Quantum to classical transition from the cosmic background radiation

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We have revisited the Ghirardi-Rimini-Weber-Pearle (GRWP) approach for continuous dynamical evolution of the state vector for a macroscopic object. Our main concern is to recover the decoupling of the state vector dynamics for the center-of-mass (CM) and internal motion, as in the GRWP model, but within the framework of the standard cosmology. In this connection we have taken the opposite direction of the GRWP argument that the cosmic background radiation (CBR) has originated from a fundamental stochastic hitting process. We assume the CBR to be a clue of the Big Bang, playing a main role in the decoupling of the state vector dynamics of the CM and internal motion. In our model, instead of describing a continuous spontaneous localization (CSL) of a system of massive particles as proposed by Ghirardi, Pearle, and Rimini [Phys. Rev. A **42**, 78 (1990)] the Itô stochastic equation accounts for the intervention of the CBR on the system of particles. Essentially, this approach leads to a precursor of the master equation for both the CBR and particle degrees of freedom. The violation of the principle of energy conservation characteristic of the CSL model is avoided as well as the additional assumption on the size of the GRWP's localization width necessary to reach the decoupling between the collective and internal motions. Moreover, realistic estimation for the decoherence time, exhibiting an interesting dependence on the CBR temperature, is obtained. From the formula for the decoherence time it is possible to analyze the transition from micro- to macrodynamics in both the early hot universe and the nowadays cold one. The entropy of the system under decoherence is analyzed and the emergent ''pointer basis'' is discussed. In spite of not having imposed a privileged basis, in our model the position still emerges as the preferred observable as in the CSL model.

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I. INTRODUCTION

In the last decade several proposals to modify the standard Hamiltonian dynamics, ranging from master equations to stochastic quantum mechanics, have been advanced to try to set up a unified description for microscopic and macroscopic physical phenomena. In the pioneer work by Ghirardi, Rimini, and Weber [1], *quantum mechanics with spontaneous localization* (QMSL), the state vector collapse, leading from quantum to classical dynamics results from the instantaneous action of a spontaneous random hitting process. Such a Poisson process is described by a ''localization'' operator, a Gaussian function acting on each microscopic constituent of any system. The localization operator carries two free parameters; a mean frequency λ and a localization width $\alpha^{-1/2}$, understood as new constants of nature (the *spontaneous localization* is argued to be a fundamental physical process). Through these basic assumptions the QMSL consists in an explicit model allowing a unified description for microscopic and macroscopic systems. It forbids the occurrence of linear superposition of states localized in far away spatial regions and induces a dynamics that agree with the predictions of classical mechanics.

Pursuing the program of the QMSL model, Diosi [2] presented an interesting connection between the original Ghirardi-Rimini-Weber (GRW) hitting process and a modified Schrödinger equation. Another significant achievement concerning a dynamical reduction model, a stochastic equation for physical ensemble, was reported by Gisin [3]. Next, Pearle [4] described the QMSL model through an Itô stochastic differential equation. Basically, Pearle replaced the Poisson process of instantaneous hits in the GRW model by a Markov process described as a stochastic modification of the Schrödinger equation, so that a continuous evolution of the state vector was accomplished. By considering a specific choice of the operators defining the Markov process (expressed in terms of creation and annihilation operators), Ghirardi, Pearle, and Rimini [5] have described the mechanism known as *continuous spontaneous localization* (CSL) of systems of identical particles (the QMSL model has consistency only in the case of systems of distinguishable particles).

Other investigations dealing with dynamical reduction models have recently been considered [6], among them it is worth mentioning the model for *intrinsic* decoherence proposed by Milburn [7]. While in the Ghirardi-Rimini-Weber-Pearle (GRWP) model the addition of stochastic terms in the Schrödinger evolution automatically destroys the quantum coherence of the physical properties of the system that attain a macroscopic level, the modification of the Liouville equation proposed by Milburn destroys the coherence even at microscopic level.

In the CSL model the Itô stochastic equation for the evolution of the state vector reads

$$d|\psi\rangle=\left(-\frac{i}{\hbar}H dt+dh-\frac{1}{2}\overline{(dh)^2}\right)|\psi\rangle, \qquad (1)$$

where dh is a linear self-adjoint operator, whose random

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fluctuation may increase or decrease the norm of the state vector. Using the Itô formula (with the notation $|d\psi\rangle \equiv d|\psi\rangle$),

$$d\|\psi\|^2=\langle\psi|d\psi\rangle+\langle d\psi|\psi\rangle+\overline{\langle d\psi|d\psi\rangle}, \tag{2}$$

it is easy to see that Eq. (1) does not conserve the norm of $|\psi\rangle$. Thus, the introduction of a norm conserving nonlinear process is mandatory. This process, whose random operator depends on the state vector, reads

$$d|\phi\rangle=\left(-\frac{i}{\hbar}Hdt+dh_\phi-\frac{1}{2}\overline{(dh_\phi)^2}\right)|\phi\rangle. \tag{3}$$

Now, it is necessary to distinguish between *raw* [Eq. (1)] and *physical* [Eq. (3)] ensembles of state vectors to correctly understand the effect of the non-Hamiltonian terms. To this end a precept is adopted, namely, that the square norm of each (unnormalized) state vector represents the weight associated with that (normalized) state vector in the ensemble coming from the Itô stochastic equation [4,5]. This precept is a generalization of the GRW assumption that the frequency of hits is proportional to the squared norm of the state vector. Therefore, in the GRW prescription the quantum theory prediction about the associated probabilities in a measurement process is recovered. By considering such a precept for the physical ensemble, the linearity of the *raw* equation and the Markov nature of the Itô stochastic process leads to the *physical* stochastic differential equation for the N-particle state vector

$$d|\Psi_N\rangle=\left(-\frac{i}{\hbar}Hdt+\mathbf{Z}\cdot d\mathbf{B}-\frac{1}{2}\gamma\mathbf{Z}^\dagger\cdot\mathbf{Z}dt\right)|\Psi_N\rangle, \tag{4}$$

where $\mathbf{Z}\equiv\{Z_i\}$ are operators on the Hilbert space of the system and the set of random operators $\mathbf{B}\equiv\{B_i\}$ is characterized through a real Wiener process, satisfying the following means and correlations over the ensemble

$$\overline{dB_i}=0, \quad \overline{dB_idB_j}=\gamma\delta_{ij}dt. \tag{5}$$

The statistical operator $\rho_N=\overline{|\Psi_N\rangle\langle\Psi_N|}$ of the *physical* ensemble and its evolution equation are directly obtained from Eq. (4); using the Itô calculus in evaluating $d\rho_N/dt$ one gets

$$\frac{d\rho_N}{dt}=-\frac{i}{\hbar}[H,\rho_N]+\gamma\mathbf{Z}\rho_N\cdot\mathbf{Z}^\dagger-\frac{\gamma}{2}\{\mathbf{Z}^\dagger.\mathbf{Z},\rho_N\}, \tag{6}$$

which is exactly the Lindblad [8] form for the generator of a quantum dynamical semigroup.

In the present work our main concern is to achieve *the decoupling between the state vector dynamics of the center-of-mass (CM) and internal motion of a system of particles*. In the GRWP model this *decoupling* results from a hypothesis of spontaneous localization of the system's wave function due to a fundamental stochastic hitting process on the particles, which induces an *increase of total mean energy of the Universe* claimed to be the origin of the cosmic background radiation (CBR). Contrary to this argument, in the present work we assume the point of view of standard cosmology: the present CBR is a clue that the universe began its expansion from a Big Bang [10]. This assumption is introduced with the purpose to avoid the unconventional increase of the total mean energy of the universe. Formally, we hypothesize that the state vector, the Hamiltonian H, and operators $\mathbf{Z},\mathbf{Z}^\dagger$ in Eq. (4) represent both the system of particle and CBR radiation; the set of random functions $\{B_i\}$ describes the intervention of the CBR on the system and substitutes the spontaneous localization process. Instead of elaborating on the formal microscopic problem of the interaction of a system with a reservoir [9], we assume *ad hoc* that the evolution of the system of particles, under the influence of the CBR, is described by an Itô equation having stochastic coupling parameters.

Therefore, in the present *conservative* continuous reduction model (the total energy of system plus CBR is conserved) we argue that (1) the increase or decrease of the system's mean energy is attributed to the CBR; (2) the positional space is not privileged with respect to the momentum space, as required when the localization operator is involved; (3) we do not claim for an *additional assumption* to decouple the collective and internal motion, namely, the width parameter $\alpha^{-1/2}\sim10^{-5}$ cm in the CSL model; (4) as mentioned above, more admissible results are obtained for decoherence times, while in the CSL model the value 10^{-7} s obtained for a system of particles to undergo from quantum to classical dynamics seems to be too large (as well as the localization width $\alpha^{-1/2}\sim10^{-5}$ cm also seems too large when considering typical atomic distances of about 10^{-8} cm, or even superposition of the center-of-mass coordinate different by more than about $\alpha^{-1/2}$ [11]). Finally, (5) instead of the two free parameters required in the GRWP model ($\alpha^{-1/2}$ and the mean frequency λ), the random function describing the interaction between the system and the CBR carries just a single strength parameter with dimension of inverse of time. In fact, the coupling constant of the CBR photons to the N-particle system, as the strength parameter in the GRWP model, defines the inverse of a characteristic time, which is associated to the net effect of the random pseudo-''potential'' dh [12]. Also, as in the GRWP model, our strength parameter is small such that nothing changes in the Hamiltonian dynamics of a single particle even in the case in which it has an extended wave function [5].

Finally, we mention that Joos and Zeh [13], have previously argued that scattering of photons even at a relatively low temperature can induce the localization of the wave packet of a macroscopic system. So, their treatment, based on a master equation proposed by Wigner [14], suggests that the intergalactic cold CBR cannot simply be neglected [15]. The model here presented goes exactly on this point, i.e., we consider the process of random scattering of the CBR photons by a system of particles as responsible for the superselection rules and the micro- to macrotransition of its dynamical description. In this way, despite inducing the superselection rules the CBR also induces the mechanism of separating the center-of-mass (CM) coordinate from the internal motion. Besides, we present a brief cosmological analysis of our results, discussing the roles played by both the CBR temperature and the number of particles of the system, in its way from quantum to classical dynamics, as the universe evolved from a hot to a cold state.

In Sec. II we briefly review the GRWP model presenting its main achievements. In Sec. III we construct our model: beginning from an Itô stochastic equation we derive a pre-master equation for a system of N particles and the CBR. Tracing over the CBR degrees of freedom we obtain a master equation for the system of particles only and in Sec. IV we show that structurally it shows exactly the Lindblad form. In Sec. V we estimate the coupling parameter and in Sec. VI we estimate the decoherence time for the system of particles. In Sec. VII we show that at low temperature limit our master equation and the GRWP Itô equation are equivalent, thus this last one is a particular situation of the former; these equations allow the decoupling of the state vector dynamics into two separate equations, one for the CM and the other for the internal motion. In Sec. VIII we calculate the entropy and analyze the problem of selection of a preferred basis. Finally, in Sec. IX we present a summary and conclusions.

II. GHIRARDI-RIMINI-WEBER-PEARLE MODEL

As explained in the Introduction, in the CSL model the random operator dh contains in its definition the length parameter $\alpha^{-1/2}$ and a strength parameter ζ, which is related to the mean hitting frequency λ. In this section we present a brief review of the CSL model as a class of Markov processes in Hilbert space [5]. We will consider a system of N identical particles so that the localization operator must involve globally the whole set of particles in order to preserve the symmetry properties of the wave function [16]. For this purpose let us consider the creation and annihilation field operators $a^{\dagger}(\mathbf{q},s)$, $a(\mathbf{q},s)$ of a particle at the point $\mathbf{q}$ in some reference frame with spin component s, satisfying the canonical commutation or anticommutation relations. From these operators a locally averaged number density operator is defined as

$$N(\mathbf{x})=\left(\frac{\alpha}{2\pi}\right)^{3/2}\sum_{s}\int d^3\mathbf{q}\exp\left[-\frac{1}{2}\alpha(\mathbf{q}-\mathbf{x})^2\right]\times a^{\dagger}(\mathbf{q},s)a(\mathbf{q},s). \tag{7}$$

The operator $N(\mathbf{x})$ is self-adjoint and its commutator for different values of $\mathbf{x}$ vanishes. The total number operator is defined as $N=\int d^3\mathbf{x}N(\mathbf{x})$, and the symmetrized (antisymmetrized) states containing n particles at the indicated positions,

$$|\mathbf{q},s\rangle=\mathcal{N}a^{\dagger}(\mathbf{q}_1,s_1)\cdots a^{\dagger}(\mathbf{q}_n,s_n)|0\rangle, \tag{8}$$

constitutes the normalized common eigenvectors related to the eigenvalue equation $N(\mathbf{x})|\mathbf{q},s\rangle=n_{\mathbf{x}}|\mathbf{q},s\rangle$, with

$$n_{\mathbf{x}}=\left(\frac{\alpha}{2\pi}\right)^{3/2}\sum_{i=1}^{N}\exp\left[-\frac{1}{2}\alpha(\mathbf{x}-\mathbf{q}_i)^2\right]. \tag{9}$$

Applying [5,16] the stochastic process established by Eq. (4) to a system of identical particles and considering the locally averaged density operator defined by Eq. (7), one gets the physical stochastic nonlinear differential equation for the state vector as

$$d|\psi_N\rangle=\left[-iHdt+\int d^3\mathbf{x}N(\mathbf{x})dB(\mathbf{x})-\frac{1}{2}\zeta\int d^3\mathbf{x}N^2(\mathbf{x})dt\right]|\psi_N\rangle, \tag{10}$$

where the Wiener process $B(\mathbf{x})$ satisfies

$$\overline{dB(\mathbf{x})}=0, \tag{11a}$$

$$\overline{dB(\mathbf{x})dB(\mathbf{y})}=\zeta\delta^3(\mathbf{x}-\mathbf{y})dt. \tag{11b}$$

From Eq. (10) the evolution equation of the N-particle statistical operator obtained from Itô calculus reads

$$\frac{\partial\rho_N}{\partial t}=-i[H,\rho_N]+\zeta\int d^3\mathbf{x}N(\mathbf{x})\rho_N N(\mathbf{x})-\frac{1}{2}\zeta\left\{\int d^3\mathbf{x}N^2(\mathbf{x}),\rho_N\right\} \tag{12}$$

and it can be checked that taking $\lambda=\zeta(\alpha/4\pi)^{3/2}$, Eq. (12) reduces to the correspondent equation for a single particle considered in the QMSL model.

To discuss the physical implications of the modified dynamical equation (10), the separation of the CM motion will be made. If $\mathbf{Q}$ is the CM coordinate of the system and $\tilde{\mathbf{q}}_i$ its internal coordinates (measured from the CM of the particles), one can define the particle coordinates as

$$\mathbf{q}_i=\mathbf{Q}+\tilde{\mathbf{q}}_i(\{\mathbf{r}_i\}), \tag{13}$$

where $\{\mathbf{r}_i\}$ represents a set of $3N-3$ independent variables. In the GRWP model the set $\{\mathbf{r}_i\}$ does not contain macroscopic variables. As a consequence, assuming that the Hamiltonian can be written as $H=H_Q+H_{r_i}$, we consider the wave function

$$\phi(\mathbf{q},s)=\Psi(\mathbf{Q})\chi(\mathbf{r}_i,s), \tag{14a}$$

$$\chi(\mathbf{r}_i,s)=\begin{pmatrix}A\\B\end{pmatrix}\Delta(\mathbf{r}_i,s), \tag{14b}$$

where the symbol $\binom{A}{B}$ specifies the symmetrization or antisymmetrization of the internal coordinate wave function. Under the assumption that the length parameter $\alpha^{-1/2}$ is such that the internal wave function $\Delta(\mathbf{r}_i,s)$ is sharply peaked around the value $\mathbf{r}_{i0}$ of $\mathbf{r}$ (with respect to $\alpha^{-1/2}$), the action of the operator $N(\mathbf{x})$ on the wave function (14a) turns out to be

$$N(\mathbf{x})\Psi(\mathbf{Q})\chi(\mathbf{r}_i,s)=F(\mathbf{Q}-\mathbf{x})\Psi(\mathbf{Q})\chi(\mathbf{r}_i,s), \tag{15}$$

with

$$F(\mathbf{Q}-\mathbf{x})=\sum_i \left(\frac{\alpha}{2\pi}\right)^{3/2}\exp\left\{-\frac{1}{2}\alpha[\mathbf{Q}+\tilde{\mathbf{q}}_i(\mathbf{r}_0)-\mathbf{x}]^2\right\}. \tag{16}$$

Therefore, the operator $N(\mathbf{x})$ acts only on Ψ and the separately normalized wave functions Ψ and χ satisfy the equations

$$d\Psi=\left[-iH_Q dt+\int d^3\mathbf{x}F(\mathbf{Q}-\mathbf{x})dB(\mathbf{x}) -\frac{1}{2}\zeta\int d^3\mathbf{x}F^2(\mathbf{Q}-\mathbf{x})dt\right]\Psi, \tag{17a}$$

$$d\chi=-iH_{r_i}\chi dt. \tag{17b}$$

By assuming a large enough length parameter and an internal wave function, which is independent of the macroscopic variables, the internal motion decouples as in the absence of the stochastic terms in Eq. (10). From this fact, the reduction rates, which are characteristic of the GRWP theory together with the position and momentum spreading, can be obtained. In particular, in the positional representation of Eq. (12), it is possible to verify with the help of the macroscopic density approximation and the sharp scanning approximation [5], that the macroscopic frequency associated to the system of identical particles is

$$\Gamma=\zeta D_0 n_{out}. \tag{18}$$

Here a homogeneous macroscopic body of density D_0 was considered and n_{out} is the number of particles of the body at position $\mathbf{Q}'$ that do not lie in the volume occupied by the body at position $\mathbf{Q}''$. In the case of distinguishable particles, one gets the direct result

$$\lambda_{CM}=n\lambda, \tag{19}$$

n being the total number of particles, so that for a typical macroscopic number $n\approx 10^{23}$, one obtains $\lambda_{CM}\approx 10^{-7}s$, as mentioned above.

The position and momentum spreading obtained from the approximations leading to Eq. (18), are written as

$$\langle Q_i^2\rangle=\langle Q_i^2\rangle_s+\zeta\delta_i\frac{\hbar^2}{6M^2}t^3, \tag{20a}$$

$$\langle P_i^2\rangle=\langle P_i^2\rangle_s+\frac{1}{2}\zeta\delta_i\hbar^2 t, \tag{20b}$$

where the suffix s indicates the Schrödinger evolution, and

$$\delta_i=\int d^3\mathbf{y}\left(\frac{\partial F(\mathbf{y})}{\partial y_i}\right)^2. \tag{21}$$

Now, using the macroscopic density approximation applied to the identical particles system, Eq. (16) is modified to

$$F(\mathbf{Q}-\mathbf{x})=\int d^3\tilde{\mathbf{y}}D(\tilde{\mathbf{y}})\left(\frac{\alpha}{2\pi}\right)^{3/2}\exp\left[-\frac{1}{2}\alpha(\mathbf{Q}+\tilde{\mathbf{y}}-\mathbf{x})^2\right], \tag{22}$$

where $D(\mathbf{y})$ is the number of particles per unit volume in the neighborhood of the point $\mathbf{y}=\mathbf{Q}+\tilde{\mathbf{y}}$. The evaluation of the factor δ_i for the case of a homogeneous macroscopic box containing the N particles through the Eq. (22) gives the result [5]

$$\delta_i=(\alpha/\pi)^{1/2}D_0^2 S_i, \tag{23}$$

where S_i is the transversal section of the macroscopic box.

From Eq. (20b) it is evident that the momentum variance implies that the CM energy increases per unit time as

$$\frac{\Delta E}{t}=\frac{\zeta\delta_i\hbar^2}{M}\sim 10^{-32}(\mathrm{g\ cm\ s^{-1}})S_i\ \ \mathrm{cm}^{-2}, \tag{24}$$

with the GRWP choice $\alpha^{-1/2}\sim 10^{-5}$ cm together with $D_0\sim 10^{24}$ cm^{-3}. From the requirement that the macroscopic frequency associated to the system of identical particles Eq. (18) is exactly the same as for distinguishable particles Eq. (19), GRWP have chosen $\zeta\sim 10^{-30}$ cm^3 s^{-1}.

III. DECOHERENCE FROM THE COSMIC BACKGROUND RADIATION

Our approach uses the stochastic dynamical equation (4), where we identify the continuous component (in frequency space) of the operator responsible for the interaction of the N-particle system to the CBR as

$$\mathbf{Z}(\Omega)\equiv\sum_{k=1}^{N}[A(\Omega)\mathbf{a}_k^\dagger+A^\dagger(\Omega)\mathbf{a}_k], \quad \mathbf{a}_k=(a_{k,x},a_{k,y},a_{k,z}). \tag{25}$$

where

$$\mathbf{a}_k=\frac{1}{\sqrt{2\hbar m\omega}}(m\omega\mathbf{q}_k+i\mathbf{p}_k), \tag{26}$$

and $\mathbf{a}_k^\dagger$ is its Hermitian conjugate ($[a_{k,i},a_{k',j}^\dagger]=\delta_{k,k'}\delta_{i,j}, i=x,y,z$), $\mathbf{q}_k$ and $\mathbf{p}_k$ are, respectively, position and momentum operators of the kth particle of mass m. $\hbar\omega$ is a characteristic energy of the system of particles associated to the quantum fluctuation of the CM. The operators $A^\dagger(\Omega),A(\Omega)$ stand for the creation and annihilation of a quantum of energy $\hbar\Omega$ from the environment. The coupling parameter is defined by the continuous stochastic Wiener process $\mathbf{B}(\Omega)$ satisfying

$$\overline{d\mathbf{B}(\Omega)}=0, \tag{27a}$$

$$\overline{dB_i(\Omega)dB_j(\Omega')}=\gamma(\Omega)\delta_{i,j}\delta(\Omega-\Omega')dt, \tag{27b}$$

with $\gamma(\Omega)=\Lambda\Gamma(\Omega)$ accounting for a strength parameter Λ and a frequency distribution function $\Gamma(\Omega)$. Note that $\Gamma(\Omega)$ refers to the effective frequency distribution of the CBR photons, which interact with the system of particles at energy around $\hbar\omega$. We next consider the system of particles and CBR interacting almost resonantly with Lorentzian spectrum

$$\Gamma(\Omega)=\frac{1}{\pi}\frac{\tau_c}{\tau_c^2(\Omega-\omega)^2+1}. \tag{28}$$

In view of Eq. (28) it follows from the Fourier transform of Eq. (27b) that

$$\overline{dB_i(t)dB_j(t')}=\frac{\Lambda}{2\pi}e^{i\omega(t-t')}e^{-(t-t')/\tau_c}dt, \tag{29}$$

where the correlation time τ_c defines the memory time over which the stochastic function changes appreciably. From Eq. (29) we conclude that when considering τ_c extremely short, i.e., much less than all other times of interest (evolution of the particle system) so that in a good approximation $\overline{dB_i(t)dB_j(t')}\sim\delta(t-t')dt$, the system is Markovian. Through Eqs. (27a) and (27b) the physical stochastic differential equation (4) reads

$$d|\Psi_{N+CBR}\rangle=\Bigg\{-\frac{i}{\hbar}H_{N+CBR}dt+\int d\Omega\sum_{k=1}^{N}[A(\Omega)\mathbf{a}_k^\dagger+A^\dagger(\Omega)\mathbf{a}_k]\cdot d\mathbf{B}(\Omega)-\frac{\Lambda}{2}\int d\Omega\Gamma(\Omega)\times\left[\sum_{k=1}^{N}(A(\Omega)\mathbf{a}_k^\dagger+A^\dagger(\Omega)\mathbf{a}_k)\right]^2dt\Bigg\}\times|\Psi_{N+CBR}\rangle. \tag{30}$$

It must be emphasized that Eq. (30) describes the evolution of the state vector of system of N particles and CBR differently from the stochastic differential equation in the CSL model. The Hamiltonian H_{N+CBR} in this equation describes the free evolution of the system of particles and CBR, while the two remaining terms account for the stochastic interaction between the CBR and the particles.

By defining both, the Wiener process $d\mathbf{B}$ and the operator $\mathbf{Z}$ depending on the CBR frequency space, the positional space will not be anymore privileged with respect to the momentum space, as occurs in the CSL model. We now proceed to the separation of the CM motion of the modified dynamical equation (30). The substitution of the operators $\mathbf{a}_k^\dagger,\mathbf{a}_k$ as position and momentum operators $\mathbf{p}_k,\mathbf{q}_k$, permits us to express Eq. (30) in terms of the CM coordinates $\mathbf{Q}=(1/N)\Sigma_k\mathbf{q}_k$ and $\mathbf{P}=\Sigma_k\mathbf{p}_k$ as

$$d|\Psi_{N+CBR}\rangle=\Bigg\{-\frac{i}{\hbar}H_{N+CBR}dt+\int d\Omega[A(\Omega)\mathbf{X}^\dagger+A^\dagger(\Omega)\mathbf{X}]\cdot d\mathbf{B}(\Omega)-\frac{\Lambda}{2}\int d\Omega\Gamma(\Omega)\times[A(\Omega)\mathbf{X}^\dagger+A^\dagger(\Omega)\mathbf{X}]^2dt\Bigg\}|\Psi_{N+CBR}\rangle. \tag{31}$$

where the operator $\mathbf{X}$ accounting for the macroscopic object reads

$$\mathbf{X}=\frac{1}{\sqrt{2\hbar m\omega}}(Nm\omega\mathbf{Q}+i\mathbf{P}), \tag{32}$$

while $\mathbf{X}^\dagger$ is its Hermitian conjugate. These operators satisfy the commutation relation $[X_i,X_j^\dagger]=N\delta_{i,j}\hat{1}$. As mentioned earlier the coupling constant of the interaction between the CBR and the system of particles defines a characteristic time Λ^{-1}, which is associated to the net effect of the random ''pseudopotential'' described by the last two terms on the right-hand side of Eq. (31).

As the stochastic operator in Eq. (31) automatically acts only on the joint wave vector of the CM degree of freedom and the CBR $|\Psi_{CM+CBR}\rangle$, the separately normalized state vectors $|\Psi_{CM+CBR}\rangle$ and $|\phi_{\{\mathbf{r}_i\}}\rangle$, the latter for the internal degrees of freedom, satisfy the equations

$$d|\Psi_{CM+CBR}\rangle=\Bigg[-\frac{i}{\hbar}H_{CM+CBR}dt+\int d\Omega[A(\Omega)\mathbf{X}^\dagger+A^\dagger(\Omega)\mathbf{X}]\cdot d\mathbf{B}(\Omega)-\frac{\Lambda}{2}\int d\Omega\Gamma(\Omega)\times[A(\Omega)\mathbf{X}^\dagger+A^\dagger(\Omega)\mathbf{X}]^2dt\Bigg]|\Psi_{CM+CBR}\rangle, \tag{33a}$$

$$d|\phi_{\{\mathbf{r}_i\}}\rangle=-iH_{\{\mathbf{r}_i\}}|\phi_{\{\mathbf{r}_i\}}\rangle dt. \tag{33b}$$

It should be noted that the above Eqs. (33a) and (33b), different from those in the CSL model [Eqs. (17a) and (17b)], involve also the CBR degrees of freedom. As will be shown later, the present approach in the low-temperature limit allows us to obtain separately the normalized wave functions for the system of particles, $|\Psi_{CM}\rangle$ and $|\phi_{\{\mathbf{r}_i\}}\rangle$, satisfying equations similar to those in the CSL. Next, from Eqs. (6) and (31) the statistical operator $\rho_{N+CBR}=\overline{|\Psi_{N+CBR}\rangle\langle\Psi_{N+CBR}|}$ reads

$$\frac{d\rho_{N+CBR}}{dt}=-\frac{i}{\hbar}[H_{N+CBR},\rho_{N+CBR}]-\frac{\Lambda}{2}\int d\Omega\Gamma(\Omega)[\{[A(\Omega)\mathbf{X}^\dagger+A^\dagger(\Omega)\mathbf{X}]^2,\rho_{N+CBR}\}-2[A(\Omega)\mathbf{X}^\dagger+A^\dagger(\Omega)\mathbf{X}]\cdot\rho_{N+CBR}[A(\Omega)\mathbf{X}^\dagger+A^\dagger(\Omega)\mathbf{X}]], \tag{34}$$

which is a precursor to the master equation in that it contains operators from both the N-particles system and the CBR, allowing to calculate correlations between operators of a system of particles and CBR. However, since we only have at our disposal the statistical properties of the CBR field, the obvious procedure is to trace over the CBR degrees of freedom, considered thermalized at temperature T, which leads to the reduced density operator of the system of particles only, containing the average number of photons of the CBR as a parameter.

Back to Eqs. (29), when the following assumptions are met (i) a short correlation time τ_c ($\ll\Lambda^{-1}$), leading to the Markovian approximation; ii) the interaction between the system of particles and CBR sufficiently small (exactly the purpose at hand), the density operator of the global system can be written as $\rho_{N+CBR}(t)=\rho_N(t)\otimes\rho_{CBR}(t)+\rho_{correl}(t)$, where the correlation term ρ_{correl} can be neglected [17]. By considering the thermalized CBR density operator $\rho_{CBR}=\exp[-\beta H_{CBR}(A^{\dagger},A)]/\mathrm{Tr}\{\exp[-\beta H_{CBR}(A^{\dagger},A)]\}$, with $\beta=k_BT$, k_B being the Boltzmann's constant and T the CBR temperature, we find the master equation for the N-particle system

$$\frac{d\rho_N}{dt}=-\frac{i}{\hbar}[H_N,\rho_N]-\frac{\Lambda}{2}\int d(\Omega)\Gamma(\Omega)\{[\mathbf{X}^{\dagger}\cdot,\mathbf{X}\rho_N]+[\rho_N\mathbf{X}^{\dagger}\cdot,\mathbf{X}]+\langle n\rangle_{\Omega}([\mathbf{X}^{\dagger}\cdot,[\mathbf{X},\rho_N]]+[\mathbf{X}\cdot,[\mathbf{X}^{\dagger},\rho_N]])\}, \tag{35}$$

where ρ_N is the *reduced density operator* of the N-particle system and $\langle n\rangle_{\Omega}=1/[\exp(\beta\hbar\Omega)-1]$ is the thermal averaged photon number.

As time goes on, it is expected that the stochastic coupling induces the N-particle system to a thermal equilibrium with the CBR. By evaluating the rate of energy change between the system and the CBR we shall estimate the strength parameter Λ and improve our understanding about the nature of this stochastic coupling. In order to estimate the energy mean-value let us consider the mean value of a generic dynamical variable $\mathcal{V}$ whose equation of motion is obtained through Eq. (35) as

$$\frac{d\langle\mathcal{V}\rangle}{dt}=-\frac{i}{\hbar}\mathrm{Tr}([\mathcal{V},H_N]\rho_N)-\frac{\Lambda}{2}\int d(\Omega)\Gamma(\Omega)\mathrm{Tr}\{[[\mathcal{V},\mathbf{X}^{\dagger}]\cdot\mathbf{X}+\mathbf{X}^{\dagger}\cdot[\mathbf{X},\mathcal{V}]+\langle n\rangle_{\Omega}([[\mathcal{V},\mathbf{X}^{\dagger}]\cdot,\mathbf{X}]+[[\mathcal{V},\mathbf{X}]\cdot,\mathbf{X}^{\dagger}])]\rho_N\}, \tag{36}$$

By applying Eq. (36) to the position and momentum variables consecutively, we observe that not only the pure Schrödinger evolution is modified but also the results from the CSL model, such that the equations of motion become

$$\frac{d\langle\mathbf{Q}\rangle}{dt}=\frac{1}{M}\langle\mathbf{P}\rangle-\frac{1}{2}N\Lambda\langle\mathbf{Q}\rangle, \tag{37a}$$

$$\frac{d\langle\mathbf{P}\rangle}{dt}=-\frac{1}{2}N\Lambda\langle\mathbf{P}\rangle. \tag{37b}$$

These equations lead to the results $\langle\mathbf{P}\rangle_t=\exp(-\frac{1}{2}N\Lambda t)\langle\mathbf{P}\rangle_s$ and $\langle\mathbf{Q}\rangle_t=\exp(-\frac{1}{2}N\Lambda t)\langle\mathbf{Q}\rangle_s$, where the subscript s indicates the pure Schrödinger evolution: $\langle\mathbf{P}\rangle_s=\langle\mathbf{P}\rangle_{t=0}$ and $\langle\mathbf{Q}\rangle_s=\langle\mathbf{Q}\rangle_0+\langle\mathbf{P}\rangle_{t=0}/Mt$. For $\mathcal{V}=\mathbf{Q}^2,\mathbf{Q}\cdot\mathbf{P}+\mathbf{P}\cdot\mathbf{Q}$ and $\mathbf{P}^2$ successively, the equations of motion for the mean values become, respectively,

$$\frac{d\langle\mathbf{Q}^2\rangle}{dt}=\frac{1}{M}\langle\mathbf{Q}\cdot\mathbf{P}+\mathbf{P}\cdot\mathbf{Q}\rangle-N\Lambda\langle\mathbf{Q}^2\rangle+\frac{3\hbar\Lambda}{2m\omega}\int d\Omega\Gamma(\Omega)(1+2\langle n\rangle_{\Omega}), \tag{38a}$$

$$\frac{d\langle\mathbf{Q}\cdot\mathbf{P}+\mathbf{P}\cdot\mathbf{Q}\rangle}{dt}=\frac{2}{M}\langle\mathbf{P}^2\rangle-N\Lambda\langle\mathbf{Q}\cdot\mathbf{P}+\mathbf{P}\cdot\mathbf{Q}\rangle, \tag{38b}$$

$$\frac{d\langle\mathbf{P}^2\rangle}{dt}=-N\Lambda\langle\mathbf{P}^2\rangle+\frac{3N^2\Lambda m\hbar\omega}{2}\int d\Omega\Gamma(\Omega)(1+2\langle n\rangle_{\Omega}), \tag{38c}$$

which differ from the pure Schrödinger evolution since $\Lambda\neq0$.

IV. MASTER EQUATION AND ITÔ DYNAMICS

It will be useful to be reminded of the conventional treatment of the problem of interaction of an N-particle system with the reservoir (R). Under the Hamiltonian $H=H_N+H_R+V$, V being the interaction between both systems, the reduced density operator of the N-particle system, $\rho_N(t)=\mathrm{Tr}_R[\rho_N(t)]$, evolves, up to the second order in the interaction, according to the generalized master equation [18]

$$\frac{d\rho_N(t)}{dt}=-\frac{i}{\hbar}[H_N,\rho_N(t)]-\frac{1}{\hbar^2}\mathrm{Tr}_R\int_0^t[V,e^{-iL_0(t-t')}[V,\rho_N(t')\rho_R]]dt', \tag{39}$$

where $L_0(\cdot)\equiv[H_N+H_R,\cdot]$ is the Liouvillian operator of the free Hamiltonian. The second term in Eq. (39), acting as a source of noise for the system and also as a sink (or source) of energy, is responsible for the irreversibility of the process and the loss of coherence in $\rho_N(t)$. As such, the Itô calculus is justified when the stochastic terms are introduced into the Schrödinger equation. So, the CBR is responsible for the variation of the mean energy of the system and the increase of entropy. As shown by Isar *et al.* [19], choosing conveniently the interaction term V it is possible to obtain Eq. (35) (the Lindblad form) from Eq. (39).

It is worth noting that the master equation (35) can be written as

$$\frac{d\rho_N(t)}{dt}=-\frac{i}{\hbar}[H_N,\rho_S(t)]+\sum_{n=1}^{2}\mathcal{S}[c_n]\rho_N(t), \tag{40}$$

where the superoperator $\mathcal{S}[c_n]$ is defined as

$$\mathcal{S}[c_n]\rho_N=c_n\cdot\rho_Nc_n^{\dagger}-\frac{1}{2}\{c_n^{\dagger}\cdot c_n,\rho_N\}, \tag{41}$$

with $c_1=[\Lambda\int d\Omega\Gamma(\Omega)\langle n\rangle_\Omega]^{1/2}\mathbf{X}^\dagger$ and $c_2=[\Lambda\int d\Omega\Gamma(\Omega)(1+\langle n\rangle_\Omega)]^{1/2}\mathbf{X}$. Written as in Eq. (40) our master equation resembles the Lindblad form for the decay of a mode of the eletromagnetic field inside a cavity [20].

In summary, we have assumed *ad hoc* that the evolution of the system of particles in its way from quantum to classical dynamics, under the influence of the CBR, is described by an Itô stochastic equation. However, here we showed that the usual master equation formalism can be viewed as a subdynamics of the Itô dynamics, without any need to use perturbation methods as is done in the conventional derivation.

V. STRENGTH PARAMETER

Back to the equations of motion (38), their solutions are

$$\langle\mathbf{Q}^2\rangle=\langle\mathbf{Q}^2\rangle_s e^{-N\Lambda t}-\frac{3\mathcal{I}\hbar\omega}{M}\left[\frac{t}{N\Lambda}\left(1-\frac{N\Lambda t}{2}\right)e^{-N\Lambda t}-\left(\frac{1}{N^2\Lambda^2}+\frac{1}{2\omega^2}\right)(1-e^{-N\Lambda t})\right], \tag{42a}$$

$$\langle\{\mathbf{Q},\mathbf{P}\}\rangle=\langle\{\mathbf{Q},\mathbf{P}\}\rangle_s e^{-N\Lambda t}-3\mathcal{I}\hbar\omega\left[te^{-N\Lambda t}-\frac{1}{N\Lambda}(1-e^{-N\Lambda t})\right], \tag{42b}$$

$$\langle\mathbf{P}^2\rangle=\langle\mathbf{P}^2\rangle_s e^{-N\Lambda t}+\frac{3\mathcal{I}Nm\hbar\omega}{2}(1-e^{-N\Lambda t}), \tag{42c}$$

where

$$\langle\mathbf{Q}^2\rangle_s=\langle\mathbf{Q}^2\rangle_0+\frac{1}{M}\left(\langle\{\mathbf{Q},\mathbf{P}\}\rangle_0 t+\frac{1}{M}\langle\mathbf{P}^2\rangle_0 t^2\right), \tag{43a}$$

$$\langle\{\mathbf{Q},\mathbf{P}\}\rangle_s=\langle\{\mathbf{Q},\mathbf{P}\}\rangle_0+\frac{2}{M}\langle\mathbf{P}^2\rangle_0 t, \tag{43b}$$

$$\langle\mathbf{P}^2\rangle_s=\langle\mathbf{P}^2\rangle_0. \tag{43c}$$

The effect of the CBR temperature is present in the integral $\mathcal{I}=\int d\Omega\Gamma(\Omega)(1+2\langle n\rangle_\Omega)$. It is worth noting that the time evolution of the operators in Eqs. (42) does not show the additive property with respect to the Schrödinger terms as obtained in the CSL model. As a consequence, Eq. (42c) differs from the corresponding one in the CSL model, Eq. (20b), because instead of the diffusion inducing a steady increase of the mean value of the kinetic energy, the present model exhibits, asymptotically, thermalization due to the CBR,

$$\langle K\rangle=(\langle K\rangle_s-K_{eq})e^{-N\Lambda t}+K_{eq}, \tag{44}$$

where the equilibrium kinetic energy reads $K_{eq}=3\mathcal{I}\hbar\omega/4$. So, ω is a characteristic frequency proportional to the thermalized mean kinetic energy of the CM.

As mentioned above, in the CSL model the localization of a single particle of the system is sufficient to localize the whole system; as a consequence, the CM energy increases linearly with the "interaction" parameter $N\Lambda t$. However, from Eq. (44) we conclude that the stochastic coupling accounts for a CM energy, which grows or decays exponentially with $N\Lambda t$, depending on the negative or positive value for $\langle K\rangle_s-K_{eq}$, respectively.

In order to estimate the strength parameter Λ, from Eq. (44) we assume that the relaxation time follows from the relation $(\langle K\rangle_s-K_{eq})e^{-N\Lambda\tau_R}\sim K_{eq}$, so that

$$\Lambda\approx\frac{1}{N\tau_R}\ln\left(\frac{\langle K\rangle_s-K_{eq}}{K_{eq}}\right). \tag{45}$$

For a system of $N\approx10^{23}$ particles initially at room temperature the equipartition energy theorem gives a mean kinetic energy $\langle K\rangle_s\sim10^9$ erg. The integral $\mathcal{I}$ accounting for the effect of the temperature of the CBR has been estimated in the Appendix for $\beta\tau_c\ll\hbar$, with $\omega\tau_c\lesssim1$. The result $\mathcal{I}\sim1+2\langle n\rangle_\omega$ holds for both low- and high-frequency regimes. So, we find for the equilibrium energy at the low-frequency regime ($\hbar\omega\ll k_BT$, so that $\langle n\rangle_\omega\sim k_BT/\hbar\omega$), $K_{eq}\sim k_BT\sim10^{-16}$ erg. At the high-frequency regime ($\hbar\omega\gg k_BT$), the equilibrium energy obeys $K_{eq}\gg k_BT$. (We are referring to low- and high-frequency regimes since the present CBR temperature, $T\approx3$ K, is assumed). Taking K_{eq} at the low-frequency regime (in fact, due to the ln function, choosing K_{eq} in low or high frequency will not change appreciably the value of Λ), and the relaxation time τ_R of the order of the age of the universe, about 10^{16} s (what seems to be reasonable when considering, as obtained below, such a small coupling of the system to the CBR), we get

$$\Lambda\approx10^{-38}\ \mathrm{s}^{-1}, \tag{46}$$

a value to be compared with the above-mentioned coupling in the CSL model $\zeta\sim10^{-30}$ cm^3 s^{-1}. Thus, the parameter Λ is of the order of the upper limit of the excitation rate for nucleons estimated by Pearle and Squires [21], by comparison with a neutrino-induced process. As already pointed out, such a value hardly affects the dynamics of a microscopic particle.

VI. WAVE-PACKET REDUCTION RATES

Back to Eq. (35), in the CM positional representation, the density matrix $\rho_N(\mathbf{Q},\mathbf{Q}')$ evolves according to the differential equation

$$\frac{\partial\rho_N(\mathbf{Q},\mathbf{Q}',\mathbf{t})}{\partial t}=\left\{-\frac{\hbar}{2iM}\left(\frac{\partial^2}{\partial\mathbf{Q}^2}-\frac{\partial^2}{\partial\mathbf{Q}'^2}\right)-\mathcal{D}\left[(\mathbf{Q}-\mathbf{Q}')^2-\frac{\hbar^2}{(M\omega)^2}\left(\frac{\partial}{\partial\mathbf{Q}}+\frac{\partial}{\partial\mathbf{Q}'}\right)^2\right]-\frac{1}{2}N\Lambda\left[\left(\mathbf{Q}\cdot\frac{\partial}{\partial\mathbf{Q}'}+\mathbf{Q}'\cdot\frac{\partial}{\partial\mathbf{Q}}\right)-1\right]\right\}\times\rho_N(\mathbf{Q},\mathbf{Q}',\mathbf{t}). \tag{47}$$

The first term on the right-hand side comes from the commutator in Eq. (35), the terms multiplied by the diffusion constant $\mathcal{D}=NM\Lambda\omega(1+2\langle n\rangle_\omega)/4\hbar$ (as well as the remaining terms, which are independent of temperature) account for the fluctuations (or random kicks) and for the energy changes due to the stochastic coupling, respectively.

To analyze the wave-packet reduction rates we will not consider Eq. (47) in detail, since the effect of the second term on quantum superposition will be of much greater interest [22]. For a brief estimation of the off-diagonal matrix elements Eq. (47) will decay exponentially as

$$\langle\mathbf{Q}|\rho_S(t)|\mathbf{Q}'\rangle=e^{-\zeta t}\langle\mathbf{Q}|\rho_S(0)|\mathbf{Q}'\rangle, \tag{48}$$

where $\zeta=\mathcal{D}(\Delta\mathbf{Q})^2$ and $(\Delta\mathbf{Q})^2=(\mathbf{Q}-\mathbf{Q}')^2$. It follows from Eq. (48) that the quantum coherence of a macroscopic system will disappear on a decoherence time scale

$$\tau_D\approx\frac{1}{\mathcal{D}(\Delta\mathbf{Q})^2}=\frac{1}{(1+2\langle n\rangle_\omega)}\frac{\hbar}{NM\Lambda\omega(\Delta\mathbf{Q})^2}. \tag{49}$$

Analyzing Eq. (49) in terms of the CBR temperature, it is interesting to note that in the low-temperature limit (the present universe, $T\sim3$ K), i.e., $\langle n\rangle_\omega\rightarrow0$, the number of particles N plays a crucial role in the decoherence process induced by the CBR. In the high-temperature limit, i.e., $\langle n\rangle_\omega\rightarrow\infty$ (the early universe in the present model), we conclude that Eq. (49) leads from quantum to classical physics even in a system composed by a small number of particles. This is a key result, which helps support the assumptions considered in the present model.

Let us now estimate the decoherence time for both a macroscopic and a microscopic object in the present universe, i.e., $T\sim3$ K. In order to compare our results with that presented in the literature, we consider the low-frequency regime, such that Eq. (49) reduces to

$$\tau_D\approx\frac{1}{\mathcal{D}(\Delta\mathbf{Q})^2}=\frac{\hbar^2}{2N\Lambda Mk_BT(\Delta\mathbf{Q})^2}. \tag{50}$$

By considering a system of N ($\sim10^{23}$) hydrogen atoms with mass $M\approx1$ g and separation $\Delta\mathbf{Q}\approx1$ cm, quantum coherence would be destroyed in $\tau_D\approx10^{-24}$ s. Such a value turns to be significantly smaller than the one obtained by GRWP, $\lambda_{CM}\approx10^{-7}$ s, Eq. (19), and comparable with that obtained through the linear response model of Caldeira and Leggett (CL) [23], where, also at the low-frequency regime, $\tau_D/\tau_R\approx\hbar^2/2mk_BT(\Delta\mathbf{Q})^2$, τ_R being a relaxation time. For the above-mentioned system of N atoms, and assuming $\tau_R\approx10^{16}$ s, as we have done to obtain Λ, Eq. (45), we get from CL model $\tau_D\approx10^{-23}$ s. So, Eq. (49), and consequently Eq. (50), arise from a theory that, despite assuring the essential character of the GRWP model, gives a more realistic value for the decoherence time of a macroscopic object.

As far as a microscopic object is concerned, for example, a single atom, $m\approx10^{-24}$ g on atomic scale $\Delta\mathbf{Q}\approx10^{-8}$ cm, we observe the persistence of quantum coherence since $\tau_D\approx10^{41}$ s. Finally, we note that when considering a tiny Weber bar [24,22], $\Delta\mathbf{Q}\approx10^{-19}$ m, at cryogenic temperatures, $T\approx10^{-3}$ K, we also observe the persistence of quantum coherence from Eq. (49), as should be expected.

Back to Eq. (48), when interpreting the exponential damping factor ζ by the light of the CSL model [Eqs. (18) and (19)], we conclude that the strength Λ plays the role of a microscopic frequency hitting parameter.

VII. CM AND INTERNAL MOTION

By construction we assumed that the CBR acts only on the CM coordinates of the system of particles. Such an assumption automatically decouples the dynamics of the collective and internal motions in the master equation (35). Next, we show that even the vector state dynamics for the CM and the internal motion decouple, as in the CSL model. Of course, our analysis will be restricted to the low-temperature limit where, as obtained in Eq. (49), the macroscopic character of the system becomes really important due to the number of particles N. In this limit Eq. (35) simplifies to

$$\frac{d\rho_N}{dt}=-\frac{i}{\hbar}[H_N,\rho_N]+\Lambda\mathbf{X}\cdot\rho_N\mathbf{X}^\dagger-\frac{\Lambda}{2}\{\mathbf{X}^\dagger\cdot\mathbf{X},\rho_N\}. \tag{51}$$

The stochastic differential equation for the state vector of the system of particles, which leads to Eq. (51), can be written as

$$d|\Psi_N\rangle=\left(-\frac{i}{\hbar}H_Ndt+\mathbf{X}\cdot d\mathbf{W}-\frac{\Lambda}{2}\mathbf{W}^\dagger\cdot\mathbf{W}^dt\right)|\Psi_S\rangle, \tag{52}$$

now with the Wiener process $\overline{dW_i}=0$, $\overline{dW_idW_j}=\Lambda\delta_{ij}dt$.

The assumption made in the CSL model, that the set $\{\mathbf{r}_i\}$ in Eq. (13) does not contain macroscopic variables, implies that the state vector for the macroscopic object factorizes as $\Psi_N(\{\mathbf{q}_k\})=\psi_{CM}(\mathbf{Q})\phi_{int}(\{\mathbf{r}_i\})$. The additional assumption that the CM motion is decoupled from the internal degrees of freedom means that the Hamiltonian must be written as a sum of two terms, $H_N=H_{CM}+H_{int}$ [5]. Under these assumptions the Itô calculus, $\overline{d\Psi_N}=d(\psi_{CM}\phi_{int})=(d\psi_{CM})\phi_{int}+\psi_{CM}(d\phi_{int})+(d\psi_{CM})(d\phi_{int})$, shows that the wave functions $\psi_{CM}(\mathbf{Q})$ and $\phi_{int}(\mathbf{r}_i)$, similar to Eqs. (17a) and (17b), satisfy equations

$$d|\psi_CM\rangle=\left(-\frac{i}{\hbar}H_{CM}dt+\mathbf{X}\cdot d\mathbf{W}-\frac{\Lambda}{2}\mathbf{W}^\dagger\cdot\mathbf{W}dt\right)|\Psi_{CM}\rangle, \tag{53a}$$

$$d|\phi_{int}\rangle=-\frac{i}{\hbar}H_{int}|\phi_{int}\rangle dt. \tag{53b}$$

The stochastic terms do not affect the internal structure of the system of particles, i.e., nothing changes in the Schrödinger dynamics of microscopic particles. It is worth noting that in the CSL model the additional assumption of a large enough localization width parameter (besides an internal wave function independent of macroscopic variables) is necessary to decouple the dynamics of ψ_{CM} from ϕ_{int}. In fact, as shown in Ref. [11], a width parameter of order of atomic size leads to the breakdown of the translational symmetry of the system and the interaction between the CM and the relative coordinates (i.e., $H = H_{CM} + H_{int} + V$), has to be taken into account. However, in the present model, since we have assumed that the CBR acts only on the CM coordinates of the system of particles, no additional conjectures were requested about the random operator $\mathbf{Z}(\Omega)$, Eq. (25), to achieve the remarkable result of the CM decoupling from the internal motion, as if the stochastic terms in Eq. (35) were absent. The operator $\mathbf{Z}(\Omega)$ has thus the advantage of not needing additional conjectures about the width parameter of the localization process.

VIII. DECOHERENCE AND ENTROPY

The decoherence process resulting from the interaction of the state vector for a macroscopic object with the CBR can be quantified by the rate of increase of either the linear or the statistical entropy. In terms of the density matrix, the statistical entropy, a measure of our ignorance, is defined as [25] $\mathcal{S}_s = -\mathrm{Tr}[\rho \ln(\rho)]$ (the subscript s refers to *statistical*). This definition does not require that the system be in a thermal equilibrium state. Alternatively, a good measure of the loss of purity for states of an evolving open system is based on the increase of the linear entropy (subscript l) [26]

$$\mathcal{S}_l = \mathrm{Tr}(\rho - \rho^2). \tag{54}$$

Next, we estimate the rate of increase of the linear entropy through the evolution of the density matrix given in the operator form by Eq. (47). Considering a weak strength parameter ($\Lambda \approx 0$) and the state vector remaining approximately pure ($\mathrm{Tr}\rho^2 \approx 1$), up to first order in Λ we obtain

$$\dot{\mathcal{S}}_l = 4\mathcal{D}\left(\langle (\Delta \mathbf{Q})^2 \rangle + \frac{1}{(Nm\omega)^2} \langle (\Delta \mathbf{P})^2 \rangle \right), \tag{55}$$

where $\langle (\Delta \mathbf{Q})^2 \rangle$ and $\langle (\Delta \mathbf{P})^2 \rangle$, obtained from Eqs. (42a)–(43c), stand for the variances of the position and momentum operators and can be rewritten as function of their initial values $\langle \mathbf{Q} \rangle_0$ and $\langle \mathbf{P} \rangle_0$.

In order to better understand the rate of increase of the linear entropy in Eq. (55), it is worth comparing it with that obtained by Zurek [26] who used the linear response model of Caldeira and Leggett [9] (in the high-temperature limit). With the above approximations Zurek obtained $\dot{\mathcal{S}}_l = 4\mathcal{D}\langle (\Delta \mathbf{Q})^2 \rangle$ (for a single oscillator), so that the rate of increase of linear entropy (in quantum Brownian motion) is proportional to the dispersion in position coordinate only—the preferred observable singled out by the interaction Hamiltonian. In our approach, from Eq. (55) we observe that no preferred observable emerges from the dynamic equation (35) (the dispersion in momentum is also present), contrary even to the CSL model where the position representation is taken from the outset as privileged. However, for a large number of particles ($N \gg 1$), Eq. (55) indicates that the dispersion in momentum is considerably smaller when compared with that in position which, in this situation, emerges as the preferred observable.

In the weak-coupling limit we integrate Eq. (55) replacing the general evolution in Eq. (35) by the free von Neumann equation to obtain

$$\mathcal{S}_l = 4\mathcal{D}\left[\left(\langle (\Delta \mathbf{Q})^2 \rangle_0 + \frac{1}{(Nm\omega)^2} \langle (\Delta \mathbf{P})^2 \rangle_0 \right) t + \frac{1}{2M} \langle \Delta \{\mathbf{Q},\mathbf{P}\} \rangle_0 t^2 + \frac{1}{3M^2} \langle (\Delta \mathbf{P})^2 \rangle_0 t^3 \right], \tag{56}$$

with $\langle \Delta\{\mathbf{Q},\mathbf{P}\}\rangle \equiv \langle \{\mathbf{Q},\mathbf{P}\}\rangle - 2\langle \mathbf{Q}\rangle\langle \mathbf{P}\rangle$. The dispersions appearing in Eq. (56) are computed for the pure initial state.

Back to the preferred basis problem, recall that Zurek considered the free Heisenberg equations for the oscillator operators (P,Q) and obtained the linear entropy $2\mathcal{D}[\langle (\Delta Q)^2 \rangle_0 + 1/(Nm\omega)^2 \langle (\Delta P)^2 \rangle_0]$ $(N=1)$, averaged over one oscillator period. So, this result corresponds only to the coefficient for the linear time dependence in Eq. (56), where additional terms as square and cubic time-dependent behavior also take place. Such a behavior indicates that, in spite of the large number of particles, for large times the momentum plays an important role in the problem of the preferred observable because we have considered the free motion of an N-particle system instead of a single harmonic oscillator.

IX. SUMMARY AND CONCLUSIONS

In the GRWP model of continuous dynamical reduction of the state vector it is assumed that each microscopic constituent of a system of N particles is subject to a sudden collapse due to a spontaneous random hitting process consisting in a localization of the wave function of the particle within an appropriate range [5]. In what turns to be a remarkable result the localization of a single constituent of the system of particles is sufficient to localize the whole system. Such a spontaneous localization, considered as a fundamental physical process, induces a steady increase of the mean energy value of the physical system and so the increase in temperature per unit time of the universe. When taking into account that the age of the universe is about 10^{16} s, the GRWP model leads to a total temperature increase from the beginning of the universe of 10^{-3} K, a value claimed to be comparable with the cosmic background radiation (CBR) of 3 K.

In the present model for continuous dynamical reduction, also based in a stochastic differential equation describing a Markovian evolution of state vectors, the random hitting process in GRWP model is substituted by the intervention of the CBR. Such a strategy is intended to maintain (i) the principle of conservation of energy, and (ii) the claim that the universe originated from the Big Bang leaving the CBR as a signature. In (i) the increase or decrease of the CM mean energy of the system of N particles is subject to a stochastic interaction with the CBR, which acts as a reservoir. In (ii), taking the opposite direction to the GRWP argument (which claims that the present temperature of the universe comes from the increase of the total energy arising from the random hitting process), we propose that the CBR temperature plays an important role in the reduction of the N-particle wave packet. So, we assumed, in agreement with the standard cosmology, that the Universe has originated from a hot state and cooling during its expansion, with decreasing mean photon energy. The Planck law for the thermal average boson number in CBR, indeed the best blackbody known, has recently been tested by the COBE satellite [27]. The temperature of the CBR, decreasing as the mean photon energy decreases due to the cosmic expansion, makes the mass of the system increasingly more important for the transition from quantum to classical description. On this basis one can argue that the quantum nature of the Universe becomes increasingly important as it is cooling. In fact, for the early Universe, the number of particles does not play a fundamental role in estimating the decoherence time, where higher temperatures (by itself) turn the system from micro- to macrodynamics. However, as the universe becomes cooler the number of particles becomes increasingly important.

Moreover, the present model leads to realistic results for decoherence times. While in the GRWP model the value 10^{-7} s obtained for a system of particles to go from micro- to macrodynamics seems to be too large, the value 10^{-24} s here obtained for a system of N atoms in the low-frequency regime is comparable to the decoherence time obtained from the Caldeira-Leggett model.

As mentioned, whereas the GRWP model requires a *privileged positional space*, in the present model, by construction, the stochastic operator acts on the CBR spectrum, carrying the same status for both the position and the momentum space. The GRWP's result, the wave function collapse of a single particle induces the collapse of the wave function of the whole system, was obtained exactly from the choice of the position as a preferred basis. The same result follows from our model without the choice of the position as a preferred basis. However, it has to be mentioned that in spite of attributing the same status for the position and the momentum space, when analyzing the entropy under the process of decoherence, the position coordinate still emerges as a preferred basis when considering a system with a large number of particles N. So, the preferred basis is directly related to the number of particles in the system.

Another interesting feature is that we do not claim for an *additional assumption* to decouple collective from internal motion as the required large width parameter $\alpha^{-1/2} \sim 10^{-5}$ cm in the GRWP model. The random operator $\mathbf{Z}(\Omega)$ here assumed, besides being a more conventional choice since it is associated to a reservoir (CBR), leads to the advantage of decoupling the CM and internal motion without additional assumption beyond that usually assumed for a reservoir.

The random operator describing the interaction between the system and the CBR carries only one parameter, the strength Λ, instead of the two free parameters, as required in the GRWP model ($\alpha^{-1/2}$ and the mean frequency λ). In our model, the coupling of the CBR to the system, proportional to Λ, corresponds to the random pseudopotential dh [12] of the GRWP model. As well as the parameter λ in GRWP model, our Λ is weak enough in the sense that it does not affect the dynamics of a unique particle, even in the case in which its wave function is spatially spread [5].

Finally, we point out that the Itô equation is not derived from a physical picture of the background and associated scattering processes of the CBR by the system of particles. Instead of considering a particular interaction and choosing some specific particle property sensible to the electric and magnetic field of the CBR, we approached the problem by modeling the interaction by a stochastic coupling, such that the dynamics could be described by an Itô equation. We have considered an effective strength parameter Λ accounting for all kind of light-particle scattering processes. We also stress that our precursor (34) to the master equation (with respect to the particles) still has information on both, the system of particles and the CBR, since it contains operators of both subsystems. This approach is different from the usual one where for getting a master equation it is necessary to trace over the environment degrees of freedom, as is done in the theories of Joos and Zeh and Caldeira-Leggett or even in quantum optics. In our model it is possible to calculate correlations between observables of both subsystems. However, we have to get rid of CBR degrees of freedom, Eq. (35), just because the available information on the CBR subsystem is sparse, consisting of the blackbody radiation distribution function at 3 K. Thus the master equation (35) expressed in the CM positional representation, Eq. (49), incorporates the similar equations obtained in both theories, Joos and Zeh and Caldeira-Leggett. The main difference between the three approaches stem in the nature of the diffusion constant (DC): In Joos and Zeh the DC originates from the scattering of electromagnetic waves by small objects; in Caldeira-Leggett it comes from the fluctuations arising from energy dissipation of the system of interest to a thermal reservoir. In our model the DC originates from the stochastic interaction between N particles of mass m and the CBR at temperature T.

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APPENDIX: CALCULATION OF INTEGRAL $\mathcal{I}$

Due to the normalized Lorentzian spectrum [Eq. (28)], the integral accounting for the temperature of the CBR reads $\mathcal{I} = 1 + 2\int d\Omega \Gamma(\Omega)\langle n\rangle_\Omega$. Now, since the Planck's distribution $\langle n\rangle_\Omega$ diverges when Ω goes to zero, the same occurs to the remaining integral $\int d\Omega \Gamma(\Omega)\langle n\rangle_\Omega$. However, as usual, we assume that the spectrum $\Gamma(\Omega)$ has its maximum far away from zero in order to cancel the divergence coming from $\langle n\rangle_\Omega$. In what follows we are going to estimate under which conditions this approximation is valid.

After the transformations $\Omega\tau_c = x$ and $\gamma = \hbar/k_B T\tau_c$, the remaining integral reads

$$\frac{1}{\pi}\int_{-\infty}^{+\infty}dx\frac{1}{[x-(\omega\tau+i)][x-(\omega\tau_c-i)]}\frac{1}{e^{\gamma x}-1}, \quad \text{(A1)}$$

which can be solved in the complex space through Jordan's lemma, leading to the result

$$2i\left\{\frac{1}{2i}\frac{1}{e^{\gamma(\omega\tau_c+i)}-1}+\frac{1}{\gamma}\sum_{n=0}^{\infty}\left(1-\frac{1}{2}\delta_{n,0}\right)\right.$$
$$\left.\times\frac{1}{\left[\omega\tau+i\left(1-\frac{2\pi n}{\gamma}\right)\right]\left[\omega\tau_c-i\left(1+\frac{2\pi n}{\gamma}\right)\right]}\right\}. \quad \text{(A2)}$$

It can be shown that the imaginary term coming from the above result is zero. Now, denoting $\gamma=p/\xi$, where the parameter p is equal to $\hbar\omega/k_BT$ whereas $\xi=\omega\tau_c$, the real term coming from (A2), reads

$$\frac{\cos(\xi/p)e^{\xi}-1}{e^{\xi}[e^{\xi}-2\cos(\xi/p)]+1}$$
$$-8\pi\frac{p^3}{\xi^2}\sum_{n=1}^{\infty}\frac{n}{[1+p^2-(2\pi np/\xi)^2]+(4\pi np^2/\xi)^2}. \quad \text{(A3)}$$

For large n the second term of (A3) reduces to

$$\sim\frac{\xi^2}{p}\sum_{n=1}^{\infty}\frac{1}{n^3}. \quad \text{(A4)}$$

The analysis of the above result will be restricted to the condition $\xi/p\ll 1$, with $\xi\lesssim 1$, under which the sum in (A4) can be disregarded (since even $\xi^2/p\ll 1$), and the first term in (A3) gives us $1/(e^{\hbar\omega/k_BT}-1)$, in a way that the Lorentzian distribution $\Gamma(\Omega)$ acts practically as a δ function $[\delta(\Omega-\omega)]$. In fact, the limit $\xi\lesssim 1$, leads to the condition $\omega\lesssim\tau_c^{-1}$, so that the frequency can be taken far away from zero since, as discussed above, we are considering an extremely short correlation time (Markovian approximation). Under such a condition it is expected that the Lorentzian function Γ acts indeed as a δ function, which means that the action of the reservoir over the system of particles is restricted to the oscillators whose frequencies are closely related to ω. So, the problem of how far ω has to be from zero, in order to eliminate the divergence coming from Planck's distribution when $\omega\to 0$, depends exactly on the Lorentzian height in its maximum. Moreover, the condition $\xi/p\ll 1$, with $\xi\lesssim 1$, holds for both the low- and high-frequency regimes. When $\xi\sim 1$ (so that $\omega\sim\tau_c^{-1}$), we get the high-frequency regime $\hbar\omega\gg k_BT$, whereas for $\xi\ll 1$ even the low-frequency regime is allowed. For the latter case we have to assure that $0\ll\omega\ll\tau_c^{-1}$, not only to get rid of the divergence arising from $\langle n\rangle_\omega$, but also to hold the assumption of highly excited oscillations of the CBR leading to the Markovian approximation. Summarizing, under the conditions established above we get the result $\mathcal{I}\sim 1+2\langle n\rangle_\omega$, which holds for the high- and low-frequency regime.

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