[2] J. Hadamard, *The Psychology of Invention in the Mathematical Field*, Princeton University Press, Princeton, NJ, 1945; reprinted by Dover, New York, 1954.

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Numerical Analysis of Systems of Ordinary and Stochastic Differential Equations. *S. S. Artemiev and T. A. Averina.* VSP BV, Utrecht, the Netherlands, 1997. $122.50. vii+176 pp., hardcover. ISBN 90-6764-250-9.

Recently, there has been very considerable interest in developing effective numerical methods for the solution of stochastic ordinary differential equations. However, many of the books in this area have clearly been written for those working in stochastics and probability theory and not for numerical analysts. Since the language used in these two fields is quite different, numerical analysts have been at a disadvantage. While this book is not aimed at filling this gaping void it is nevertheless clearly written and concise and written in such a way as to be readily understandable by researchers working in the area of computational science and university students in their fourth year of study.

The monograph is divided into just two chapters (which I believe to be too concise a division), with the first chapter focusing on the deterministic case and the second chapter focusing on stochastic ordinary differential equations. The first chapter represents approximately one-quarter of the material of the book and is not a strength of this book. This first chapter discusses stiffness in some detail and focuses in the main on Rosenbrock methods and their order and stability properties. There is no attempt to place these methods in a general framework or even to introduce in a general way the classes of Runge–Kutta and linear multistep methods. Furthermore, the study of the order conditions is done in a haphazard way, with no attempt to do a general study in terms of B-series and rooted trees. Some numerical comparisons are done with packages such as GEAR, EPISODE, and LSODE. The feel of this chapter is that the content is between 10 and 20 years out-of-date, and there is really little that is interesting here. This is emphasized by the set of references for this chapter.

Chapter 2, on the other hand, is much better. The material is up-to-date, interesting, and reasonably well presented in 15 sections (which is too many sections for a single chapter). The focus is again on the development of a family of Rosenbrock methods, but this time for stochastic differential equations (SDEs) which may or may not be stiff.

The first three sections give background theory in probability, statistics, stochastics and SDEs, and Fokker–Planck theory. The strength of these sections is that the reader is not overburdened with highly technical mathematics; rather, the writers concentrate on concepts. Sections 4 and 5 focus on linear systems, problems with either additive or multiplicative noise, and stability analyses for the problem. The authors define stiffness for linear SDEs, but there is no real treatment of the nonlinear case.

Sections 6, 7, and 8 focus on the convergence behavior of the Euler–Maruyama method and certain classes of implicit Rosenbrock methods. There is a very good discussion of why the Euler–Maruyama method converges to the Itô solution of an SDE and not the Stratonovich solution. A simple form of the Taylor series expansion for SDEs is given, but as in the deterministic case, no general formalism is given. Furthermore, the treatment of order is mainly given for the one-Wiener process case, and the implicit methods constructed are implicit only in the drift. There is a good discussion on the simulation of the stochastic integrals needed in the higher order methods.

Sections 9 and 10 discuss and analyze the mean square stability properties of the Rosenbrock methods introduced in this monograph, and while this is insightful, there is very little on the multi-Wiener process case.

The final five sections focus on implementational issues. In section 11 a variable step implementation of a stochastic Rosenbrock method is given and some numerical implementations are carried out. This section is worrying, as there is no attempt to maintain the same Wiener path between different simulations. Thus it is virtually impossible in this approach to compare the behavior of different simulations. Variable step implementations for SDEs are, I believe, much more difficult than is suggested here. The remaining sections focus on a variety of interesting problems such as automated control, Kalman–Bucy filters, motion of an aircraft in a turbulent atmosphere, a doubly connected van der Pol oscillator, and a stock price model.

In summary, I would recommend this book to numerical analysts as an introduction to the numerical solution of SDEs. There are a number of issues that are not discussed in full depth, especially in the multi-Wiener process case, but the focus on concepts rather than obscure mathematical details is significant. There is, however, little of real interest in the deterministic segment of this book. In addition, the list of only 109 references is overly brief, and nearly a quarter of these are by the authors themselves. For a small monograph of 176 pages, the price of $122.50 seems very high, and the book would certainly benefit from having an index—there is none.

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Stability by Liapunov's Matrix Function Method with Applications. *By A. A. Martynyuk.* Marcel Dekker, New York, 1998. $150.00. x+276 pp., hardcover. Pure and Applied Mathematics. Vol. 214. ISBN 0-8247-0191-7.

In an 1892 paper (an English translation may be found in [8]), the Russian mathematician Liapunov introduced a novel approach to stability analysis. This approach consists of finding a scalar function, now called the Liapunov function, which depends on the system variables and satisfies certain preset conditions. Liapunov showed that this function can be used to deduce the stability properties of solutions to the equations governing the system. A simple example is the energy in a dissipative system, where the fact that the energy must decrease over time can be used to show that certain equilibrium solutions of the system must be stable.

Since its introduction, many monographs [1, 2, 4, 5, 9, 10] have been written describing the mathematical theory behind the Liapunov function method of stability analysis and examining the difficulties of constructing Liapunov functions for specific systems. Recently, researchers have explored ways of extending this method to larger classes of systems and stability problems. Extensions include using more general ways of measuring distance [6] and defining more general Liapunov functions [7]. This book describes an extension of this latter type, where one uses a matrix-valued function of the system variables instead of the usual scalar function. The goal of this approach is to separate the variables into groups, isolating those on which the strongest conditions are needed, so that conditions on other variables may be weakened.

This book is quite self-contained, including an introductory chapter which gives an overview of stability theory and the Liapunov function method of stability analysis. This overview uses excerpts from and commentary on Liapunov's own work, which I found particularly interesting. (Similar material is found in another book coauthored by Martynyuk [3].) The chapter also provides a fairly extensive review of the literature and a summary of recent developments. The second chapter contains the core material of the book, giving a complete mathematical development of the matrix Liapunov function method of stability analysis. Chapters 3 and 4 extend this method to singularly perturbed and stochastic systems, respectively. These chapters follow a similar format, beginning with a solid introduction to the class of systems to be studied before continuing to build on the results of Chapter 2, and finishing with example applications, some motivated by real physical systems. The final chapter contains examples, taken from engineering and biological applications, where the stability results using a standard Liapunov function can be improved or extended by the use of a matrix Liapunov function.

This could be a useful book for two audiences. For those interested in the theory of Liapunov functions for its own sake, the