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Macroscopic many-qubit interactions in superconducting flux qubits

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Superconducting flux qubits are considered to investigate macroscopic many-qubit interactions. Many-qubit states based on current states can be manipulated through the current-phase relation in each superconducting loop. For flux qubit systems comprised of N-qubit loops, a general expression of low-energy Hamiltonian is presented in terms of low-energy levels of qubits and macroscopic quantum tunnelings between the many-qubit states. Many-qubit interactions classified by *Ising-type or tunnel* exchange interactions can be observable experimentally. Flux qubit systems can provide various artificial-spin systems to study many-body systems that cannot be found naturally.

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It is believed that electrons are interacting in a pair. The interaction is called two-body interaction. Understanding the many-electron effects is one of the most important researches in condensed-matter physics. Normally, rich many-electron physics has been revealed by the two-body interactions of spin pairs in solid-state materials. In a strongly correlated electronic system, however, a low-energy Hamiltonian can involve more than three spin interactions.[1–4] Such multiple-spin interactions are known to play a significant role in strongly correlated systems. Some examples include the magnetism of solid ^{3}He, the high T_c superconductivity, and so on.[2,3] Moreover, it is well known theoretically that higher dimensional systems can be mapped to the multiple-spin interaction Hamiltonian of one-dimensional chain unlikely found naturally.[5]

The last decade has seen rapidly developing advanced material technologies that make it possible to investigate previously inaccessible quantum systems for quantum information and computation in solid-state systems. Especially, coherent manipulation of quantum states in tunable superconducting devices has enabled to demonstrate macroscopic qubits[6–8] and entangled states of qubits.[9–11] Experimentally, it has been shown that, in terms of pseudospins, different types of exchange interactions between two artificial spins such as an Ising interaction for charge qubits[9] and flux qubits[10,12] and an XY interaction for phase qubits[11] can be realized and controlled by the system parameters. Although different types of solid-state qubit systems have revealed such artificial-spin exchange interactions, multiple artificial-spin interactions have not been demonstrated yet. This Brief Report aims to discuss, in a general framework, how artificial-multiple-spin interactions are possible and realizable in superconducting qubit systems. Flux qubit systems are shown to have an intrinsic property which is multiple artificial-spin interactions. Moreover, controllable system parameters in the flux qubit systems enable to create various types of artificial-spin systems even though a flux qubit system is fabricated experimentally. Then, the man-made superconducting structures of flux qubits can offer experimental simulators for many-body physics in association with multiple-spin interactions.

In a superconductor, the macroscopic wave function can be written as $\psi(\mathbf{r})=\sqrt{n^*}e^{i\varphi(\mathbf{r})}$, where n^* and $\varphi(\mathbf{r})$ are the density and phase of Cooper pairs, respectively. $\psi(\mathbf{r})$ describes the behavior of the entire ensemble of Cooper pairs in the superconductor. The supercurrent density in electromagnetic field is given by

$$\mathbf{J}=\frac{q^*n^*}{m^*}[\hbar\nabla\varphi(\mathbf{r})-q^*\mathbf{A}(\mathbf{r})], \quad (1)$$

where q^* and m^* are, respectively, the charge and the mass of Cooper pairs. Then, the current states of flux qubit loops are influenced by the variations of the phase $\varphi(\mathbf{r})$ across Josephson junctions and the vector potential $\mathbf{A}(\mathbf{r})$. A change of current state in a qubit loop results in a change of current states in other qubit loops because: (i) the change in Josephson-junction phases in superconducting loops coupling qubits mediates the change in the current states of all qubits and (ii) the circulating current in the qubit produces the induced magnetic flux that influences on all other qubits. In experiments, several ways to make two- or four-flux qubits interacting have been employed. Disconnected superconducting loops, as the indirect way, are coupled inductively[10,12] by means of the induced magnetic flux. Other direct ways are to introduce connecting superconducting loops,[13–15] which is called phase coupling. Consequently, many-flux qubits defined by current states can interact all together, which can be observable in experiments.

We present a general expression of N-qubit Hamiltonian describing low-energy physics. The Hamiltonian is determined by the low-level energies and the tunneling amplitudes between N-qubit states in the flux qubit systems. We define two types of many-qubit exchange interactions originating from the energy differences of many-qubit states and the macroscopic quantum tunneling between the states. Further, it is shown that a specific coupling scheme enables to map flux qubit systems into many-body systems.

We consider a general model including the inductive and phase coupling ways. The N-flux qubit systems are composed of N-qubit loops with N' loops connecting the qubit

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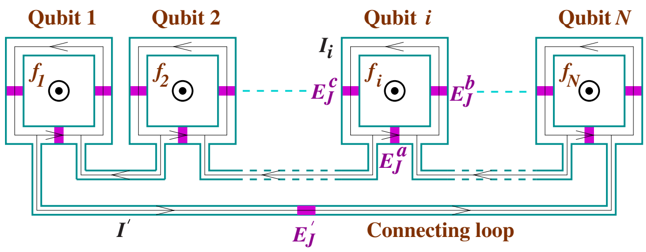

FIG. 1. (Color online) An N-flux qubit system with one connecting loop ($N'=1$). The N superconducting loops are connected by a connecting loop interrupted by a Josephson junction E_J'. In each qubit loop, the diamagnetic (paramagnetic) current states assigned by $|\downarrow\rangle(|\uparrow\rangle)$ are superposed, which makes the loop being regarded as a qubit. $\odot$ (oppositely $\otimes$) denote the directions of the applied and induced magnetic fields $f_i=\Phi_i/\Phi_0$ in the qubit loop i. $I_i(I')$ stand for the currents in the qubit i (connecting) loop. E_J's are the Josephson coupling energies of the Josephson junctions in the connecting and qubit loops. The fluxoid quantization in the connecting loop gives rise to the boundary condition connecting the phases φ_i across each Josephson junction. Both the mutual inductances and the fluxoid quantization make it possible to realize many-qubit interactions in the N-flux qubit system. The many-qubit interactions are defined in the text.

loops (Fig. 1). Unprimed (primed) indices will indicate qubit (connecting) loops. The charging energy of Josephson junctions in the $N(N')$ qubit (connecting) loops is given by

$$\mathcal{H}_C=\frac{1}{2}\left(\frac{\Phi_0}{2\pi}\right)^2\left(\sum_{i=1}^{N}\sum_{\alpha}C_i^{\alpha}\dot{\varphi}_{i\alpha}^2+\sum_{i'=1}^{N'}\sum_{\alpha'}C_{i'}^{\alpha'}\dot{\varphi}_{i'\alpha'}^2\right), \quad (2)$$

where $C(C')$ is the capacitance of the Josephson junctions in the qubit (connecting) loops. $\Phi_0=h/2e$ is the unit flux quantum. $\alpha(\alpha')$ rely on the number of Josephson junctions in a qubit (connecting) loop. The inductive energy is given by

$$\mathcal{H}_L=\sum_{i,j=1}^{N}\frac{1}{2}[L^{(ij)}+\delta_{ij}L_K^{(i)}]I_iI_j+\sum_{i,i'=1}^{N,N'}\mathcal{L}^{(ii')}I_iI'_{i'}$$
$$+\sum_{i',j'=1}^{N'}\frac{1}{2}[L'^{(i'j')}+\delta_{i'j'}L_K'^{(i')}]I'_{i'}I'_{j'}, \quad (3)$$

where $I_i(I'_{i'})$ are the circulating currents in the qubit (connecting) loop $i(i')$. $L^{(ii)}=L_S^{(i)}$ is the self-inductance for the qubit loop i. For $i\neq j$, $L^{(ij)}$ is the mutual inductance between the qubits i and j. $L_K^{(i)}$ is the kinetic-inductance[13,16,17] in the qubit loop i. Similarly, $L_S'^{(i')}$, $L_K'^{(i')}$, and $L'^{(i'j')}$ are denoted for the connecting loops. $\mathcal{L}^{(ii')}$ is the mutual inductance between the qubit loop i and the connecting loop i'. Finally, the Josephson energy of the junctions is given by

$$\mathcal{H}_J=\sum_{i=1}^{N}\sum_{\alpha}2E_{Ji}^{\alpha}\sin^2\frac{\varphi_i^{\alpha}}{2}+\sum_{i'=1}^{N'}\sum_{\alpha'}2E_{Ji'}'^{\alpha'}\sin^2\frac{\varphi_{i'}^{\alpha'}}{2}, \quad (4)$$

where E_J's are the Josephson energy of junctions in the qubit and connecting loops.

By integrating Eq. (1) along the closed path in the ith loop, the fluxoid quantization gives the boundary conditions,

$$L_K^{(i)}I_i/\Phi_0=n_i-\frac{1}{2\pi}\sum_{\alpha}\varphi_i^{\alpha}-f_i, \quad (5)$$

where φ_i^{α} is the phase across the Josephson junction α, n_i is an integer, and $f_i=f_{\rm ext}^{(i)}+f_{\rm ind}^{(i)}$ consists of an external and induced magnetic fields, i.e., $f_{\rm ext}^{(i)}=\Phi_i/\Phi_0$ and $f_{\rm ind}^{(i)}=\Sigma_{j=1}^{N}L^{(ij)}I_j/\Phi_0+\Sigma_{i'=1}^{N'}\mathcal{L}^{(ii')}I'_{i'}/\Phi_0$. Similarly, the boundary conditions in the connecting loops can be given. From the boundary conditions, the total energy can be re-expressed as a function of the phases $\{\varphi_i\}$ and their time derivatives $\{\dot{\varphi}_i\}$.

The number of Cooper pairs n and the phase of wave function φ are noncommuting variables, i.e., $[\varphi,n]=i$, such that the canonical momentum P_φ can be introduced as $P_\varphi=n\hbar=-i\hbar\partial_\varphi$,[18] where $n=q/2e$ with the charge from the Josephson relation, $q=C(\Phi_0/2\pi)\dot{\varphi}$. When the charging energy is much smaller than the Josephson energy, the phase is well defined while the number is strongly fluctuating. The charging energy $\mathcal{H}_C(\{\dot{\varphi}_i\})$ plays a role of kinetic energy for a particle in an effective potential defined by $U(\{\varphi_i\})=\mathcal{H}_L(\{\varphi_i\})+\mathcal{H}_J(\{\varphi_i\})$.

In the three-Josephson-junction qubit loops ($\alpha\in\{a,b,c\}$) with $E_{Ji}^{b,c}=E_{Ji}$ and $\varphi_i^{b,c}=\varphi_i$, the effective potential has the 2^N local minima corresponding to the 2^N basis, $\{|m_1,\cdots,m_N\rangle\}$, of the N qubits with $m_i=\uparrow$ and $\downarrow$ for $i=1,\cdots,N$. The values of $\{\varphi_i\}$ at the local minimum corresponding to the state $|m_1,\cdots,m_N\rangle$ are denoted by $\{\varphi^0_{i;m_1\cdots m_N}\}$. Then, $\{\varphi^0_{i;m_1\cdots m_N}\}$ determines the current state of flux qubit i by the current-phase relation.

In the low-energy limit, one can employ a tight-binding approximation in which the 2^N states of N qubits correspond to 2^N-lattice sites. In the 2^N basis $\{|m_1,\cdots,m_N\rangle\}$, the low-energy N-qubit Hamiltonian matrix can be written as

$$\mathcal{H}_N=\sum_{j_1,\cdots,j_N\in\{0,x,y,z\}}C_{j_1\cdots j_N}\boldsymbol{\sigma}_1^{j_1}\otimes\cdots\otimes\boldsymbol{\sigma}_N^{j_N}, \quad (6)$$

where $\boldsymbol{\sigma}^0(\boldsymbol{\sigma}^{x,y,z})$ are the identity (Pauli) matrices. The coefficients are obtained by

$$C_{j_1\cdots j_N}=\frac{1}{2^N}{\rm Tr}\,[\boldsymbol{\sigma}_1^{j_1}\otimes\cdots\otimes\boldsymbol{\sigma}_N^{j_N}\mathcal{H}_N]. \quad (7)$$

The diagonal components of the Hamiltonian matrix are the level energies $E_{m_1,\cdots,m_N}$ at the local minima $\{\varphi^0_{i;m_1\cdots m_N}\}$. The level energies are given by

$$E_{m_1,\cdots,m_N}=\frac{\hbar}{2}\sum_{i=1}^{N}\omega_{i;m_1,\cdots,m_N}+U(\{\varphi^0_{i;m_1\cdots m_N}\}), \quad (8)$$

where the characteristic oscillating frequencies are $\omega^2_{i;m_1,\cdots,m_N}=\frac{1}{M_i}\frac{\partial^2}{\partial\varphi_i^2}U(\{\varphi_i\})|_{\{\varphi^0_{i;m_1,\cdots,m_N}\}}$ with an effective mass $M_i=(\frac{\Phi_0}{2\pi})^2C_{\rm eff}^{(i)}$ and effective capacitance $C_{\rm eff}^{(i)}$ in the harmonic-oscillator approximation.[19]

Generally, the macroscopic tunneling processes between any two many-qubit states are possible due to the quantum fluctuation originating from the kinetic energy. The off-diagonal components are the macroscopic quantum tunneling amplitudes, i.e.,

$$t:|m_1',\cdots,m_N'\rangle \Leftrightarrow |m_1,\cdots,m_N\rangle, \tag{9}$$

for the tunneling between the two states, $|m_1',\cdots,m_N'\rangle$ and $|m_1,\cdots,m_N\rangle$. The tunneling amplitudes can be calculated by the well-known numerical methods such as WKB approximation, instanton method, and Fourier grid Hamiltonian method.[20] The tunneling process, $|\uparrow\uparrow\uparrow\cdots\uparrow\rangle \Leftrightarrow |\downarrow\uparrow\uparrow\cdots\uparrow\rangle$, describes the first pseudospin flip. Such a tunneling process that describes only one pseudospin flip among the N qubits is called *single-qubit tunneling* t_1. If the N qubits are flipped for tunneling, the tunneling processes can be called *N -qubit tunneling* t_N, e.g., $|\downarrow\uparrow\uparrow\cdots\downarrow\rangle \Leftrightarrow |\uparrow\downarrow\downarrow\cdots\uparrow\rangle$. Normally, single-qubit tunneling amplitudes are much larger than other multiple-qubit ones. However, when a multiple-qubit tunneling amplitude is larger than single-qubit one, the multiple-qubit tunneling processes can play an important role in determining the property of eigenstates of the system.[21,22]

Actually, Eq. (6) describes any N-qubit system including all types of many-qubit interactions. Let us expand the low energy N-qubit Hamiltonian matrix in terms of qubit interactions,

$$\mathcal{H}_N = H_0 + \sum_i H_1^{(i)} + \sum_{i<j} H_2^{(ij)} + \sum_{i<j<k} H_3^{(ijk)} + \cdots + H_N^{(1\cdots N)}, \tag{10}$$

where $H_0=(1/2^N)\,\mathrm{Tr}\,[\mathcal{H}_N]$ and the qubits are described by $H_1^{(i)}=\varepsilon_i \boldsymbol{\sigma}_i^z + t_1^{(i)}\boldsymbol{\sigma}_i^x$ with the energy difference $2\varepsilon_i$ and the tunneling amplitude $t_1^{(i)}$ between the two states of the qubit i. Qubit interactions are denoted by two-qubit interactions $H_2^{(ij)}$, three-qubit interactions $H_3^{(ijk)}$, and so on. Then, the N-qubit interaction is presented by

$$H_N^{(1\cdots N)} = \sum_{j_1,\cdots,j_N\in\{x,y,z\}} C_{j_1\cdots j_N}\boldsymbol{\sigma}_1^{j_1}\otimes\cdots\otimes\boldsymbol{\sigma}_N^{j_N}. \tag{11}$$

We define the *N -qubit exchange coupling constant* as

$$J_{z\cdots z}^{(N)} = C_{z\cdots z} = \frac{1}{2^N}\,\mathrm{Tr}\,[\boldsymbol{\sigma}_1^z\otimes\cdots\otimes\boldsymbol{\sigma}_N^z\mathcal{H}_N], \tag{12}$$

which has a form of the *Ising-type exchange interaction* for N qubits. For other terms of the N-qubit interaction, the coefficients of the terms can be called *N -qubit tunnel exchange coupling constants*, e.g.,

$$J_{x\cdots y\cdots z}^{(N)} = C_{x\cdots y\cdots z} = \frac{1}{2^N}\,\mathrm{Tr}\,[\boldsymbol{\sigma}_1^x\otimes\cdots\boldsymbol{\sigma}_i^y\cdots\otimes\boldsymbol{\sigma}_N^z\mathcal{H}_N], \tag{13}$$

since the off-diagonal components of the Hamiltonian matrix result from the hopping (tunneling) between the sites (states).

For two-qubit systems, the two-qubit interaction is given by $H_2^{(12)}=\sum_{j\in\{x,y,z\}}J_{jj}^{(2)}\boldsymbol{\sigma}_1^j\otimes\boldsymbol{\sigma}_2^j$, where $J_{xx}^{(2)}=-(t_2^a+t_2^b)/2$, $J_{yy}^{(2)}=(t_2^a-t_2^b)/2$, and $J_{zz}^{(2)}=(E_{\uparrow\uparrow}-E_{\uparrow\downarrow}-E_{\downarrow\uparrow}+E_{\downarrow\downarrow})/4$. The two-qubit tunneling amplitudes, t_2^a and t_2^b, describe the tunneling processes: (i) $|\uparrow\uparrow\rangle\Leftrightarrow|\downarrow\downarrow\rangle$ in the parallel pseudospin states and (ii) $|\uparrow\downarrow\rangle\Leftrightarrow|\downarrow\uparrow\rangle$ in the antiparallel pseudospin states. As expected, the exchange coupling constant $J_{zz}^{(2)}$ is the energy difference between the parallel and antiparallel pseudospin states. The two-qubit tunnelings contribute to the pseudospin exchange interaction. Then, $H_2^{(12)}$ has a form of XYZ model for two pseudospins. $t_2^a \ll t_2^b$ gives an XXZ pseudospin model and, for $J_{zz}^{(2)}=0$, i.e., $E_{\uparrow\uparrow}+E_{\downarrow\downarrow}=E_{\uparrow\downarrow}+E_{\downarrow\uparrow}$, an XY pseudospin model. For $t_2^{a,b}\ll J_{zz}^{(2)}$, $H_2^{(12)}$ becomes an Ising pseudospin model. This shows that various types of pseudospin models can be realized by manipulating the system parameters.

Next, for comparison, let us consider a two-qubit interaction of three-qubit system given by $H_2^{(12)}=\sum_{j\in\{x,y,z\}}J_{jj0}^{(2)}\boldsymbol{\sigma}_1^j\otimes\boldsymbol{\sigma}_2^j+J_{xz0}^{(2)}\boldsymbol{\sigma}_1^x\otimes\boldsymbol{\sigma}_2^z+J_{zx0}^{(2)}\boldsymbol{\sigma}_1^z\otimes\boldsymbol{\sigma}_2^x$, where $J_{xx0}^{(2)}=-(\overline{t}_2^a+\overline{t}_2^b)/4-(\underline{t}_2^a+\underline{t}_2^b)/4$, $J_{yy0}^{(2)}=(\overline{t}_2^a-\overline{t}_2^b)/4+(\underline{t}_2^a-\underline{t}_2^b)/4$, and $J_{zz0}^{(2)}=(E_{\uparrow\uparrow\uparrow}-E_{\uparrow\downarrow\uparrow}-E_{\downarrow\uparrow\uparrow}+E_{\downarrow\downarrow\uparrow})/8+(E_{\uparrow\uparrow\downarrow}-E_{\uparrow\downarrow\downarrow}-E_{\downarrow\uparrow\downarrow}+E_{\downarrow\downarrow\downarrow})/8$. Here, $\overline{t}(\underline{t})$ denotes the two-qubit tunnelings for the up (down) state of the third pseudospin. Compared to the two-qubit interaction in two-qubit systems, interestingly, there are two extra tunnel exchange coupling terms, $J_{xz0}^{(2)}$ and $J_{zx0}^{(2)}$, mediated by the single-qubit tunnelings.

In the three-qubit interaction $H_3^{(123)}$, the three-qubit exchange coupling constant is given by $J_{zzz}^{(3)}=(E_{\uparrow\uparrow\uparrow}-E_{\uparrow\downarrow\uparrow}-E_{\downarrow\uparrow\uparrow}+E_{\downarrow\downarrow\uparrow})/8-(E_{\uparrow\uparrow\downarrow}-E_{\uparrow\downarrow\downarrow}-E_{\downarrow\uparrow\downarrow}+E_{\downarrow\downarrow\downarrow})/8$. Also, the single- and two-qubit tunnelings as well as the three-qubit tunnelings give rise to the three-qubit tunnel exchange coupling constants. Especially, if the three-qubit tunnelings are stronger than the two-qubit tunnelings, the ground state can be in a Greenberger-Horne-Zeilinger state and if the two-qubit tunnelings are stronger than the three-qubit tunnelings, a W state can be generated in an excited state.[22]

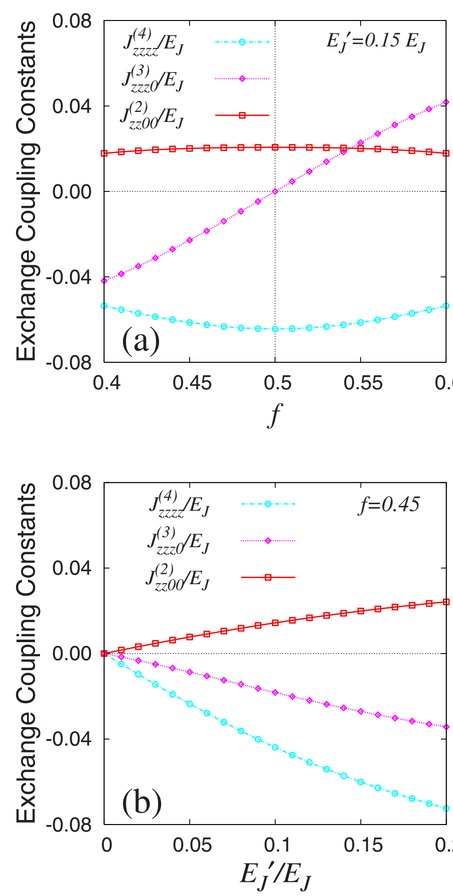

FIG. 2. (Color online) Multiple-qubit exchange coupling constants, $J_{zzzz}^{(4)}$, $J_{zzz0}^{(3)}$, and $J_{zz00}^{(2)}$, in the four qubits (N=4) as a function of (a) the applied magnetic field $f=f_{\rm ext}^{(i)}$ $(i=1,\ldots,4)$ for $E_J'=0.15E_J$ and (b) the Josephson energy E_J' for f=0.45. Other parameters are $n_i=n'=0$ and $E_J^\alpha=E_J(\alpha\in\{a,b,c\})$.

To explore many-qubit interactions explicitly, let us consider a specific multiple-qubit system in Fig. 1. For simplicity, the inductances are assumed to be very small and then the inductive energy can be negligible. The boundary conditions for the qubit loops and the connecting loop are reduced to $2\varphi_i+\varphi_i^a=2\pi[n_i-f_{\rm ext}^{(i)}]$ and $\varphi'=2\pi n'-\Sigma_{i=1}^N\varphi_i^a$, respectively. The effective potential is given as

$$U((\varphi_i))=\sum_{i=1}^N\left\{4E_J\sin^2\frac{\varphi_i}{2}+2E_J^a\sin^2\pi\left[n_i-f_{\rm ext}^{(i)}-\frac{\varphi_i}{\pi}\right]\right\}+2E_j'\sin^2\pi\left\{n'-\sum_{i=1}^N\left[n_i-f_{\rm ext}^{(i)}-\frac{\varphi_i}{\pi}\right]\right\}. \quad (14)$$

For the four-qubit system ($N=4$), we plot the exchange coupling constants as a function of $f=f_{\rm ext}^{(i)}$ and E_J' in Figs. 2(a) and 2(b), respectively. At the coresonance point, $f=0.5$, the three-qubit interaction disappears while the two- and four-qubit interaction strengths reach their maximum values in Fig. 2(a). The sign of the three-qubit interaction is changed from negative for $f<0.5$ to positive for $f>0.5$. As E_J' increases, the two-, three-, and four-qubit interactions increase monotonically in Fig. 2(b).

Interestingly, the four-qubit interaction is stronger than the two- and three-qubit interactions; that is, $J_{zzzz}^{(4)}\approx 3J_{zz00}^{(2)}$. Also, the three-qubit interaction can be stronger than the two-qubit interaction for a certain applied magnetic field. This result seems to be counterintuitive. However, for an N-qubit system, the result can be understood from Eq. (5) as well as the boundary condition of the connecting loop without the assumption, $L_K'I'/\Phi_0=n'-(1/2\pi)(\varphi'+\Sigma_{i=1}^N\varphi_i^a)-f_{\rm ind}'$, where $f_{\rm ind}'=L'I'/\Phi_0+\Sigma_{i=1}^N\mathcal{L}_M^{(i)}I_i/\Phi_0$ with the mutual inductance $\mathcal{L}_M^{(i)}$. When one superconducting loop couples all qubit loops, all qubits are interconnected through the effective flux $f_{\rm eff}\equiv(1/2\pi)\Sigma_{i=1}^N\varphi_i^a$ as well as $f_{\rm ind}'$. Normally, the induced flux is much smaller than the effective flux, i.e., $f_{\rm ind}^{(i)},f_{\rm ind}'\ll f_{\rm eff}$, so that much stronger many-qubit interaction for the effective flux than for the induced magnetic flux can be expected. Therefore, if the N-qubit interaction is much stronger than other quit interactions, $\mathcal{H}_N\approx H_N^{(1\ldots N)}$ can map higher dimensional systems.[5]

We also considered two more models: (i) For N qubits inductively coupled without any connecting loop, multiple-qubit interactions are intrinsically involved but their strengths are very weak, as for instance of $N=4$, $J_{zzzz}^{(4)}\approx 10^{-6}J_{zz00}^{(2)}$ in the parameters of Ref. 12. If the two-qubit interactions are much stronger than other multiple-qubit interactions, $\mathcal{H}_N\approx\Sigma_{i<j}H_2^{(ij)}$. Then, an N qubits inductively coupled can be a many-spin system in which one artificial-spin interact with all other artificial spins by the two-body interactions. (ii) For the model of Ref. 13, the multiple-qubit interactions behave similarly to the model of Fig. 1. In the same parameter values with Fig. 2(a), however, this model gives $J_{zzzz}^{(4)}\approx 0.17J_{zz00}^{(2)}$. The two models show that the four-qubit interaction is smaller than the two-qubit interactions for the four-qubit systems. In general, hence, many-qubit interactions are dependent on specific experimental setups and on varying the system parameters. Various types of artificial-spin systems can be prepared in flux qubit systems. Therefore, it is possible to explore a many-spin system realized in flux qubit systems.

We investigated many-qubit interactions in the superconducting flux qubit systems. There are two types of many-qubit exchange interactions: one is similar to the Ising spin interaction; and the other types of exchange interactions are due to macroscopic quantum tunnelings between the many-qubit states. Various types of many-qubit interactions can be realized experimentally in flux qubit systems. Moreover, an experimental setup can be provided to study many-spin systems that can be mapped into many-flux qubit systems.

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