Highly effective way in five-axis sculptured surfaces machining using flat-end cutter

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Abstract: This paper applied the concept of “contact” in Differential Geometry into the machining of the sculptured surface. I presented the contact principle of the machining of complicated surfaces, using the circumference circle of the cylindrical cutter to sweep the curved surface instead of ball-end mill. This is highly effective method. In this paper an theory for machining complicated surface is presented. By using a flat-end mill instead of ball-end mill, and adjusting the axis relate to the surface, the two surfaces, The swept surface and the required surface, has the same curvature, up to as high as 3th order.

Keywords: flat-end mill; Sculptured surface; Contact

1 Introduction

Sculptured surface are used in a wide variety of application in the automotive, aerospace and shipbuilding industries, such as turbine blades, impeller and marine propellers. The original design concept is often embodied in a physical model, perhaps sculpted from clay by a skilled artisan, from which measurement data is scanned. Surfaces are fitted to the scanned data, and a mathematically precise description is then available for subsequent steps in the product-design process. The surfaces are usually mathematically defined by parametric forms such as the well known Bezier, B-spleen and NURBS type. Typically, a design is composed of number of parametric surface patches.

Despite advanced machining technology, a 3D sculptured surface is generally machined by a ball end mill in a 3-axis machine tool.. It is subsequently round and polished to create the required surface finish. Unfortunately, the finishing process is time-consuming, and it decreases the form accuracy. Researchers have produced various milling processes to reduce the milling and polishing time and to improve the accuracy of the finished surface. Miyazawa and Takada[1] predicted the surface roughness when a curved surface is machined by a ball end mill, and suggested a micro milling process in which the path interval is very small. In this case, however, high productivity cannot be achieved, because the cutting edge cannot run at its highest cutting speed. Moreover, since the cutting speed decreases to zero near the axis of rotation, the cutter literally rubs away material during the machining of a low-curvature surface or downward machining[2,3] As a result, it is not possible to achieve a high productivity and a good surface finish with this method. To overcome these difficulties, many researchers have used flat end mill or a face mill in multi-axis machine tools for the machining of 3D surfaces. In these cases, the material is always cut at the periphery of the cutter at maximum speed. As a result, productivity and the surface roughness improve when parts with low curvature, such as turbine blades, propellers and aircraft-structure components, are machined.[4,5] However, a multi-axis machine tool has many disadvantages, including expensive machinery and an expensive controller, difficulty in generating CL data and NC code.

In theory, the 5-axis machining of sculptured surfaces offers many advantages over 3-axis machining, including faster metal-removal rate, improved surface finish, and the elimination of hand finishing. In practice, 5-axis machining comes from a number of drawbacks, which are mostly related to gouging avoidance. Susan X..Li and Robert B.Jerard[6] presented the method to generate the gouge-free, nonisoparametric 5-axis tool path across composite surface. Their research can decrease the time required to progress from the design data to the machining of the dies and moulds used in product manufacture.

The method to generating “optimal cutter-location data for 5-axis NC contour milling from given cutter-contact data is very impotent. A few 5-axis machining software systems have been reported, and most of the commercial CAD/CAM systems supported, 5-axis machining capabilities, but very little has been published concerning machining efficiency. The two major issues in 5-axis milling are: how to generate cutter-contact path to give 2 minimum machining times, and how to generate CL data from the given cutter-contact paths to give minimum cusp heights. The first issue is addressed in Reference [7], in which the possibility of reducing machining time by fitting the tool trajectory to the surface shape . The second issue is researched by B.K.chil [8], in his paper, the

cutter-location data-optimization problem is formulated as a 2D constrained minimization problem. The cutter orientation angles consisting of the tile angle and yaw angle are used as decision variables. An analytic expression for approximate cusp heights is derived as a function of the two angles and be used as a measure of optimality.
This paper creatively applied the concept of “contact” in Differential Geometry into the machining of the sculptured surface, presented the third order contact principle of the machining of complicated surfaces, using the circumference circle of the flat-end mill (as shown in Fig.1) to sweep the curved surface instead of ball-end mill. This is the highly effective method. Compared with other references, [1-18] the theory of this paper is more restrict, going deep into the parameter of third order instead of the second order parameter, suitable to both the primary surface of analytical geometry and the Computer-generated surface of the Computation geometry, have definite procedure of calculation, and easy to solve the equation.
This paper applied the concept of “contact” in Differential Geometry into the machining of the sculptured surface, presented the 3th order contact principle of the machining of complicated surfaces, using the circumference circle of the flat-end mill (as shown in Fig.1) to sweep the curved surface instead of ball-end mill.

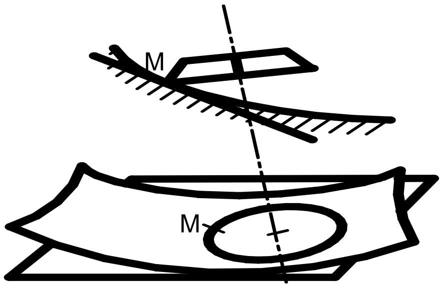

Fig.1 Contact between circle and surface

This is the highly effective method. Compared with other references, [1-18] the theory of this paper is more restrict, going deep into the parameter of 3th order instead of the second order parameter, suitable to both the primary surface of analytical geometry and the Computer-generated surface of the Computation geometry, have definite procedure of calculation, and easy to solve the equation.
This paper attempts to present a method for machining with a flat-end cutter, where the cutter meets the surface with 3rd order contact. While this would be an interesting result worthy research, The method is to establish a local coordinate system, then rewrite the problem relative to the local coordinate system, where sets up a system of implicit equations to solve for 3rd order contact, then uses the Maclaulin Series as a means of transforming parametric equations to implicit equations, gives an example, and conclude.

2 Contact between sculptured surface and circumference of circle

Suppose we have a surface

$$\Sigma:\square(x,y,z) = 0$$

And a curve

$$x = x(\theta) \quad y = y(\theta) \qquad z = z(\theta)$$

establish a function□

$$\Phi(\theta)\ \square = (x(\theta),y(\theta),z(\theta))$$

if at one point M(θ□θ_0), the difference of function $\Phi(\theta)$

$$\dot{\Phi}(\theta_0)=0$$

$$\ddot{\Phi}(\theta_0)=0\ldots\ldots$$

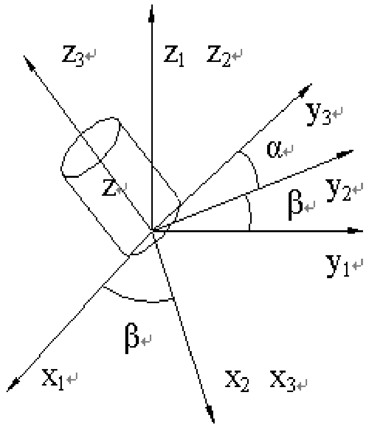

Fig.2 The coordinate system

$$\Phi^{(N)}(\theta_0)=0$$

(N=1,2 ,3……) N is the non-negative integer

while $\Phi^{(N+1)}(\theta_0) \neq 0$, then we say that, the curve and the surface are in Nth contact with each other .The point must be on the surface, and the N must be positive. The higher the order, the closer the contact is. There are several different expression of this concept in the differential geometry, but the basic ideal is the same.

3 The establishment of the coordinate system

To describe the relative motion of a cutter with respect to the work piece or the surface, we establish the following coordinate systems:

Let O=xyz be the reference frame moving together with the surface, the surface being machined can be expressed in this system as:

$$(x, y ,z)=0$$

Let $O_3=x_3y_3z_3$ be the reference frame moving together with cutting tool, as shown in Fig.2, the coordination transformation matrix :

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \square \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \\ z_C \end{bmatrix}$$

$$\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ \sin\alpha\sin\beta & \cos\alpha & -\sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \sin\alpha & \cos\alpha\cos\beta \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \\ z_C \end{bmatrix} \qquad (1)$$

The circumference circle of the milling cutter expressed in this system as a parametric curve:

$$x_3=R\sin\theta$$
$$y_3=R\cos\theta \qquad (2)$$
$$z_3=0$$

4 Machining movement equation and its solution

Transform the equation of the circumference circle of the tool into the stationary system:

$$x = R\cos\beta\sin\theta + x_C$$
$$y = R\sin\alpha\sin\beta\sin\theta + R\cos\alpha\cos\theta + y_C \quad (3)$$
$$z = R\cos\alpha\sin\beta\sin\theta + R\sin\alpha\cos\theta + z_C$$

When in 5-axis machining, choose a series of machined point (x_M, y_M , z_M) from the surface in proper order, the coordination of the corresponding point on the circle are:

$$x_0 = x_M \qquad y_0 = y_M \quad z_0 = z_M \quad (4)$$

or:

$$R\cos\beta\sin\theta + x_C = x_M$$
$$R\sin\alpha\sin\beta\sin\theta + R\cos\alpha\cos\theta + y_C = y_M$$
$$R\cos\alpha\sin\beta\sin\theta + R\sin\alpha\cos\theta + z_C = z_M$$

Since the points (x_M, y_M , z_M) is the points on the surface F(x,y,z)=0, so that the equation F(x, y , z)=0 can be satisfied naturally. The 3th order contact condition are:

$$\dot{F}(x_0, y_0, z_0)|_{\theta \mathfrak{s} \theta_0} = 0$$
(5)

$$\ddot{F}(x_0, y_0, z_0)|_{\theta \mathfrak{s} \theta_0} = 0$$
(6)

$$\dddot{F}(x_0, y_0, z_0)|_{\theta \mathfrak{s} \theta_0} = 0$$
(7)

in equations (4)-(7) there are six equations, from which we can solve out six unknown quantities : x_c ,y_c ,z_c ,θ,α and β. While in machining we need x_c ,y_c ,z_c ,αand β(five axis) simultaneity NC control.

When 4.5-axis NC control system is used, we do not know the point being machined before hand. In every travel, let z_c to be constant, we have

$$\square(x_0 \square y_0 \square z_0) = 0$$
(8)

$$\dot{F}(x_0, y_0, z_0) = 0$$
(9)

$$\ddot{F}(x_0, y_0, z_0) = 0 \quad (10)$$

$$\dddot{F}(x_0, y_0, z_0) = 0 \quad (11)$$

in four equations (8)-(11) there are x_c, y_c,θ, α and βfive unknown quantities, among which one is the parametrical variation, let, say, x_c to be the parametrical variation, the equations are solvable. When machining, we need x_c, y_c, α, β(four coordination) simultaneity movement. After a travel finished, change z_c, then calculate the next travel.

5 Several other forms of solution

In engineering, most sculptured surfaces are expressed in the form of parameters:

$$x = x(u,v);\ y = y(u,v);\ z = z(u,v)$$

How to transform it into the implicit equation? We can use the concept of approximates surface. Assume that there is a surface, the equation of which is:

$$z = f(x,y)$$

use the Maclaulin Series, the function z=f(x,y) can be developed in its adjacent region of point M□

$$z = f(0,0) + f_x x + f_y y + \frac{1}{2} f_{xx} x^2 + f_{xy} xy + \frac{1}{2} f_{yy} y^2 + \ldots$$

Omit the items of higher than third order, since f_x□f_y□0, f_{xx}□κ_x, f_{xy}□τ, f_{yy}□κ_y, here κ_x, κ_y is the normal curvature along the x and y direction respectively, andτis the deflect curvature, so that

$$z = \frac{1}{2} f_{xx}\, x^2 + f_{xy}\, xy + \frac{1}{2} f_{yy}\, y^2$$

and $$F(x, y, z) = z - \frac{1}{2} f_{xx}\, x^2 - f_{xy}\, xy - \frac{1}{2} f_{yy}\, y^2$$

(12)

There is a drawback using the circumference circle of the cylindrical cutter, it is easy for cutting edgy to wear out. In most occasion, the cutter is substituted by the annular cutter. Since the central line of the annular cutter is still a circle, so that we can use the following method: first, generate an offset surface of the surface being machined. It is called assumed surface. According to the properties of the offset conjugate surface, when the assumed surface l contact with the central circle of the annular cuter, the real surface contact with the annular cutter. As shown in Fig.3.

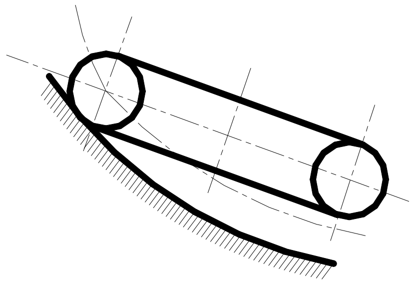

Fig.3 Use the annual cutter

6 Example

There is a curved surface as showed in Fig.4. The equation of the surface is:

$$x = u\cos v - \left(a + u\sqrt{(a+u)/(a-u)}\right)\sin v$$
$$y = u\sin v + \left(a + u\sqrt{(a+u)/(a-u)}\right)\cos v$$
$$z = -pv$$

a=10mm, p=100mm, The radius of the cutter R=16mm,Using the 5-axis NC machining, The calculation results of the coordination of the points being machined, the coordination of the center of the cutter, the direction vector of the axis of cutter x_M, y_M, z_M, x_C, y_C, z_C, l, m, n, is listed as following table1.

Table 1 The result of calculation

x_M	y_M	z_M	x_C	y_C	z_C	l	m	n
-1.0836	9.0281	4.0513	-5.449	23.360	5.616	-0.823	-0.0823	-0.568
2.8434	12.8091	7.8203	-1.126	24.500	11.223	-0.915	-0.0086	-0.404
5.6800	20.8212	12.7506	4.779	29.814	13.203	-0.992	0.060	-0.109
7.2685	28.2757	17.2575	8.262	37.447	13.044	-0.997	0.006	0.079
7.9884	33.8884	20.7825	8.025	43.410	12.859	-0.993	-0.097	0.074

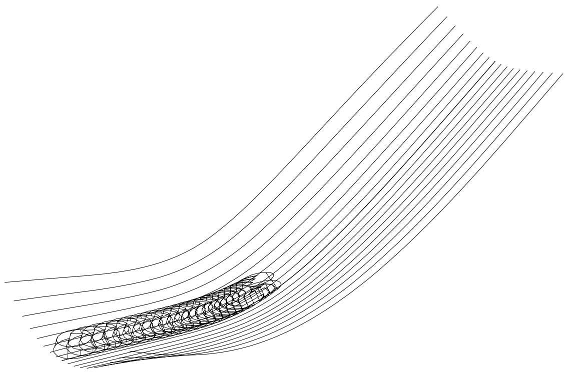

Fig.4. Illustration of the computed posture

With the method of third order contact mentioned above, the number of tool path are 5, while the ball-end miller is used, the times of travels are 53. So that with our advanced method, the times of travel can be reduced dramatically.

7 CONCLUSION

This paper opened a new field in machining the complicated surface, based on a relatively more rigid mathematical basis. The theory presented here is relatively more systematical. Since the lack of theoretical guidance, in the former research, people have to try in machining many times. Such case will be changed. The movement of the cutter determined by this method is definite, and the residual is the smallest while the times of travel is the fewest. The criterion is simple, the calculation is easy.

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