Multiphoton Transitions between Energy Levels in a Current-Biased Josephson Tunnel Junction

A. Wallraff,* T. Duty,† A. Lukashenko, and A.V. Ustinov

Physikalisches Institut III, Universität Erlangen-Nürnberg, D-91058 Erlangen, Germany

(Received 13 June 2002; published 23 January 2003)

The escape of a current-biased Josephson tunnel junction from the zero-voltage state in the presence of weak microwave radiation is investigated experimentally at low temperatures. The measurements of the junction switching current distribution indicate the macroscopic quantum tunneling of the phase below a crossover temperature of $T^\star \approx 280\,\mathrm{mK}$. At temperatures below $T^\star$ we observe both single-photon and *multiphoton* transitions between the junction energy levels by applying microwave radiation in the frequency range between 10 and 38 GHz to the junction. These observations reflect the anharmonicity of the junction potential containing only a small number of levels.

DOI: 10.1103/PhysRevLett.90.037003 PACS numbers: 74.50.+r, 03.67.Lx, 32.80.Rm, 85.25.Cp

At low temperatures and small damping the dynamics of a current-biased Josephson junction is governed by the macroscopic quantum mechanics of the superconducting phase difference across the junction (see, e.g., Refs. [1,2], and references therein). Macroscopic quantum tunneling of the phase [3], energy level quantization [3,4] and the effect of dissipation [5] have been studied in detail [2]. Josephson junctions are solid-state quantum devices fabricated with integrated circuit technology. Their parameters can be adjusted in a wide range and can be well controlled. Josephson junction circuits have been proposed [6] and recently successfully tested [7] as qubits in quantum information processing [8].

In this Letter we present the first experimental evidence of multiphoton transitions between the ground and the first excited state of the macroscopic quantum system formed by the current-biased Josephson junction. Such processes, well known in quantum optics but unexplored in macroscopic quantum mechanics, may be used to manipulate quantum circuits containing Josephson junctions at fractions of the fundamental transition frequency. Using lower frequencies may substantially simplify the microwave engineering and qubit circuit integration. At the same time, multiphoton mediated processes may lead to decoherence originating from coupling to excited states. Multiphoton processes are relevant for any superconducting qubit implementation using Josephson junctions as nonlinear elements.

The experiments presented here have been performed below the crossover temperature $T^\star$ [9], where the escape of the junction from a metastable state is dominated by quantum tunneling from the quantized energy levels. Using a high resolution measurement [10] of the switching current [11], we detect the multiphoton absorption by monitoring the decay of the junction from the zero voltage to the finite voltage state.

In the Stewart-McCumber model [12], the current-biased small Josephson junction is modeled as a particle of mass m_ϕ moving in an external washboard potential $U^\phi = -E_J(\gamma\phi + \cos\phi)$, see Fig. 1, according to the equation of motion $m_\phi\ddot{\phi} + m_\phi(RC)^{-1}\dot{\phi} + \partial U^\phi/\partial\phi = 0$. Here, the phase difference ϕ across the junction represents the position of the particle. The particle mass is given by $m_\phi = C(\Phi_0/2\pi)^2$, where C is the junction capacitance and Φ_0 is the flux quantum. $E_J = \Phi_0 I_c/2\pi$ is the Josephson coupling energy with the critical current of the junction I_c determining the depth of the potential. The applied bias current I normalized as $\gamma = I/I_c$ determines the tilt of the potential and the junction resistance R causes the damping proportional to the coefficient $1/RC$.

In the absence of thermal or quantum fluctuations and for $\gamma < 1$, the junction is in the zero-voltage state, corresponding to the particle being localized in the potential well. At finite temperatures $T > 0$, the particle may escape from the well at bias currents $\gamma < 1$ by thermally activated processes [11,13] or by quantum tunneling through the barrier [3]. The rate at which both processes occur depends on the barrier height $U_0^\phi = 2E_J[\sqrt{1-\gamma^2} - \gamma\arccos(\gamma)] \approx E_J\,4\sqrt{2}/3\,(1-\gamma)^{3/2}$, the

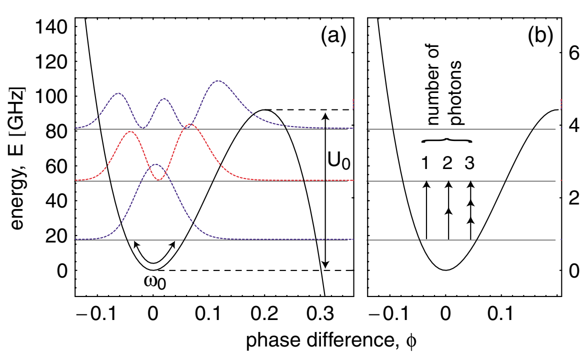

FIG. 1 (color online). (a) The Josephson junction energy $U^\phi(\phi)$ calculated for the experimental parameters at the bias $\gamma = 0.995$ (solid line). Numerically calculated energy levels (dotted lines) and the squared wave functions (dashed lines) are shown. (b) Multiphoton transitions between the ground state and the first excited state.

oscillation frequency of the particle at the bottom of the well $\omega_0^\phi = [U''^\phi(\phi_0)/m_\phi]^{1/2} = \omega_p(1-\gamma^2)^{1/4}$, see Fig. 1, and the damping. Here $\omega_p = (2\pi I_c/\Phi_0 C)^{1/2}$ is the Josephson plasma frequency. At temperatures below $T^\star$ [9] the quantum tunneling rate dominates the thermal activation rate. The quantization of the energy of oscillations of the phase at the bottom of the well, see Fig. 1(a), has been observed both below [3] and above $T^\star$ [4].

The experiments presented here were performed using a high quality $5 \times 5\ \mu\mathrm{m}^2$ tunnel junction fabricated on an oxidized silicon waver using a standard $\mathrm{Nb/Al-AlO}_x\mathrm{/Nb}$ trilayer process. The junction had a critical current density of $j_c \approx 1.1\ \mathrm{kA/cm}^2$, an effective capacitance of $C \approx 1.6$ pF and a subgap resistance of $R > 500\ \Omega$ at $T < 2.0$ K determined from its dc current voltage characteristic. For these sample parameters the expected energy level separation is larger than 100 GHz at zero bias. The level width is small relative to the level spacing, due to the small intrinsic damping of the junction and its moderate coupling to the electromagnetic environment.

The sample was mounted in an rf-tight sample box on the cold finger of a dilution refrigerator. The dc-bias leads were filtered with π-type feedthrough filters at room temperature, RC filters at the 1 K pot of the refrigerator and thermocoax filters at the sample box in order to reduce external electromagnetic interference. A microwave signal was fed into the sample cell via a superconducting semirigid coaxial cable. To reduce the relative level of noise in the microwave signal, several stages of cold attenuators of a total of -40 dB were used. We have verified that the power of all harmonics and subharmonics was at least 80 dB below the fundamental frequency power. For the switching current measurements [11], the current was ramped up at a constant rate of $\dot{I} = 0.245$ A/s with a repetition rate of 500 Hz. The switching current was determined by a measurement of the time delay between the zero crossing of the bias current and the appearance of a voltage across the junction [10].

The switching current distribution $P(I)$ of the sample in the absence of microwaves was measured in the temperature range between 4.2 K and 25 mK. These measurements [10] indicate the thermal activation of the phase at high temperatures followed by a crossover to quantum tunneling at the predicted [9] temperature of $T^\star = \hbar\omega_0^\phi/2\pi k_B \approx 280$ mK evaluated using the mean switching current.

Microwaves in the frequency range between 10 and 38 GHz were applied to the sample. While monitoring the $P(I)$ distributions of the junction, the microwave power $P_{\mu w}$ was swept from low values, at which the $P(I)$ distribution is not changed by the microwaves, to higher values for each chosen frequency. At negligibly small microwave powers the $P(I)$ distribution is essentially determined by the unperturbed quantum tunneling of the phase from the ground state of the well. If the microwave power is increased to substantially populate the excited level, the $P(I)$ distribution becomes double peaked. This double-peak structure smoothly varies with $P_{\mu w}$, as shown for $\nu = 36.554$ GHz in Fig. 2(a). Further increasing the power, only the pronounced resonant peak is visible in the distribution. At this level of power the populations of the ground and the first excited state are equal, but the tunneling rate from the excited state is exponentially larger than that from the ground state. Because of the resonance excitation of transitions between the two levels, the switching current distribution at this level of power is *more narrow* than in the absence of microwaves. This fact proves that the measured $P(I)$ distribution in the absence of microwaves is not limited by noise in our experimental setup and that below $T^\star$ the escape indeed occurs due to quantum tunneling through the barrier.

The bias current at which the resonant peak in the $P(I)$ distribution appears depends strongly on the microwave frequency. Most strikingly, we observe resonant peaks at similar or the same bias current for very different microwave frequencies. In Figs. 2(b) and 2(c), two representative density plots of the switching current distributions versus the applied microwave power are shown for the microwave frequencies 36.554 and 18.399 GHz. For both

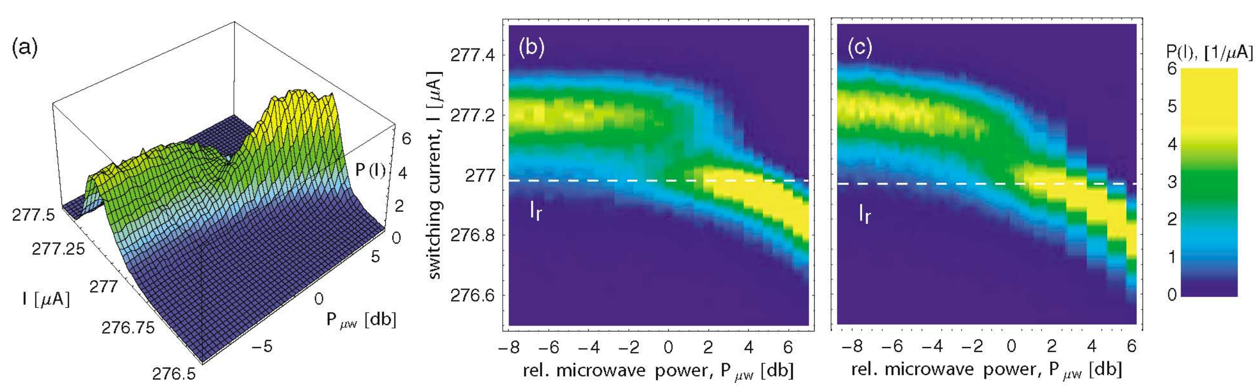

FIG. 2 (color online). (a) 3D plot and (b) density plot of the measured $P(I)$ distribution versus the applied microwave power $P_{\mu w}$ at $\nu = 36.554$ GHz and $T = 100$ mK. (c) Experimental data at 18.399 GHz for the same temperature. The switching probability $P(I)$ is color coded as indicated by the scale.

frequencies the resonant peaks appear at almost identical bias currents. Both sets of data show the *pronounced narrowing of the distribution* at the resonance.

The resonant bias currents I_r, defined as the current at which the escape of the phase is maximally enhanced by the microwaves as indicated in Figs. 2 and 4, were extracted for all measured microwave frequencies ν, see Fig. 3. It is clearly observed that the resonances fall into different groups as indicated by the dashed lines. We find that ratios of the resonance frequencies ν for a fixed current I_r are given with high accuracy by ratios m/n of two small integer numbers n and m, suggesting that the observed effect is related to multiphoton transitions between energy levels of the phase.

In parabolic approximation of the potential, one expects the energy level separation in a small Josephson junction to scale with the applied bias current as $\Delta E = \hbar\omega_p[1 - (I_r/I_c)^2]^{1/4}$. Therefore resonances with the external applied microwaves are expected to appear for $n\nu = \Delta E$, where n is the number of photons absorbed in the transition between two energy levels. Such multiphoton transitions between neighboring energy levels are quantum mechanically allowed [14] in the anharmonic potential for the phase of a Josephson junction due to the large diagonal matrix elements of the excited states. In Fig. 3 all data are fitted to the single formula $(1/n)\,\nu_p[1 - (I_r/I_c)^2]^{1/4}$, with $\nu_p = 116\,\mathrm{GHz}$, $I_c = 278.45\,\mu\mathrm{A}$, and $n = 1, \ldots, 5$. The agreement between the experimental data and this simple formula for the resonance condition is excellent. The effective capacitance of the junction calculated from the fitted zero bias plasma frequency and the critical current is found to be $C = 1.61\,\mathrm{pF}$. We note that the corresponding classical harmonic resonances, arising from the nonlinearity of the junction, have been observed experimentally [15] at large applied microwave power and at high temperatures.

In Fig. 4, the escape rate $\Gamma(I)$ reconstructed from the $P(I)$ distribution is plotted for a range of microwave powers for (a) single- and (b) two-photon absorption. In both cases, at low values of $P_{\mu w}$ the escape rate is a monotonic function of the bias current. With increasing $P_{\mu w}$ a clear resonance develops in the escape rate. The resonant current I_r is indicated in the plot.

The enhancement $[\Gamma(P_{\mu w}) - \Gamma(0)]/\Gamma(0)$ of the escape rate in the presence of microwaves of power $P_{\mu w}$ is plotted for both processes in the insets of Figs. 4(a) and 4(b). $P_{\mu w}$ was chosen to result in a maximum enhancement of roughly ten in both cases. The data are fitted to a Lorentzian line shape [3] (solid line in Fig. 4). From the width $\delta\nu$ of the single-photon resonance we obtain the quality factor $Q \approx \nu/\delta\nu = 45 \pm 5$, which characterizes the junction embedded in its electromagnetic environment. For the two-photon process it is observed that the linewidth is approximately a factor of 2 larger than for the single-photon process. For both processes it is observed that the line width of the transition is independent of the microwave power. This indicates that it is entirely limited by the lifetime of the excited state and that a coherent broadening of the resonance was not observable in these measurements. The enhancement of the escape rate due to microwave radiation increases approximately linearly with $P_{\mu w}$ for the single-photon process, whereas it increases roughly as $P_{\mu w}^2$ for the two-photon process.

We have compared our experimental results with the predictions of the Larkin-Ovchinnikov theory [16]. The bias-current dependent escape rate $\Gamma(I)$ due to tunneling from all possible energy levels was calculated using a master equation approach, considering the occupation of the energy levels in the presence of microwaves at $T = 100\,\mathrm{mK}$. The energy levels and matrix elements were determined using the approximations introduced in

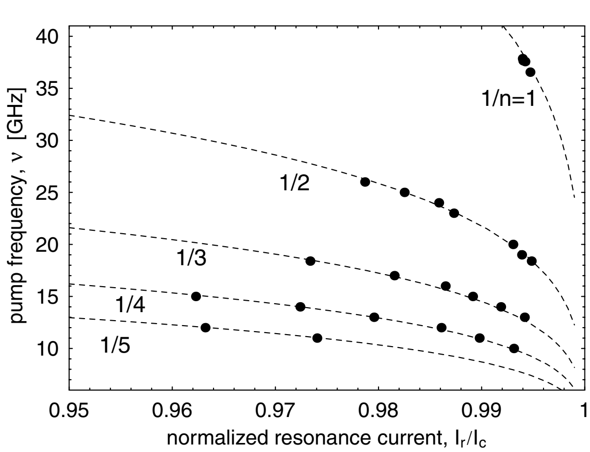

FIG. 3. Applied microwave frequency ν versus normalized resonant bias current I_r/I_c (dots). Dashed lines are a fit of the data to $\nu = (1/n)\,\nu_p[1 - (I_r/I_c)^2]^{1/4}$.

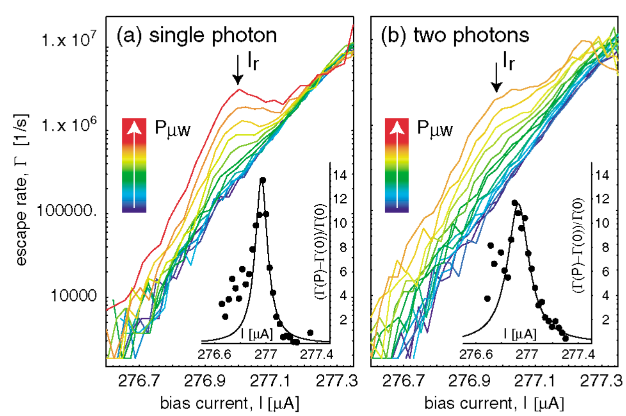

FIG. 4 (color online). Experimental escape rate $\Gamma(I)$ for (a) single-photon and (b) two-photon absorption. Different curves correspond to $P_{\mu w}$ being increased (see arrow) from zero to a value at which the maximum enhancement $\Gamma[P_{\mu w} - \Gamma(0)]/\Gamma(0)$ is approximately 10. The resonance current I_r is indicated by an arrow. The insets show the enhancement of the escape rate $[\Gamma(P_{\mu w}) - \Gamma(0)]/\Gamma(0)$ at the largest displayed value of $P_{\mu w}$. Symbols are data, solid lines are fits to a Lorentzian line shape.

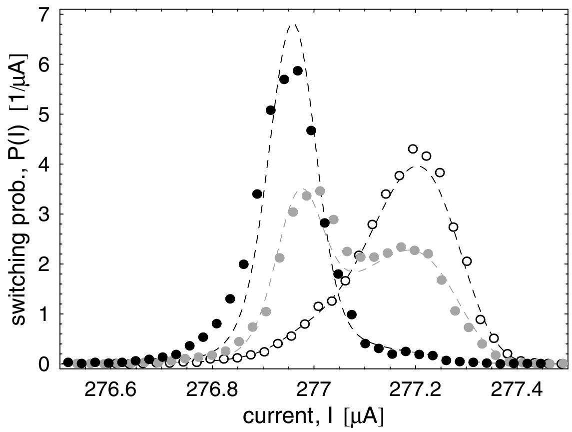

FIG. 5. $P(I)$ distributions in the presence of microwave radiation at $\nu = 36.554$ GHz for $P_{\mu w} = -7.5$ dB (open points), 1 dB (gray points), and 4 dB (black points) measured at 100 mK. Dashed curves are calculated according to Larkin-Ovchinnikov theory.

Refs. [16,17], which we found to be consistent with our direct numerical solutions of the Schrödinger equation for this problem. For single-photon processes the microwave-induced transition rates between nearest-neighbor levels j are given by [16,17] $W^{\mu w}_{j,j+1} \propto P_{\mu w}\Gamma_j[(2\pi\nu - \hbar^{-1}\Delta E_{j,j+1})^2 + \Gamma_j^2/4]^{-1}$, where Γ_j is the inverse "lifetime" of the $j+1 \rightarrow j$ transition. In Fig. 5, the measured switching current distributions for the single-photon absorption are fitted to the calculated distributions at different microwave powers for a junction capacitance of $C = 1.27$ pF, a critical current of $I_c = 278.23\,\mu$A and an effective resistance of $R = 180\,\Omega$. The capacitance obtained in this fit is about 20% lower than that obtained from the plasma frequency. We suppose this discrepancy to be due to a microwave-induced level shift which is to be expected due to the large diagonal matrix elements of the excited states. The theory well explains the power dependence of $P(I)$, strengthening the claim that the resonances are due to the microwave-induced transition of the phase from the ground state to the first excited state in the well.

In the presented experiments we have found evidence for the microwave-induced multiphoton transitions between quantized energy levels of the phase in a current-biased Josephson junction. This process can be used as an alternative way for manipulating the quantum state of a superconducting qubit. At the same time it presents an additional source of decoherence which needs to be considered in such systems.

We are grateful for discussions with B. Birnir, M. H. Devoret, M. Fistul, S. Han, J. E. Mooij, G. Schön, A. Shnirman, and F. Wilhelm. We acknowledge the partial financial support by the Deutsche Forschungsgemeinschaft (DFG) and by D-Wave Systems Inc.

*Current address: Department of Applied Physics, Yale University, New Haven, CT 06520.
Electronic address: andreas.wallraff@yale.edu
†Present address: Department of Microelectronics and Nanoscience, MC2, Chalmers University of Technology and Göteborg University, S-41296 Gothenburg, Sweden.

[1] A. J. Leggett, in *Percolation, Localization, and Superconductivity* (Plenum Press, New York, 1984).
[2] M. H. Devoret, D. Esteve, C. Urbina, J. Martinis, A. Cleland, and J. Clarke, in *Quantum Tunneling in Condensed Media* (North-Holland, Amsterdam, 1992).
[3] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).
[4] P. Silvestrini, V. G. Palmieri, B. Ruggiero, and M. Russo, Phys. Rev. Lett. **79**, 3046 (1997).
[5] A. N. Cleland, J. M. Martinis, and J. Clarke, Phys. Rev. B **37**, 5950 (1988).
[6] M. F. Bocko, A. M. Herr, and M. J. Feldman, IEEE Trans. Appl. Supercond. **7**, 3638 (1997); J. E. Mooij, T. P. Orlando, L. Levitov, L. Tian, C. H. van der Wal, and S. Lloyd, Science **285**, 1036 (1999); Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[7] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999); J. R. Friedman, V. Patel, W. Chen, S. K. Tolpygo, and J. E. Lukens, Nature (London) **406**, 43 (2000); C. H. van der Wal, A. C. J. ter Haar, F. K. Wilhelm, R. N. Schouten, C. J. P. M. Harmans, T. P. Orlando, S. Lloyd, and J. E. Mooij, Science **290**, 773 (2000); D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002); Y. Yu, S. Han, X. Chu, S.-I. Chu, and Y. Wang, Science **296**, 889 (2002).
[8] C. H. Bennett and D. P. DiVincenzo, Nature (London) **404**, 247 (2000).
[9] H. Grabert, P. Olschowski, and U. Weiss, Phys. Rev. B **36**, 1931 (1987).
[10] A. Wallraff, A. Lukashenko, C. Coqui, T. Duty, and A. V. Ustinov, cond-mat/0204527.
[11] T. Fulton and L. Dunkleberger, Phys. Rev. B **9**, 4760 (1974).
[12] W. C. Stewart, Appl. Phys. Lett. **12**, 277 (1968); D. E. McCumber, J. Appl. Phys. **39**, 3113 (1968).
[13] V. Ambegaokar and B. I. Halperin, Phys. Rev. Lett. **22**, 1364 (1969).
[14] C. Cohen-Tannoudji, B. Diu, and F. Laloe, *Quantum Mechanics* (John Wiley & Sons, New York, 1977), Vol. 2.
[15] A. J. Dahm, A. Denestein, T. F. Finnegan, D. N. Langenberg, and D. J. Scalapino, Phys. Rev. Lett. **20**, 859 (1968); C. K. Bak, B. Kofoed, N. F. Pedersen, and K. Saermark, J. Appl. Phys. **46**, 886 (1975).
[16] A. I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP **64**, 185 (1986).
[17] P. Kopietz and S. Chakravarty, Phys. Rev. B **38**, 97 (1988); K. S. Chow and V. Ambegaokar, Phys. Rev. B **38**, 11 168 (1988).