Violations of Bell inequalities for measurements with macroscopic uncertainties: What it means to violate macroscopic local realism

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We suggest a method to test the premise of ''macroscopic local realism'' that is sufficient to derive Bell inequalities when measurements of photon numbers are only accurate to an uncertainty of order n photons, where n is macroscopic. Macroscopic local realism is only sufficient to imply, in the context of the original Einstein-Podolsky-Rosen argument, fuzzy ''elements of reality'' that have a macroscopic indeterminacy. We show therefore how the violation of local realism in the presence of macroscopic uncertainties implies the failure of macroscopic local realism. Quantum states violating this macroscopic local realism are presented.

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I. INTRODUCTION

There is increasing evidence for the failure of ''local realism'' as defined originally by Einstein, Podolsky, and Rosen [1], Bohm [2], and Bell [3,4]. For certain correlated quantum systems, Einstein, Podolsky, and Rosen argued that local realism is sufficient to imply that the results of measurements are predetermined. These predetermined ''hidden variables'' (called ''elements of reality'' by Einstein, Podolsky, and Rosen) exist to describe the value of a physical quantity, whether or not the measurement is performed, and as such are not part of a quantum description. Bell later showed that the predictions of quantum mechanics for certain ideal quantum states could not be compatible with such hidden variable theories.

It is now widely accepted therefore, as a result of Bell's theorem and related experiments [5], that local realism must be rejected. However, the rejection of local realism implied by these results is at the most microscopic level of a single photon, in the sense that the hidden variables (or ''elements of reality'') and the experimental results of measurements involved must be defined to the precision of one photon or better in order to prove local realism invalid. The failure of local realism in microscopic systems has long been associated with the existence of entangled quantum superposition states. In microscopic systems there can only be superpositions of states microscopically distinct.

Little is known about the validity of local realism in more ''macroscopic experiments,'' where experimental uncertainties are larger (becoming macroscopic) in size, in an absolute sense. At least there has to our knowledge been no formulation of a Bell-type theorem or a related experimental investigation for such a situation. Previous works [6–8] suggestive of incompatibilities of local realism in macroscopic systems have considered the case where the measurements are performed with perfect accuracy and are thus not examples of macroscopic experiments as we have defined them here.

Generally, it is thought that true macroscopic quantum effects come about from the quantum superpositions of states that are macroscopically distinguishable [9,10], often referred to as ''Schrödinger cat'' states. This point has been much discussed. Leggett and Garg [10] have shown the incompatibility of such macroscopic quantum states with the combined premises of ''macroscopic realism'' and ''macroscopic noninvasive measurability.'' They, however, considered macroscopic quantum superposition states at a single location only, and did not introduce the premise of locality.

There has been much interest and debate over whether or not ''Schrödinger cat'' states can truly exist. The existence, for which there is now experimental evidence [11–13], of ''Schrödinger cat'' states would appear to be closely linked to the question of the validity of local realism at the more macroscopic level we have described. One would suspect that a violation of local realism, which is evident in an experiment where uncertainties are large, would be due more to entangled macroscopic superpositions than microsuperpositions.

In this paper, we begin (in Sec. II) by defining the physical premise of ''macroscopic local realism'' [14] so as to identify the peculiar features of the macroscopically entangled quantum states in a way that is independent of the quantum formulation. Macroscopic local realism is only sufficient to assign, to a system, predetermined elements of reality (or hidden variables) that are intrinsically macroscopic, in that they have a macroscopic indeterminacy in their values.

Suppose our ''Schrödinger's cat'' is correlated with a second system spatially separated from the cat, for example, a gun used to kill the cat. Let us suppose a gun that has been fired implies a dead cat; a gun that has not been fired implies a cat that is alive. We can predict the result for a measurement of the cat (whether dead or alive), without disturbing the cat, by a measurement of the gun. Macroscopic local realism is the premise used to imply the existence of an element of reality for the cat. The element of reality in this case is a variable that assumes one of two values: one value corresponding to the ''dead'' state and the other value corresponding to the ''alive'' state. The assignment of this element of reality then means that the cat is always either ''dead'' or ''alive,'' regardless of whether or not it is being observed or measured. Macroscopic local realism is used in this case because the two possible results of measurement of the cat, ''alive'' and ''dead,'' are macroscopically distinct. To summarize, the rejection of macroscopic local realism in this example means that we cannot think of the cat as being either dead or alive, even though we can predict the ''dead'' or ''alive'' result of ''measuring'' the cat, without disturbing the cat, by measuring the correlated spatially separated second system, which in this case is the gun. The rejection of macroscopic local realism is a more startling result than, and is not implied by, the rejection of local realism as indicated by Bell's theorem.

In Sec. III we point out that for situations where the possible results are all macroscopically distinguishable, we need only to assume the strong premise of macroscopic local realism in order to derive the Bell inequalities. We then focus attention on the more general case where the results of measurement may be microscopically separated. We show that with the addition of macroscopic classical noise sources that model a macroscopically imprecise measurement, one may derive the Bell inequalities using only the premise of macroscopic local realism. Thus if a violation of a Bell inequality is maintained in the presence of macroscopic uncertainties in the measurement process, we have direct evidence for an incompatibility with macroscopic local realism.

In Sec. IV, quantum states are presented that show a violation of Bell's inequality with such macroscopic noise, thus indicating an incompatibility of the predictions of quantum mechanics with the very strict form of macroscopic local realism we have defined. We believe this is a very different result, although some preliminary results presented in this paper have been published previously [15]. Such a test of macroscopic local realism provides an avenue to focussing on the peculiar macroscopic nonlocal aspects of the ''macroscopic entangled quantum state.''

The application of Bell inequality theorems, and the effect of noise on the violations predicted, to situations where many photons fall on a detector is relevant to the question of whether or not tests of local realism can be conducted in the experiments such as those performed by Smithey *et al.* [16]. Here correlation of photon numbers between two spatially separated but very intense fields is sufficient to give ''squeezed'' noise levels. In these high-flux experiments, detection losses can be relatively small, allowing for the possibility of the violation of a strong Bell inequality, but noise that limits the resolution of the photon number measurement can be large in absolute terms, as compared to traditional Bell inequality experiments, which involve photon counting with low incident photon numbers.

II. DEFINITIONS OF MACROSCOPIC LOCAL REALISM

In the original argument of Einstein, Podolsky, and Rosen (EPR) [1], ''local realism'' is defined in the following way. For ''realism,'' it is sufficient to state that if one can predict with certainty the result of a measurement of a physical quantity at A, without disturbing the system A, then the results of the measurement were predetermined and one has an ''element of reality,'' corresponding to this physical quantity. The element of reality is a variable that assumes one of the set of values that are the predicted results of the measurement. Locality postulates that measurements at B cannot disturb A in any way. Taken together with realism then, as defined above, for ''local realism'' it is sufficient to imply that if one can predict the result of a measurement at A, by making a simultaneous measurement at B, then the result of the measurement at A is a predetermined property of the system A. In the case of perfect correlation and perfect measurements, the predetermined value (the element of reality) for any individual system A will have zero uncertainty, since we can determine it precisely by measurements on B, and because all orders of change to system A as a result of the measurement at B are excluded by locality.

Macroscopic local realism may be defined as a premise stating the following. This meaning and definition of macroscopic local realism has been previously introduced in references Refs. [14] and [15] in line with the original EPR argument, and its experimental realization for continuous variables introduced by Ou *et al.* [17]. If one can predict the result of a measurement at A by performing a simultaneous measurement on a spatially separated system B, then the result of the measurement at A is predetermined but described by an element of reality that has an indeterminacy in each of its possible values, so that only values macroscopically different from those predicted are excluded. We note that the meaning of ''predict'' in the above definition could be loosened to allow for an uncertainty in the prediction, as one would have in macroscopic experiments that incorporate measurement uncertainties.

Macroscopic local realism incorporates two assumptions. We define a ''macroscopic locality,'' which states that measurements at a location B cannot instantaneously induce changes of a macroscopic magnitude (for example, the dead to alive state of a cat, or a change between macroscopically different photon numbers) in a second system A spatially separated from B. Locality in its entirety, as used originally by EPR and Bell, postulates that measurements at B cannot disturb A in any way. We expect that our definition of a macroscopic order of locality is equivalent to postulating that locality will always appear to be satisfied where measurement uncertainties do not enable resolution of results that differ by a microscopic or mesoscopic number of photons.

The second assumption incorporated by macroscopic local realism is the assumption of a ''macroscopic realism,'' since a macroscopic local realism implies elements of reality with (up to) a macroscopic indeterminacy. Suppose an element of reality may be symbolized by the variable x, where x can take on numerical values $x_1, x_2, \ldots$. For microscopic realism, these values are specified to a microscopic level. For macroscopic realism, these values have a macroscopic indeterminacy, by this meaning that one can only exclude values for the associated physical variable that are macroscopically different from the values $x_1, x_2, \ldots$. We see that if x_1, x_2 are only microscopically distinct, they are in this case no longer distinguished by different hidden variable values.

The notion of realism is exclusive of ''quantum superposition states'' in the following sense. If a physical quantity for an ensemble of systems is attributed an element of reality x as above, then the element of reality for each individual system will take on one of the values $x_1, x_2, \ldots$. This value is the result of the measurement of the physical quantity, should it be performed. This element of reality picture is different from the standard quantum picture of a system being in a ''quantum superposition'' of two states of different x_i. According to a standard quantum mechanics interpretation, an individual system described by such a superposition cannot be thought of as being in one or the other of the two states prior to measurement. If the values of the element of reality are defined with zero uncertainty, then the element of reality theory excludes (or is different in its interpretation from) a ''quantum superposition'' of states x_i and $x_i+\delta$ where δ is nonzero.

We consider the existence of an element of reality that is only macroscopically specified, having values that can only be specified not to be macroscopically different from a value x. This macroscopic realism description says nothing about the possibility of superpositions of states microscopically or mesoscopically different from x. Macroscopic local realism cannot exclude the possibility of quantum superpositions of states microscopically or mesoscopically different, with respect to the physical quantity represented by the element of reality. We can, however, exclude the possibility of the quantum superpositions of states with macroscopically different values for the physical quantity concerned.

Since it says nothing about microscopic systems, macroscopic local realism is a less restrictive premise than ''local realism'' used in its entirety. Local realism in its full sense can define elements of reality with values having no uncertainty and therefore can exclude the possibility of quantum superpositions of states with all separations (microinclusive to macroinclusive) in the relevant variable.

III. BELL INEQUALITIES WITH NOISE: TESTS OF MACROSCOPIC LOCAL REALISM

Our proposed experiment to test macroscopic local realism is depicted in Fig. 1, where $\hat{a}_\pm$ and $\hat{b}_\pm$ are boson operators for outgoing fields, generated from a suitable source to be discussed in Sec. IV, at the spatially separated locations A and B, respectively. We define the Schwinger spin operators

$$\hat{S}_x^A=(\hat{a}_+^\dagger\hat{a}_-+\hat{a}_-^\dagger\hat{a}_+)/2,$$

$$\hat{S}_y^A=(\hat{a}_+^\dagger\hat{a}_- -\hat{a}_-^\dagger\hat{a}_+)/2i, \tag{1}$$

$$\hat{S}_z^A=(\hat{a}_+^\dagger\hat{a}_+-\hat{a}_-^\dagger\hat{a}_-)/2.$$

Similar operators $\hat{S}_x^B,\hat{S}_y^B,\hat{S}_z^B$ are defined for the modes at B. We measure simultaneously at A and B the Schwinger spin operators

$$\hat{S}_\theta^A=\hat{S}_x^A\cos\theta+\hat{S}_y^A\sin\theta, \tag{2}$$

and

$$\hat{S}_\phi^B=\hat{S}_x^B\cos\phi+\hat{S}_y^B\sin\phi, \tag{3}$$

respectively.

In Fig. 1(a) the measurement at A is performed with phase shift θ and a beam splitter to produce $\hat{c}'_\pm=[\hat{a}_+\pm\hat{a}_-\exp(-i\theta)]/\sqrt{2}$, followed by photodetection. At B modes, $\hat{d}'_\pm=[\hat{b}_+\pm\hat{b}_-\exp(-i\phi)]/\sqrt{2}$ are similarly generated. The possible outcomes for the photon number $\hat{c}'^\dagger_+\hat{c}'_+$ (and $\hat{d}'^\dagger_+\hat{d}'_+$) are 0,1, . . . in integer steps. The spin values for $\hat{S}_\theta^A$ and $\hat{S}_\phi^B$ are then given by the photon-number differences $\hat{n}_\theta^A=2\hat{S}_\theta^A=\hat{c}'^\dagger_+\hat{c}'_+-\hat{c}'^\dagger_-\hat{c}'_-$ and $\hat{n}_\phi^B=2\hat{S}_\phi^B=\hat{d}'^\dagger_+\hat{d}'_+-\hat{d}'^\dagger_-\hat{d}'_-$.

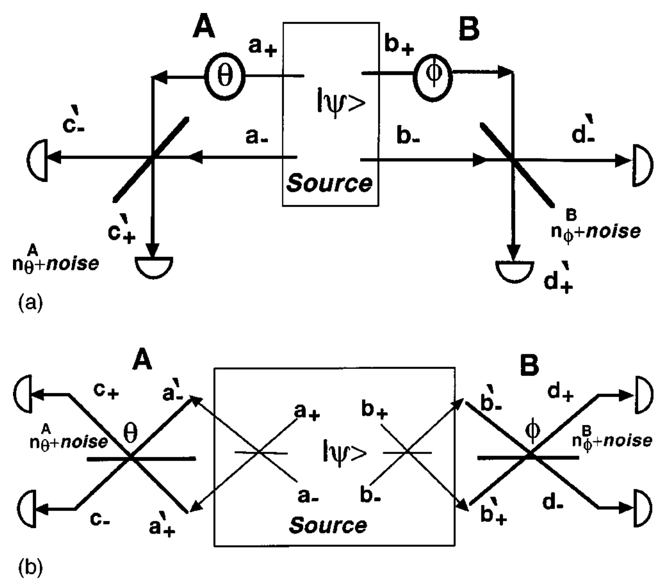

FIG. 1. Schematic representation of our proposed test of macroscopic local realism. (a) Measurement of spin operators $\hat{S}_\theta^A$ and $\hat{S}_\phi^B$. This measurement scheme is equivalent to balanced homodyne detection of the quadrature phase amplitudes $\hat{X}_\theta^A$ and $\hat{X}_\phi^B$ of the fields $\hat{a}_-,\hat{b}_-$, in the limit of large α,β. In the proposed experiment, $\hat{a}_-,\hat{b}_-$ are of low intensity while $\hat{a}_+,\hat{b}_+$ are intense coherent-state $|\alpha\rangle$ ''local oscillator'' fields. In this experiment, large intensities are incident on each of the photodiode detectors. (b) Importantly in this alternative arrangement, the fields $\hat{a}_\pm$ are first combined using a beam splitter and phase shift so that both outgoing fields $\hat{a}'_\pm$ incident on the measuring apparatus are macroscopic. The measurement apparatus is depicted here by the beam splitter with variable angle θ, although a polarizer may also be possible for suitable states. A similar arrangement occurs at B. In this experiment the entire boxed apparatus may be considered the source. The measured quantity in terms of the $\hat{a}_\pm,\hat{b}_\pm$ fields is still $\hat{S}_\theta^A$ and $\hat{S}_\phi^B$ as above in (a).

Alternatively in Fig. 1(b), the $\hat{a}_\pm$ are first combined [18] through a beam splitter and then phase shifted, to give outgoing fields $\hat{a}'_-=(\hat{a}_--\hat{a}_+)/\sqrt{2}$ and $\hat{a}'_+=i(\hat{a}_-+\hat{a}_+)/\sqrt{2}$. These may now be considered system fields, upon which the measurement $\hat{n}_\theta^A=2\hat{S}_\theta^A=\hat{c}_+^\dagger\hat{c}_+-\hat{c}_-^\dagger\hat{c}_-$ is made through the transformation (with polarizer or beam splitter) $\hat{c}_+=\hat{a}'_+\cos\theta/2+\hat{a}'_-\sin\theta/2$ and $\hat{c}_-=\hat{a}'_+\sin\theta/2-\hat{a}'_-\cos\theta/2$ followed by photodetection. Figure 1(b) depicts a measurement $\hat{S}_z^{A'}\cos\theta+\hat{S}_y^{A'}\sin\theta$ made on system operators $\hat{a}'_\pm$, but is the same measurement depicted in Fig. 1(a) for the fields $\hat{a}_\pm$. We use similar definitions $\hat{S}_x^{A'}$, $\hat{S}_y^{A'}$, and $\hat{S}_z^{A'}$ for the Schwinger operators in terms of $\hat{a}'_\pm$. Similar transformations are defined for the measurement at B. We present this scheme because, for the particular choice of quantum state discussed in Sec. IV, it ensures both fields $\hat{a}'_{\pm}$ incident on the measurement apparatus (polarizer) can be macroscopic. This arrangement then is crucial in providing a test of macroscopic realism.

We classify the result of our measurement as $+1$ if the result for the photon number difference measurement $\hat{n}_{\theta}^{A}$ or $\hat{n}_{\phi}^{B}$ is positive or zero, and -1 otherwise. The results at B are classified similarly. We build up the following probability distributions: $P_{+}^{A}(\theta)$ for obtaining $+1$ at A; $P_{+}^{B}(\phi)$ for obtaining $+1$ at B; and $P_{++}^{AB}(\theta,\phi)$, the joint probability of obtaining $+1$ at both A and B.

We first consider the predictions as given by the original definition of local realism (local hidden variables) used by Einstein-Podolsky-Rosen, Bell, and Clauser-Horne [3,4]. The probability of obtaining $+1$ for S_{θ}^{A} is expressed as

$$P_{+}^{A}(\theta)=\int \rho(\lambda)p_{+}^{A}(\theta,\lambda)d\lambda. \tag{4}$$

The probability of obtaining $+1$ for S_{ϕ}^{B} is

$$P_{+}^{B}(\phi)=\int \rho(\lambda)p_{+}^{B}(\phi,\lambda)d\lambda. \tag{5}$$

The joint probability for obtaining $+1$ for both of the simultaneous measurements with θ at A and ϕ at B is

$$P_{++}^{AB}(\theta,\phi)=\int \rho(\lambda)p_{+}^{A}(\theta,\lambda)p_{+}^{B}(\phi,\lambda)d\lambda. \tag{6}$$

Here $p_{+}^{A}(\theta,\lambda)$ is the probability for getting the result $+1$ given the hidden variables λ; $p_{+}^{B}(\phi;\lambda)$ is the probability for getting the result $+1$ given λ; while $\rho(\lambda)$ is the probability distribution for the hidden variables λ.

It is well known [3,4] that one can derive the following "strong" Bell-Clauser-Horne inequality from the assumptions of local realism made so far:

$$S=\frac{P_{++}^{AB}(\theta,\phi)-P_{++}^{AB}(\theta,\phi')+P_{++}^{AB}(\theta',\phi)+P_{++}^{AB}(\theta',\phi')}{P_{+}^{A}(\theta')+P_{+}^{B}(\phi)} \leqslant 1. \tag{7}$$

To date, this "strong" inequality has not been violated in any experiment, because of the poor detection inefficiencies that occur in photon counting experiments. It is well documented that it is possible to derive, with the assumption of additional premises, a weaker form of the Bell inequality that has been violated in photon counting experiments where detection losses are high. In this paper, however, we restrict our attention to the strong inequalities that do not require additional assumptions. Our proposed experiments involve photodiode detectors that have high efficiencies and therefore allow for the possibility of a strong violation of local realism.

In deriving the Bell inequalities, one specifies a probability $p_{+}^{A}(\theta,\lambda)$ for getting the result $+1$ as opposed to -1 given the hidden variables λ. If the results $+1$ and -1 are always macroscopically different, it becomes apparent that one need only assume "macroscopic local realism" as opposed to local realism in its entirety to obtain the Bell inequalities. This is because in assuming the independence of this probability $p_{+}^{A}(\theta,\lambda)$ on ϕ, we need only assume a macroscopic locality, that the measurement at B does not disturb the system at A in a macroscopic way to make the change from $+1$ to -1. The elements of reality need only be specified "macroscopically"; that is, they can have a macroscopic indeterminacy in their values and still adequately represent the distinct outcomes of measurement. We can add certain (though not all) perturbations of a macroscopic size (in photon number) to the values predicted by the "elements of reality" and not change the final form of the Bell inequality.

The violation of the Bell inequality (7), where the possible results of all relevant measurements (for all relevant angles θ and ϕ) are macroscopically distinct, would be firm confirmation of an incompatibility with macroscopic local realism. To our knowledge no such violation has yet been demonstrated.

In order to test for macroforms of local realism in more general situations (where the possible results are not always macroscopically separated), we propose to add local classical noise sources to the final readout stage of each of the measurement processes, at A and B. We will assume that the result for the photon number difference $\hat{n}_{\theta}^{A}$ or $\hat{n}_{\phi}^{B}$ at A and B, respectively, is of the form $n+\mathcal{N}$, where n is the result of the measurement in the absence of the noise and $\mathcal{N}$ is a local classical noise term. The noise terms at A and B are independent, modeling a local physical source of noise, and as such always satisfy locality, the noise added at A, for example, being independent of the experimental choice of the angle ϕ at B.

We will derive a Bell inequality based on the premise of macroscopic local realism alone by showing that the addition of this classical noise to the final measurement result can alter the premises needed to derive the Bell inequality. We first define the probability $P_{ij}^{0,AB}(\theta,\phi)$ for obtaining results $i/2$ and $j/2$, respectively, upon joint measurement of S_{θ}^{A} at A, and S_{ϕ}^{B} at B, in the absence of the applied noise. The i and j are then results for the photon-number differences $\hat{n}_{\theta}^{A}$ or $\hat{n}_{\phi}^{B}$, respectively. In terms of a local hidden variable description, this probability is given by

$$P_{ij}^{0,AB}(\theta,\phi)=\int \rho(\lambda)p_{i}^{A}(\theta,\lambda)p_{j}^{B}(\phi,\lambda)d\lambda. \tag{8}$$

We next outline how the assumption of local realism, as defined originally by EPR, implies the hidden variable description (8) above. This is in order to postulate how the above expression is modified if one makes only the macroscopic local realism assumption.

A perfect correlation between measurement results at A and B is predicted to be possible for some quantum states. For such situations, it is possible to predict precisely the result of a measurement at A by performing a particular measurement at B. We are able to deduce [3], assuming local realism and following the reasoning of EPR as outlined in

Sec. II, the existence of a set of ''elements of reality,'' m_θ^A and m_ϕ^B, one for each subsystem at A and B, and one for each choice of measurement angle, θ or ϕ, at A or B, respectively. The m_θ^A assumes one of a set of definite values, this value giving the result of the measurement θ at A should it be performed. The set m_θ^A, m_ϕ^B forms a set of hidden variables λ for the system.

More generally, there will be a reduced correlation between measurements performed at A and B. This is generally so for the case where measurements incorporate macroscopic uncertainties. Local realism still allows us to deduce the existence of an element of reality (we will call it m_θ^A) for the photon-number difference at A, with measurement angle θ at A, since we can make a prediction of the result at A without disturbing the system at A, under the locality assumption. This prediction is based on a measurement performed at B. In this case, however, the element of reality m_θ^A becomes ''fuzzy.'' The ''values'' that the element of reality can assume do not form a set of definite numbers with zero uncertainty, but rather a set of distributions, one for each possible result m at B, which we label by $m_\theta^A = m$. The distribution labeled by the element of reality m_θ^A assuming the value m gives the probability of a result for the measurement θ at A should it be performed. It is independent of ϕ, the experimenter's choice of angle at B, if a simultaneous measurement at B should be performed. One can apply similar reasoning to deduce the existence of a set of indeterminate elements of reality m_ϕ^B.

The assumption of ''local realism''then justifies the local hidden variable description used in Eq. (8), and Eqs. (4)–(6), above. Local realism implies that the system is always in a state corresponding to a particular value for each of the elements of reality m_θ^A and m_ϕ^B. The whole set of ''elements of reality'' m_θ^A and m_ϕ^B form a set of ''hidden variables'' that can be attributed to the system at a given time. Common notation symbolizes the complete set of hidden variables by λ, and the underlying joint probability distribution $p(m_\theta^A, m_\phi^B)$ becomes $\rho(\lambda)$. The probabilities $\rho(\lambda)$ for the hidden variables are predetermined, and do not depend on the experimental choice of θ and ϕ. For each such state λ there is a probability $p_n^A(\theta,\lambda)$ that the result of a θ measurement at A will be n. In the case with perfect correlation, the ''elements of reality'' give precise values for the result of the photon number measurement. Suppose the result m at B correlates with n at A. Then we have $p_n^A(\theta,\lambda)=1$ if $\lambda = m_\theta^A = m$, and is zero otherwise. More generally, we have imperfect correlation and ''fuzzy'' elements of reality, meaning that this $p_n^A(\theta,\lambda)$ assumes a finite variance as discussed above.

We focus attention on the distribution $p_i^A(\theta,\lambda)$, the probability of getting a photon number i for measurement at A with angle θ, given that the system is in a hidden variable state λ. The independence of $p_i^A(\theta,\lambda)$ on ϕ is based on the locality assumption used in its entirety, that the experimenter's choice of measurement angle at B cannot (instantaneously) change the result of the measurement at A in any way. With macroscopic local realism the locality condition is relaxed, allowing the conditional distributions $p_i^A(\theta,\lambda)$ to become nonlocal, that is, to have an explicit dependence on the experimental angle ϕ. The locality condition is relaxed, however, only up to the level of M photons, where M is not macroscopic, by maintaining that the measurement at B cannot instantaneously change the result at A by an amount exceeding M photons.

By relaxing the locality assumption up to M photons, the elements of reality m_θ^A (deduced by way of the EPR argument) even in situations of perfect correlation will automatically have a distribution $p_i^A(\theta,\phi,\lambda)$, which is no longer a δ function, though the distribution will be zero for values of i exceeding the value of m_θ^A by greater than M photons. This is because we can no longer exclude the possibility of changes to the result of photon number measurements at A by an amount of up to M photons, due to the measurement at B.

Similarly, in the case of imperfect correlation, the ''fuzziness'' of the elements of reality as given by the conditional distribution $p_i^A(\theta,\lambda)$ is increased by an amount whose upper limit is determined by the value of M and which may depend on ϕ. Now we must consider the prediction for Eq. (8) as given by macroscopic local realism. The elements of reality deduced using macroscopic local realism cannot give predictions for the results of measurement that are macroscopically different from those predicted from the elements of reality deduced using local realism. Where our predicted result for a measurement at A is i' using local realism, macroscopic local realism allows the result to be $i'+m_A$ where m_A can be any number not macroscopic. Importantly, while i' is not dependent on the choice ϕ for a simultaneous measurement at B, the value m_A can be. We therefore introduce the macroscopic locality assumption into the expression (8) for the probabilities in terms of the hidden variables in the following manner. We assume that the conditional probability $p_i^A(\theta,\lambda)$ in Eq. (8) takes the form of the following convolution (where M is a integer that is not macroscopic):

$$p_i^A(\theta,\phi,\lambda) = \sum_{m_A=-M}^{+M} p_{m_A}^{A,NL}(\theta,\phi,\lambda)\, p_{i'=i-m_A}^{A,L}(\theta,\lambda). \quad (9)$$

[We similarly relax the locality assumption for $p_i^B(\phi,\lambda)$, allowing for a dependence on θ, and introduce a $p_i^B(\phi,\theta,\lambda)$ defined in a similar fashion.] The original local probability distribution $p_{i'}^{A,L}(\theta,\lambda)$, as would be specified through local realism, may be convolved with a microscopic or mesoscopic nonlocal probability function $p_{m_A}^{A,NL}(\theta,\phi,\lambda)$. The local specification, which is not dependent on the experimental choice of angle ϕ at B, gives a (local) probability distribution $p_{i'}^{A,L}(\theta,\lambda)$ for obtaining i' photons at A, but the prediction is only correct to within $\pm M$ photons. These (local) distributions form the fuzzy ''macroscopic elements of reality.'' The probability distribution for an actual result $i = i' + m_A$ at A is determined by the further nonlocal perturbation term $p_{m_A}^{A,NL}(\theta,\phi,\lambda)$, which gives the probability of a further change of m_A photons. The nonlocal term is necessary because macroscopic local realism allows for the possibility that the measurement at B instantaneously changes the result at A by M or less photons, where M is not macroscopic. The only restriction is that the nonlocal distribution does not provide macroscopic perturbations, so that the probability of getting a nonlocal change outside the range $m_A = -M, \ldots, +M$ is zero. Equivalently, we must have (and similarly for terms with B)

$$\sum_{m_A=-M}^{M} p_{m_A}^{A,NL}(i',\theta,\phi,\lambda)=1. \tag{10}$$

We now wish to obtain an expression for the measurable probabilities $P_{++}^{AB}(\theta,\phi)$ in the presence of the local noise terms, in terms of the $P_{ij}^{0,AB}(\theta,\phi)$. We introduce noise distribution functions at each of A and B, and define probabilities such as $P^A(\mathcal{N}\geqslant x)$, such that the $\mathcal{N}$ at A is greater than or equal to the value x. A probability $P^B(\mathcal{N}\geqslant x)$ is defined similarly, for the noise term at B. The final measured probability in the presence of noise is expressed as

$$P_{++}^{AB}(\theta,\phi)=\sum_{i,j=-\infty}^{\infty} P_{ij}^{0,AB}(\theta,\phi)P^A(\mathcal{N}\geqslant -i)P^B(\mathcal{N}\geqslant -j). \tag{11}$$

We write the predictions for this expression in terms of the hidden variable theory by substituting the macroscopic locality assumption (9) into the hidden variable prediction (8) for $P_{ij}^{0,AB}(\theta,\phi)$. We get

$$P_{++}^{AB}(\theta,\phi)=\sum_{i,j=-\infty}^{\infty}\int \rho(\lambda)\left[\sum_{m_A=-M}^{M} p_{m_A}^{A,NL}(i',\theta,\phi,\lambda)p_{i'=i-m_A}^{A,L}(\theta,\lambda)\sum_{m_B=-M}^{M} p_{m_B}^{B,NL}(j',\phi,\theta,\lambda)p_{j'=j-m_B}^{B,L}(\phi,\lambda)\right] \times d\lambda\, P^A(\mathcal{N}\geqslant -i)P^B(\mathcal{N}\geqslant -j). \tag{12}$$

Recalling $i=i'+m_A$ and $j=j'+m_B$, we change the $i, \quad j$ summation to one over $i', \quad j'$ to get

$$P_{++}^{AB}(\theta,\phi)=\sum_{i',j'=-\infty}^{\infty}\int \rho(\lambda)p_{i'}^{A,L}(\theta,\lambda)\left\{\sum_{m_A=-M}^{M} p_{m_A}^{A,NL}(i',\theta,\phi,\lambda)P^A[\mathcal{N}\geqslant -(i'+m_A)]\right\}p_{j'}^{B,L}(\phi,\lambda) \times\left\{\sum_{m_B=-M}^{M} p_{m_B}^{B,NL}(j',\phi,\theta,\lambda)P^B[\mathcal{N}\geqslant -(j'+m_B)]\right\}d\lambda. \tag{13}$$

At this point we introduce the following assumption regarding the macroscopic nature of the noise term $P^A(\mathcal{N}\geqslant x)$: the increase or decrease of x by an amount of up to M photons gives only a negligible change to the probability that the noise is of size x or greater, $P^A[\mathcal{N}\geqslant -(i'+m_A)]\approx P^A(\mathcal{N}\geqslant -i')$ and similarly for the noise term at B. This gives us

$$\sum_{m_A=-M}^{M} p_{m_A}^{A,NL}(i',\theta,\phi,\lambda)P^A[\mathcal{N}\geqslant -(i'+m_A)]\approx P^A(\mathcal{N}\geqslant -i')\sum_{m_A=-M}^{M} p_{m_A}^{A,NL}(i',\theta,\phi,\lambda). \tag{14}$$

Clearly this is only valid for noise that is macroscopic in size (recalling that M is a number that is not macroscopic). With assumption (10) we get the simplification to obtain a final form

$$P_{++}^{AB}(\theta,\phi)=\sum_{i',j'}\int \rho(\lambda)p_{i'}^{A,L}(\theta,\lambda)p_{j'}^{B,L}(\phi,\lambda)d\lambda\, P^A(\mathcal{N}\geqslant -i')P^B(\mathcal{N}\geqslant -j'). \tag{15}$$

This prediction of the hidden variable theory is now given in a (local) form like that of Eq. (6). A similar study of the expressions for the marginal probabilities leads to (local) expressions like those of Eqs. (4) and (5), and the Bell inequalities (7) therefore readily follow. The noise terms $\mathcal{N}$, which add a macroscopic uncertainty to the photon number result, alter the premises needed to derive the Bell inequality. We need only to assume macroscopic local realism to derive the inequalities (7) in the presence of macroscopic noise terms. Therefore violation of these Bell inequalities in the presence of truly macroscopic noise terms would be evidence of a failure of macroscopic local realism.

IV. QUANTUM STATES VIOLATING BELL INEQUALITIES WITH MACROSCOPIC NOISE: PREDICTED FAILURE OF MACROSCOPIC LOCAL REALISM

We present a quantum state that shows violations of Bell inequalities in the presence of macroscopic noise. By the above arguments, this state then is evidence of macroscopic local realism.

$$|\psi\rangle=[I_0(2r_0^2)]^{-1/2}\sum_{n=0}^{\infty}\frac{(r_0^2)^n}{n!}|n\rangle_{a_-}|n\rangle_{b_-}|\alpha\rangle_{a_+}|\beta\rangle_{b_+}. \tag{16}$$

Here I_0 is a modified Bessel function. The fields $\hat{a}_+$ and $\hat{b}_+$ are in coherent states $|\alpha\rangle_{a_+}$ and $|\beta\rangle_{b_+}$, respectively, and we allow α, β to be real and large. $|n\rangle_k$ is a Fock state for field k. The fields $\hat{a}_-$ and $\hat{b}_-$, often referred to as signal and idler fields, respectively, are microscopic and generated in a pair-coherent state with $r_0=1.1$. Pair-coherent states were considered originally by Agarwal and co-workers [19]. They might potentially be generated using nondegenerate parametric oscillation (as suggested by Reid and Krippner [19] and explored in the recent work by Gilchrist and Munro [19]) in a limit where one-photon losses are negligible, or some similar process, as modeled by the following Hamiltonian in which coupled two-photon signal-idler loss dominates over linear single-photon loss:

$$H=i\hbar E(\hat{a}_-^\dagger\hat{b}_-^\dagger-\hat{a}_-\hat{b}_-)+\hat{a}_-\hat{b}_-\hat{\Gamma}^\dagger+\hat{a}_-^\dagger\hat{b}_-^\dagger\hat{\Gamma}. \quad (17)$$

The coherent states for $\hat{a}_+$ and $\hat{b}_+$ would be derived from the laser pump for the oscillator. Here E represents a coherent driving parametric term that generates signal-idler pairs, while $\hat{\Gamma}$ represents reservoir systems that give rise to the coupled signal-idler loss. The Hamiltonian preserves the signal-idler photon number difference operator $\hat{a}_-^\dagger\hat{a}_- - \hat{b}_-^\dagger\hat{b}_-$, of which the quantum state (16) is an eigenstate, with an eigenvalue of zero. We note the analogy here to the single mode "even" and "odd" coherent superposition states $N_\pm^{1/2}(|\alpha\rangle \pm |-\alpha\rangle)$ {where α is real and $N_\pm^{-1}=2[1\pm\exp(-2|\alpha|^2)]$} which are generated by the degenerate form (put $\hat{a}_-=\hat{b}_-$) of the Hamiltonian (17). These states for large α are analogous to the famous "Schrödinger-cat" states [9,10] and have been recently experimentally explored [11–13]. We point out later other choices of $|\psi\rangle$ possible.

To model noise we allow $\mathcal{N}$ to be a random noise term with a Gaussian distribution of standard deviation σ. An example of a noisy photon-number measurement is the photodiode detection of very large intensities, such as those used in the experiments of Smithey *et al.* [16]. The photocurrent is processed electronically in a way that adds noise to the final output current, giving a final imprecision in the photon number measurement. Although percentage detection efficiencies are high for diode detectors, detection inefficiencies can also create a potentially large absolute noise term that also limits the resolution of the photon number measurement.

Violations of the Bell inequality (7), for the state (16), in the absence of noise are shown in Fig. 2, curve (a). The effect of adding increasing noise is to reduce the value of S until eventually the violation is lost, at a cutoff noise value σ_c, as shown in Fig. 3. In Fig. 2, curve (b) shows this cutoff value σ_c (the maximum noise still allowing a violation of the Bell inequality) versus α. We note the linear dependence of σ_c on α ($\sigma_c=0.26\alpha$). In the limit of larger α this cutoff noise σ_c then becomes macroscopic. Violations of fixed magnitude ($S\rightarrow 1.057$ as $\alpha\rightarrow\infty$) are still possible for increasingly larger absolute noise, simply by increasing α.

The asymptotic behavior in the large α,β limit is crucial to determining whether macroscopic local realism will be violated, and it is understood by replacing the boson operators $\hat{a}_+$ and $\hat{b}_+$ by classical amplitudes α and β, respectively. We see that $\hat{S}_\theta^A$ from Eq. (2) can be expressed as $\hat{S}_\theta^A=[\hat{a}_+^\dagger\hat{a}_-\exp(-i\theta)+\hat{a}_+\hat{a}_-^\dagger\exp(i\theta)]/2=\alpha\hat{X}_\theta^A/2$, and similarly $\hat{S}_\phi^B=\beta\hat{X}_\phi^B/2$, where $\hat{X}_\theta^A=\hat{a}_-\exp(-i\theta)+\hat{a}_-^\dagger\exp(i\theta)$ and $\hat{X}_\phi^B=\hat{b}_-\exp(-i\phi)+\hat{b}_-^\dagger\exp(i\phi)$. The $\hat{X}_\theta^A$ and $\hat{X}_\phi^B$ are the quadrature phase amplitudes of the fields $\hat{a}_-$ and $\hat{b}_-$, respectively. We then see that the photon-number measurements $2\hat{S}_\theta^A$ and $2\hat{S}_\phi^B$ give results in the large α,β limit corresponding numerically to the scaled quadrature phase amplitudes $\alpha\hat{X}_\theta^A$ and $\beta\hat{X}_\phi^B$, respectively.

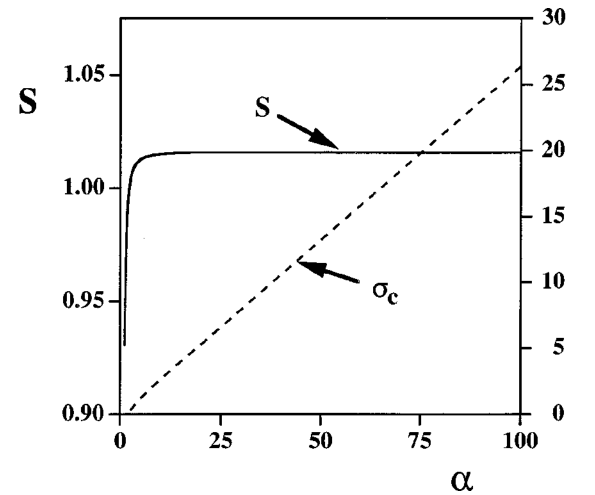

FIG. 2. S vs α for $\theta=0,\phi=-\pi/4,\theta'=\pi/2,\phi'=-3\pi/4,\alpha=\beta$ for the quantum state (16) with no noise present. The dashed line gives the maximum noise σ_c still giving a violation of the Bell inequality (7) for the above parameters vs α. Macroscopic values are possible with increasing α.

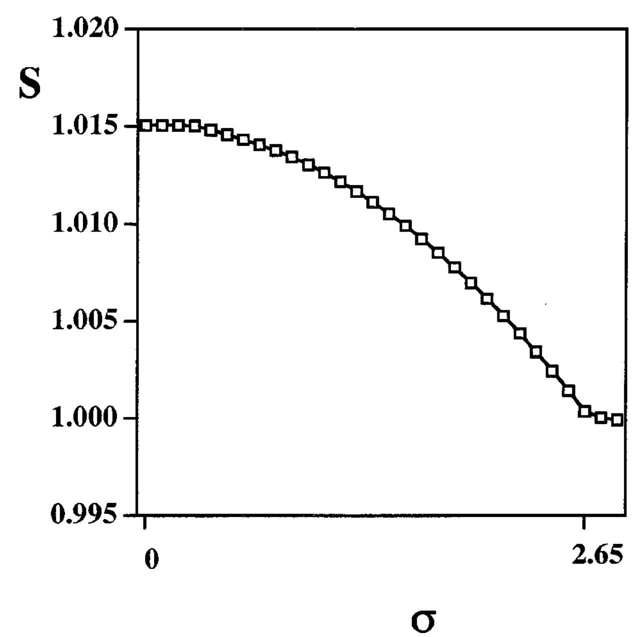

FIG. 3. S vs the noise parameter σ, for $\theta=0,\phi=-\pi/4,\theta'=\pi/2,\phi'=-3\pi/4,\alpha=\beta$ for the quantum state (16), where $\alpha=10$.

Figure 1(a) in fact shows for large α,β the experimental arrangement for balanced homodyne detection [20], a technique commonly used to measure quadrature phase amplitudes. In Fig. 1(a) the homodyne scheme measures the quadrature phase amplitudes $\hat{X}_\theta^A$ and $\hat{X}_\phi^B$, of the fields $\hat{a}_-$ and $\hat{b}_-$. The large intensity fields $\hat{a}_+$ and $\hat{b}_+$ are the ''local oscillator'' fields usually considered to be classical amplitudes α,β. Violations of Bell inequalities (7) (failure of local realism) for precisely these asymptotic quadrature phase amplitude measurements have recently been shown by Gilchrist and co-workers [21], the value of $S=1.0157$ presented in these quadrature phase amplitude calculations indeed corresponding to our large α limit (Fig. 2).

Calculations [21,22] that model the addition of noise to the quadrature phase amplitude measurements $\hat{X}_\theta^A,\hat{X}_\phi^B$ reveal violations of the Bell inequality to be lost at the cutoff value of $\sigma_0=0.26$. This asymptotic result allows us to make a prediction of the effect of noise (in the large α limit) on the full photon-number calculation presented in Fig. 2. The detected photon-number difference is given as

$$\hat{n}_\theta^A=2\hat{S}_\theta^A=\hat{c}_+^\dagger\hat{c}^+-\hat{c}_-^\dagger\hat{c}_-=\alpha\hat{X}_\theta^A. \tag{18}$$

Noise of size $\mathcal{N}$ added to the photon-number difference $\hat{n}_\theta^A$ result is equivalent to noise of size $\mathcal{N}/(\alpha)$ added to the signal quadrature phase amplitude $\hat{X}_\theta$ result. The noise in the photon-number difference is scaled by a factor of α, the local oscillator amplitude. Therefore the cutoff value $\sigma_0=0.26$ will correspond to a cutoff noise value of $\sigma_c=\alpha\sigma_0$ in the measurement of photon-number difference $\hat{n}_\theta^A=2\hat{S}_\theta^A$, confirming the linear behavior shown in Fig. 2, and the prediction that is made from this that it is possible to obtain macroscopic noise values while still obtaining a contradiction with local realism. This property then is a predicted contradiction of quantum mechanics with macroscopic local realism as we have defined it.

Detection inefficiencies will also contribute to a noise in the final result for the measurement, though in this case the noise will not be Gaussian. Noise caused by detector losses is often modeled by a beam-splitter interaction immediately prior to photodetection. The field to be detected, say, $\hat{c}'_+$, is taken to be an input to a beam splitter. The second input to the beam splitter $\hat{a}_{vac+}$ is considered to be a vacuum. The output

$$\hat{c}'_{L+}=\sqrt{\eta}\hat{c}'_++\sqrt{1-\eta}\hat{a}_{vac+}, \tag{19}$$

where η is the overall efficiency factor, is then taken to be the effective detected field. A similar effective field $\hat{c}'_{L-}$ is constructed for the second detector, used to measure $\hat{c}'_-$, at location A, and a second vacuum input $\hat{a}_{vac-}$ is defined. The detected photon-number difference is now given as

$$\hat{n}_\theta^A=\hat{c}'^\dagger_{L+}\hat{c}'_{L+}-\hat{c}'^\dagger_{L-}\hat{c}'_{L-}=\eta\alpha\hat{X}_{L\theta}^A, \tag{20}$$

where

$$\hat{X}_{L\theta}^A=\eta\hat{X}_\theta^A+(\sqrt{1-\eta}/\sqrt{2})(\hat{X}_{\theta,vac+}+\hat{X}_{\theta,vac-}), \tag{21}$$

and the terms $\hat{X}_{\theta,vac\pm}$ are quadrature phase amplitudes for the independent $+$ and $-$ vacuum modes representing the input fields $\hat{a}_{vac+}$ and $\hat{a}_{vac-}$, respectively. Additional terms that give negligible contributions with large α have been omitted. We see how loss (described by $\eta<1$) causes a noise term $(\sqrt{1-\eta}/\sqrt{2})$ $(X_{\theta,vac+}+X_{\theta,vac-})$ in the signal quadrature phase amplitude. Because of the factor $\eta\alpha$, this term can be large enough to give potentially macroscopic absolute noise values in photon numbers for the photon number difference measurement. Violations of the Bell inequality considered by Gilchrist and co-workers [21] have been shown to be obtainable in the presence of detector losses ($\eta\approx0.98$). We see from the above analysis that this will correspond for sufficiently large α to a macroscopic absolute noise term in the photon number measurements. Thus we have a second situation where violations of a Bell inequality are predicted possible in the presence of large absolute detector noise, this prediction indicating an incompatibility of quantum mechanics with macroscopic local realism.

We can deduce from our asymptotic (large α,β) study other states $|\psi\rangle$ that will give a failure of macroscopic local realism. Any state $|\psi\rangle$ that shows a failure of local realism for measurements $\hat{X}_\theta^A$ and $\hat{X}_\phi^B$ on fields $\hat{a}_-$ and $\hat{b}_-$ will also show a violation of macroscopic local realism, provided α, β are large. This follows because there will always be a finite noise cutoff σ_0, meaning that a failure of local realism is possible for noise values less than σ_0. For large enough α,β this cutoff will correspond to a macroscopic noise cutoff value $\sigma_c=\alpha\sigma_0$ in the photon number measurement $\hat{n}_\theta^A$ (and similarly for measurement $\hat{n}_\phi^B$). This is an important point since other states violating local realism for quadrature phase amplitude measurements, either by way of a Bell inequality or by way of the Greenberger-Horne-Zeilinger phenomenon, have recently been predicted [21]. This greatly increases the scope for a practical violation of macroscopic local realism.

A failure of local realism in the presence of macroscopic noise terms (as we have predicted here for states showing failure of local realism for quadrature phase amplitude measurements) is not typical. Consider as a source for the outgoing fields $\hat{a}'_\pm$, pictured in Fig. 1(b), the following higher spin state, which has been studied in much detail by Mermin and Drummond and others [6–8]. It is well known that this state gives a violation of Bell inequalities for large N, and is often considered to be an example of a violation of a ''macroscopic local realism'':

$$|\varphi\rangle=\frac{1}{N!(N+1)^{1/2}}(\hat{a}'^\dagger_+\hat{b}'^\dagger_++\hat{a}'^\dagger_-\hat{b}'^\dagger_-)^N|0\rangle|0\rangle. \tag{22}$$

Yet a study of the behavior of the violation of the Bell inequality (7) with respect to noise added to the final photon number measurements gives a cutoff noise limit that is microscopic for large incident photon number N. This effect is

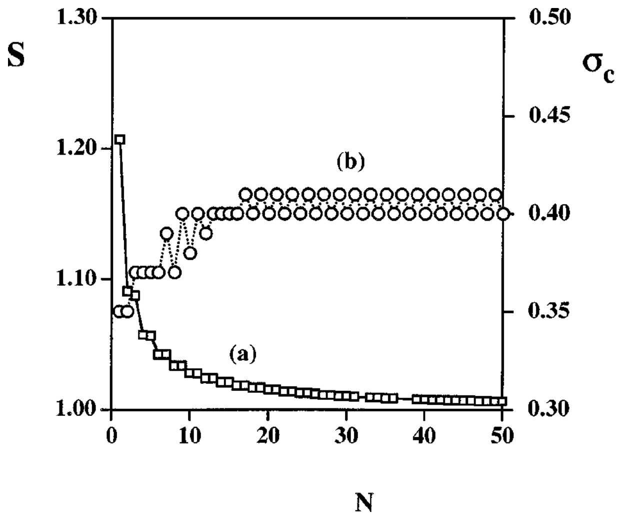

FIG. 4. Line (a) gives S versus N, for the quantum state (22) with no noise present. Here we have selected the following relation between the angles: $\phi-\theta=\theta'-\phi=\phi'-\theta'=\psi$ and $\phi'-\theta=3\psi$ and optimized S with respect to ψ. Line (b) gives the maximum noise σ_c still giving a violation of the Bell inequality (7) for the above parameters. In this case the cutoff noise σ_c remains microscopic for large N.

plotted in Fig. 4. This is in contrast to our state (16), which gives a macroscopic cutoff noise value in the limit of large α.

It may be asked how a macroscopic claim can be made from the predictions discussed in this paper, given that the signal field $\hat{a}_-$ is microscopic. It is noted in response to this question that, although the field $\hat{a}_-$ is itself microscopic, the physical quantity measured, and to which the elements of reality relate, is the combined Schwinger operator $\hat{S}_\theta^A$. The results for this measurement have a macroscopic range and can tolerate increasing levels of (absolute) noise.

However, it is crucial that the macroscopic nature of our result is clarified in the arrangement of Fig. 1(b). Here the field $\hat{a}_-$ is combined with the field $\hat{a}_+$ to produce the macroscopic fields $\hat{a}'_\pm$, prior to the experimenter's selection of the angle θ. These outgoing macroscopic fields $\hat{a}'_\pm$ may then be regarded as the system at A. In this situation, both fields $\hat{a}'_\pm$ incident on the measurement apparatus, depicted by a polarizer (or beam splitter) with the choice of θ in Fig. 1(b), are macroscopic. (A similar description applies to the fields at B.)

An important point is that the combining of fields, which comes about as part of the state preparation, can be clearly distinguished from amplification, which comes after the selection of θ, as part of the measurement process. This second-mentioned amplification comes about in all experiments, but does not imply that one can deduce ''macroscopic elements of reality'' as we have defined them here. The ''element of reality'' is a variable whose values refer to a physical quantity defined for a system, for example, the position of a particle. In the context of the Einstein-Podolsky-Rosen and Bell arguments, the ''system'' (for example, the particle or photon field) has a well-defined meaning independent of the measuring apparatus (polarizer or beam splitter phase-shift combinations) and associated amplification. A macroscopic element of reality is a variable whose possible values are defined only with a macroscopic uncertainty. The value for the element of reality and its associated uncertainty have a clear meaning, and can be readily classified as macroscopic or not macroscopic. For example, the uncertainty in the measured value for the position of a particle can be microscopic regardless of an amplified final readout value.

V. CONCLUSIONS

Our claim therefore is that earlier work [7,8] suggestive of violations of local realism at a macroscopic level must be interpreted carefully before claiming a loss of local realism at a ''macroscopic'' level. The failure of a Bell inequality in cases where the photon number can be macroscopic but where measurement resolution is perfect may not automatically imply the failure of a macroscopic local realism, as we have defined it.

In summary, we have considered the concept of orders of local realism, from macro- through mesoscopic to microscopic, which apply to experiments with an increasing precision of measurement. Macroscopic local realism excludes the possibility of macroscopic changes to a system B occurring as a result of events that occur simultaneously at a spatially separated system A. This is as opposed to local realism used in its entirety, right down to the most microscopic level, which excludes all orders of change.

We have derived Bell inequalities which, if violated in experiments with a limited resolution of photon numbers, will imply a failure of these less restrictive forms of local realism. We claim that the proven failure, if ever achievable, of this macroscopic local realism is conclusive evidence that the ''startling'' properties apparently attributed to ''entangled Schrödinger cat'' states are inescapable. A class of quantum states (those showing a violation of local realism for quadrature phase amplitudes) with this property has been proposed.

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