The die was mounted on a commercial Toshiba 250 ton cold chamber high pressure die casting machine and 502 castings of the A380 alloy and 328 castings of the AZ91D alloy were produced. Precise measurements (resolution 1 μm) were made of the position of the piston tip that injects the molten metal into the cavity. Piston velocity was calculated from these measurements. Pressure within the die cavity was measured using commercially available piezo-electric quartz pressure transducers which were designed for use in molten metal contact at temperatures up to 700 °C and pressures of 200MPa. The pressure sensors have been shown to be an effective tool to measure in-cavity pressure during die casting.[15] The die is modified to incorporate these sensors such that the measuring surface of each sensor is flush with the die cavity surface. The heat transfer sensors that were also incorporated into the die were used to detect the arrival of the liquid alloy and measure its temperature. The temperature and pressure sensors were located opposite each other (Fig. 2) so that measurements from each sensor could be directly correlated.

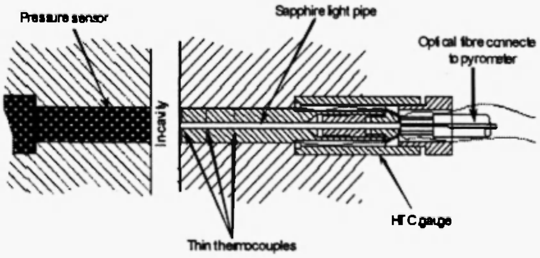


Fig. 2. Vertical cross section of the die, showing the Heat Transfer Coefficient gauge (HTC gauge) and pressure sensor.

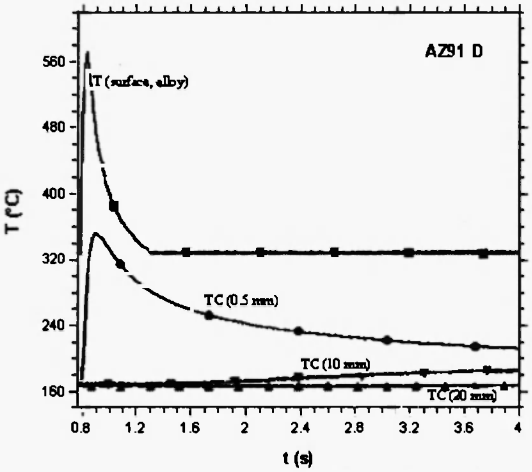


Fig. 3. Typical temperature data measured for the magnesium alloy by both the light pipe pyrometric chain and the thermocouples contained in the heat transfer sensor. The symbols mark only one data point in every 50 or 75 of the total number of data points that were collected. The melt temperature was 700 °C, shot speed was 1.54 ms^{-1} and the nominal intensification pressure as 80 MPa.

3. Results

Unprocessed data obtained from the heat transfer sensor is shown in Fig. 3 which shows the temperature variation with time for the aluminium alloy. The curves labeled T_{alloy} refer to the temperature of the casting surface at both the rib and gate positions obtained by the lightpipe/pyrometric chain. The lower temperature curves correspond to the die temperature measurements obtained from the thermocouples situated at different depths from the die cavity surface (0.5, 9.5 and 20 mm – described in detail in [13]). When a thermocouple is further away from the interface, it is colder and has a slower response, as would be expected from the diffusion of a short peak of heat according to Fourier's Law.

Fig. 4 shows the temperature data after analysis by the inverse model. The top curves are the interfacial heat flux density q and the heat transfer coefficient h versus time. The lower set of curves are the die temperatures as measured and as recalculated with the $q(t)$ data as a boundary condition. This analysis gives information about the die surface temperature and also about the reliability of the temperature calculation obtained using the inverse model. For the aluminium alloy the die surface temperature never reached 500°C, heat flux densities peak at around 16 MWm^{-2} and the heat transfer coefficient reaches close to 90 $kWm^{-2}K^{-1}$. The estimations give rise to residuals of less than 10°C (typically below 3°C in the first 5s) on the die temperatures around 10mm from the surface. The temperatures close to the surface were used to derive q and so have much lower residuals. For the magnesium alloy AZ91 the die surface temperature never reaches 440°C, peak heat flux densities are around 11-17 MWm^{-2} and the peak heat transfer coefficient is close to 100 $kWm^{-2}K^{-1}$ at the gate and 85 $kWm^{-2}K^{-1}$ at the rib position.

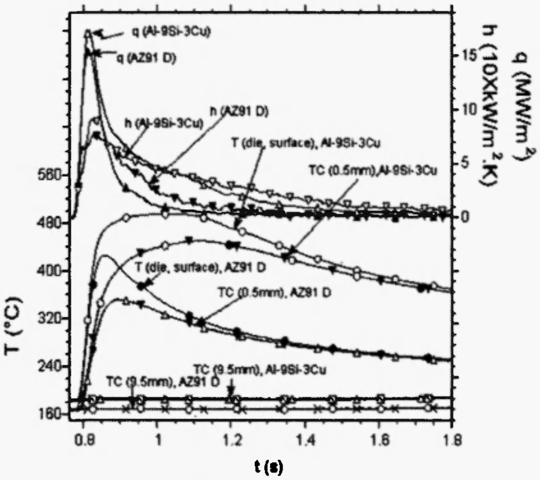
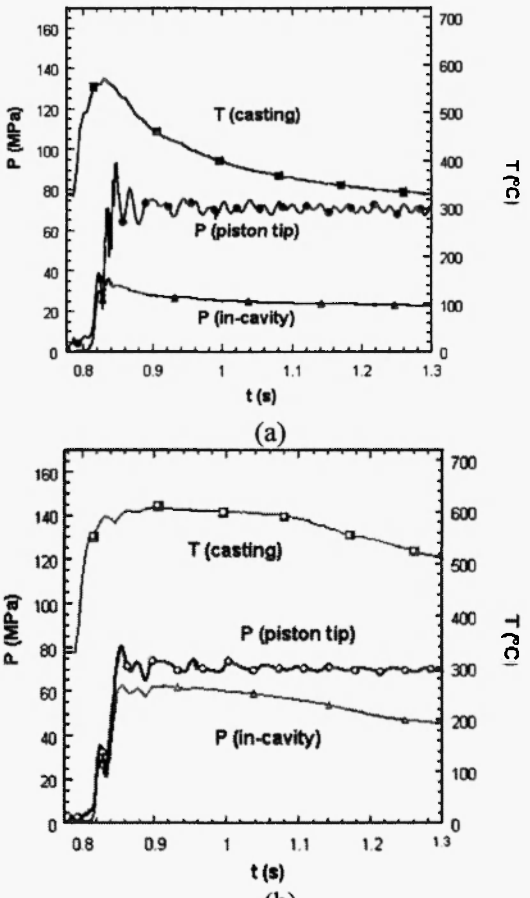


Fig. 4. Results derived from inverse modelling evaluations of the data shown in Fig. 3.

It is worth noticing that the peak value of heat transfer coefficient is fairly comparable for the two alloys. So the heat transfer at the beginning is about the same. But because the volumetric latent heat of AZ91 is much smaller than that for Al-9Si-3Cu (A380), the casting solidifies and cools down much quicker as discussed before. This results in a more rapid decrease of h for AZ91 as the quality of the casting-die contact degrades rapidly. As a result the heat flux density also decreases quickly, while the temperature gap at the interface remains constant at around 50°C (after 1.3 s, the temperature of the casting surface for the AZ91 alloy measured by the pyrometer reaches its lower limit of 320°C).

The final consequence of this rapid evolution of the interfacial heat transfer is that the maximum die surface temperature reached for AZ91 (about 440°C) is lower than that reached for the Al-9Si-3Cu alloy (around 500°C). Similarly the temperature within the die measured by the thermocouple installed 0.5 mm from the die surface reaches 350°C and 450°C for AZ91 and A380 respectively.



(a)

(b)

Fig. 5. Temperatures and in-cavity pressures during cavity fill and pressure intensification (a) Magnesium alloy AZ91 and (b) Aluminium alloy A380.

Fig. 5 shows a set of curves covering only the die filling and pressure intensification stages. The information contained in this figure provides an illustration of the important details that can be obtained using this approach to understanding the complex solidification environment. The figure shows clearly the relationship between pressure and temperature and their evolution with time for the two alloys. The maximum heat flux density corresponds exactly to the slowing of the piston speed (cavity fill). At this very same moment, the pressures at the tip and in the cavity show a slight peak corresponding to completion of the cavity filling process but before the pressure intensification stage has commenced. The intensification pressure is applied at 0.85s after the beginning of the movement of the piston at the start of the cavity filling stage for the aluminium alloy (Fig. 5(a)) and 0.83 s for the magnesium alloy (Fig. 5(b)), The improved efficiency of heat transfer due to the rapid filling of the die cavity with liquid metal is much higher than any benefits that may be achievable due to pressure application when the liquid is static.

4. Discussion

This new knowledge presented in this paper is the result of the application of new experimental measurement techniques and inverse modelling methods to the determination of heat transfer during the high pressure die casting of both magnesium and aluminium alloys resulting in highly accurate heat transfer data. In particular the application of an inverse model to the highly accurate temperature information obtained by the thermocouple arrays and pyrometer provide detailed knowledge of the thermal characteristics occurring during solidification in these alloys. The accurate temperature information was achieved through the application of an array of fine rapid-response thermocouples integrated in one sensor along with the light pipe/pyrometric chain which have enabled the accurate measurement of both the die and metal surface temperatures through the entire die casting cycle. The rapid response time of the sensors (less than 20 ms) and their sensitivity (±0.1°C) along with the application of an inverse method,[2-4] have enabled the accurate determination of both the heat transfer coefficient and heat flux along with their variation with time during the die filling and intensification stages of the high pressure die casting process. The relevance, accuracy and reproducibility of the processed data from the sensor has been confirmed elsewhere.[13] Based on the analysis undertaken by Dour et al.,[13] the precision in evaluation of heat flux density and die surface temperature is 3%. By calibrating the pyrometric chain according to a black body and alloy emissivity the accuracy of the casting temperature measurements in the investigations discussed in the paper following the analysis of Dour et al.,[13] was found to be 5% . The precision in evaluation of the heat transfer coefficient was found to be 15% at the beginning of solidification and 30% at the end of the process.

The heat transfer coefficient values reported here have wide application in the modelling of the development of microstructure during solidification of light alloys along with the development of engineering software used to predict the filling and solidification of light alloys during high pressure die casting. The present work enables a much higher resolution of the evolution with time of heat flux and heat transfer coefficient than any measurements previously reported for high pressure die casting.

The results presented here show that there is a slight different in the peak value of heat transfer for both alloys as a result of the difference in thermal conductivity of the magnesium alloys compared to aluminum alloys since it has been previously determined during investigations on the Thermal Contact Resistance during solid-solid contact that the effective thermal conductivity at the interface (derived from the thermal conductivities of the two contacting bodies) can play a significant role in determining the peak value of the Thermal Contact Resistance.[16I In addition the evolution of the temperature of the casting strongly depends on its latent heat. Therefore the heat transfer coefficient and as a consequence, the heat flux density at the casting-die interface tend to decrease much more rapidly for magnesium alloys. The application of the accurate temperature measurement techniques along with the inverse modeling techniques to determination of heat transfer values during solidification of light alloys in de casting has shown that the peak value of heat transfer coefficient (h) is more dependent on interface characteristics such as die surface roughness, contact conditions and some process parameters than on latent heat and that the peak of heat flux density depends mostly on the peak value of h and the initial temperature difference between the die and the casting surface.

5. Conclusions

An inverse model has been applied to determine the interfacial heat transfer coefficient (h) and heat flux (q) during the high pressure die casting of magnesium alloy AZ91 and an Al-9%Si-3%Cu alloy. The effectiveness of the inverse modeling technique in this application is associated with the accuracy of the temperature data input into the model which was obtained using a new measurement and analysis technique incorporating infra-red probes and thermocouple arrays that accurately determine both casting and die surface temperatures.

The measurement methods and analysis techniques presented here provide a powerful illustration of the application of an inverse modeling technique to a materials engineering application enabling the determination of the most accurate heat transfer coefficient and heat flux data obtained to date.

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