Mathematics Education Research Journal 2000, Vol. 12, No. 3, 303-320

Reshaping Teacher and Student Roles in Technology-Enriched Classrooms

Merrilyn Goos
The University of Queensland
Peter Galbraith
The University of Queensland
Peter Renshaw
The University of Queensland
Vince Geiger
Hillbrook Anglican School

This paper draws on data from a three-year longitudinal study of secondary school classrooms to examine pedagogical issues in using technology resources in mathematics teaching—in particular, graphics calculators and overhead projection panels that allow screen output to be viewed by the whole class. We theorise four roles for technology in relation to such teaching and learning interactions—master, servant, partner, and extension of self—and illustrate this taxonomy with observational data from five senior secondary mathematics classrooms. Our research shows how technology can facilitate collaborative inquiry during both small group interactions and whole class discussions when students use their calculators and the overhead projection panel to share their mathematical understanding.

At the beginning of the last decade, it was predicted that technologies such as computers and graphics calculators would have a major impact on the teaching and learning of secondary school mathematics (Barrett & Goebel, 1990; Demana & Waits, 1990). Some of these predictions were concerned with opportunities for enhancing student learning—for example, by enabling connections to be made between algebraic, graphical, and numeric representations of mathematical concepts. It was also anticipated that technology would bring about changes in the roles of teachers and students, in that teachers would act as facilitators of student discussion and collaborative exploration with peers (Heid, Sheets, & Matras, 1990). The potential for technology to enrich mathematics teaching and learning is recognised by curriculum authorities and professional bodies (Australian Association of Mathematics Teachers, 1996; Australian Education Council, 1991; National Council of Teachers of Mathematics, 1989, 1991), and is evident in the increasing acceptance of graphics calculators for use in high stakes examinations (Jones & McCrae, 1996).

Research in mathematics education over the last decade has begun to address the nature of these new technologies and their effects on learning and teaching. Although overall findings concerning the predicted benefits for students' learning have been somewhat inconclusive (Lesmeister, 1997; Maldonado, 1998; Penglase & Arnold, 1996), many studies have reported that the use of technology has a positive effect on students' attitudes towards mathematics, understanding of function and graphing concepts, and spatial visualisation skills (Portafoglio, 1998; Weber, 1999; see also Penglase & Arnold, 1996). However, it seems that less is known about how the availability of technology, especially graphics calculators and their peripheral devices, has affected teaching approaches (Penglase & Arnold, 1996). While some studies have found changes in classroom dynamics leading to a less teachercentred and more investigative environment (Simonsen & Dick, 1997), it appears that negotiation of such a pedagogical shift is mediated not only by teachers' mastery of the technology itself, but also by their personal philosophies of mathematics and mathematics education (Simmt, 1997; Tharp, Fitzsimmons, & Ayers, 1997).

The purpose of this paper is to consider some pedagogical issues in using graphics calculator technology in mathematics teaching, arising from a three-year longitudinal study of secondary school classrooms. In particular, we examine interactions between teachers and students, amongst students themselves, and between humans and technology, in order to investigate the extent to which different participation patterns provide opportunities for students to engage constructively and critically with mathematical ideas. We begin by providing an overview of pedagogical approaches in technology augmented learning, and introduce the notion of emergence in relation to the role of technology in mathematics classrooms. This is followed by an outline of the theoretical perspective that has guided our classroom research, and results from the study that illustrate four metaphors for relationships between teachers, students, and technology. Although the initial discussion refers to technology resources in general, the focus of the paper is on teachers' and students' use of a graphics calculator and the overhead projection unit (an LCD panel that allows the calculator screen to be projected for whole class viewing).

Technology-Augmented Learning

As technology has been increasingly imported into educational settings, so the variety of pedagogies associated with its use has increased. Olsen (1999) describes one of the most extensive examples of technology used to provide automated instruction, noting how politicians visiting Virginia Tech's Mathematics Emporium, a 58 000 square foot (5000 m^2) computer classroom,

> see a model of institutional productivity; a vision of the future in which machines handle many kinds of undergraduate teaching duties—and universities pay fewer professors to lecture. On weekdays from 9 am to midnight dozens of tutors and helpers stroll along the hexagonal pods on which the computers are located. They are trying to spot the students who are stuck on a problem and need help. (p. 31)

The approach is openly driven by economic rationalism, and the educational assumptions behind the programme may be inferred from the comment that students must take dozens of quizzes each semester. The driving philosophy is clearly within the transmissive model of teaching-learning. Ramsden (1997) has acknowledged the impact of such inherited traditions on the use of technology by referring to an instinct for teachers to begin by looking for electronic ways of doing familiar jobs previously done by textbooks and lectures. Similarly Thorpe (1997), in examining teaching behaviours and attitudes towards technology, found that computer technology was being used essentially to enhance preferred teaching methods—that is, the technology, although freely available, was utilised in a conservative way.

However, Ramsden (1997) also introduces a more positive note, observing the attraction of technology for educators who want to give their students more power and welcome technology as a liberating opportunity. Typical is the approach of Templer, Klug, and Gould (1998), who shared an explicit conviction that the computer should be used to allow the student to explore and investigate mathematical concepts. They saw being involved in a technology-based mathematics course as a dynamic experience in many senses—not least, in the way that mathematics itself can become a subject for experiment and conjecture. They also acknowledge the fundamental philosophical issue that underlies the eventual choice of a pedagogical approach and indeed all programs of change: Does technology represent an arena for exploration of mathematical ideas, or a channel for the transmission of knowledge?

Emergent Properties

Ramsden (1997) argues that while a technology cannot be used for a purpose that is patently unsuited to its design, emergent uses should be productively sought. These are uses that no one (including the designers) could have predicted, and the space of these is vast and unexplored. Shneiderman, Borkowski, Alavi, and Norman (1998) describe settings in which teachers have evolved personal styles in using an elaborately fitted out "electronic classroom": New patterns of interaction emerged which, while retaining characteristics of personal styles, have in common a more collaborative approach. Each of these entailed combining the technology resources and human interaction to develop methods that were not obviously the precinct of the hardware design. One emergent property noted was the role technology played in changing the communication structure by providing alternative and parallel channels for students to contribute to discussions and provide feedback, both privately (on individual computer screens) and publicly (on a class screen). This equaliser role was evidenced in the contributions of "quiet" students who would not participate in conventional classroom dialogue, but who eagerly shared comments through an electronic interface.

Emergence in the sense described here is a product of the interaction between human and technological agencies. In consequence, it is not surprising that those who have been involved with some of the most innovative uses of technology (Shneiderman et al., 1998) are among the most definite in rejecting the teacher-replacement concept, the very antithesis of the concept of emergence:

> While technology can be wonderfully empowering for teachers and students, the relationship between human beings is still the heart of the educational process … key function of a university or school setting is to encourage the tie between teachers and students: technology can support and strengthen relationships, but never create or replace them. (p. 24)

While the above discussion refers mainly to computer technology, the arguments concerning teaching and learning may be applied equally well to the use of more recent innovations such as graphics calculators and peripherals—although the portability of calculators adds another dimension in that students may own and attain intellectual intimacy with these devices. To date, research on graphics calculators and pedagogical change in mathematics education has tended to investigate (a) the effects of different instructional strategies (both with and without technology) and (b) teacher attitudes towards technology. That is, technology has been treated as a variable in the interpretation of results (Penglase & Arnold, 1996). This contrasts with our own research programme, whose theoretical orientation explicitly addresses technology usage as an integral component of the learning environment. The next section outlines the theoretical assumptions underpinning our research, which applies sociocultural models of learning to pedagogical practices in technology rich classrooms.

A Sociocultural Perspective on Teaching and Learning with Technology

Sociocultural perspectives on learning emphasise the socially and culturally situated nature of mathematical activity, and view learning as a collective process of enculturation into the practices of mathematical communities (Goos, Galbraith, & Renshaw, 1999). The classroom, as a community of mathematical practice, supports a culture of sense making where students learn by immersion in the practices of the discipline. Rather than relying on the teacher as an unquestioned external authority, students in such classrooms are expected to defend and critique ideas by proposing justifications, explanations, and alternatives.

A central claim of sociocultural theory is that human action is mediated by cultural tools and is fundamentally transformed in the process (Wertsch, 1985). The rapid development of computer and graphics calculator technology provides numerous examples of how such tools transform mathematical tasks and their cognitive requirements (Goos, 1998). However, tools are not limited to physical artefacts, but also include "concepts, structures of reasoning, and the forms of discourse that constrain and enable interactions within communities (Resnick, Pontecorvo, & Säljö, 1997, p. 3). The emergent properties of technology in promoting new forms of classroom interaction highlight this tool-mediated aspect of learning. Within particular knowledge communities, then, tools are cultural resources that re-organise, rather than amplify, cognitive processes through their integration into human practices. From this perspective, learning is not simply the accumulation of mental structures, but a process of appropriating the cultural tools recognised by a community of practice. Participation in such classroom communities represents a challenge for learners to move past their established capabilities towards new forms of reasoning and action.

The Research Study

The research reported here forms part of a three-year (1998-2000) longitudinal study in which we investigated the role of technology in facilitating students' exploration of mathematical ideas and in mediating teacher-student and student-student interaction. Data collection involved five senior secondary mathematics classrooms drawn from three co-educational schools (two government and one independent) in a large Australian city. The students who participated in the study were in Years 11 and 12 and were taking either Mathematics B (an introductory calculus and statistics subject) alone or in combination with Mathematics C (an advanced subject usually chosen by students intending to pursue further study of mathematics at tertiary level). While the syllabuses for both subjects did not mandate the use of graphics calculators and computers, teachers were encouraged to make use of technology wherever appropriate.

At least one lesson every week was observed and videotaped, but more frequent classroom visits were scheduled if the teacher planned a technology-intensive approach to a topic. Audiotaped interviews with individuals and groups of students were conducted at regular intervals to examine the extent to which technology was contributing to students' understanding of mathematics and how technology changing the teacher's role in the classroom. At the beginning and end of each year, students also completed a questionnaire on their attitudes towards technology and its role in learning mathematics (see Geiger, 1998). For further details on the methodology, see Goos et al. (1999).

Analysis of technology-focused interactions has been framed by four metaphors we have developed to form a taxonomy of sophistication with which teachers and students work with technology. We draw on classroom observation and videotape data from 1998 and 1999 to illustrate these metaphors with respect to both teacher and student roles. The teachers involved will be referred to as Steve, Chris, Jack, and Brian.

Reshaping Teacher Roles

In developing the potential of technology-enriched learning, the role of the teacher in making use of technology is clearly crucial. We theorise four roles for the interaction between teacher and technology.

Technology as master. Here the teacher is subservient to the technology and is able to employ only such features as are permitted either by limited individual knowledge or force of circumstance. This is clearly the role promoted in large-scale transmissive programs, where course organisation imposes the relationship (Olsen, 1999). However, it may also occur in classrooms where teachers have individual autonomy if, following a training program, pressure to use technology results in implementation dominated by whatever basic skill has been acquired without consideration of any impact beyond the present (Stuve, 1997).

Technology as servant. Here the user may be knowledgeable with respect to the technology but uses it only in limited ways to support preferred teaching methods (Thorpe, 1997). The technology is not used in creative ways to change the nature of activities in which it is used. For example, just as a graphics calculator can be restricted to the purpose of producing fast reliable answers to routine exercises, an overhead projection panel may be limited to providing a medium for a teacher to demonstrate output to the class.

Technology as partner. Here the teacher has developed an affinity with both the class and the teaching resources available. Technology is used creatively in an endeavour to increase the power that students collectively exercise over their learning, rather than the teacher exercising power over the students (Templer, Klug, & Gould, 1998). This occurs both in the use of mathematically-based technology (e.g., graphics calculators) for the purpose of enhancing individual prowess and in the use of communications technology to enhance the quality of learning through sharing, testing, and reworking mathematical understandings.

For example, instead of functioning as a transmitter of teacher input, an overhead projection panel may be a used to engender otherwise non-existent participation (Shneiderman et al., 1998) or to act as a medium for the presentation and examination of alternative mathematical conjectures. A defining characteristic of this metaphor for technology is that "the locus of control never passes from user to machine" (Templer, Klug, & Gould, 1998).

Technology as extension of self. This, the highest level of functioning, may presently be only rarely in evidence. Here powerful and creative use of both mathematical and communications technology forms as natural a part of a teacher's repertoire as do fundamental pedagogical and mathematical skills. Writing courseware to support and enhance an integrated teaching program would be an example of operating at this level.

It can be noted that these modes of operating are not necessarily tied to the level of mathematics taught or to the sophistication of technology available. Simple mathematics and basic technologies are sufficient to provide a context for highly creative teaching and learning.

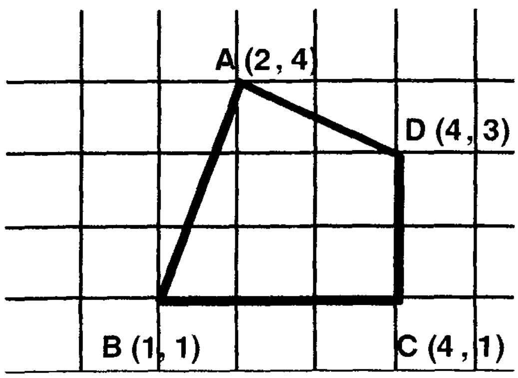
Classroom Examples

Use of the metaphors of *master, servant, partner,* and *extension of self* does not imply that teachers remain attached to a single mode of working with technology. In fact, some of our most interesting observations reveal variations on these themes—for example, where teachers may be in transition between different modes of operating or where the modes themselves are instantiated in unexpected ways. Below we outline some classroom examples of instructional use of graphics calculators and overhead projection panels to illustrate these variations.

Master and servant. Jack admitted minimal expertise in using the graphics calculator, but countered his own lack of confidence by calling on a recognised student "expert" to demonstrate calculator procedures via the overhead projection panel. This student owed his expertise to having completed a "train the trainer" program offered by the calculator company. While the teacher lacked personal autonomy in the use of technology (suggesting that technology had the role of *master*), he nevertheless retained control of the lesson agenda through the medium of the student presenter—often to the extent of providing the mathematical commentary and explanations accompanying the student's silent display (thus suggesting that technology was being used as a *servant*). Even when the student instructed the class on calculator keystrokes, the teacher's voice could still be heard in the student's articulation of carefully controlled, step-by-step procedures consistent with the teacher's preferred methods. Ultimate authority rested with the teacher, who remained reluctant to allow students to use technology to explore mathematical territory that was unfamiliar or outside the immediate lesson topic. Nevertheless, this teacher's actions could be interpreted as movement towards greater student participation, albeit through working with technology in the servant mode.

Intelligent servant. Brian tended to use the calculator and OHP panel as conventional instructional devices. For example, he used the panel like an electronic blackboard for demonstrating calculator operations, which students then checked against their own working. However, emergent uses of the technology in conjunction with other material resources were apparent. One simple example involved projecting the calculator display onto a whiteboard that simultaneously acted as a screen and a writing surface, thus enabling the teacher to interact with, highlight, and modify aspects of the calculator's output by writing on the screen image projected onto the whiteboard.

An even more creative approach integrated technology with concrete aids, thus enhancing even further the graphics calculator's capacity for linking multiple representations of a concept. For example, in a Year 11 lesson on matrix transformations, students were supplied with the worksheet in Figure 1. The teacher physically demonstrated the results of several matrix transformations using transparent grid paper, plastic cutout polygons, and the overhead projector. The students then investigated further by placing their own polygons on grid paper and recording the coordinates of the vertices before and after transformation—with graphics calculators taking care of the matrix calculations. While the technology was still used to support the teacher's preferred approach involving hands-on activities, the calculators and projection panel were exploited in novel ways that retained effective features of more conventional instruction.



Place a plastic polygon on the grid so that each vertex lies on an integer co-ordinate position. This shape can be manipulated to any number of integer positions.

Consider each vertex as a vector; e.g., A (2, 4) can be thought of as $\mathbf{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

From the list of 2 × 2 matrices below choose one. Apply this matrix to the vertex vectors and re-position your polygon to the new co-ordinates. Try and identify what each matrix transformation does geometrically. Follow up by trying to identify from the arithmetic elements of the matrices why your polygons were transformed in the way they were.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{pmatrix} \quad \mathbf{H} = \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{pmatrix}$$

Figure 1. Matrix transformation task (student worksheet).

Creative partnerships. Vignettes from two additional project classrooms illustrate how technology can have a liberating effect on both teachers and students. Chris, the teacher in the first classroom, had basic but growing competence with graphics calculators and was willing to try out calculator operations only partly understood. One of our visits to his Year 11 classroom prompted him to use the overhead projection panel for the first time in a lesson situation. Students were set the task of investigating various transformations of the functions $y = x^2$, $y = 1/x$, and $y = |x|$. Instead of using the projection panel to control the students' activities by requiring them to reproduce teacher-demonstrated transformations, the teacher shifted the locus of control to the students by assigning different functions to small groups and inviting them to use the OHP to present their findings to the whole class. Consequently, the LCD panel acted as a communication medium that enabled students to explain and defend their own conjectures.

In our view, the liberating potential of the overhead projection panel represents one of the most significant emergent properties of technology in mathematics classrooms, as control is shared between the machine and its student users. Of course, such presentations may parallel the teacher's actions if students simply display their solutions. However, we have observed instances of knowledge production and repair where partial solutions were shared and completed with whole class input; in other cases, previously unnoticed errors in the student-presenter's work were identified and corrected by peers. Such was the case in Steve's classroom. Steve was already an expert and innovative user of technology and fosters similar expertise in his students—not through detailed instructions on keystrokes, but by providing tasks that require students to use the calculator intelligently.

A brief vignette involving programming illustrates this point. The students (Year 12, Mathematics C) were asked to program their calculators to find the angle between two three dimensional vectors $\mathbf{r}_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$, given by the formula $\vartheta = cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1||\mathbf{r}_2|}\right) = cos^{-1}\left(\frac{ad + be + cf}{\sqrt{a^2 + b^2 + c^2}\sqrt{d^2 + e^2 + f^2}}\right)$. The teacher provided only minimal instruction in basic programming techniques, expecting students to consult more knowledgeable peers for assistance. Volunteers then demonstrated their programs via the overhead projection panel. The students noted that there was a wide variation in the programs they had produced, as shown in Figure 2.

This public inspection of student work also revealed programming errors that were subsequently corrected by other members of the class. For example, the class disputed the answer obtained by executing the program shown in the first screen of Figure 3. Following the instructions of fellow students, the presenter scrolled down through the program and replaced the plus sign with a multiplication sign between the two bracketed terms in the denominator (Figure 3, second screen). The output of this amended program was again challenged by his audience, one of whom located the offending element of the program where multiplication instead

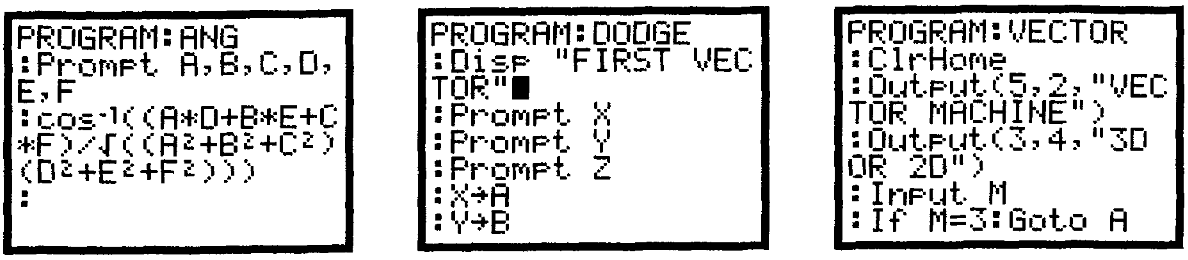


Figure 2. First lines of three student programs for computing the angle between two vectors.

of addition signs had been entered in the second term of the denominator. The presenter made this correction (Figure 3, third screen) and executed the program once more. The appearance of the correct answer was greeted with cheers and applause from his classmates.

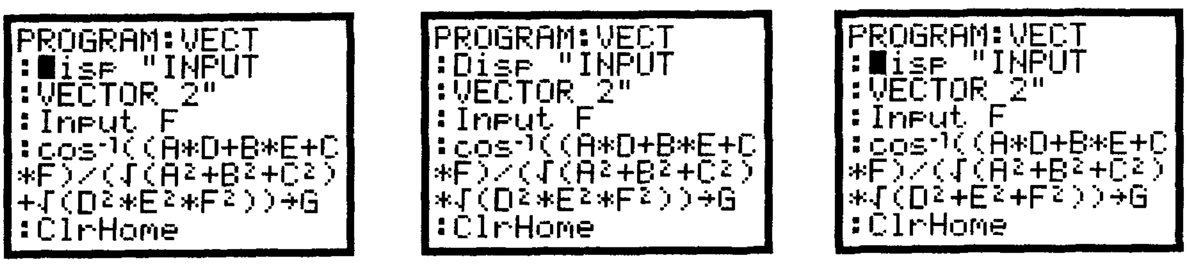


Figure 3. Correcting errors in a student program.

The day after the lesson, individual students were interviewed to discover how they thought programming could help them with mathematics. All acknowledged that a program could save time with calculations, but they also commented that writing programs required thorough understanding of the underlying mathematical concepts. When asked whether programming could further one's understanding, one student referred to the benefits of extending his own repertoire by comparing programs written by different people for the same task—an opportunity afforded by the public sharing of knowledge in the previous day's lesson.

Reshaping Student Roles

The above examples are representative of the qualitatively different models of teaching that have been recorded in the research project classrooms. Similarly, the metaphors of *master, servant, partner,* and *extension of self* can be used to analyse students' interaction with technology.

Technology as master. Here the student is subservient to the technology—a relationship that can be induced by either technological or mathematical dependence. If the complexity of usage is formidable, student activity will be confined to those limited operations over which they have technical competence. Alternatively, if necessary mathematical understanding is absent, the student is reduced to blind consumption of whatever output is generated, irrespective of its accuracy or worth.

Technology as servant. In this role, technology is used as a reliable timesaving replacement for mental or pen-and-paper computations. The tasks of the mathematics classroom remain the same, but now they are facilitated by a fast mechanical aid. Unlike the previous category, the user is in control and instructs the technology as an obedient but dumb assistant. Trust in the reliability of the servant means that the output is regarded as authoritative, although the discerning user will continue to monitor reasonableness of outcome against the possibility of keying errors.

Technology as partner. Here a rapport has developed between the user and the technological device—which may even be addressed in human terms. A graphics calculator, for example, becomes a friend to go exploring with rather than merely a producer of results. The user is still in control, but there is appreciation of the fact that calculator-generated outcomes cannot be blindly accepted but need to be judged against mathematical criteria. Explorations, for example in graphical work, lead to situations where the output needs to be checked against the known properties of related graphical forms. It is possible for the calculator to be misleading, and a feature of its use in this mode is the way in which the respective authorities of mathematics and technology are balanced.

Technology as extension of self. This highest level of functioning involves users incorporating technological expertise as an integral part of their mathematical repertoire. Here, powerful use of calculators and computers forms an extension of the user's mathematical prowess. Rather than existing as a third party, a calculator may be used to share and support mathematical argumentation on behalf of the individual—as when students share and compare graphical output as part of their own contribution to a solution process. The technology becomes as much a part of the user's catalogue of resources as tabulated information and mathematical know-how inside the head.

As with the corresponding teacher roles, there is no necessary connection between these successively higher forms of technological functioning and the level of mathematics involved or the grade level.

Classroom Examples

In classroom communities of mathematical practice, interactions between students and technology may have characteristics of the servant, partner, and extension-of-self roles. The *servant* role, where technology is treated as a reliable means of obtaining results necessary to progress a line of development, is often appropriate. Note that understanding remains paramount even in this role. For example, one student maintained that technology "*makes* you understand, because you have to understand the maths before you can do it on the calculator". Technology as a *partner* is important during individual or group exploratory work, and in widening the options in problem-solving situations. Technology as an *extension of self* enters at the highest levels of mathematical activity, where the emphasis is on forms of argumentation and other characteristics of the discipline of mathematics. We illustrate the latter two roles for technology with vignettes from Steve's classroom.

Interacting with technology. A series of Year 11 lessons on matrices given by Steve was observed and videotaped in order to analyse students' use of technology in learning this new subject matter. Matrix algebra was not taught as a series of algorithms, but instead was developed by presenting students with life-related problems from which matrix representations and manipulations arose naturally. For example, students were introduced to the Leontief Input-Output Model of an economy as a real-life application of matrices and used as a context for developing understanding of the inverse of a matrix. Figure 4 shows one such problem on which the students worked with the aid of their graphics calculators.

An economy with the four sectors manufacturing, petroleum, transportation, and hydroelectric power has the following technology matrix:

$$T=\begin{pmatrix} 0.15 & 0.18 & 0.3 & 0.1 \\ 0.22 & 0.12 & 0.37 & 0 \\ 0.09 & 0.3 & 0.11 & 0 \\ 0.27 & 0.05 & 0.07 & 0.1 \end{pmatrix}$$

Find the production matrix if all the entries in the demand matrix are 200.

Solution

Let D = demand matrix (consumer demand), T = technology matrix (linking input and output), and P = production matrix (how much is produced).

Hence TP = internal consumption matrix (how much output is consumed by system).

Production must satisfy both consumer demand D and the system's internal needs TP.

$$\begin{aligned} D &= P - TP \\ &= (I-T)P \\ (I-T)^{-1}D &= (I-T)^{-1}(I-T)P \\ (I-T)^{-1}D &= IP \\ (I-T)^{-1}D &= P \end{aligned}$$

$$D=\begin{pmatrix} 200 \\ 200 \\ 200 \\ 200 \end{pmatrix} \qquad P=(I-T)^{-1}D=\begin{pmatrix} 579.25 \\ 572.31 \\ 476.21 \\ 464.83 \end{pmatrix}$$

Figure 4. Leontief matrix problem and solution.

The following lesson excerpt illustrates how three students, Nerida, Helen, and Edward, interacted with each other and with their calculators as they tackled the Leontief problem.

Nerida: Yeah, it's the identity of ... the inverse of the *I* take away *T* is supposed to be a four by four, and the four by four times the one column one ... the answer's got to be four rows, one column.
Helen: I got four columns, one row! (holds up her calculator for Nerida to see) Look, I know I got that. Is that right?
Nerida: (inspects Helen's calculator screen) I haven't done it like that.
Helen: What did you get, Edward?
Edward: (still pressing buttons) Ah, just give me a minute.

Nerida faltered for a moment in describing the inverse of the identity matrix, but recovered to reason out that the answer must be a matrix with four rows and one column.

Nerida: (to herself) Row, column. (balances calculator on her head as she thinks)
Helen: (to Nerida, looking at her calculator screen) How did you get that? (no reply, issues general plea to whole class) Has anyone done it?
Nerida: Yeah, in about two seconds. (to herself) Give it a name. What was the other one called? Three by one is 200, 200, 200 (entering elements of demand matrix) Okay!
Edward: (to Nerida) All righty, what have you got?
Nerida: Hang on, I got it! (verbalises keystrokes) D times 'Kan' (label for her matrix) (groans and lowers head to desk)
Helen: What happened?
Nerida: (reading dejectedly from calculator screen) "Dimension problem"!
Helen: Did you go 200 down that way or across?
Nerida: Down.
Helen: I've got to check that.

In this exchange, Nerida interacted with the calculator almost as with a human partner, verbalising the menu choices and keystrokes and responding with despair when the calculator returned an error message.

Helen: Edward ... (he is not listening, talking to another student) Edward! (Helen taps his arm) What did you get?
Edward: (turns back to Helen) This! (indicating calculator screen) I wrote all that (indicating pen and paper notes) to get that!
Helen: (inspects Edward's screen carefully, compares with her own) Oh my God, oh my God! I got it right!
Edward: (grinning at Helen's excitement) Happy now?
Helen: (jumping up and down) Yes, very happy!
Edward: Good!

Note the intense emotional response engendered by Helen's involvement with the task and the technology—apparent in her surprise and delight as she "got it right!"

Helen's attention then turned to her friend Nerida, who was still unable to recognise her error in keying only three entries for the demand matrix instead of four.

Nerida: (still trying to identify the source of her error) Maybe my inverse is wrong.

Edward: (to Nerida, wanting to help) So what did you get? What did you get?
Helen: (to Nerida) What did you get for your inverse?
Nerida: (dejected) It tells me there's a dimension error, and I don't know why.
Edward: Did you get that? (passing his calculator to Nerida so she can look at his working)
Helen: (also showing her calculator working to Nerida) It should be that.
Nerida: (comparing her working with the other two screens) That's what I had!
Helen: So then you ...
Nerida: (puzzled, comparing screens with Edward) Is that what you have? It's exactly the same as mine!
Helen: Yeah, and you times that by 200, 200, 200 *down* (referring to demand matrix)
Nerida: (sudden insight) Oh hang on ... That should be *four* ... Oh God!
Helen: What did you do?
Nerida: I didn't do four 200s!
Helen: Oh you big dork! (Nerida and Helen laugh) You've only got three 200s! (referring to number of entries in demand matrix—there should be four rows, not three)
Nerida: (chastened) God I'm a moron! (talking to her calculator as she presses buttons) Second, quit. Now ... (re-does the calculation. Asks Helen) Did you get that?
Helen: (inspects Nerida's screen) Yeah! (Nerida jumps up from her seat in delight)
Edward: (to Nerida) Look at mine.
Nerida: (goes over to Edward) Did you get this? (Edward holds both calculators up side by side, compares his screen with Nerida's. Nerida pulls his hair when he deliberately hesitates in replying.)
Edward: (with cheeky grin) Yes!
Nerida: Thank you!

In this lesson, the intended role for technology is that of a *servant*; for example, in the context of learning about matrices, one student commented that technology can "speed up the process of messy calculations and help you concentrate on the whole problem". However, the segment primarily illustrates how technology can become a surrogate *partner* as students verbalise their thinking in the process of locating and correcting an error. Even the most faithful transcript cannot do justice to the students' actions, gestures, tone of voice, and facial expressions in situations such as this. They clustered around their calculators, holding them up side by side to compare the working on the screens, sometimes passing them back and forth to emphasise points as they spoke. Nerida even imagined her calculator spoke to her: "It tells me there's a dimension error". Not only did the calculator output provide a stimulus for peer discussion, but the students also invested the technology with human qualities—an interesting counterpoint to the claim of Shneiderman et al. (1998) that educational use of technology can strengthen human relationships but never replace them.

Extending the mathematical self. A final example demonstrates the level of sophistication with which students can integrate a variety of technological resources into the construction of a mathematical argument. The task, which came from a unit of work designed to introduce students to iteration as one of the central ideas in chaos theory, required them to use iterative methods to find the approximate roots of the equation $x^3 - 8x - 8 = 0$ (see Goos, 1998, for a fuller description of this task). The equation is expressed in the form $x = F(x)$, and a first approximation to the solution is obtained by estimating the point of intersection of the curves $y = x$ and $y = F(x)$. The solution is conveniently done using a two-column spreadsheet, with one column containing the input x-values and the second column containing the outputs $F(x)$—with each $F(x)$-value becoming the input for the subsequent iteration. Figure 5 shows the calculation when $F(x) = x^3/8 - 1$. Cell B4 contains the formula =(1/8)*((A4)^3)-1and cell A5 contains =B4, both these formula then being copied down into the other cells in these columns.

	A	B	C	D	E
1	x^3-8x-8=0 rearranged as x=x^3/8-1				
2					
3	x	F(x)			
4	-1.5	-1.421875			
5	-1.421875	-1.35933065			
6	-1.35933065	-1.31396797			
7	-1.31396797	-1.28357265			
8	-1.28357265	-1.26434517			
9	-1.26434517	-1.25264282			
10	-1.25264282	-1.24569243			
11	-1.24569243	-1.24162534			
12	-1.24162535	-1.23926640			

Figure 5. Spreadsheet method for solving equation by iteration.

Depending on the way in which the original equation is rearranged and the initial value chosen, the iteration may converge on a solution or generate increasingly large outputs and hence no solution. After some trial and error, Steve's students were systematic in testing initial values and persistent in searching for rearrangements that would yield all three roots (-2, -1.236, and 3.236). Table 2 shows some of the formulae and initial values which they investigated. Note that the spreadsheet program they were using, Claris Works 2.0, could not evaluate the cube root of negative numbers. The students found a way around this problem using the absolute value function.

Goos (1998) found that students attempting this task quickly discovered that they could create an alternative, graphical, representation of the problem using graphing software. Plotting the graphs of $y = x$ and $y = F(x)$ enabled students to make a realistic first approximation to the roots of the equation. However, the students participating in the present study chose to use their calculators to reproduce the graphs initially plotted on the computer, as this enabled them to view the spreadsheet (computer screen) and graph (calculator screen) simultaneously.

Table 2
Some Formulae and Initial Values Investigated in Finding the Roots of $x^3 - 8x - 8 = 0$

Initial value	Formula (algebraic and spreadsheet forms)		
	$x^3/8 - 1$ (1/8)*((A2)^3)-1	$(8x + 8)^{1/3}$ (8*A2+8)^(1/3)	$-\lvert 8x + 8 \rvert^{1/3}$ – ABS(8*A2+8)^(1/3)
$x < -2$	no solution	not defined	–2
$-2 < x < -1.236$	–1.236	not defined	–2
$-1.236 < x < -1$	–1.236	not defined	–2
$-1 < x < 3.236$	–1.236	3.236	not valid
$x > 3.236$	no solution	3.236	not valid

This is a challenging task, and students rarely find all three roots without some prompts from the teacher. When one group of students in Steve's class did so, the teacher made a spur of the moment decision to ask them to present their solution to the whole class via the laptop computer and data projector. With no time for rehearsal, the students shared the tasks of operating the computer keyboard, data projector remote control (which permits scrolling and zooming independently of the computer) and laser pen, while coordinating their explanations (supplemented with calculator output) and answering questions from their peers. Mathematical and communications technologies were smoothly incorporated into their unfolding argument, and were used to link different representations of the equation-solving task and to clarify and elaborate on points raised by fellow students and the teacher.

Conclusion

In their review of research on the use of graphics calculators in mathematics education, Penglase and Arnold (1996, p. 85) concluded that "approaches to teaching and learning which emphasise problem solving and exploration, and within which students actively construct and negotiate meaning for the mathematics they encounter, find in this new technology a natural and mathematically powerful partner". While the findings presented here suggest that this is the case in some of the classrooms participating in our research program, the relationship between technology usage and teaching/learning environments is not one of simple cause and effect. The metaphors of *master, servant, partner,* and *extension of self* are intended to capture some of the diversity of teacher and student interactions in technology-rich classrooms.

It seems natural for teachers to use new technologies such as graphics calculators and overhead projection panels in ways that are consistent with preferred teaching methods. However, teaching with technology need not—and perhaps should not—simply be a matter of grafting this tool onto existing pedagogical practices. For example, likening the graphics calculator to a portable computer, or the overhead projection panel to an electronic blackboard, obscures important qualitative differences between old and new technologies and may limit the scope of what teachers and students are able to achieve in the classroom.

In contrast, research that seeks out emergent (i.e., unplanned, unanticipated) uses of technology reveals that the calculator and projection panel are not passive or neutral objects, since these technologies are actively re-shaping human interactions and interactions between humans and the technology itself. For example, even though the graphics calculator is designed as a personal mathematical tool it can facilitate social interaction and sharing of knowledge. This personalisation of technology is evident in the way that the graphics calculator is incorporated almost as a human partner into face-to-face discussions between students. A more public form of interaction occurs when students are invited to present their calculator work to the whole class via an overhead projection panel. When control of the discussion is handed over to students, the panel is no longer simply a presentation device—instead, it becomes a discourse tool that mediates interaction between students at a whole class level. This is a class-wide form of collaborative inquiry that is facilitated by the public display and interrogation of mathematical ideas.

Introducing new mathematical and communication technologies into classrooms can change the ways that knowledge is produced. Implicit in these changes are a number of challenges for teachers, the most obvious of which involves becoming familiar with the technology itself. While this aim will remain a significant professional development focus (as Penglase & Arnold, 1996, point out), more attention needs to be directed to the inherent mathematical and pedagogical challenges in technology-enhanced classrooms if the goal of a problem-solving and investigative learning environment is to be realised. For example, placing graphics calculators in the hands of students gives them the power and freedom to explore mathematical territory that may be unfamiliar to the teacher; and for many teachers, this challenge to their mathematical expertise and authority is something to be avoided rather than embraced. Perhaps the most significant challenge for teachers lies in orchestrating collaborative inquiry to share control of the technology with students. The research reported in this paper has begun to consider such emergent uses of technology in re-shaping social interaction patterns in mathematics classrooms.

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Authors

Merrilyn Goos, Graduate School of Education, The University of Queensland, QLD 4072. E-mail: <m.goos@mailbox.uq.edu.au>.

Peter Galbraith, Graduate School of Education, The University of Queensland, QLD 4072. E-mail: <p.galbraith@mailbox.uq.edu.au>.

Peter Renshaw, Graduate School of Education, The University of Queensland, QLD 4072. E-mail: <p.renshaw@mailbox.uq.edu.au>.

Vince Geiger, Hillbrook Anglican School, Enoggera QLD 4051. E-mail: <vincent@gil.com.au>.