Random antiferromagnetic spin-$\frac{1}{2}$ chains with competing interactions

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We study disordered antiferromagnetic spin-$\frac{1}{2}$ chains with nearest- and further-neighbor interactions using the real-space renormalization-group method. We find that the system supports two different phases, depending on the ratio of the strength between nearest-neighbor and further-neighbor interactions as well the bond randomness strength. For weak further-neighbor coupling the system is in the familiar random singlet phase, while stronger further-neighbor coupling drives the system to a large spin phase similar to that found in the study of random antiferromagnetic-ferromagnetic spin chains. The appearance of the large spin phase in the absence of ferromagnetic coupling is due to the frustration introduced by further-neighboring couplings, and is unique to the disordered chains.

I. INTRODUCTION

One-dimensional quantum spin systems have been of interest to physicists for many years. This is not only because these systems have been good testing grounds for various theoretical techniques and approximations but also because they exhibit a wealth of fascinating low-energy physics. Among various intriguing phenomena of these systems, the interplay between quantum fluctuation and disorder has attracted considerable recent attention. The most thoroughly studied model in this context is the random antiferromagnetic (AF) spin-$\frac{1}{2}$ chain with nearest-neighbor interaction. It has been shown,[1] using the celebrated real-space renormalization-group (RSRG) method,[2,3] that the low-energy physics of the model is controlled by the random singlet (RS) fixed point of the RSRG and is universal. Among the universal properties of the random singlet phase are the uniform spin susceptibility: $\chi \sim 1/T \ln^2 T$, and the disorder-averaged spin-spin correlation function $\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \sim (-1)^{i-j}/(i-j)^2$. The RSRG method (with proper extensions) has also been applied with considerable success to a number of other disordered spin chain models (all with nearest-neighbor interaction only),[4–13] as well as two-leg spin ladders.[14–16]

In the present work we study random AF spin-$\frac{1}{2}$ chains with nearest- *and* further-neighbor couplings, using the RSRG method. Our motivation comes from the following considerations. First of all, as mentioned above, existing theoretical studies have been focusing on models with nearest-neighbor couplings only; the renormalization-group (RG) flow equations of the couplings are relatively simple in this case which allows, for example, exact analytical solution of the fixed point in the case of random AF spin-$\frac{1}{2}$ chains.[1] In real physical systems, on the other hand, further-neighbor couplings are always present, and in certain cases they can even be quite strong. There are a few promising experimental realizations of materials that exhibit nontrivial next-nearest-neighbor interactions. One of the examples of real physical systems that may meet the criteria is $CuGeO_3$.[17–21] Studies on this system have revealed that the angle of the Cu-O-Cu bond is close to 90°. This will induce a competition of antiferromagnetic superexchange between the Cu ions mediated by the oxygen ion and ferromagnetic direct exchange between the Cu ions. As a result the nearest-neighbor superexchange interaction is weakened and hence it is expected that the next-nearest-neighbor interactions which arise from the Cu-O-O-Cu path cannot be neglected. The strength of the second-neighbor bonds can also be controlled by applying pressure to such systems. Masuda and co-workers[22] studied the effect of pressure on highly Mg-doped $CuGeO_3$ and found that the frustration is enhanced as the pressure is increased. Another example of material that exhibits nontrivial second-neighbor interaction is $Cu_6Ge_6O_{18-x}H_2O$ studied by Hase and co-workers.[23] Thus, motivated by these experimental realizations, we study the effects of next-nearest-neighbor interactions, and in particular, the stability of the RS fixed point against their presence.

Secondly, nearest-neighbor models have no frustration. Further-neighbor interactions, on the other hand, can introduce frustration, and this is known to lead to new physics and phases in the case of pure chains. For example, it is known in the case of a spin-$\frac{1}{2}$ chain with nearest- and next-nearest-neighbor couplings (J_1 and J_2), that there are two different phases depending on the ratio between the two.[24–28] For zero or small J_2/J_1, the system is in a gapless (critical) phase with power-law spin-spin correlation, while for larger J_2/J_1 the system spontaneously dimerizes and opens a gap in the excitation spectrum, and the spin-spin correlation becomes short range. In the special case of $J_2/J_1 = 1/2$, which is the so-called Majumdar-Ghosh model, the ground state of the system is known exactly; they are collections of neighboring spins forming singlet pairs over either even or odd nearest-neighbor bonds.[29–32] It is thus of interest to study how frustration affects the physics of disordered chains, and whether new phases can be stabilized by it.

Our results can be summarized as follows. We find that there are two phases in the model we are considering, controlled by the ratio of the strength of nearest-neighbor and next-nearest-neighbor interactions and the strength of bond randomness. The RS phase is found to be stable against weak further-neighbor couplings; in this case the strength of further-neighbor couplings (as measured by the strength of nearest-neighbor couplings) flows to zero as the energy scale decreases, thus the low-temperature properties of the system

are still controlled by the RS fixed point. For strong enough further-neighbor couplings, on the other hand, the RS phase becomes unstable and the system is driven into another phase which is controlled by large effective spins at low energies. We find that in this phase the system is still dominated by effective nearest-neighbor interactions at low energy; however, the effective couplings can be either antiferromagnetic or ferromagnetic, with random distributions. We conclude that this phase is the same as that found in random AF-ferromagnetic (F) spin chain systems with nearest-neighbor interactions only, studied by Westerberg *et al.*[5] The physical origin of the appearance of effective ferromagnetic couplings is the frustration introduced by further-neighbor couplings.

The remainder of the paper is organized as follows. In Sec. II we introduce the model we study and discuss the application of the RSRG method to this model. Results of our numerical studies on the model are presented in Sec. III. In Sec. IV we summarize our findings and make connections with previous works that are related to our studies.

II. THE MODEL

We consider the AF spin-$\frac{1}{2}$ chain described by the following Hamiltonian:

$$H = \sum_{i=1}^{N-1} J_i S_i \cdot S_{i+1} + \sum_{i=1}^{N-2} K_i S_i \cdot S_{i+2}, \quad (1)$$

where N is the number of spins on the chain, S_i is a spin-$\frac{1}{2}$ operator at the ith site, and the positive couplings J_i and K_i are distributed randomly according to some probability distributions which will be described in more detail in the next section. The Hamiltonian in Eq. (1) consists of two terms, where the first term describes nearest-neighbor interactions between the spins and the second term describes next-nearest-neighbor (nnn) interactions. The schematic diagram of the system described by the Hamiltonian (1) is depicted in Fig. 1(a). We mostly focus on chains with nn and nnn couplings in this paper, but some results of chains with couplings beyond nnn will also be presented.

We use the real-space renormalization-group method to study the Hamiltonian (1). The application of this method to AF spin-$\frac{1}{2}$ chains with nn couplings only is well known. The basic idea is to isolate the strongest bond in the system, decimate it, and calculate the effective interactions generated between what were the third-nearest neighbors. The key simplifying features in this case are that the generated interactions are always antiferromagnetic, and they connect only nearest-neighbor spins (after the two spins coupled by the strongest bond are removed).

Appropriate extensions of the original RG scheme need to be included in order to study the present model with further-neighbor couplings properly. First we notice that the coordination number, i.e., the number of spins coupled to a given spin, grows as the energy scale is lowered so we need to keep track of the structure of the system. This is in contrast to the AF spin-$\frac{1}{2}$ chain with nn couplings where the coordination number is always two. Second, as we see later in the paper, effective ferromagnetic couplings may be generated at
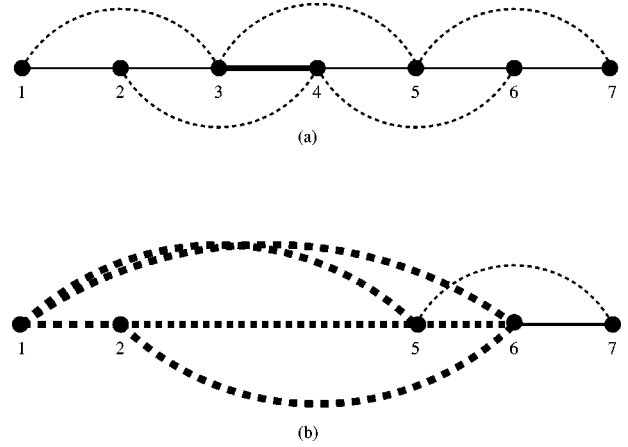

FIG. 1. (a) Schematic diagram for the AF spin-$\frac{1}{2}$ chain given by the Hamiltonian (1). In addition to the nearest-neighbor couplings between the spins, we also include the next-nearest-neighbor couplings represented by the dashed lines. Here the strongest bond is represented by the thick bold line. (b) The renormalization scheme after the strongest bond is decimated. The thick dashed lines are the renormalized couplings.

certain stages as the RSRG method is carried out in the presence of antiferromagnetic nnn couplings. The formation of ferromagnetic couplings allows the possibility of generating effective spins with sizes larger than one-half, so we need to extend the RG rules to incorporate arbitrary spin sizes and coupling signs. Let us discuss these in more detail. Consider spin 3 and 4 in Fig. 1(a), which are coupled by the strongest bond, and other spins in the system that couple to at least one of them. Due to the presence of nnn couplings, we have a six-spin problem instead of a four-spin problem for a given pair of spins coupled by the strongest bond. The Hamiltonian for the six-spin problem is given by

$$H = H_0 + H_I, \quad (2)$$

where

$$H_0 = J_{34} S_3 \cdot S_4,$$

$$H_I = J_{23} S_2 \cdot S_3 + J_{45} S_4 \cdot S_5 + J_{13} S_1 \cdot S_3$$
$$+ J_{35} S_3 \cdot S_5 + J_{24} S_2 \cdot S_4 + J_{46} S_4 \cdot S_4, \quad (3)$$

where J_{ij} is the antiferromagnetic coupling between S_i and S_j. We have shown in our previous work on spin ladders[15] that to the second-order perturbation calculation, H_I only generates pairwise interactions among the spins and hence it is only necessary to include a pair of spins coupled to the two spins connected by the strongest bond when we consider the effective interaction between them, i.e., we just have to consider four-spin clusters for a given segment which contains the strongest bond. Let us consider the most complicated four-spin cluster where a given spin is coupled to three other spins as depicted in Fig. 2. The renormalized coupling between two spins in the cluster, say, spins 2 and 5, is given by
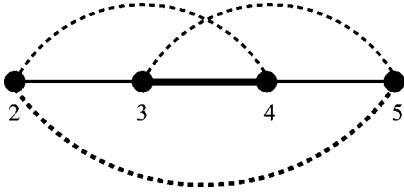
FIG. 2. The most complicated structure of a four-spin cluster where a given spin is coupled to the other three spins.

$$\tilde{J}_{25}=J_{25}+\frac{1}{2J_{34}}(J_{23}J_{45}+J_{24}J_{35}-J_{23}J_{35}-J_{24}J_{45})$$

$$=J_{25}+\frac{1}{2J_{34}}(J_{23}-J_{24})(J_{45}-J_{35}),\qquad(4)$$

where $\tilde{J}_{ij}$ is the *renormalized* coupling between S_i and S_j, and J_{ij} is the *original* bond between S_i and S_j. Examining Eq. (4), we can see that some of the contributions to the renormalized coupling from second-order processes are *ferromagnetic*. The overall sign of the total interaction between the second and fifth spins will be determined by the relative strength between the antiferromagnetic nearest-neighbor and next-nearest-neighbor bonds. In general if the nnn couplings are very weak compared to the nn couplings then the ferromagnetic interactions will not appear. This is quite different from what we found in the study of the ladder where effective ferromagnetic interactions appear as soon as the RG is applied to the system. Due to the possibility of the appearance of ferromagnetic couplings at some step of the RG, it is necessary to generalize the RG procedure to include arbitrary spin sizes and coupling signs. The discussion on how this is done has been spelled out in great detail in our earlier work on spin ladders.[15] We carry out the numerical calculation using the rules described in previous paragraphs and present the results in the next section.

III. NUMERICAL RESULTS

We present numerical results for spin chains with nn and nnn interactions with the total number of spins up to 60 000. We search for the bond with the largest gap, Δ_0, which is defined as the gap between the ground state and the first excited state, decimate it, and calculate the effective interactions among the remaining spins. The procedure is repeated until the number of spins left is about 1% of the original number of spins in the system. We use 100 samples and take the disorder average over all these samples in all our calculations. The nearest-neighbor bonds are chosen to be distributed randomly according to the power-law probability distribution

$$P_{nn}(J_i)=(1-\alpha)J_i^{-\alpha},\quad 0<J_i<1,\qquad(5)$$

where the power-law exponent $\alpha<1$ parametrizes the randomness strength; the larger the α, the stronger the randomness. The reason for choosing a power-law form is because for the random spin-$\frac{1}{2}$ chain, the fixed-point distribution is known to be in the power-law form. So by choosing initial

distributions in the power-law form, we expect to start closer to the fixed point and hence reduce the necessity to use a larger system size.

We consider two different ways of generating the nnn bonds. First we consider nnn bonds which are completely correlated with the nn bonds, where next-nearest-neighbor bond K_i is determined from the nn bonds through the following relation:

$$K_i=\Lambda\frac{J_iJ_{i+1}}{\Omega_0},\qquad(6)$$

where Λ is a parameter introduced to control the strength of next-nearest-neighbor interactions and Ω_0 is the cutoff of the initial nearest-neighbor bonds distribution, which is 1. In the limit $\Lambda\to0$, the AF spin-$\frac{1}{2}$ chain with nearest-neighbor interactions only is recovered. Equation (6) comes from the following consideration. The interactions between two spins come from the overlap integral of the electron wave functions which are bound to the atoms sitting on the lattice sites. In general, the wave function decays exponentially at large distances, and so does the overlap integral. Let us consider three electrons sitting on different lattice sites labeled 1, 2, and 3. For two electrons separated by a distance R, the typical interaction would have the form $J\sim e^{-R/a}$, where a is a length scale of order of the size of the wave function. Based on this picture, the interaction between the first and third spins, which is basically the overlap integral between the first and third spins, can be written as $J\sim e^{-(R_3-R_1)/a}$, where R_3 and R_1 are measured with respect to some reference point. This relation can be rewritten as

$$J\sim e^{-(R_3-R_2)/a}e^{-(R_2-R_1)/a}\propto J_2J_1,\qquad(7)$$

where J_i is the overlap integral between S_i and S_{i+1}. Hence, it is reasonable to model the correlation as the product of two nearest-neighbor bonds as shown in Eq. (6). We focus mostly on this type of further-neighbor coupling, and unless stated otherwise, the results presented below are for this type of further-neighbor coupling. For comparison, we have also studied cases in which the nnn couplings are an uncorrelated case with the nn couplings, i.e., the nnn bonds are distributed randomly in the system, independent of the distribution of the nn bonds. We choose the distribution to be in a power-law form with the same exponent, but a different cutoff Λ:

$$P_{nnn}(K_i)=\frac{1-\alpha}{\Lambda^{1-\alpha}}K_i^{-\alpha},\quad 0<K_i<\Lambda.\qquad(8)$$

Again Λ parametrizes the strength of nnn couplings. As we see later in the paper, while the topology of the phase diagrams is the same for these two cases, there is huge quantitative differences in the position of the phase boundary.

As we carry out the RSRG method numerically, we monitor the appearance and proliferation of large effective spins in the system. We plot the sample-averaged fraction of spins larger than one-half as a function of energy scale, Δ_0, in Fig. 3. The left panel of Fig. 3 shows how the formation of large effective spins evolves as the energy scale, Δ_0, is lowered by fixing $\alpha=0$ and varying nnn bond strength controlled by Λ,
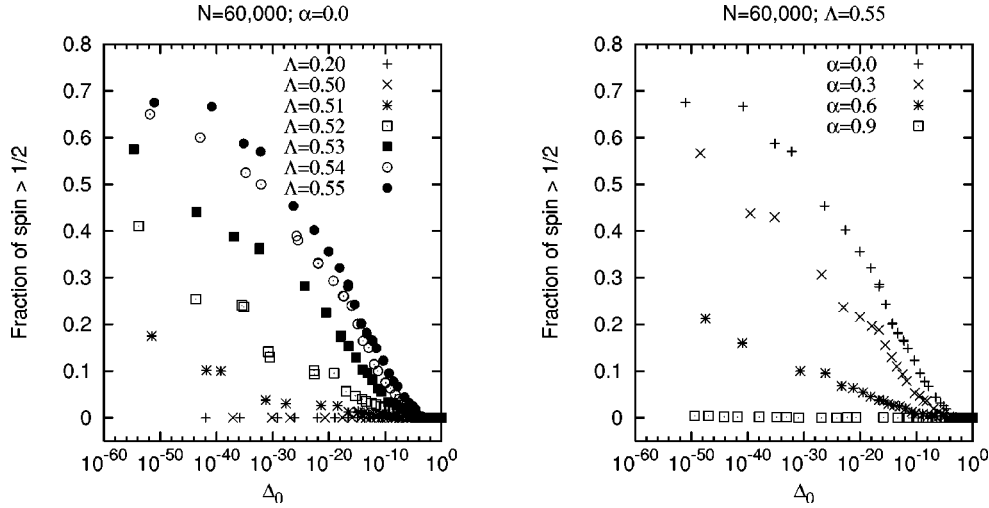
FIG. 3. The sample-averaged fraction of spins larger than one-half as a function of energy scale, Δ_0. The error bars are about the size of the data points shown in the figure. The left panel shows how the fraction of spins larger than one-half for $\alpha=0.0$ changes as Λ is varied and the right panel shows the change for $\Lambda=0.55$ as α is varied. Both are calculated for N=60 000. Strong enough next-nearest-neighbor interactions will drive the system into a new phase controlled by large effective spins. All calculations are done with correlated next-nearest-neighbor bonds given in Eq. (6).

while the right panel shows the change by fixing $\Lambda=0.55$ and varying α. Let us analyze the left panel of Fig. 3. It is very clear that, for fixed α, different antiferromagnetic nnn bond strength will lead to different scenarios in the low-energy limit. For weak enough Λ (in the regime where $\Lambda<0.5$) we do not find spin sizes other than one-half; not only do we never find any spin larger than one-half but also we never find any ferromagnetic bonds in this regime. The situation drastically changes when we tune the strength of anti-ferromagnetic nnn bonds up to 0.55 where we can see clearly that large effective spins dominate in the low-energy limit and drive the system into a new phase. This can be understood in the following way. For weak enough nnn bonds, these interactions are always suppressed by the presence of nn bonds. We have explained in Eq. (4) that the ferromagnetic bond will appear if the nnn bonds are strong enough to overcome the nn bonds. Apparently for $\Lambda<0.5$, the nnn bonds are too weak to compete with nn bonds so we never see the emergence of ferromagnetic interactions in the system. On the other hand, for $\Lambda>0.55$, the antiferromagnetic nnn bonds are strong enough to overcome the nn bonds and allow the appearance of ferromagnetic bonds which in turn will drive the system into a new phase controlled by large effective spins.

The right panel of Fig. 3 shows another study of how large effective spins appear in the system by varying the disorder strength α for fixed $\Lambda=0.55$. We find that the formation of large effective spins is suppressed as the bond disorder gets stronger. This also has a simple explanation. With increasing bond disorder strength, the probability of finding weak nn bonds increases. This will give us even weaker nnn bonds because of the correlation between a next-nearest-neighbor bond with two nearest-neighbor bonds, as given by Eq. (6). These weak nnn bonds cannot compete with the nn bonds which in turn will suppress the formation of ferromagnetic bonds in the system. Based on this view, we

can understand why large effective spins are more difficult to form in the regime where the bond disorder is strong. So for strong enough bond disorder, no ferromagnetic bonds will appear due to the fact that nnn bonds cannot compete with nn bonds and the system will remain in the random singlet (RS) phase.

The appearance of a new phase can also be deduced from plotting sample-averaged χT as a function of temperature where the temperature is associated with the energy scale, Δ_0. We plot this in Fig. 4 where in the left panel α is fixed and Λ is varied, whereas in the right panel Λ is fixed and α is varied. χT in the RS phase is well known to be given by $1/\ln^2 T$. For fixed $\alpha=0$, we can see increasing deviations from $1/\ln^2 T$ with increasing strength for Λ which gives us a clear indication that the system is driven away from the RS phase; for $\lambda>0.5$ instead of falling as $1/\ln^2 T$, χT appears to approach a constant in the low-T limit. The explanation for this behavior is similar to the discussion in the previous paragraph. Strong enough nnn bonds will allow the appearance of ferromagnetic bonds which in turn form large effective spins in the low-energy limit. These strongly correlated effective spins govern the susceptibility of the system at low temperature. The susceptibility in this phase has a different origin from the susceptibility for the RS phase where the contribution comes from the undecimated half spins. The same situation is encountered when Λ is fixed and α is varied, as shown in the right panel of Fig. 4. The deviations are more significant for small α. This is consistent with our discussion in the previous paragraph that for strong enough bond disorder, the system remains in the RS phase because the overall strength of nnn bonds is much weaker than that of nn bonds. This is indeed what we see in our numerical results, that χT for bigger $\alpha(>0.6)$ is closer to the value for the RS phase $1/\ln^2 T$.

We have established that there exist two phases in the system. The transition from one phase to another is con-
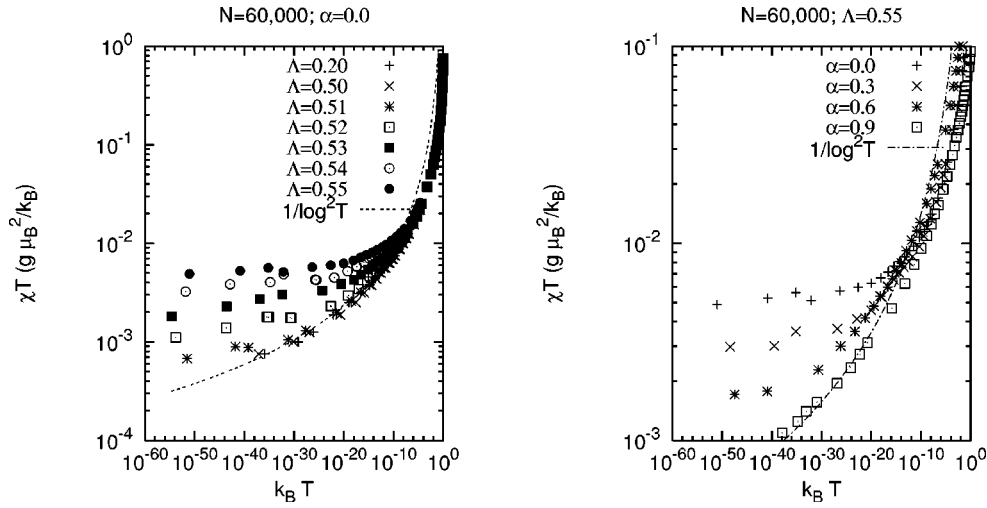
FIG. 4. The sample-averaged χT as a function of parameters of the model, α and Λ. The error bars are about the size of the data points. For strong enough correlated next-nearest-neighbor interactions, given in Eq. (6), the susceptibilities behave differently from $1/T \ln^2 T$. The contribution to the susceptibilities comes from large effective spins formed at low temperature.

trolled by the strength of bond disorder α and the strength of nnn bonds Λ. For $\alpha = 0$ and $\Lambda < 0.5$ the system remains in the RS phase while for $\Lambda > 0.55$ the system is driven into the new phase. We have already seen that the new phase is controlled by large effective spins in the low-energy limit. Is there any other parameter we can use to study the nature of the new phase? We address this question by studying the ratio of nn bond strength to nnn bond strength in the two phases, as shown in Fig. 5. It is found that on either side of the phase boundary, nearest-neighbor bonds always dominate further-neighbor bonds.[33] Now we have a more complete

picture of the new phase found in the system. The new phase is controlled by large effective spins in the low-energy limit and the dominant interactions come from the nearest-neighbor bonds only. These nearest-neighbor interactions consist of both antiferromagnetic and ferromagnetic bonds. These results suggest that in the low-energy limit, spin chains with antiferromagnetic nn and sufficiently strong nnn interactions behave just like random antiferromagnetic-ferromagnetic spin chains, including a Curie susceptibility discussed earlier. This brings us to the conclusion that the new phase found in the system we are studying is the same
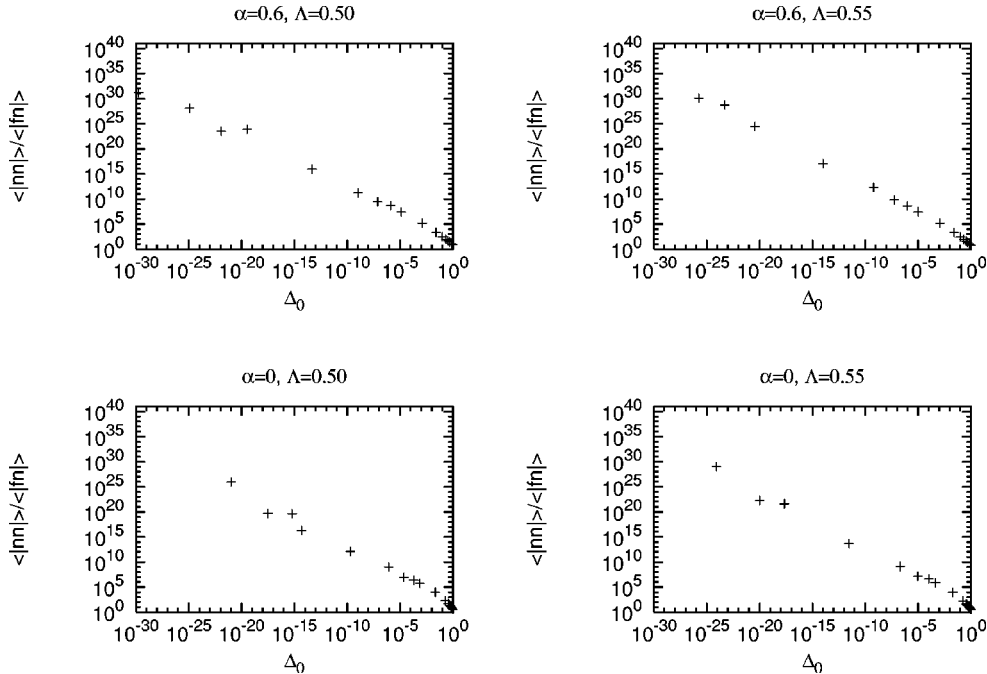

FIG. 5. The sample-averaged ratio of the strength of the nearest-neighbor bonds to the strength of the bonds that are beyond nearest neighbor as a function of energy scale. It is clear from the plot that in either side of the phase, the interactions are dominated by nearest-neighbor bonds only. We use the correlated next-nearest-neighbor interactions defined in Eq. (6).
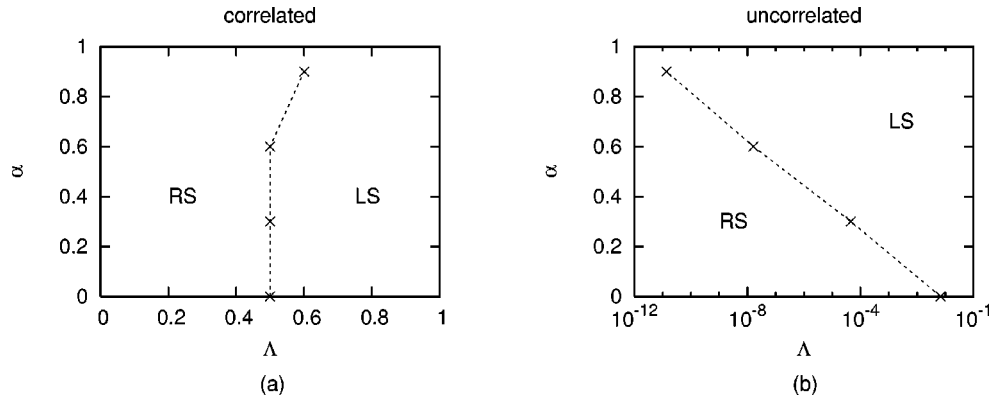
FIG. 6. (a) The numerically determined phase diagram for spin chains with competing interactions between nearest-neighbor and next-nearest-neighbor interactions. The nnn interactions are correlated with the nn interactions (see text). (b) The numerically determined phase diagram for spin chains with uncorrelated nnn interactions. In both cases α denotes the strength of the bond randomness and Λ represents the strength of the next-nearest-neighbor interactions. The crosses in both figures represent numerical calculations. The dashed lines are drawn by connecting the data points to illustrate the phase boundary more clearly.

as the large spin phase found in the random antiferromagnetic-ferromagnetic spin chains in the low-energy limit. The numerically determined phase diagram for spin chains with random antiferromagnetic nn and nnn bonds is shown in Fig. 6.

The left panel of this figure shows the phase diagram for the correlated next-nearest-neighbor bonds as given by Eq. (6) whereas the right panel shows the diagram for uncorrelated next-nearest-neighbor bonds. In both cases we find that the system supports only two phases, which are the random singlet phase and the large spin phase. There are some differences in the phase boundaries in these two cases. First, the trend on how the phase boundaries change as we vary α and Λ is different for the correlated and uncorrelated next-nearest-neighbor bonds. For the correlated case, Λ stays constant as we increase α from 0 to 0.6 and tends to increase for α larger than 0.6. For the uncorrelated one, Λ decreases with increasing α. Secondly, the magnitude of critical Λ for the uncorrelated nnn couplings is much smaller than that for the correlated case, by as much as ten orders of magnitude for α close to 1.

We believe that these differences can be understood as follows. For the uncorrelated case we assign a probability distribution function for the nnn bonds whose cutoff is determined by Λ, and the bonds are generated independent of the configuration of the nn bonds. Although in general the strength of the nnn bonds is much weaker than that of nn bonds when Λ is small, due to the uncorrelated nature of the way in which they are generated, there is a small probability that the next-nearest-neighbor coupling is actually stronger than the nearest-neighbor one in some regions of the system. As we have explained earlier in the text, the overall sign of the total interaction generated by the RG between two spins depends heavily on the relative strength of the antiferromagnetic nn and nnn bonds; thus such rare events can lead to the generation of ferromagnetic bonds, which in turn may proliferate as the energy scale lowers. In the correlated case, on the other hand, such rare events are greatly suppressed by the correlation between nn and nnn bonds. We also know that α parametrizes the width of the distribution; for a given Λ in

the uncorrelated case, the bigger the α is, the wider the distributions for both the nn and nnn bond distributions, thus the larger the probability of the rare events discussed above, and the more likely ferromagnetic couplings get generated. On the other hand this effect is again suppressed for the case of correlated nnn bonds, due to the way in which we parametrize their strength; the larger the α, the smaller the overall strength of the nnn bonds due to the way in which they are generated.

As discussed earlier, the appearance of effective ferromagnetic couplings is a consequence of competition between nearest- and further-neighbor couplings, or frustration. We have also studied spin chains with further-neighbor interactions that do *not* introduce frustration to the system. This is done by introducing ferromagnetic next-nearest-neighbor bonds or antiferromagnetic third-nearest-neighbor bonds. The ferromagnetic next-nearest-neighbor bonds and antiferromagnetic third-nearest-neighbor bonds are generated in the way discussed at the beginning of this section, i.e., the bonds are generated through Eq. (6). We present our results for this particular system in Fig. 7.

The upper panels of Fig. 7 show the sample-averaged plot of the strength of nearest-neighbor interactions compared to the strength of further-neighbor interactions and the fraction of spins with sizes larger than one-half as a function of the energy scale, Δ_0, for the system with ferromagnetic next-nearest-neighbor bonds. We choose to fix $\alpha = 0$ and to vary Λ to see how the ratio changes as the energy scale is lowered. We find that the nearest-neighbor interactions always dominate over further-neighbor interactions at all energy scales. The evolution of the spin sizes as the energy scale is lowered is also studied here. The result shows that no spin having a size larger than one-half is found in the system. Based on these results we conclude that the presence of ferromagnetic next-nearest-neighbor bonds does not drive the system into a new phase. The couplings are dominated by antiferromagnetic bonds which suppress the formation of effective spins larger than one-half at low energy. In the low-energy limit the system stays in the RS phase. The lower
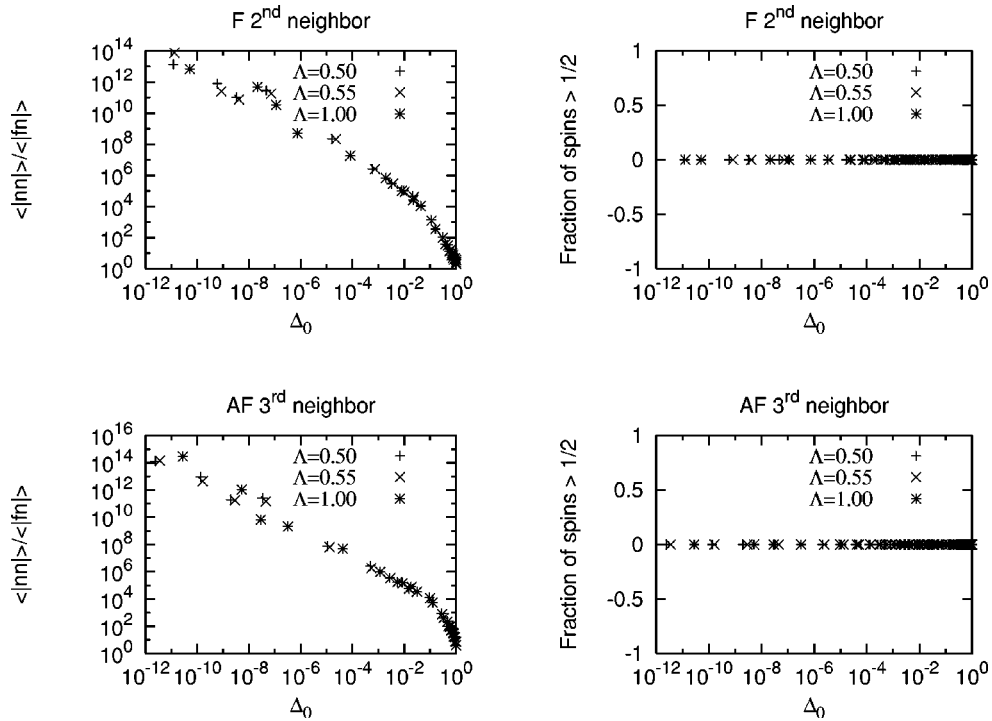
FIG. 7. The sample-averaged ratio of the strength of the nearest-neighbor bonds to the strength of the bonds that are beyond nearest neighbor and the fraction of spins larger than one-half as a function of the energy scale for the model with no frustration introduced into the system. Two types of interactions which do not generate frustration, i.e., ferromagnetic second neighbors and antiferromagnetic third neighbors, are introduced into the system. The upper two panels show the calculation for a model in which ferromagnetic second-neighbor interactions are introduced into the system while the lower two panels show the calculations for antiferromagnetic third-neighbor interactions. All graphs are calculated for $\alpha = 0$ but with varying Λ. It is clear from the plot that the interactions are dominated by nearest-neighbor bonds only, regardless of the value of Λ, and there is no formation of effective spins whose sizes are larger than one-half.

panels of Fig. 7 show the plot of the ratio between the strength of nearest- and further-neighbor bonds and the fraction of spins with sizes larger than one-half as a function of the energy scale, Δ_0, for the system with antiferromagnetic third-nearest-neighbor bonds. We also fixed $\alpha = 0$ and vary Λ for this case. The results are the same for those with ferromagnetic next-nearest-neighbor bonds. These results give us a strong indication that the system stays in the RS phase. We can thus conclude that nonfrustrating further-neighbor bonds act as irrelevant perturbations in the low-energy limit, and hence the system stays in the RS phase.

IV. SUMMARY AND DISCUSSION

In this paper we have used the real-space renormalization-group method to study random antiferromagnetic spin-$\frac{1}{2}$ chains, with both nearest- and further-neighbor interactions. We find that the system supports two phases, the random singlet phase and the large spin phase. The latter is only stabilized by sufficiently strong further-neighbor couplings that compete with the nearest-neighbor couplings, so that there is frustration in the system.

The real-space renormalization-group procedure is quantitatively accurate only when the initial distributions of the couplings are broad. We believe, however, that our conclusion remains valid even if the initial distribution of couplings

is not broad. In the case of nearest-neighbor coupling only, Doty and Fisher[34] showed that weak bond randomness is a relevant perturbation that immediately destabilizes the Luttinger liquid fixed point that describes the gapless phase of the pure chain, and bond randomness *grows* as the energy scale lowers, eventually bringing the system to the random singlet fixed point. Their arguments remain valid even in the presence of further-neighbor couplings, as long as they are not strong enough to destabilize the gapless phase in the absence of bond randomness. On the other hand when they are strong enough to put the pure system in the gapped phase with spontaneous dimerization, one of us[35] showed that the dimerized phase is *also unstable* against weak randomness, since randomness nucleates solitons and destroys spontaneous dimerization; the low-energy degrees of freedom are the half spins carried by the solitons, with random interaction with broad distribution (due to the fluctuation of intersoliton distance, etc.). Depending on whether the coupling between these spins is purely AF or both F and AF, the systems can be in either one of the two phases we find here. We thus conclude that these are the only two phases the system supports in the presence of any amount of bond randomness.[36]

The frustration-induced ferromagnetic coupling and the resultant large spin formation has been discussed in a different context.[10] In that work Yang and Bhatt studied spin-1 chains with random AF nearest-neighbor bonds, with both quadratic and biquadratic couplings on each bond. It was

shown that even though overall each individual bond is AF, as long as in some of the bonds the quadratic and biquadratic couplings have opposite tendencies (i.e., one is AF and the other F), effective ferromagnetic couplings may be generated at low energy, and the large spin phase stabilized. In this case the bonds are frustrated due to the competition between quadratic and biquadratic couplings on the *same* bond. Thus the phenomenon of frustration-induced large spin formation, although never seen in pure systems, may actually be rather generic in disordered systems.

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