

Can Signal-To-Noise Be Improved by Heterodyne Detection Using an Amplitude Squeezed Local Oscillator?

In “Sub-Shot-Noise-Limited Optical Heterodyne Detection Using an Amplitude-Squeezed Local Oscillator” by Li *et al.* [1], it is claimed that increased signal sensitivity (higher signal-to-noise) is achieved in a novel optical heterodyne detection arrangement by using a squeezed local oscillator. This is a surprising result because (as noted in the first paragraph on p. 5226 of the Letter) ultimately the sensitivity in a standard heterodyne measurement is limited by vacuum fluctuations entering with the signal. How can this new system reduce these fluctuations without also reducing the signal?

The experimental results show that indeed the signal emerges on a squeezed noise floor. From this it is inferred that the signal-to-noise has been increased over what would have been detected using a standard setup. However, no direct comparison of signal-to-noises is made. Does the signal emerging on a squeezed noise floor necessarily imply increased signal-to-noise?

Their theoretical result [Eq. (2), Ref. [1]] is based on the assumption that the signal field passes unchanged through the nonlinear crystal (fourth paragraph, second page, Ref. [1]). Given that the action of the squeezer is to deamplify amplitude fluctuations, is the signal really unaffected?

These questions can be answered via a rigorous quantum mechanical calculation. Here we present the results of such a calculation. Unfortunately, the calculation shows that no increase of signal-to-noise is possible with this arrangement. An outline of the theory follows. As in the experiment, the signal (at the second harmonic frequency) and local oscillator (at the fundamental) are combined on a dichroic mirror. The signal makes a single pass through the nonlinear crystal. Since the detection frequency (7.5 MHz) is well below the repetition rate of the laser (80 MHz), a cw treatment of the fluctuations is justified. For simplicity we assume that a cavity, resonant with the local oscillator, is present around the crystal. This allows the mean field approximation to be made. This does not change the physics of the setup, provided we consider only frequencies well within the cavity linewidth. The intensity fluctuation spectrum of the output field for such a situation has been calculated by various authors [2–4] and is given by

$$V_{\text{out}} = \frac{8\chi\gamma + (\chi - \gamma)^2 V_s}{(3\chi + \gamma)^2}, \quad (1)$$

where γ is the linear loss rate of the fundamental from the cavity through the input-output mirror, while χ describes the nonlinear loss rate of the fundamental through conversion to second harmonic. The spectrum is normalized to

the quantum limit such that shot noise equals 1. Scattering losses have been neglected and the local oscillator noise, which is uncorrelated with the signal, has been assumed shot-noise limited. The heterodyned spectrum of the signal is given by

$$V_s(\omega) = n_s(\omega) + 1, \quad (2)$$

where $n_s(\omega)$ is the signal photon number at a rf, ω , with respect to the local oscillator and the “1” is the vacuum fluctuation contribution (shot noise). Consider first the case of very low nonlinearity, i.e., $\gamma \gg \chi$. From Eq. (1) we find $V_{\text{out}} = V_s$. The signal-to-noise is n_s , limited by the vacuum noise entering with the signal. This is equivalent to the result obtained with balanced heterodyne detection, or indeed to mixing the signal with a local oscillator on a strongly asymmetric beam splitter. Now consider what happens with a very strong nonlinear interaction, i.e., $\chi \gg \gamma$. We now find $V_{\text{out}} = (1/9)V_s$. The noise floor of V_s has been strongly squeezed, but of course so has the actual signal, such that the signal-to-noise remains n_s . Thus, though the signal appears on a squeezed noise floor, there is no increase in signal-to-noise. The squeezing achieved in the experiment is consistent with a nonlinearity of $\chi/\gamma = 0.45$. In fact, for such conditions, Eq. (1) predicts a 90% reduction in signal-to-noise.

In summary, we have shown that, even though the heterodyne detection scheme of Li *et al.* results in a subquantum limit noise floor (as demonstrated in their experiment), this does *not* imply an increased signal-to-noise for their measurement. We note that this situation is fundamentally different from other schemes (see Refs. [11–14] in Ref. [1]) where the vacuum modes at the actual signal measurement site are replaced with modes which are squeezed or exhibit nonclassical correlations.

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