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Improving detectors using entangling quantum copiers

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We present a detection scheme that, using imperfect detectors, and imperfect quantum copying machines (which entangle the copies), allows one to extract more information from an incoming signal than with the imperfect detectors alone.

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Copying machines in general use two approaches. One of the extreme cases is a classical copying machine, where measurements (destructive or nondestructive) are made on the original state, the results of which are then fed as parameters into some state preparation scheme that attempts to construct a copy of the original. This approach obviously allows one to generate an arbitrary amount of copies, possibly all identical to each other. The opposite extreme is a fully quantum copying machine, which by some process that is unseen by external observers, creates a fixed number of copies, usually destroying the original in the process. Naturally in a realistic situation, noise will additionally degrade the quality of the copies, and copiers that utilize both of the processes above are obviously also possible.

Ignoring for now the matter of the inevitable noise, the exact state of the original can only be determined with certainty by some measurement if all the possible states of the original are mutually orthogonal. In all other situations, any classical copying machine must have a finite probability of producing imperfect copies. In fact, by the well-known no-cloning theorem [1,2] the same can be said of quantum copying machines. If the possible states of the original are not mutually orthogonal, there is no quantum copier that will always make perfect copies. So one might ask what good *are* quantum copiers, then? Well, the obvious answer is that for the situation where the possible originals are not orthogonal, often quantum copiers can create better copies than classical ones. Some examples are the UQCM for unknown qubits [3], or other copiers for two non-orthogonal qubits [4].

While this promises the possibility of many applications of quantum copying in the future, few specific examples of uses for a quantum copier have been considered so far. When discussing practical applications, quantum copiers have mainly been put forward as something to be defended against by quantum cryptography schemes. This article presents a analysis of a possible application of quantum copiers: using them to improve detection efficiencies.

We first note that in practice one always has restricted detector resources. In particular, this article treats the situation where the best available detectors have some efficiency less than 1. As an example system, consider the case where one of a set of possible input states is to be distinguished by a measurement scheme, using (some number of identical) imperfect detectors. One also has some (identical) quantum copiers that can act on the possible input states. At first, let us suppose that the possible input states are mutually orthogonal, and that one has somehow acquired perfect quantum copiers for this set of states. Assume the copiers destroy the original, and produce two copies for simplicity. Then, an obvious way to take advantage of the copiers is to send the originals through a quantum copier, before trying to detect both copies separately (depicted in Fig. 1). This basically gives one a second chance to distinguish the input state, if the detection at the first copy fails.

Consider a very simplified model of photodetection using this measurement scheme. Suppose one has perfect copiers, and noiseless photodetectors of efficiency η. That is, the probability of a count on the detector is η if a photon is incident, and 0 otherwise. With the copier set up as in Fig. 1, if any of the detectors register a count, one can with certainty conclude that a photon was incident. So, if a photon *is* incident, the probability of finding it is

$$P^{(1)}_{\mathrm{count|photon}}=\eta+(1-\eta)\eta \tag{1}$$

as opposed to just η with no copier, because one gets a "second chance" at detection. On the other hand, if no count is registered, then the probability that no photon was incident is

$$P^{(1)}_{\mathrm{nophoton|nocount}}=\frac{1-p}{1-\eta p(2-\eta)}, \tag{2}$$

where p is the probability that a photon is incident on average, irrespective of the measurement result. The expression of Eq. (2) is always greater than $1-p/1-\eta p$, which is the

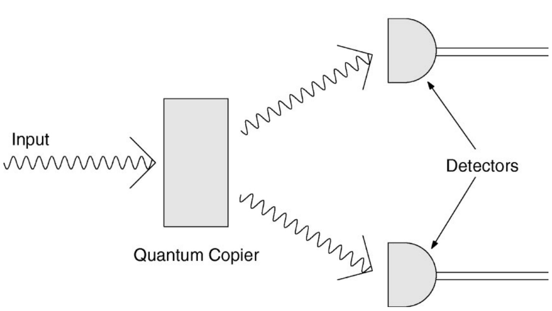

FIG. 1. Basic detection scheme using imperfect detectors, and a quantum copier.

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probability if no copier is used. This increase reflects the added confidence that comes from both detectors failing to register the photon.

We note that using quantum copiers and not classical ones is vital. A classical copier would have to rely on the same imperfect photodetectors, and would actually *reduce* the detection efficiency, since to detect a photon at one of the two copy detectors, one must have been first detected at the copier. This gives $P^{(1)}_{\mathrm{count|photon}} = \eta^2(2-\eta)$, which is always less than or equal to η, a result achieved without any copiers at all.

Detection with the help of perfect quantum copiers, as briefly discussed above, is all very well, but what happens when the equipment used is noisy, and not 100% efficient? Consider the following, more realistic, model of photodetection. The possible states that are to be distinguished are the vacuum $|0\rangle$ and single photon $|1\rangle$ states. The *a priori* probability that the input state is a photon is p. A generalized measurement on some state $\hat{\rho}$ can be modeled by a positive operator-valued measure (POVM) $\{\hat{A}_i\}$ [5,6] described by a set of n positive operators $\hat{A}_i$, such that $\Sigma_{i=1}^n \hat{A}_i = \hat{I}$, where $\hat{I}$ is the identity matrix in the Hilbert space of $\hat{\rho}$ (and of the $\hat{A}_i$). The probability of obtaining the ith result, by measuring on a state $\hat{\rho}$, is then

$$P_i = \mathrm{Tr}[\hat{\rho}\hat{A}_i]. \tag{3}$$

Now suppose the photodetectors at one's disposal are noisy and have quantum efficiency η. The effect of these can be modeled by the POVM

$$\hat{A}_+ = \eta|1\rangle\langle 1| + \eta\xi|0\rangle\langle 0|, \tag{4a}$$

$$\hat{A}_- = (1-\eta)|1\rangle\langle 1| + (1-\eta\xi)|0\rangle\langle 0|, \tag{4b}$$

where the operator $\hat{A}_+$ represents a count, and the operator $\hat{A}_-$ the lack of one. The parameter $\xi\in[0,1)$ controls the amount of noise. That is, $\xi\eta$ is the probability that the photodetector registers a spurious (''dark'') count when no photon is incident.

We will model the quantum copier as one that has a probability ε of working correctly and producing perfect copies. Otherwise, the parameter $\mu\in[-1,1]$ determines (in a somewhat arbitrary way) what is produced. This can be written

$$\hat{\rho}_1 = |1\rangle|d\rangle\langle 1|\langle d| \to \varepsilon|1\rangle|1\rangle\langle 1|\langle 1| + (1-\varepsilon)\hat{\rho}_N = \hat{\rho}_1^1, \tag{5a}$$

$$\hat{\rho}_0 = |0\rangle|d\rangle\langle 0|\langle d| \to \varepsilon|0\rangle|0\rangle\langle 0|\langle 0| + (1-\varepsilon)\hat{\rho}_N = \hat{\rho}_0^1, \tag{5b}$$

where $|d\rangle$ is a dummy state, which is fed into the copier, and becomes the second copy. It is included here to preserve unitarity in the perfect copying case $\varepsilon=1$. The state produced upon failure of the copier, $\hat{\rho}_N$ is independent of the original, and is given by

$$\hat{\rho}_N = (1-|\mu|)\frac{\hat{I}}{4} + \begin{cases} \mu, & |1\rangle|1\rangle\langle 1|\langle 1| \quad \text{if } \mu>0, \\ |\mu|, & |0\rangle|0\rangle\langle 0|\langle 0| \quad \text{if } \mu\leq 0. \end{cases} \tag{6}$$

Here, $\frac{1}{4}\hat{I}$ is the totally random mixed state. So, for $\mu=0$ a totally random noise state is produced upon failure to copy, for $\mu=-1$ vacuum, for $\mu=1$ photons in both copies, and for intermediate values of μ a linear combination of the three cases mentioned.

This model [Eq. (5)] of the copier is an extension (to allow for inefficiencies) of the Wootters-Zurek copier, which has been extensively studied [1,3]. In the ideal case ($\varepsilon=1$), with the dummy input state in the vacuum ($|d\rangle=|0\rangle$), the transformation is

$$|0\rangle|0\rangle\to|0\rangle|0\rangle, \qquad |1\rangle|0\rangle\to|1\rangle|1\rangle. \tag{7}$$

This transformation can be implemented by the simplest of all quantum logic circuits, the single controlled-not gate. These have recently begun to be implemented for some systems (although admittedly not for single-photon systems), and are the subject of intense ongoing research, because of their application to quantum computing. This means that similar schemes to the one considered here may become experimentally realizable in the foreseeable future. We also point out that the transformation (7) can be also considered an ''entangler'' rather than a copier. Consider its effect on the photon-vacuum superposition state

$$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\to\frac{1}{\sqrt{2}}(|0\rangle|0\rangle+|1\rangle|1\rangle). \tag{8}$$

This correlation between the copies is an essential property for the detection scheme presented here to be useful—otherwise one could not combine the results of the different detector measurements to better infer properties of the original. We will now examine how we determine whether the copying scheme we are proposing is more efficient.

Let us now consider the total amount of information about the input state that is contained in the measurement results. This is the (Shannon) mutual information I_m per input state between some observer A who knows with certainty what the original states are (perhaps because they were prepared by that observer), and another observer B who has access to the measurement results of the detection scheme. This can be readily evaluated from the expression [7–9]

$$I_m = \sum_{i,j} P_{j|i}P_i \log_2\frac{P_{j|i}}{P_j} \tag{9}$$

where i ranges over the number of possible input states, and j over the number of possible detection results. P_i are the *a priori* probabilities that the ith input state entered the detection scheme, $P_{j|i}$ is the probability that the jth the detection result was obtained given that the ith state was input, and P_j is the marginal probability that the jth detection result was obtained overall.

This mutual information has very concrete meaning even though, in general, B can never be actually certain what any particular input state was. It is known that by using appropriate block-coding and error-correction schemes, A can transmit to B an amount of *certain* information that can come arbitrarily close to the upper limit I_m imposed by the detection probabilities. In other words, I_m is the maximum amount of information that A and B can share using a given detection scheme, if they are cunning enough. It follows then that the detection scheme that gives a greater information content about the initial state I_m, will be the potentially more useful one. The authors have actually shown that the Wootters-Zurek copier is the optimal quantum broadcaster of information when the information is decoded one symbol at a time [10], and this will be discussed in a future paper.

From expression (9) it can be seen that I_m depends on the *a priori* input probabilities (the parameter p in the cases considered here). This leads one to surmise that (at least in general) various detection schemes may do relatively better or worse depending on how frequently the input is a photon. This is in fact found to be the case. However, in what follows, we will concentrate mainly on the $p=1/2$ case of equiprobable photons and vacuum, since this is the situation that allows the maximum amount of information to be encoded in the original message, and so is in some ways the most basic case.

If the new detection scheme gives mutual information content $I_m(\varepsilon,\eta,\mu,\xi,N,p)$ per input state, then $\eta^e(I_m(\varepsilon,\eta,\mu,\xi,N,p))$ is defined as the efficiency of a noiseless detector that would give the same mutual information content if it were used by itself in the basic scheme with no copiers, i.e.,

$$I_m(\cdot,\eta^e,\cdot,0,0,p)=I_m(\varepsilon,\eta,\mu,\xi,N,p). \tag{10}$$

η^e is a one-to-one, monotonically increasing function of I_m, and so if (and only if) some detection scheme increases η^e, it also increases the mutual information; thus η^e and I_m are equivalent for ranking detection schemes in terms of effectiveness. η^e also has the advantage that for some cases of the new copier-enhanced detection scheme it is independent of the photon input probability p.

Now it is time to ask the question: For what parameter values does the copier-enhanced detection scheme provide more information about the initial states than using a single detector? Consider first the simplest case of interest, where there are no spurious (dark) counts in the photodetectors $(\xi=0)$, and one has a copier of efficiency ε that produces vacuum upon failure $(\mu=-1)$. This will give some idea about the relationship between the detector and copier efficiencies required, leaving the effects of noise for later consideration.

As mentioned previously, in this situation the effective efficiency is independent of p, and with one layer of copiers $(N=1)$ it is found to be given by the simple expression

$$\eta^e_{(1)}=\varepsilon[1-(1-\eta)^2]. \tag{11}$$

Since this is independent of p, introducing a second lot of copiers is equivalent to replacing η in the above expression by $\eta^e_{(1)}$, i.e., $\eta^e_{(n+1)}=\varepsilon[1-(1-\eta^e_{(n)})^2]$. In fact, in the limit of never-ending amounts of copiers, the effective efficiency approaches

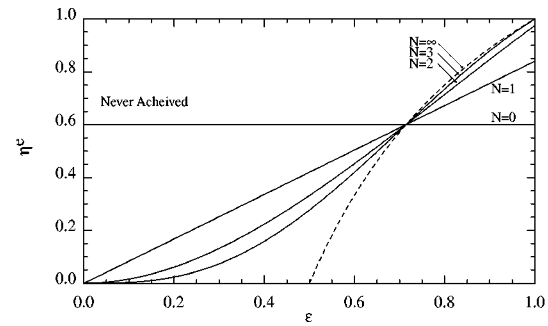

FIG. 2. Equivalent efficiency η^e as a function of copier efficiency ε and number of levels of copiers N when detector efficiency is $\eta=0.6$, and both detectors and copiers are noiseless $(\xi=0,\mu=-1)$. Results for $N=0$ to $N=3$ are shown as solid lines, and the limit of what can be achieved is shown as a dashed line. Regions beyond the $N=0$ and $N\rightarrow\infty$ cases are not achievable with noiseless copiers. The results are independent of the photon input probability p, in this case.

$$\lim_{N\rightarrow\infty}\eta^e=2-\frac{1}{\varepsilon}. \tag{12}$$

One finds that effective efficiency is improved (over $\eta^e=\eta$) by the copier scheme whenever

$$\varepsilon>\frac{1}{2-\eta}. \tag{13}$$

Since no random noise is introduced by either copier or detector, improvement is achieved whenever more copiers are added, to arbitrary order N. A few things of interest to note:

(i) The copier efficiency required is always above η and above 1/2.

(ii) A gain in efficiency can be achieved even with quite poor copiers—for relatively small detector efficiencies η (which occur for photodetection in practice), the copier efficiency required is only slightly above half

(iii) For very good detectors, to get improvement, the copier efficiency ε has to be slightly larger than the detector efficiency η.

(iv) For low efficiencies, the relative gain in efficiency can be very high, and can reach approximately 2^N for very poor detectors and very good copiers.

To examine how much improvement can be achieved in more detail, consider when the efficiency of the detectors is $\eta=0.6$. This is a typical efficiency for a pretty good single-photon detector at present. This is shown by the solid lines in Fig. 2. Note how quite large efficiency gains are achievable even when the copier efficiency is slightly over the threshold useful value of $\epsilon=0.714$ [from Eq. (13)], and how adding more copiers easily introduces more gains at first, but after three levels of copiers, adding more becomes a lot of effort for not much gain.

To conclude it can be seen that when one is restricted to using imperfect detectors (as is always the case), more efficiency of detection can be gained by employing entangling quantum copiers such as a controlled-not gate. In fact if the efficiency of the detectors is far from 100% (such as in single-photon detection) the copier does not have to be very efficient itself, and significant gains in detection can still be made. We note that although a detailed analysis was carried out for the case of single-photon detection, the basic scheme can be readily generalized to other types of detectors.

From Eq. (13), it can be seen that to be useful, the quantum copiers must be successful with an efficiency ε over 50% and somewhat greater than the detector efficiency η. It is not generally clear how feasible this is for various physical systems or measurement schemes that one might wish to employ. With current technology it is often still easier to make measurements on a system, rather than entangling it with other known systems; however, this varies from measurement to measurement and from system to system. The physical processes involved in measurement and quantum copying are often quite different: the former requires creating a correlation between a quantum system and a macroscopic pointer, whereas the latter involves creating quantum entanglement between two similar microscopic states. Efficient detection depends on correlating the system with its environment in a strong, yet controlled way, whereas quantum copying depends on isolating the system from its environment. One thus supposes that the usefulness of a scheme such as the one outlined here will depend on the system and measurements in question, due to the relative ease of implementing detection and controlled quantum evolution in those systems.

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