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Free-field realization of the $osp(2n|2n)$ current algebra

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The $osp(2n|2n)$ current algebra for a *generic* positive integer n at general level k is investigated. Its free-field representation and corresponding energy-momentum tensor are constructed. The associated screening currents of the first kind are also presented.

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I. INTRODUCTION

The interest in conformal field theories (CFTs) [1,2] related to superalgebras has grown over the past ten years because of their applications in physics areas ranging from string theory [3,4] and logarithmic CFTs [5,6] (for a review, see e.g. [7,8], and references therein) to modern condensed matter physics [9–14]. In particular, the Wess-Zumino-Novikov-Witten (WZNW) models associated with the supergroups $GL(n|n)$ and $OSP(2n|2n)$ stand out as an important class of CFTs due to the fact that they have vanishing central charge and primary fields with negative dimensions [10,11,15–17]. However, unlike their bosonic versions, the WZNW models on supergroups are far from being understood [17] (for references therein and some recent progress, see [18]), largely due to technical reasons such as indecomposability of the operator product expansion (OPE), appearance of logarithms in correlation functions, and continuous modular transformations of the irreducible characters [19].

On the other hand, the Wakimoto free-field realizations of current algebras (or affine algebras [20]) [21] have been proven to be powerful in the study of the WZNW models on bosonic groups [22–27]. The free-field realizations of bosonic current algebras have been extensively investigated [28–36]. However, to our knowledge, explicit free-field expressions for current algebras associated with superalgebras have so far been known only for some isolated cases: those associated with superalgebras $gl(2|1)$ and $gl(2|2)$ [37–40], $gl(m|n)$ [41], and $osp(1|2)$ and $osp(2|2)$ [31,38]. In particular, explicit free-field expressions of the $osp(2n|2n)$ current algebra for generic n are still lacking due to the fact that it practically would be very involved (if not impossible) to apply the general procedure developed in [32,38] to the current algebra with a large n.

The recent studies of the random bond Ising model have revealed that an appropriate algebraic framework for studying multispecies Gaussian disordered systems at criticality is based on the $osp(2n|2n)$ current algebra with the positive integer n related to the species [10,12,14]. The explicit free-field expressions of the $osp(2n|2n)$ current algebra will enable one to explicitly construct correlation functions [2,22,25] of the disordered systems at the critical points.

In this paper, motivated by its great applications to both string theory and condensed matter physics, we investigate the $osp(2n|2n)$ current algebra associated with the $OSP(2n|2n)$ WZNW model at general level k for a *generic* positive integer n. Based on a particular order introduced in Ref. [42] for the roots of (super)algebras, we work out the explicit expression of the differential realization of $osp(2n|2n)$. We then apply the differential realization to construct explicit free-field representation of the current algebra. This representation provides the Verma modules of the algebra.

This paper is organized as follows. In Sec. II, we briefly review the definitions of finite-dimensional superalgebra $osp(2n|2n)$ and the associated current algebra, which also introduces our notation and some basic ingredients. In Sec. III, after constructing explicitly the differential operator realization of $osp(2n|2n)$, we construct the explicit free-field representation of the $osp(2n|2n)$ current algebra at a generic level k. In Sec. IV, we construct the free-field realization of the corresponding energy-momentum tensor by the Sugawara construction. In Sec. V, we construct the free-field realization of the associated screening currents. Section VI is for conclusions. In the appendix, we give the matrix realizations associated with the defining representation for all generators of $osp(2n|2n)$.

II. NOTATION AND PRELIMINARIES

Let us fix our notation for the underlying nonaffine superalgebra $osp(2n|2n)$ for a positive integer n. $osp(2n|2n)$ is a $\mathbb{Z}_2$-graded simple superalgebra with a dimension $d = 8n^2$. Let $\{E_i|i = 1, \dots, d = 8n^2\}$ be a basis of $osp(2n|2n)$ with a definite $\mathbb{Z}_2$ grading and denoting the corresponding grading by $[E_i]$. The generators $\{E_i\}$ satisfy (anti)commutation relations

$$[E_i, E_j] = \sum_{m=1}^{8n^2} f_{ij}^m E_m, \qquad (2.1)$$

where f_{ij}^m are the structure constants of $osp(2n|2n)$. Here and throughout, we adopt the convention $[a, b] = ab - (-1)^{[a][b]}ba$, which extends to inhomogenous elements through linearity.

One can introduce a nondegenerate and invariant supersymmetric metric or bilinear form (E_i, E_j) for $osp(2n|2n)$ by (A19). Then the $osp(2n|2n)$ current algebra [or affine algebra $osp(2n|2n)_k$ [20]] is generated by the currents $E_i(z)$ associated with the generators E_i of $osp(2n|2n)$. The current algebra at a general level k obeys the following OPEs [2]:

$$E_i(z)E_j(w) = k\frac{(E_i, E_j)}{(z-w)^2} + \frac{\sum_{m=1}^{d} f_{ij}^m E_m(w)}{(z-w)}, \quad i, j = 1, \ldots, d, \tag{2.2}$$

where f_{ij}^m are the structure constants in (2.1) and d is the dimension of $osp(2n|2n)$, i.e. $d = 8n^2$. The aim of this paper is to construct explicit free-field realizations of the $osp(2n|2n)$ current algebra with a generic positive integer n at a generic level k.

Alternatively, one can use the associated root system to label the generators of $osp(2n|2n)$ as follows [43]. Because of the fact that the rank of $osp(2n|2n)$ is $2n$, let us introduce $2n$ linear-independent vectors: $\{\delta_i | i = 1, \ldots, n\}$ and $\{\epsilon_i | i = 1, \ldots n\}$. These vectors are endowed a symmetric inter product such that

$$\begin{aligned} (\delta_m, \delta_l) &= \delta_{ml}, \qquad (\delta_m, \epsilon_i) = 0, \\ (\epsilon_i, \epsilon_j) &= -\delta_{ij}, \qquad i, j, m, l = 1, \ldots, n. \end{aligned} \tag{2.3}$$

The set of roots of $osp(2n|2n)$ [or $D(n, n)$], denoted by Δ, can be expressed in terms of $\{\delta_i, \epsilon_j | i, j = 1, \ldots, n\}$ as

$$\Delta = \{\pm\epsilon_i \pm \epsilon_j, \pm\delta_m \pm \delta_l, \pm 2\delta_l, \pm\delta_l \pm \epsilon_i\}, \quad i \neq j, \quad m \neq l, \tag{2.4}$$

while the set of even roots denoted by $\Delta_{\bar{0}}$ and the set of odd roots denoted by $\Delta_{\bar{1}}$ are given, respectively, by

$$\begin{aligned} \Delta_{\bar{0}} &= \{\pm\epsilon_i \pm \epsilon_j, \pm\delta_m \pm \delta_l, \pm 2\delta_l\}, \\ \Delta_{\bar{1}} &= \{\pm\delta_l \pm \epsilon_i\}, \qquad i \neq j, \qquad m \neq l. \end{aligned} \tag{2.5}$$

The distinguished simple roots are

$$\begin{aligned} \alpha_1 &= \delta_1 - \delta_2, \ldots, \qquad \alpha_{n-1} = \delta_{n-1} - \delta_n, \\ \alpha_n &= \delta_n - \epsilon_1, \qquad \alpha_{n+1} = \epsilon_1 - \epsilon_2, \ldots, \\ \alpha_{2n-1} &= \epsilon_{n-1} - \epsilon_n, \qquad \alpha_{2n} = \epsilon_{n-1} + \epsilon_n. \end{aligned} \tag{2.6}$$

Then the corresponding positive roots denoted by Δ_+ are

$$\delta_m - \delta_l, \qquad 2\delta_l, \qquad \delta_m + \delta_l, \qquad 1 \leq m < l \leq n, \tag{2.7}$$

$$\delta_l - \epsilon_i, \qquad \delta_l + \epsilon_i, \qquad 1 \leq i, l \leq n, \tag{2.8}$$

$$\epsilon_i - \epsilon_j, \qquad \epsilon_i + \epsilon_j, \qquad 1 \leq i < j \leq n. \tag{2.9}$$

Among these positive roots, $\{\delta_l \pm \epsilon_i | i, l = 1 \ldots, n\}$ are odd and the others are even. Moreover, associated with each positive root $\alpha \in \Delta_+$, there are a raising operator E_α which altogether spans the subalgebra $(osp(2n|2n))_+$, a lowering operator F_α which altogether spans the subalgebra $(osp(2n|2n))_-$, and a Cartan generator H_α which altogether spans the Cartan subalgebra $\mathbf{h}$. Then one has the Cartan-Weyl decomposition of $osp(2n|2n)$

$$osp(2n|2n) = (osp(2n|2n))_- \oplus \mathbf{h} \oplus (osp(2n|2n))_+.$$

Hereafter, we adopt the convention that

$$E_i \equiv E_{\alpha_i}, \qquad F_i \equiv F_{\alpha_i}, \qquad i = 1, \ldots, 2n. \tag{2.10}$$

We remark that the $\mathbb{Z}_2$ grading of the generators associated with the simple roots and the Cartan subalgebra are

$$[E_n] = [F_n] = 1, \qquad [E_i] = [F_i] = 0, \quad \text{for } i \neq n, \tag{2.11}$$

$$[g] = 0, \qquad \forall\, g \in \mathbf{h}. \tag{2.12}$$

The matrix realization of the generators associated with all roots of $osp(2n|2n)$ is given in the appendix, from which one may derive the structure constants f_{ij}^m of the algebra in (2.1) for the particular choice of the basis.

III. FREE-FIELD REALIZATION OF THE $osp(2n|2n)$ CURRENTS

A. Differential operator realization of $osp(2n|2n)$

Let us introduce a bosonic coordinate ($x_{m,l}$, $\bar{x}_{m,l}$, x_l, $y_{i,j}$, or $\bar{y}_{i,j}$ for $m < l$ and $i < j$) with a $\mathbb{Z}_2$-grading zero: $[x] = [\bar{x}] = [y] = [\bar{y}] = 0$ associated with each positive even root (respectively, $\delta_m - \delta_l$, $\delta_m + \delta_l$, $2\delta_l$, $\epsilon_i - \epsilon_j$, or $\epsilon_i + \epsilon_j$ for $m < l$ and $i < j$), and a fermionic coordinate ($\theta_{l,i}$ or $\bar{\theta}_{l,i}$) with a $\mathbb{Z}_2$-grading one: $[\theta] = [\bar{\theta}] = 1$ associated with each positive odd root (respectively, $\delta_l - \epsilon_i$ or $\delta_l + \epsilon_i$). These coordinates satisfy the following (anti)commutation relations:

$$\begin{aligned} [x_{i,j}, x_{m,l}] &= 0, \qquad [\partial_{x_{i,j}}, \partial_{x_{m,l}}] = 0, \\ [\partial_{x_{i,j}}, x_{m,l}] &= \delta_{im}\delta_{jl}, \end{aligned} \tag{3.1}$$

$$\begin{aligned} [\bar{x}_{i,j}, \bar{x}_{m,l}] &= 0, \qquad [\partial_{\bar{x}_{i,j}}, \partial_{\bar{x}_{m,l}}] = 0, \\ [\partial_{\bar{x}_{i,j}}, \bar{x}_{m,l}] &= \delta_{im}\delta_{jl}, \end{aligned} \tag{3.2}$$

$$[x_m, x_l] = 0, \qquad [\partial_{x_m}, \partial_{x_l}] = 0, \qquad [\partial_{x_m}, x_l] = \delta_{ml}, \tag{3.3}$$

$$\begin{aligned} [y_{i,j}, y_{m,l}] &= 0, \qquad [\partial_{y_{i,j}}, \partial_{y_{m,l}}] = 0, \\ [\partial_{y_{i,j}}, y_{m,l}] &= \delta_{im}\delta_{jl}, \end{aligned} \tag{3.4}$$

$$[\bar{y}_{i,j}, \bar{y}_{m,l}] = 0, \qquad [\partial_{\bar{y}_{i,j}}, \partial_{\bar{y}_{m,l}}] = 0, \qquad [\partial_{\bar{y}_{i,j}}, \bar{y}_{m,l}] = \delta_{im}\delta_{jl}, \tag{3.5}$$

$$[\theta_{i,j}, \theta_{m,l}] = 0, \qquad [\partial_{\theta_{i,j}}, \partial_{\theta_{m,l}}] = 0, \qquad [\partial_{\theta_{i,j}}, \theta_{m,l}] = \delta_{im}\delta_{jl}, \tag{3.6}$$

$$[\bar{\theta}_{i,j}, \bar{\theta}_{m,l}] = 0, \qquad [\partial_{\bar{\theta}_{i,j}}, \partial_{\bar{\theta}_{m,l}}] = 0, \qquad [\partial_{\bar{\theta}_{i,j}}, \bar{\theta}_{m,l}] = \delta_{im}\delta_{jl}, \tag{3.7}$$

and the other (anti)commutation relations are vanishing. Let $\langle\Lambda|$ be the highest weight vector of the representation of $osp(2n|2n)$ with highest weights $\{\lambda_i\}$, satisfying the following conditions:

$$\langle\Lambda|F_i = 0, \qquad 1 \le i \le 2n, \tag{3.8}$$

$$\langle\Lambda|H_i = \lambda_i\langle\Lambda|, \qquad 1 \le i \le 2n. \tag{3.9}$$

Here the generators H_i are expressed in terms of some linear combinations of H_α (A14)–(A16). An arbitrary vector in the corresponding Verma module is parametrized by $\langle\Lambda|$ and the corresponding bosonic and fermionic coordinates as

$$\langle\Lambda; x, \bar{x}; y, \bar{y}; \theta, \bar{\theta}| = \langle\Lambda|G_+(x, \bar{x}; y, \bar{y}; \theta, \bar{\theta}), \tag{3.10}$$

where $G_+(x, \bar{x}; y, \bar{y}; \theta, \bar{\theta})$ is given by (cf. [32,38][1])

$$G_+(x, \bar{x}; y, \bar{y}; \theta, \bar{\theta}) = (\bar{G}_{2n-1,2n}G_{2n-1,2n})\ldots(\bar{G}_{n+1,n+2}\ldots\bar{G}_{n+1,2n}G_{n+1,2n}\ldots G_{n+1,n+2}) \times(\bar{G}_{n,n+1}\ldots\bar{G}_{n,2n}G_nG_{n,2n}\ldots G_{n,n+1})\ldots(\bar{G}_{1,2}\ldots\bar{G}_{1,2n}G_1G_{1,2n}\ldots G_{1,2}). \tag{3.11}$$

Here $G_{i,j}$ and $\bar{G}_{i,j}$ are given by

$$G_{m,l} = e^{x_{m,l}E_{\delta_m-\delta_l}}, \qquad \bar{G}_{m,l} = e^{\bar{x}_{m,l}E_{\delta_m+\delta_l}}, \qquad 1 \le m < l \le n, \tag{3.12}$$

$$G_l = e^{x_lE_{2\delta_l}}, \qquad G_{l,n+i} = e^{\theta_{l,i}E_{\delta_l-\epsilon_i}}, \qquad \bar{G}_{l,n+i} = e^{\bar{\theta}_{l,i}E_{\delta_l+\epsilon_i}}, \qquad 1 \le l, i \le n, \tag{3.13}$$

$$G_{n+i,n+j} = e^{y_{i,j}E_{\epsilon_i-\epsilon_j}}, \qquad \bar{G}_{n+i,n+j} = e^{\bar{y}_{i,j}E_{\epsilon_i+\epsilon_j}}, \qquad 1 \le i < j \le n. \tag{3.14}$$

One can define a differential operator realization $\rho^{(d)}$ of the generators of $osp(2n|2n)$ by

$$\rho^{(d)}(g)\langle\Lambda; x, \bar{x}; y, \bar{y}; \theta, \bar{\theta}| \equiv \langle\Lambda; x, \bar{x}; y, \bar{y}; \theta, \bar{\theta}|g, \qquad \forall\ g \in osp(2n|2n). \tag{3.15}$$

Here $\rho^{(d)}(g)$ is a differential operator of the coordinates $\{x, \bar{x}; y, \bar{y}; \theta, \bar{\theta}\}$ associated with the generator g, which can be obtained from the defining relation (3.15). The defining relation also assures that the differential operator realization is actually a representation of $osp(2n|2n)$. Therefore it is sufficient to give the differential operators related to the simple roots, as the others can be constructed through the simple ones by the (anti)commutation relations. Using the relation (3.15) and the Baker-Campbell-Hausdorff formula, after some algebraic manipulations, we obtain the following differential operator representation of the simple generators:

$$\rho^{(d)}(E_l) = \sum_{m=1}^{l-1}(x_{m,l}\partial_{x_{m,l+1}} - \bar{x}_{m,l+1}\partial_{\bar{x}_{m,l}}) + \partial_{x_{l,l+1}}, \qquad 1 \le l \le n-1, \tag{3.16}$$

$$\rho^{(d)}(E_n) = \sum_{m=1}^{n-1}(x_{m,n}\partial_{\theta_{m,1}} + \bar{\theta}_{m,1}\partial_{\bar{x}_{m,n}}) + \partial_{\theta_{n,1}}, \tag{3.17}$$

$$\rho^{(d)}(E_{n+i}) = \sum_{m=1}^{n}(\theta_{m,i}\partial_{\theta_{m,i+1}} - \bar{\theta}_{m,i+1}\partial_{\bar{\theta}_{m,i}}) + \sum_{m=1}^{i-1}(y_{m,i}\partial_{y_{m,i+1}} - \bar{y}_{m,i+1}\partial_{\bar{y}_{m,i}}) + \partial_{y_{i,i+1}}, \qquad 1 \le i \le n-1, \tag{3.18}$$

$$\rho^{(d)}(E_{2n}) = \sum_{m=1}^{n}(2\theta_{m,n-1}\theta_{m,n}\partial_{x_m} + \theta_{m,n-1}\partial_{\bar{\theta}_{m,n}} - \theta_{m,n}\partial_{\bar{\theta}_{m,n-1}}) + \sum_{m=1}^{n-2}(y_{m,n-1}\partial_{\bar{y}_{m,n}} - y_{m,n}\partial_{\bar{y}_{m,n-1}}) + \partial_{\bar{y}_{n-1,n}}, \tag{3.19}$$

[1]It practically would be very involved (if not impossible) to apply the general procedure proposed in Refs. [32,38] for $osp(2n|2n)$ with a large n; however, our choice of $G_+(x, \bar{x}; y, \bar{y}; \theta, \bar{\theta})$ (3.11)–(3.14) allows us to obtain the explicit expressions of the differential operator realization of the algebra with a generic n [see (3.16)–(3.25) below].

$$\begin{aligned}\rho^{(d)}(F_l) = &\sum_{m=1}^{l-1}(x_{m,l+1}\partial_{x_{m,l}} - \bar{x}_{m,l}\partial_{\bar{x}_{m,l+1}}) - x_l\partial_{\bar{x}_{l,l+1}} - 2\bar{x}_{l,l+1}\partial_{x_{l+1}}\\ &+ \sum_{m=l+2}^{n}(x_{l,m}\bar{x}_{l,m}\partial_{\bar{x}_{l,l+1}} - x_{l,m}\partial_{x_{l+1,m}} - 2\bar{x}_{l,m}x_{l+1,m}\partial_{x_{l+1}} - \bar{x}_{l,m}\partial_{\bar{x}_{l+1,m}})\\ &- \sum_{m=1}^{n}(\theta_{l,m}\bar{\theta}_{l,m}\partial_{\bar{x}_{l,l+1}} + \theta_{l,m}\partial_{\theta_{l+1,m}} + 2\bar{\theta}_{l,m}\theta_{l+1,m}\partial_{x_{l+1}} + \bar{\theta}_{l,m}\partial_{\bar{\theta}_{l+1,m}})\\ &- x_{l,l+1}^2\partial_{x_{l,l+1}} + 2x_{l,l+1}x_{l+1}\partial_{x_{l+1}} - 2x_{l,l+1}x_l\partial_{x_l}\\ &+ x_{l,l+1}\Big[\sum_{m=l+2}^{n}(x_{l+1,m}\partial_{x_{l+1,m}} + \bar{x}_{l+1,m}\partial_{\bar{x}_{l+1,m}} - x_{l,m}\partial_{x_{l,m}} - \bar{x}_{l,m}\partial_{\bar{x}_{l,m}})\Big]\\ &+ x_{l,l+1}\Big[\sum_{m=1}^{n}(\theta_{l+1,m}\partial_{\theta_{l+1,m}} + \bar{\theta}_{l+1,m}\partial_{\bar{\theta}_{l+1,m}} - \theta_{l,m}\partial_{\theta_{l,m}} - \bar{\theta}_{l,m}\partial_{\bar{\theta}_{l,m}})\Big] + x_{l,l+1}(\lambda_l - \lambda_{l+1}), \qquad 1 \le l \le n-1,\end{aligned} \tag{3.20}$$

$$\begin{aligned}\rho^{(d)}(F_n) = &\sum_{m=1}^{n-1}(\theta_{m,1}\partial_{x_{m,n}} - \bar{x}_{m,n}\partial_{\bar{\theta}_{m,1}}) - x_n\partial_{\bar{\theta}_{n,1}} + \sum_{m=2}^{n}(\theta_{n,m}\partial_{y_{1,m}} - \theta_{n,m}\bar{\theta}_{n,m}\partial_{\bar{\theta}_{n,1}} + \bar{\theta}_{n,m}\partial_{\bar{y}_{1,m}})\\ &- \theta_{n,1}\sum_{m=2}^{n}(\theta_{n,m}\partial_{\theta_{n,m}} + \bar{\theta}_{n,m}\partial_{\bar{\theta}_{n,m}} + y_{1,m}\partial_{y_{1,m}} + \bar{y}_{1,m}\partial_{\bar{y}_{1,m}}) - 2\theta_{n,1}x_n\partial_{x_n} - 2\theta_{n,1}\bar{\theta}_{n,1}\partial_{\bar{\theta}_{n,1}} + \theta_{n,1}(\lambda_n + \lambda_{n+1}),\end{aligned} \tag{3.21}$$

$$\begin{aligned}\rho^{(d)}(F_{n+i}) = &\sum_{m=1}^{n}(\theta_{m,i+1}\partial_{\theta_{m,i}} - \bar{\theta}_{m,i}\partial_{\bar{\theta}_{m,i+1}}) + \sum_{m=1}^{i-1}(y_{m,i+1}\partial_{y_{m,i}} - \bar{y}_{m,i}\partial_{\bar{y}_{m,i+1}})\\ &+ \sum_{m=i+2}^{n}(y_{i,m}\bar{y}_{i,m}\partial_{\bar{y}_{i,i+1}} - y_{i,m}\partial_{y_{i+1,m}} - \bar{y}_{i,m}\partial_{\bar{y}_{i+1,m}})\\ &+ y_{i,i+1}\sum_{m=i+2}^{n}(y_{i+1,m}\partial_{y_{i+1,m}} + \bar{y}_{i+1,m}\partial_{\bar{y}_{i+1,m}} - y_{i,m}\partial_{y_{i,m}} - \bar{y}_{i,m}\partial_{\bar{y}_{i,m}}) - y_{i,i+1}^2\partial_{y_{i,i+1}} + y_{i,i+1}(\lambda_{n+i} - \lambda_{n+i+1}),\\ &1 \le i \le n-1,\end{aligned} \tag{3.22}$$

$$\begin{aligned}\rho^{(d)}(F_{2n}) = &\sum_{m=1}^{n}(\bar{\theta}_{m,n}\partial_{\theta_{m,n-1}} + 2\bar{\theta}_{m,n-1}\bar{\theta}_{m,n}\partial_{x_m} - \bar{\theta}_{m,n-1}\partial_{\theta_{m,n}}) + \sum_{m=1}^{n-2}(\bar{y}_{m,n}\partial_{y_{m,n-1}} - \bar{y}_{m,n-1}\partial_{y_{m,n}}) - \bar{y}_{n-1,n}^2\partial_{\bar{y}_{n-1,n}}\\ &+ \bar{y}_{n-1,n}(\lambda_{2n-1} + \lambda_{2n}),\end{aligned} \tag{3.23}$$

$$\begin{aligned}\rho^{(d)}(H_l) = &\sum_{m=1}^{l-1}(x_{m,l}\partial_{x_{m,l}} - \bar{x}_{m,l}\partial_{\bar{x}_{m,l}}) - \sum_{m=l+1}^{n}(x_{l,m}\partial_{x_{l,m}} + \bar{x}_{l,m}\partial_{\bar{x}_{l,m}}) - \sum_{m=1}^{n}(\theta_{l,m}\partial_{\theta_{l,m}} + \bar{\theta}_{l,m}\partial_{\bar{\theta}_{l,m}}) - 2x_l\partial_{x_l} + \lambda_l,\\ &1 \le l \le n,\end{aligned} \tag{3.24}$$

$$\rho^{(d)}(H_{n+i}) = \sum_{m=1}^{n}(\theta_{m,i}\partial_{\theta_{m,i}} - \bar{\theta}_{m,i}\partial_{\bar{\theta}_{m,i}}) + \sum_{m=1}^{i-1}(y_{m,i}\partial_{y_{m,i}} - \bar{y}_{m,i}\partial_{\bar{y}_{m,i}}) - \sum_{m=i+1}^{n}(y_{i,m}\partial_{y_{i,m}} + \bar{y}_{i,m}\partial_{\bar{y}_{i,m}}) + \lambda_{n+i}, \qquad 1 \le i \le n. \tag{3.25}$$

A direct computation shows that these differential operators (3.16)–(3.25) satisfy the $osp(2n|2n)$ (anti)commutation relations corresponding to the simple roots and the associated Serre relations. This implies that the differential representation of nonsimple generators can be consistently constructed from the simple ones. Hence, we have obtained an explicit differential realization of $osp(2n|2n)$.

B. Free-field realization of $osp(2n|2n)_k$

With the help of the differential realization given by (3.16)–(3.25), we can construct the explicit free-field representation of the $osp(2n|2n)$ current algebra at arbitrary level k in terms of $n\times(2n-1)$ bosonic β-γ pairs $\{(\beta_{i,j},\gamma_{i,j}),(\bar\beta_{i,j}\bar\gamma_{i,j}),(\beta_i,\gamma_i),(\beta'_{i,j},\gamma'_{i,j}),(\bar\beta'_{i,j}\bar\gamma'_{i,j}),1\le i<j\le n\}$, $2n^2$ fermionic $b-c$ pairs $\{(\Psi^+_{i,j},\Psi_{i,j}),(\bar\Psi^+_{i,j},\bar\Psi_{i,j}),1\le i,j\le n\}$, and $2n$ free scalar fields ϕ_i, $i=1,\ldots,2n$. These free fields obey the following OPEs:

$$\beta_{i,j}(z)\gamma_{m,l}(w)=-\gamma_{m,l}(z)\beta_{i,j}(w)=\frac{\delta_{im}\delta_{jl}}{(z-w)},\quad 1\le i<j\le n,\qquad 1\le m<l\le n, \tag{3.26}$$

$$\bar\beta_{i,j}(z)\bar\gamma_{m,l}(w)=-\bar\gamma_{m,l}(z)\bar\beta_{i,j}(w)=\frac{\delta_{im}\delta_{jl}}{(z-w)},\quad 1\le i<j\le n,\qquad 1\le m<l\le n, \tag{3.27}$$

$$\beta_m(z)\gamma_l(w)=-\gamma_m(z)\beta_l(w)=\frac{\delta_{ml}}{(z-w)},\quad 1\le m,l\le n, \tag{3.28}$$

$$\beta'_{i,j}(z)\gamma'_{m,l}(w)=-\gamma'_{m,l}(z)\beta'_{i,j}(w)=\frac{\delta_{im}\delta_{jl}}{(z-w)},\quad 1\le i<j\le n,\qquad 1\le m<l\le n, \tag{3.29}$$

$$\bar\beta'_{i,j}(z)\bar\gamma'_{m,l}(w)=-\bar\gamma'_{m,l}(z)\bar\beta'_{i,j}(w)=\frac{\delta_{im}\delta_{jl}}{(z-w)},\quad 1\le i<j\le n,\qquad 1\le m<l\le n, \tag{3.30}$$

$$\Psi^+_{m,i}(z)\Psi_{l,j}(w)=\Psi_{l,j}(z)\Psi^+_{m,i}(w)=\frac{\delta_{ml}\delta_{ij}}{(z-w)},\quad m,l,i,j=1,\ldots,n, \tag{3.31}$$

$$\bar\Psi^+_{m,i}(z)\bar\Psi_{l,j}(w)=\bar\Psi_{l,j}(z)\bar\Psi^+_{m,i}(w)=\frac{\delta_{ml}\delta_{ij}}{(z-w)},\quad m,l,i,j=1,\ldots,n, \tag{3.32}$$

$$\phi_m(z)\phi_l(w)=-\delta_{ml}\ln(z-w),\qquad 1\le m,l\le n, \tag{3.33}$$

$$\phi_{n+i}(z)\phi_{n+j}(w)=\delta_{ij}\ln(z-w),\qquad 1\le i,j\le n, \tag{3.34}$$

and the other OPEs are trivial.

The free-field realization of the $osp(2n|2n)$ current algebra is obtained by the substitution in the differential realization (3.16)–(3.25) of $osp(2n|2n)$:

$$x_{m,l}\longrightarrow\gamma_{m,l}(z),\qquad \partial_{x_{m,l}}\longrightarrow\beta_{m,l}(z),\qquad 1\le m<l\le n, \tag{3.35}$$

$$\bar x_{m,l}\longrightarrow\bar\gamma_{m,l}(z),\qquad \partial_{\bar x_{m,l}}\longrightarrow\bar\beta_{m,l}(z),\qquad 1\le m<l\le n, \tag{3.36}$$

$$x_l\longrightarrow\gamma_l(z),\qquad \partial_{x_l}\longrightarrow\beta_l(z),\qquad 1\le l\le n, \tag{3.37}$$

$$y_{i,j}\longrightarrow\gamma'_{i,j}(z),\qquad \partial_{y_{i,j}}\longrightarrow\beta'_{i,j}(z),\qquad 1\le i<j\le n, \tag{3.38}$$

$$\bar y_{i,j}\longrightarrow\bar\gamma'_{i,j}(z),\qquad \partial_{\bar y_{i,j}}\longrightarrow\bar\beta'_{i,j}(z),\qquad 1\le i<j\le n, \tag{3.39}$$

$$\theta_{l,i}\longrightarrow\Psi^+_{l,i}(z),\qquad \partial_{\theta_{l,i}}\longrightarrow\Psi_{l,i}(z),\qquad i,l=1,\ldots,n, \tag{3.40}$$

$$\bar\theta_{l,i}\longrightarrow\bar\Psi^+_{l,i}(z),\qquad \partial_{\bar\theta_{l,i}}\longrightarrow\bar\Psi_{l,i}(z),\qquad i,l=1,\ldots,n, \tag{3.41}$$

$$\lambda_j\longrightarrow\sqrt{k-2}\,\partial\phi_j(z)\qquad 1\le j\le 2n. \tag{3.42}$$

Moreover, in order that the resulting free-field realization satisfy the desirable OPEs for $osp(2n|2n)$ currents, one needs to add certain extra (anomalous) terms which are linear in $\partial\gamma(z)$, $\partial\bar\gamma(z)$, $\partial\gamma'(z)$, $\partial\bar\gamma'(z)$, $\partial\Psi^+(z)$, and $\partial\bar\Psi^+(z)$ in the expressions of the currents associated with negative roots [e.g. the last term in the expressions of $F_i(z)$; see (3.47), (3.48), (3.49), and (3.50) below]. Here we present the results for the currents associated with the simple roots:

$$E_l(z)=\sum_{m=1}^{l-1}(\gamma_{m,l}(z)\beta_{m,l+1}(z)-\bar\gamma_{m,l+1}(z)\bar\beta_{m,l}(z))+\beta_{l,l+1}(z),\qquad 1\le l\le n-1, \tag{3.43}$$

$$E_n(z)=\sum_{m=1}^{n-1}(\gamma_{m,n}(z)\Psi_{m,1}(z)+\bar\Psi^+_{m,1}(z)\bar\beta_{m,n}(z))+\Psi_{n,1}(z), \tag{3.44}$$

$$E_{n+i}(z) = \sum_{m=1}^{n} (\Psi^+_{m,i}(z)\Psi_{m,i+1}(z) - \bar{\Psi}^+_{m,i+1}(z)\bar{\Psi}_{m,i}(z)) + \sum_{m=1}^{i-1} (\gamma'_{m,i}(z)\beta'_{m,i+1}(z) - \bar{\gamma}'_{m,i+1}(z)\bar{\beta}'_{m,i}(z)) + \beta'_{i,i+1}(z),$$
$$1 \le i \le n-1, \tag{3.45}$$

$$E_{2n}(z) = \sum_{m=1}^{n} (2\Psi^+_{m,n-1}(z)\Psi^+_{m,n}(z)\beta_m(z) + \Psi^+_{m,n-1}(z)\bar{\Psi}_{m,n}(z) - \Psi^+_{m,n}(z)\bar{\Psi}_{m,n-1}(z))$$
$$+ \sum_{m=1}^{n-2} (\gamma'_{m,n-1}(z)\bar{\beta}'_{m,n}(z) - \gamma'_{m,n}(z)\bar{\beta}'_{m,n-1}(z)) + \bar{\beta}'_{n-1,n}(z), \tag{3.46}$$

$$F_l(z) = \sum_{m=1}^{l-1} (\gamma_{m,l+1}(z)\beta_{m,l}(z) - \bar{\gamma}_{m,l}(z)\bar{\beta}_{m,l+1}(z)) - \gamma_l(z)\bar{\beta}_{l,l+1}(z) - 2\bar{\gamma}_{l,l+1}(z)\beta_{l+1}(z)$$
$$+ \sum_{m=l+2}^{n} (\gamma_{l,m}(z)\bar{\gamma}_{l,m}(z)\bar{\beta}_{l,l+1}(z) - \gamma_{l,m}(z)\beta_{l+1,m}(z)) - \sum_{m=l+2}^{n} (2\bar{\gamma}_{l,m}(z)\gamma_{l+1,m}(z)\beta_{l+1}(z) + \bar{\gamma}_{l,m}(z)\bar{\beta}_{l+1,m}(z))$$
$$- \sum_{m=1}^{n} (\Psi^+_{l,m}(z)\bar{\Psi}^+_{l,m}(z)\bar{\beta}_{l,l+1}(z) + \Psi^+_{l,m}(z)\Psi_{l+1,m}(z)) - \sum_{m=1}^{n} (2\bar{\Psi}^+_{l,m}(z)\Psi^+_{l+1,m}(z)\beta_{l+1}(z) + \bar{\Psi}^+_{l,m}(z)\bar{\Psi}_{l+1,m}(z))$$
$$- \gamma^2_{l,l+1}(z)\beta_{l,l+1}(z) - \gamma_{l,l+1}(z) \sum_{m=l+2}^{n} (\gamma_{l,m}(z)\beta_{l,m}(z) + \bar{\gamma}_{l,m}(z)\bar{\beta}_{l,m}(z))$$
$$+ \gamma_{l,l+1}(z) \sum_{m=l+2}^{n} (\gamma_{l+1,m}(z)\beta_{l+1,m}(z) + \bar{\gamma}_{l+1,m}(z)\bar{\beta}_{l+1,m}(z))$$
$$- \gamma_{l,l+1}(z) \sum_{m=1}^{n} (\Psi^+_{l,m}(z)\Psi_{l,m}(z) + \bar{\Psi}^+_{l,m}(z)\bar{\Psi}_{l,m}(z)) + \gamma_{l,l+1}(z) \sum_{m=1}^{n} (\Psi^+_{l+1,m}(z)\Psi_{l+1,m}(z) + \bar{\Psi}^+_{l+1,m}(z)\bar{\Psi}_{l+1,m}(z))$$
$$+ 2\gamma_{l,l+1}(z)\gamma_{l+1}(z)\beta_{l+1}(z) - 2\gamma_{l,l+1}(z)\gamma_l(z)\beta_l(z) + \sqrt{k-2}\,\gamma_{l,l+1}(z)(\partial\phi_l(z) - \partial\phi_{l+1}(z))$$
$$+ (-k + 2(l-1))\partial\gamma_{l,l+1}(z), \qquad 1 \le l \le n-1, \tag{3.47}$$

$$F_n(z) = \sum_{m=1}^{n-1} (\Psi^+_{m,1}(z)\beta_{m,n}(z) - \bar{\gamma}_{m,n}(z)\bar{\Psi}_{m,1}(z)) - \gamma_n(z)\bar{\Psi}_{n,1}(z)$$
$$+ \sum_{m=2}^{n} (\Psi^+_{n,m}(z)\beta'_{1,m}(z) - \Psi^+_{n,m}(z)\bar{\Psi}^+_{n,m}(z)\bar{\Psi}_{n,1}(z) + \bar{\Psi}^+_{n,m}(z)\bar{\beta}'_{1,m}(z))$$
$$- \Psi^+_{n,1}(z) \sum_{m=2}^{n} (\Psi^+_{n,m}(z)\Psi_{n,m}(z) + \bar{\Psi}^+_{n,m}(z)\bar{\Psi}_{n,m}(z)) - 2\Psi^+_{n,1}(z)\bar{\Psi}^+_{n,1}(z)\bar{\Psi}_{n,1}(z)$$
$$- \Psi^+_{n,1}(z) \sum_{m=2}^{n} (\gamma'_{1,m}(z)\beta'_{1,m}(z) + \bar{\gamma}'_{1,m}(z)\bar{\beta}'_{1,m}(z)) - 2\Psi^+_{n,1}(z)\gamma_n(z)\beta_n(z) + \sqrt{k-2}\,\Psi^+_{n,1}(z)(\partial\phi_n(z) + \partial\phi_{n+1}(z))$$
$$+ (-k + 2(n-1))\partial\Psi^+_{n,1}(z), \tag{3.48}$$

$$F_{n+i}(z) = \sum_{m=1}^{n}(\Psi^{+}_{m,i+1}(z)\Psi_{m,i}(z) - \bar{\Psi}^{+}_{m,i}(z)\bar{\Psi}_{m,i+1}(z)) + \sum_{m=1}^{i-1}(\gamma'_{m,i+1}(z)\beta'_{m,i}(z) - \bar{\gamma}'_{m,i}(z)\bar{\beta}'_{m,i+1}(z)) + \sum_{m=i+2}^{n}(\gamma'_{i,m}(z)\bar{\gamma}'_{i,m}(z)\bar{\beta}'_{i,i+1}(z) - \gamma'_{i,m}(z)\beta'_{i+1,m}(z) - \bar{\gamma}'_{i,m}(z)\bar{\beta}'_{i+1,m}(z)) + \gamma'_{i,i+1}(z)\sum_{m=i+2}^{n}(\gamma'_{i+1,m}(z)\beta'_{i+1,m}(z) + \bar{\gamma}'_{i+1,m}(z)\bar{\beta}'_{i+1,m}(z)) - \gamma'_{i,i+1}(z)\sum_{m=i+2}^{n}(\gamma'_{i,m}(z)\beta'_{i,m}(z) + \bar{\gamma}'_{i,m}(z)\bar{\beta}'_{i,m}(z)) - \gamma'_{i,i+1}(z)\gamma'_{i,i+1}(z)\beta'_{i,i+1}(z) + \sqrt{k-2}\gamma'_{i,i+1}(z)(\partial\phi_{n+i}(z) - \partial\phi_{n+i+1}(z)) + (k+2(i-n-1))\partial\gamma'_{i,i+1}(z), \quad 1 \le i \le n-1, \tag{3.49}$$

$$F_{2n}(z) = \sum_{m=1}^{n}(\bar{\Psi}^{+}_{m,n}(z)\Psi_{m,n-1}(z) + 2\bar{\Psi}^{+}_{m,n-1}(z)\bar{\Psi}^{+}_{m,n}(z)\beta_m(z) - \bar{\Psi}^{+}_{m,n-1}(z)\Psi_{m,n}(z)) + \sum_{m=1}^{n-2}(\bar{\gamma}'_{m,n}(z)\beta'_{m,n-1}(z) - \bar{\gamma}'_{m,n-1}(z)\beta'_{m,n}(z)) - \bar{\gamma}'_{n-1,n}(z)\bar{\gamma}'_{n-1,n}(z)\bar{\beta}'_{n-1,n}(z) + \sqrt{k-2}\bar{\gamma}'_{n-1,n}(z)(\partial\phi_{2n-1}(z) + \partial\phi_{2n}(z)) + (k-4)\partial\bar{\gamma}'_{n-1,n}(z), \tag{3.50}$$

$$H_l(z) = \sum_{m=1}^{l-1}(\gamma_{m,l}(z)\beta_{m,l}(z) - \bar{\gamma}_{m,l}(z)\bar{\beta}_{m,l}(z)) - \sum_{m=l+1}^{n}(\gamma_{l,m}(z)\beta_{l,m}(z) + \bar{\gamma}_{l,m}(z)\bar{\beta}_{l,m}(z)) - 2\gamma_l(z)\beta_l(z) - \sum_{m=1}^{n}(\Psi^{+}_{l,m}(z)\Psi_{l,m}(z) + \bar{\Psi}^{+}_{l,m}(z)\bar{\Psi}_{l,m}(z)) + \sqrt{k-2}\partial\phi_l(z), \qquad 1 \le l \le n, \tag{3.51}$$

$$H_{n+i}(z) = \sum_{m=1}^{n}(\Psi^{+}_{m,i}(z)\Psi_{m,i}(z) - \bar{\Psi}^{+}_{m,i}(z)\bar{\Psi}_{m,i}(z)) + \sum_{m=1}^{i-1}(\gamma'_{m,i}(z)\beta'_{m,i}(z) - \bar{\gamma}'_{m,i}(z)\bar{\beta}'_{m,i}(z)) - \sum_{m=i+1}^{n}(\gamma'_{i,m}(z)\beta'_{i,m}(z) + \bar{\gamma}'_{i,m}(z)\bar{\beta}'_{i,m}(z)) + \sqrt{k-2}\partial\phi_{n+i}(z), \qquad 1 \le i \le n. \tag{3.52}$$

Here and throughout, normal ordering of free fields is implied whenever necessary. The free-field realization of the currents associated with the nonsimple roots can be obtained from the OPEs of the simple ones. We can straightforwardly check that the above free-field realization of the currents satisfies the OPEs of the $osp(2n|2n)$ current algebra: Direct calculation shows that there are at most second-order singularities [e.g. $\frac{1}{(z-w)^2}$] in the OPEs of the currents. Comparing with the definition of the current algebra (2.2), terms with first-order singularity [e.g. the coefficients of $\frac{1}{(z-w)}$] are fulfilled due to the very substitution (3.35)–(3.42) and the fact that the differential operator realizations (3.16)–(3.25) are a representation of the corresponding finite-dimensional superalgebra $osp(2n|2n)$; terms with second-order singularity $\frac{1}{(z-w)^2}$ also match those in the definition (2.2) after the suitable choice we made for the anomalous terms in the expressions of the currents associated with negative roots.

The free-field realization of the $osp(2n|2n)$ current algebra (3.43)–(3.52) gives rise to the Fock representations of the current algebra in terms of the free fields (3.26)–(3.34). These representations are, in general, not irreducible for the current algebra. In order to obtain irreducible ones, one needs certain screening charges, which are the integrals of screening currents [see (5.6), (5.7), (5.8), and (5.9) below] and performs the cohomology procedure as in [23,28,29,31]. We shall construct the associated screening currents in Sec. V.

IV. ENERGY-MOMENTUM TENSOR

In this section we construct the free-field realization of the Sugawara energy-momentum tensor $T(z)$ of the $osp(2n|2n)$ current algebra. After a tedious calculation, we find

$$
\begin{aligned}
T(z) = \frac{1}{2(k-2)}\Big\{ & -\sum_{m<l}(E_{\delta_m-\delta_l}(z)F_{\delta_m-\delta_l}(z) + F_{\delta_m-\delta_l}(z)E_{\delta_m-\delta_l}(z)) - \sum_{m<l}(E_{\delta_m+\delta_l}(z)F_{\delta_m+\delta_l}(z) + F_{\delta_m+\delta_l}(z)E_{\delta_m+\delta_l}(z)) \\
& - \sum_{l=1}^{n}[2(E_{2\delta_l}(z)F_{2\delta_l}(z) + F_{2\delta_l}(z)E_{2\delta_l}(z)) + H_l(z)H_l(z)] + \sum_{l=1}^{n}\sum_{i=1}^{n}(E_{\delta_l-\epsilon_i}(z)F_{\delta_l-\epsilon_i}(z) - F_{\delta_l-\epsilon_i}(z)E_{\delta_l-\epsilon_i}(z)) \\
& + \sum_{l=1}^{n}\sum_{i=1}^{n}(E_{\delta_l+\epsilon_i}(z)F_{\delta_l+\epsilon_i}(z) - F_{\delta_l+\epsilon_i}(z)E_{\delta_l+\epsilon_i}(z)) + \sum_{i<j}(E_{\epsilon_i-\epsilon_j}(z)F_{\epsilon_i-\epsilon_j}(z) + F_{\epsilon_i-\epsilon_j}(z)E_{\epsilon_i-\epsilon_j}(z)) \\
& + \sum_{i<j}(E_{\epsilon_i+\epsilon_j}(z)F_{\epsilon_i+\epsilon_j}(z) + F_{\epsilon_i+\epsilon_j}(z)E_{\epsilon_i+\epsilon_j}(z)) + \sum_{i=1}^{n}H_{n+i}(z)H_{n+i}(z)\Big\} \\
= & -\sum_{l=1}^{n}\left(\frac{1}{2}\partial\phi_l(z)\partial\phi_l(z) - \frac{1-l}{\sqrt{k-2}}\partial^2\phi_l(z)\right) + \sum_{i=1}^{n}\left(\frac{1}{2}\partial\phi_{n+i}(z)\partial\phi_{n+i}(z) - \frac{n-i}{\sqrt{k-2}}\partial^2\phi_{n+i}(z)\right) \\
& + \sum_{m<l}(\beta_{m,l}(z)\partial\gamma_{m,l}(z) + \bar{\beta}_{m,l}(z)\partial\bar{\gamma}_{m,l}(z)) + \sum_{l=1}^{n}\beta_l(z)\partial\gamma_l(z) + \sum_{i<j}(\beta'_{i,j}(z)\partial\gamma'_{i,j}(z) + \bar{\beta}'_{i,j}(z)\partial\bar{\gamma}'_{i,j}(z)) \\
& - \sum_{l=1}^{n}\sum_{i=1}^{n}(\Psi_{l,i}(z)\partial\Psi^+_{l,i}(z) + \bar{\Psi}_{l,i}(z)\partial\bar{\Psi}^+_{l,i}(z)). \qquad (4.1)
\end{aligned}
$$

It is straightforward to check that $T(z)$ satisfy the OPE of the Virasoro algebra

$$T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)}, \qquad (4.2)$$

with a central charge $c = 0$. The vanishing central charge of the energy-momentum tensor $T(z)$ (4.1) is a simple consequence of the fact that the superdimension of $osp(2n|2n)$ is zero. Moreover, we find that, with regard to the energy-momentum tensor $T(z)$ defined by (4.1), the $osp(2n|2n)$ currents associated with the simple roots (3.43)–(3.52) are indeed primary fields with conformal dimension one, namely,

$$
\begin{aligned}
T(z)E_i(w) &= \frac{E_i(w)}{(z-w)^2} + \frac{\partial E_i(w)}{(z-w)}, \qquad 1 \le i \le 2n, \\
T(z)F_i(w) &= \frac{F_i(w)}{(z-w)^2} + \frac{\partial F_i(w)}{(z-w)}, \qquad 1 \le i \le 2n, \\
T(z)H_i(w) &= \frac{H_i(w)}{(z-w)^2} + \frac{\partial H_i(w)}{(z-w)}, \qquad 1 \le i \le 2n.
\end{aligned}
$$

It is expected that the $osp(2n|2n)$ currents associated with nonsimple roots, which can be constructed through the simple ones, are also primary fields with conformal dimension one. Therefore, $T(z)$ is the very energy-momentum tensor of the $osp(2n|2n)$ current algebra.

V. SCREENING CURRENTS

Important objects in the application of free-field realizations to the computation of correlation functions of the CFTs are screening currents. A screening current is a primary field with conformal dimension one and has the property that the singular part of its OPE with the affine currents is a total derivative. These properties ensure that the integrated screening currents (screening charges) may be inserted into correlators while the conformal or affine Ward identities remain intact [22,25].

Free-field realizations of the screening currents may be constructed from certain differential operators [29,38] which can be defined by the relation

$$\rho^{(d)}(s_\alpha)\langle\Lambda; x, \bar{x}; y, \bar{y}; \theta, \bar{\theta}| \equiv \langle\Lambda|E_\alpha G_+(x, \bar{x}; y, \bar{y}, \theta, \bar{\theta}), \quad \text{for } \alpha \in \Delta_+. \qquad (5.1)$$

The operators $\rho^{(d)}(s_\alpha)$ ($\alpha \in \Delta_+$) give a differential operator realization of the subalgebra $(osp(2n|2n))_+$. Again it is sufficient to construct $s_i \equiv \rho^{(d)}(s_{\alpha_i})$ related to the simple roots. Using (5.1) and the Baker-Campbell-Hausdorff formula, after some algebraic manipulations, we obtain the following explicit expressions for s_i:

$$
\begin{aligned}
s_l = & \sum_{m=l+2}^{n}(-\bar{x}_{l+1,m}x_{l+1,m}\partial_{\bar{x}_{l,l+1}} + \bar{x}_{l+1,m}\partial_{\bar{x}_{l,m}} + 2x_{l+1,m}\bar{x}_{l,m}\partial_{x_l} + x_{l+1,m}\partial_{x_{l,m}}) \\
& + \sum_{m=1}^{n}(-\bar{\theta}_{l+1,m}\theta_{l+1,m}\partial_{\bar{x}_{l,l+1}} + \bar{\theta}_{l+1,m}\partial_{\bar{\theta}_{l,m}} - 2\theta_{l+1,m}\bar{\theta}_{l,m}\partial_{x_l} + \theta_{l+1,m}\partial_{\theta_{l,m}}) + x_{l+1}\partial_{\bar{x}_{l,l+1}} + 2\bar{x}_{l,l+1}\partial_{x_l} + \partial_{x_{l,l+1}}, \\
& 1 \le l \le n-1, \qquad (5.2)
\end{aligned}
$$

$$s_n = \sum_{m=2}^{n}(\bar{y}_{1,m}\partial_{\bar{\theta}_{n,m}} - \bar{y}_{1,m}y_{1,m}\partial_{\bar{\theta}_{n,1}} + y_{1,m}\partial_{\theta_{n,m}} - 2y_{1,m}\bar{\theta}_{n,m}\partial_{x_n}) - 2\bar{\theta}_{n,1}\partial_{x_n} + \partial_{\theta_{n,1}}, \tag{5.3}$$

$$s_{n+i} = \sum_{m=i+2}^{n}(\bar{y}_{i+1,m}\partial_{\bar{y}_{i,m}} - \bar{y}_{i+1,m}y_{i+1,m}\partial_{\bar{y}_{i,i+1}} + y_{i+1,m}\partial_{y_{i,m}}) + \partial_{y_{i,i+1}}, \qquad 1 \le i \le n-1, \tag{5.4}$$

$$s_{2n} = \partial_{\bar{y}_{n-1,n}}. \tag{5.5}$$

One may obtain the differential operators s_α associated with the nonsimple generators from the above simple ones. Following the procedure similar to Refs. [29,38], we find that the free-field realization of the screening currents $S_i(z)$ corresponding to the differential operators s_i is given by

$$\begin{aligned} S_l(z) = \Bigg\{ &\sum_{m=l+2}^{n}(-\bar{\gamma}_{l+1,m}(z)\gamma_{l+1,m}(z)\bar{\beta}_{l,l+1}(z) + \bar{\gamma}_{l+1,m}(z)\bar{\beta}_{l,m}(z)) + \sum_{m=l+2}^{n}(2\gamma_{l+1,m}(z)\bar{\gamma}_{l,m}(z)\beta_l(z) + \gamma_{l+1,m}(z)\beta_{l,m}(z)) \\ &+ \gamma_{l+1}(z)\bar{\beta}_{l,l+1}(z) + 2\bar{\gamma}_{l,l+1}(z)\beta_l(z) - \sum_{m=1}^{n}(\bar{\Psi}^+_{l+1,m}(z)\Psi^+_{l+1,m}(z)\bar{\beta}_{l,l+1}(z) - \bar{\Psi}^+_{l+1,m}(z)\bar{\Psi}_{l,m}(z)) \\ &- \sum_{m=1}^{n}(2\Psi^+_{l+1,m}(z)\bar{\Psi}^+_{l,m}(z)\beta_l(z) - \Psi^+_{l+1,m}(z)\Psi_{l,m}(z)) + \beta_{l,l+1}(z)\Bigg\} e^{(\alpha_l\cdot\vec{\phi}(z))/\sqrt{k-2}}, \qquad 1 \le l \le n-1, \end{aligned} \tag{5.6}$$

$$\begin{aligned} S_n(z) = \Bigg\{ &\sum_{m=2}^{n}(\bar{\gamma}'_{1,m}(z)\bar{\Psi}_{n,m}(z) - \bar{\gamma}'_{1,m}(z)\gamma'_{1,m}(z)\bar{\Psi}_{n,1}(z) - 2\gamma'_{1,m}(z)\bar{\Psi}^+_{n,m}(z)\beta_n(z)) \\ &+ \sum_{m=2}^{n}\gamma'_{1,m}(z)\Psi_{n,m}(z) - 2\bar{\Psi}^+_{n,1}(z)\beta_n(z) + \Psi_{n,1}(z)\Bigg\} e^{(\alpha_n\cdot\vec{\phi}(z))/\sqrt{k-2}}, \end{aligned} \tag{5.7}$$

$$\begin{aligned} S_{n+i}(z) = \Bigg\{ &\sum_{m=i+2}^{n}(\bar{\gamma}'_{i+1,m}(z)\bar{\beta}'_{i,m}(z) - \bar{\gamma}'_{i+1,m}(z)\gamma'_{i+1,m}(z)\bar{\beta}'_{i,i+1}(z)) + \sum_{m=i+2}^{n}\gamma'_{i+1,m}(z)\beta'_{i,m}(z) + \beta'_{i,i+1}(z)\Bigg\} e^{(\alpha_{n+i}\cdot\vec{\phi}(z))/\sqrt{k-2}}, \\ &1 \le i \le n-1, \end{aligned} \tag{5.8}$$

$$S_{2n}(z) = \bar{\beta}'_{n-1,n}(z)e^{(\alpha_{2n}\cdot\vec{\phi}(z))/\sqrt{k-2}}. \tag{5.9}$$

Here $\vec{\phi}(z)$ is

$$\vec{\phi}(z) = \sum_{i=1}^{n}(\phi_i(z)\delta_i + \phi_{n+i}(z)\epsilon_i). \tag{5.10}$$

The OPEs of the screening currents with the energy-momentum tensor and the $osp(2n|2n)$ currents (3.43)–(3.52) are

$$T(z)S_i(w) = \frac{S_i(w)}{(z-w)^2} + \frac{\partial S_i(w)}{(z-w)} = \partial_w\left\{\frac{S_i(w)}{(z-w)}\right\}, \quad i = 1, \ldots, 2n, \tag{5.11}$$

$$E_i(z)S_j(w) = 0, \qquad i, j = 1 \ldots, 2n, \tag{5.12}$$

$$H_i(z)S_j(w) = 0, \qquad i, j = 1 \ldots, 2n, \tag{5.13}$$

$$F_i(z)S_j(w) = (-1)^{[[i]]+[F_i]}\delta_{ij}\partial_w\left\{\frac{(k-2)e^{(\alpha_i\cdot\vec{\phi}(w))/\sqrt{k-2}}}{(z-w)}\right\}, \quad i, j = 1, \ldots, 2n. \tag{5.14}$$

Here $[[i]]$ is given by

$$[[i]] = \begin{cases} 1, & i = 1, \ldots, n, \\ 0, & i = n+1, \ldots, 2n. \end{cases}$$

The screening currents obtained this way are called screening currents of the first kind [30]. Moreover, the screening current $S_n(z)$ is fermionic and the others are bosonic.

VI. DISCUSSIONS

We have constructed the explicit expressions of the free-field representation for the $osp(2n|2n)$ current algebra at an arbitrary level k and the corresponding energy-momentum tensor. We have also found the free-field representation of the $2n$ associated screening currents of the first kind.

The free-field realization (3.43)–(3.52) of the $osp(2n|2n)$ current algebra gives rise to the Fock representation of the corresponding current algebra in terms of the free fields (3.26)–(3.34). It provides explicit realizations of the vertex operator construction [44,45] of representations for affine superalgebra $osp(2n|2n)_k$. Moreover, these representations are, in general, not irreducible for the current algebra. To obtain irreducible representations, one needs the associated screening charges, which are the integrals of the corresponding screening currents (5.6), (5.7), (5.8), and (5.9) and performs the cohomology analysis as in [23,28,29,31].

To fully take the advantage of the CFT method, one needs to construct its primary fields. It is well known that there exist two types of representations for the underlying finite-dimensional superalgebra $osp(2n|2n)$: typical and atypical representations. Atypical representations have no counterpart in the bosonic algebra setting, and the understanding of such representations is still very much incomplete. Although the construction of the primary fields associated with typical representations is similar to the bosonic algebra cases, it is a highly nontrivial task to construct the primary fields associated with atypical representations [46].

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APPENDIX: DEFINING REPRESENTATION OF $osp(2n|2n)$

Let V be a $\mathbb{Z}_2$-grading $4n$-dimensional vector space with an orthonormal basis $\{|i\rangle, i = 1, \ldots, 4n\}$. The $\mathbb{Z}_2$ grading is chosen as $[1] = \cdots = [2n] = 0$, $[2n+1] = \cdots = [4n] = 1$. Let e_{ij}, $i, j = 1, \ldots, n$, be an $n \times n$ matrix with entry 1 at the ith row and the jth column and zero elsewhere. The $4n$-dimensional defining representation of $osp(2n|2n)$, denoted by ρ_0, is given by the following $4n \times 4n$ matrices:

$$\rho_0(E_{\delta_m-\delta_l}) = \begin{pmatrix} e_{ml} & \\ & -e_{lm} \end{pmatrix}, \qquad \rho_0(F_{\delta_m-\delta_l}) = \begin{pmatrix} e_{lm} & \\ & -e_{ml} \end{pmatrix}, \qquad m < l, \tag{A1}$$

$$\rho_0(E_{2\delta_l}) = \begin{pmatrix} 0 & e_{ll} \\ 0 & 0 \end{pmatrix}, \qquad \rho_0(F_{2\delta_l}) = \begin{pmatrix} 0 & 0 \\ e_{ll} & 0 \end{pmatrix}, \tag{A2}$$

$$\rho_0(E_{\delta_m+\delta_l}) = \begin{pmatrix} 0 & e_{ml}+e_{lm} \\ 0 & 0 \end{pmatrix}, \qquad \rho_0(F_{\delta_m+\delta_l}) = \begin{pmatrix} 0 & 0 \\ e_{ml}+e_{lm} & 0 \end{pmatrix}, \qquad m < l, \tag{A3}$$

$$\rho_0(E_{\delta_l-\epsilon_i}) = \begin{pmatrix} & & 0 & 0 \\ & & 0 & e_{il} \\ e_{li} & 0 & & \\ 0 & 0 & & \end{pmatrix}, \qquad \rho_0(F_{\delta_l-\epsilon_i}) = \begin{pmatrix} & & e_{il} & 0 \\ & & 0 & 0 \\ 0 & 0 & & \\ 0 & -e_{li} & & \end{pmatrix}, \tag{A4}$$

$$\rho_0(E_{\delta_l+\epsilon_i}) = \begin{pmatrix} & & 0 & e_{il} \\ & & 0 & 0 \\ 0 & e_{li} & & \\ 0 & 0 & & \end{pmatrix}, \qquad \rho_0(F_{\delta_l+\epsilon_i}) = \begin{pmatrix} & & 0 & 0 \\ & & e_{il} & 0 \\ 0 & 0 & & \\ -e_{li} & 0 & & \end{pmatrix}, \tag{A5}$$

$$\rho_0(E_{\epsilon_i-\epsilon_j}) = \begin{pmatrix} e_{ij} & \\ & -e_{ji} \end{pmatrix}, \qquad \rho_0(F_{\epsilon_i-\epsilon_j}) = \begin{pmatrix} e_{ji} & \\ & -e_{ij} \end{pmatrix}, \qquad i < j, \tag{A6}$$

$$\rho_0(E_{\epsilon_i+\epsilon_j}) = \begin{pmatrix} 0 & e_{ij}-e_{ji} & \\ 0 & 0 & \end{pmatrix}, \qquad \rho_0(F_{\epsilon_i+\epsilon_j}) = \begin{pmatrix} 0 & 0 \\ -e_{ij}+e_{ji} & 0 \end{pmatrix}, \qquad i < j, \tag{A7}$$

$$\rho_0(H_{\delta_m-\delta_l}) = \begin{pmatrix} e_{mm}-e_{ll} & \\ & e_{ll}-e_{mm} \end{pmatrix}, \qquad m < l, \tag{A8}$$

$$\rho_0(H_{\delta_m+\delta_l}) = \begin{pmatrix} e_{mm}+e_{ll} & \\ & -e_{mm}-e_{ll} \end{pmatrix}, \qquad m < l, \tag{A9}$$

$$\rho_0(H_{2\delta_l}) = \begin{pmatrix} e_{ll} & \\ & -e_{ll} \end{pmatrix}, \tag{A10}$$

$$\rho_0(H_{\delta_l-\epsilon_i}) = \begin{pmatrix} e_{ii} & & & \\ & -e_{ii} & & \\ & & e_{ll} & \\ & & & -e_{ll} \end{pmatrix},$$
$$\rho_0(H_{\delta_l+\epsilon_i}) = \begin{pmatrix} -e_{ii} & & & \\ & e_{ii} & & \\ & & e_{ll} & \\ & & & -e_{ll} \end{pmatrix}, \tag{A11}$$

$$\rho_0(H_{\epsilon_i-\epsilon_j}) = \begin{pmatrix} e_{ii} - e_{jj} & \\ & e_{jj} - e_{ii} \\ & \end{pmatrix}, \qquad i < j, \tag{A12}$$

$$\rho_0(H_{\epsilon_i+\epsilon_j}) = \begin{pmatrix} e_{ii} + e_{jj} & \\ & -e_{ii} - e_{jj} \\ & \end{pmatrix}, \qquad i < j. \tag{A13}$$

We introduce $2n$ linear-independent generators H_i ($i = 1, \ldots 2n$):

$$H_l = H_{2\delta_l}, \qquad 1 \le l \le n, \tag{A14}$$

$$H_{n+i} = \tfrac{1}{2}(H_{\epsilon_i-\epsilon_j} + H_{\epsilon_i+\epsilon_j}), \quad i = 1, \ldots, n-1 \quad \text{and} \quad i < j, \tag{A15}$$

$$H_{2n} = \tfrac{1}{2}(H_{\epsilon_i+\epsilon_n} - H_{\epsilon_i-\epsilon_n}), \qquad i \le n-1. \tag{A16}$$

Actually, the above generators $\{H_i\}$ span the Cartan subalgebra of $osp(2n|2n)$. In the defining representation, these generators can be realized by

$$\rho_0(H_l) = \begin{pmatrix} e_{ll} & \\ & -e_{ll} \end{pmatrix}, \qquad l = 1, \ldots, n, \tag{A17}$$

$$\rho_0(H_{n+i}) = \begin{pmatrix} e_{ii} & & \\ & -e_{ii} & \\ & & \end{pmatrix}, \qquad i = 1, \ldots, n. \tag{A18}$$

The corresponding nondegenerate invariant bilinear supersymmetric form of $osp(2n|2n)$ is given by

$$(x, y) = \tfrac{1}{2} str(\rho_0(x)\rho_0(y)), \qquad \forall\ x, y \in osp(2n|2n). \tag{A19}$$

Here the supertrace, for any $4n \times 4n$ matrix A, is defined by

$$\mathrm{str}\,(A) = \sum_{l=1}^{4n} (-1)^{[l]} A_{ll} = \sum_{l=1}^{2n} A_{ll} - \sum_{l=2n+1}^{4n} A_{ll}. \tag{A20}$$

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