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Crease Formation in the Processing of Thin Web Material

A. Dixon and W.Y.D. Yuen

BlueScope Steel Research, P.O. Box 202, Port Kembla, 2505, Australia

Abstract: A mathematical model is developed to describe the conditions for buckling of steel strip between transport rolls due to strip camber, together with conditions necessary for the subsequent "ironing-in" of the buckle as it passes over the downstream roll. For a permanent crease to form, the buckle must be sufficiently stable so that it is prohibited by friction to spread laterally, and the stresses from the buckle defect must be large enough for plastic deformation to occur as it travels over the downstream roll. Once the conditions to produce a permanent crease are known they can be avoided in plant operations.

Keywords: buckling, creasing, elastica, thin strip, wide strip, web mechanics.

1 Introduction

Rolls are commonly used to transport and guide steel strip through processing lines. Strip defects known as "tension crease" occasionally occur in these production units, being a narrow deformation typically 30 mm wide but extending many metres along the length of the strip as shown in Figure 1. The origin of these defects is not fully understood at present, but current research [1,2] indicates that the "tension crease" is formed from a strip buckle upstream of a transport roll. The buckle is then permanently "ironed in" as the buckle passes over the roll. For a tension crease to occur, then:

- There must be sufficient transverse compressive stress upstream of the roll to form buckles. This may occur from roll misalignment [1,2] or, as is discussed here, from strip camber.
- There must be sufficient frictional stress between the strip and roll to prevent the buckle from spreading transversely, and be "ironed in" as it passes over the roll.

Good *et al* [1] and Hashimoto [2] investigated the creation of a tension crease for the cases of polyester film and coated paper-web respectively. Tension crease was experimentally produced by sufficient misalignment between the processing direction and a downstream roll. "Ripples" were initially observed upstream [2] which increased in magnitude with roll misalignment. These ripples were explained by an induced compressive stress in the transverse direction of the strip due to the roll misalignment. A model for the prediction of the onset of these buckles for misaligned rolls was developed [1,2].

An alternative mechanism for buckling is investigated here for the case of steel strip where the transverse compressive stresses is caused by camber, a term for strip curvature arising from one strip edge being longer than the other. The stability of the buckle ahead of the transport roll is then

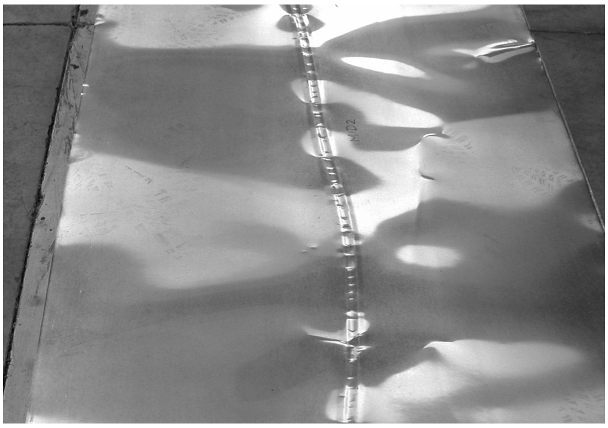


Figure 1. "Tension crease" defect in steel strip.

analysed as it passes over the roll. The buckle is modelled by the Elastica Equation, which describes the out-of-plane bending of a strip for moderate to large curvatures. Under low frictional stresses between the strip and roll, the buckle will spread out by transverse slipping as it passes over the rolls and no crease will form. Under high frictional stresses, either from large friction coefficients or high strip tensions (so that the strip tightly grips the roll), the buckle will not spread. This buckle will eventually turn into a crease if plastic deformation occurs as the strip passes over the transport roll. Predictions from this theory compare favourably with empirical guidelines used by operators. In particular tension crease increases with:

1. Lower strip thickness.
2. Increased strip width.
3. Lower strip yield stress.
4. Increased strip tensions.
5. Increased roll-strip friction.

2 Determining Camber Tolerances to Avoid Upstream Strip Buckling

A strip typically has a small degree of camber that may give rise to in-plane shear stress between rolls. For sufficient camber buckling may occur across the strip width. Consider a strip section with camber, as shown in Figure 2, with the strip exiting the upstream roll at an angle normal to the roll axis. The strip edges are normally described by a broad circular arc in an unrestrained state, but as it is fed into the downstream roll then, providing there are sufficient strip tension and no slippage between the strip and roll, the strip is bent such that it must enter the second roll at an angle perpendicular to the roll axis [1].

The resulting strip shearing stress can be approximately estimated using beam theory. Let us assume that the variation in the edge position $y(x)$ along the strip length for the unstressed cambered strip is approximately given by (see Figure 2)

$$y \approx \tfrac{1}{2}\kappa x^2, \quad \text{with the angle of the edge to the processing direction} \quad \phi \approx \frac{dy}{dx} = \kappa x \tag{1}$$

where κ is the curvature of the cambered strip. In addition, a straight strip that is bent due to a transverse force F at the second roll has an edge profile $y(x)$, and angle of the edge to the processing direction θ, given by [3]

$$y = -\frac{Fx^2}{6EI}(3l - x), \qquad \theta = \frac{dy}{dx} = -\frac{Fx}{2EI}(2l - x) \tag{2}$$

where I is the moment of inertia of the strip being bent in the transverse direction. A strip bending solution for cambered strip such that the strip enters perpendicularly to the second roll can be obtained by superposition with the angles from camber and strip bending balancing as shown in Figure 2b:

$$\theta + \phi = 0, \text{ at } x = l, \text{ or } F = 2EI\kappa / l. \tag{3}$$

The corresponding shear stress in the strip of thickness h and width W can be approximated by;

$$\tau \approx \frac{F}{hW} = \frac{2E\kappa I_y}{lWh} = \frac{E}{6}\frac{W^2\kappa}{l} \quad \text{since} \quad I = \frac{W^3 h}{12}. \tag{4}$$

If it is assumed that the normal stress in the direction of the strip width σ_y is zero [1,2] then the expressions for the principal normal stresses become

$$\sigma_1 = \tfrac{1}{2}(\sigma_x) + \sqrt{\left(\tfrac{1}{2}\sigma_x\right)^2 + \tau^2} \approx \sigma_x, \qquad \sigma_2 = \tfrac{1}{2}(\sigma_x) - \sqrt{\left(\tfrac{1}{2}\sigma_x\right)^2 + \tau^2} \approx -\frac{\tau^2}{\sigma_x} \tag{5}$$

for small shear stresses compared to the applied longitudinal stress. It should be noticed that σ_2 is compressive and, for large enough magnitudes, is capable of causing strip buckling. The resulting buckle would be parallel to the direction of σ_1, being approximately at the angle of $\alpha \approx \tau / \sigma_x$ [4] for small shears compared to the longitudinal tension, see Figure 2b.

Conditions of buckling can be found using standard theory [1,5] where the strip deformation in the out-of-plane direction w is described by

$$D\nabla^4 w = h\sigma_1 \frac{\partial^2 w}{\partial X^2} + h\sigma_2 \frac{\partial w}{\partial Y^2} \quad \text{with} \quad D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{6}$$

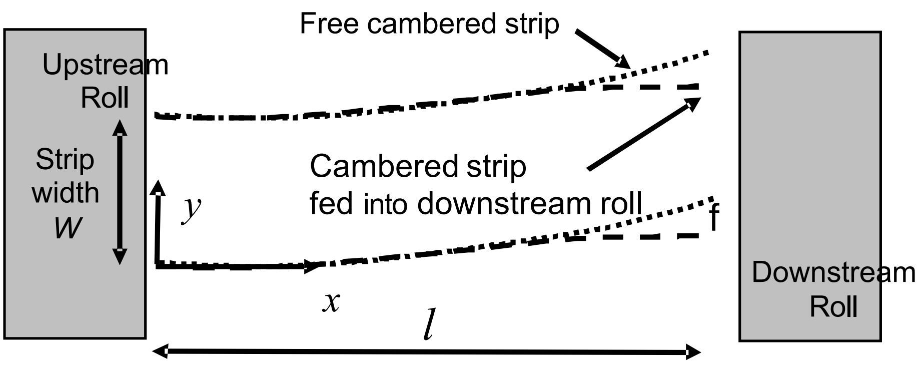
where the buckling is being described in the directions of the principal stresses (X, Y) as shown in Figure 2b, and D is the flexural rigidity with ν the Poisson's ratio.

Periodic buckles in the transverse Y direction are precursors to tension crease [2]. As we are looking for buckling solutions that are periodic across the strip width, but long in the processing direction, let us assume

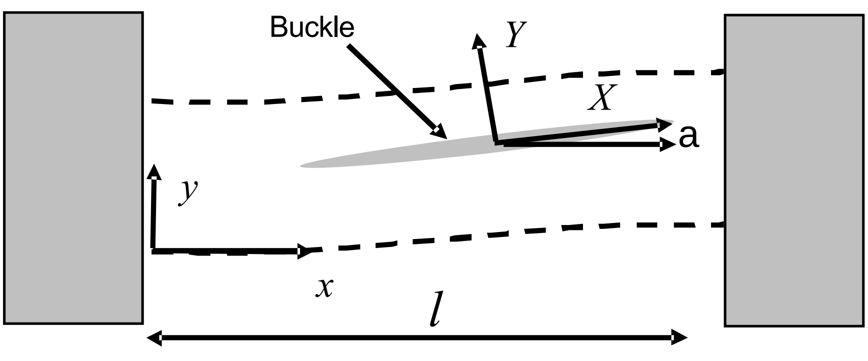
$$w(X,Y) = a_n \sin\left(\frac{\pi X}{l}\right)\sin\left(\frac{n\pi Y}{W}\right), \tag{7}$$

with n being an integer. Upon substitution into Eqn.(6) one obtains

$$-\sigma_2\left(\frac{nl}{W}\right)^2 = \sigma_e\left(1+\left(\frac{nl}{W}\right)^2\right)^2 + \sigma_1 \quad \text{with} \quad \sigma_e = \frac{\pi^2 D}{l^2}. \tag{8}$$

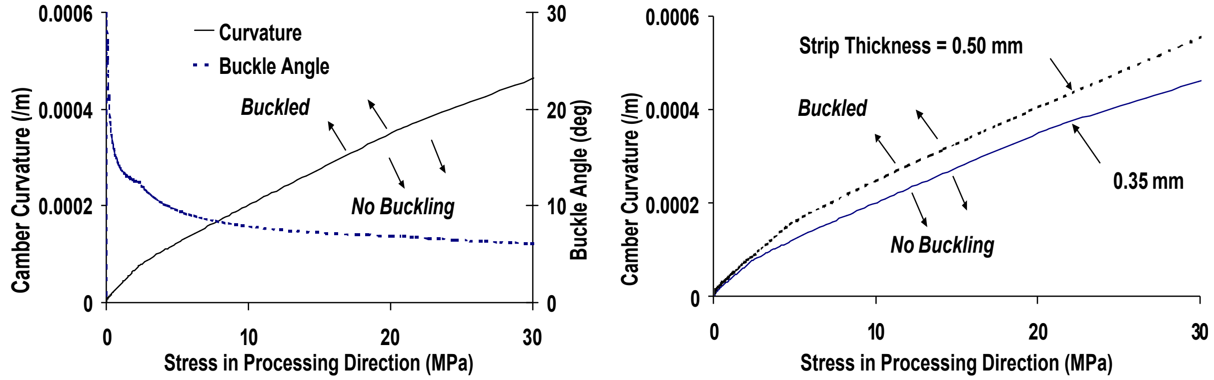


(a) Strip path between guiding rolls



(b) Formation of buckle on strip

Figure 2. Cambered strip between guiding rolls together with oblique buckle.



(a) (b)

Figure 3. (a) Predicted critical camber curvature to prevent buckling, together with the corresponding buckle angle in the processing direction. (b) Predicted camber curvature for the onset of buckling for different strip thicknesses: Strip width: 1m, distance between rolls 5.0 m.

Note that an increase in the longitudinal tensile stress $\sigma_x \approx \sigma_1$ will require a larger transverse compressive force for buckling. The value of n is chosen to minimize the necessary transverse compressive stress for buckling and leads to higher modes being chosen for higher strip tensions, giving buckles with shorter wavelengths in the width direction [1].

For a given strip tension and camber then σ_1 may be calculated, together with the value for σ_2 and the corresponding value of n for the minimum compressive force to obtain transverse buckles, from Eqn.(5) and Eqn.(8) [1]. The shear stress for buckling may then be found from Eqn.(5) and the necessary camber from Eqn.(4). These results are shown in Figure 3a where, for various applied tensions the necessary strip camber for the onset of transverse buckles is predicted, together with the angle of the buckle to the processing direction. Higher strip tensions will prevent buckling from occurring. Figure 3b shows that for thinner strip less camber is required for buckling.

3 Stability of Buckle over a Roll

Upstream buckling alone will not produce a permanent crease in the strip. For this to occur the buckle must be "ironed-in" when it passes over a roll, namely, from plastic deformation. Consider a strip, containing a localised buckle, being in contact with the roll away from the buckle. The Elastica Equation [6] describing the bending of strip from the applied moments can be approximated by:

$$D\frac{d^2\theta}{ds^2} \approx -H\theta - \frac{T_x s}{R} \tag{9}$$

for small buckle heights, where s is the distance along the strip width from the buckle centre and θ is the angle of the strip to the roll surface along its axis, as is shown in Figure 4. The term H is the horizontal component of the strip compressive force along the roll length while the second term on the right-hand side represents the downward moment into the roll from the strip curvature in the processing direction, with R being the roll radius and T_x the hoop force per unit strip width across the buckle. Away from the buckle the strip conforms to the roll so at the buckle edges:

$$s = \pm b/2 \text{ then } \theta = d\theta/ds = 0, \tag{10}$$

where b is the buckle width. For a symmetric buckle about $s = 0$ this gives the solution

$$\theta = \frac{T_x b}{2RH}\left(\frac{2s}{b} - \frac{\sin\left(\sqrt{F}s/b\right)}{\sin\left(\sqrt{F}\right)}\right), \qquad F = \frac{Hb^2}{4D}, \tag{11}$$

where F must satisfy the relation $\tan\sqrt{F}=\sqrt{F}$ for the conditions of Eqn.(10) to be satisfied. The term F is therefore not arbitrary but has discrete values corresponding to the buckling modes, with the lowest mode having $F_1 \approx 20.16$. This term relates the transverse compressive force for buckling to the strip flexural rigidity and buckle width. The strip curvature in the transverse direction κ_S and the buckle height variation w may be found from

$$\kappa_S = \frac{d\theta}{ds} \text{ and } w(s) \approx \int_{-b/2}^{s} \theta \, ds \text{ ,} \tag{12}$$

where the profiles across the buckle width for the first mode being shown in Figure 4 for a unit buckle height and normalised width. For the buckle to be stable, and not spread across the roll face, then the compressive horizontal force H must be sufficient to prevent slippage between the roll and strip. From the transverse force equilibrium equation this compressive force is maintained by friction:

$$\frac{dH}{dy} \le \mu p \quad \text{or} \quad H \le \mu p l = \mu l \sigma_x \frac{h}{R} \quad \text{as} \quad p = \sigma_x \frac{h}{R} \quad . \tag{13}$$

Here p is the pressure between the strip and roll, μ is the static friction coefficient, σ_x is the applied tension stress in the processing direction and l is the distance of the buckle from the strip edge. The greater the applied tension in the processing direction the greater the pressure between the strip and the roll. For the buckle to be stable as it travels over the roll then the frictional stress must be greater than the compressive stress necessary for buckling:

$$\mu \sigma_x l \frac{h}{R} \ge H = 4F_1 D / b^2 \tag{14}$$

for the first mode using Eqn.(11). Large friction and high applied tensions in the processing direction are therefore needed to maintain the buckled strip. The necessary conditions for the ironing-in of the buckles then become:

$$\frac{1}{\mu}\frac{E}{\sigma_x}\frac{R}{l}\left(\frac{h}{b}\right)^2 < \frac{3(1-\nu^2)}{F_1} \approx 0.135 \text{ ,} \quad \text{for steel as } D = \frac{Eh^3}{12(1-\nu^2)} \tag{15}$$

implying that thin wide strip under high tensions and high frictions are more prone to the buckles being “ironed-in” as creases when the strip passes over the roll. This is consistent with the experimental finding outlined previously. For a typical case with E = 210,000 MPa, μ = 0.2, σ_x = 20 MPa, R = 200 mm and h = 0.35 mm, Eqn.(16) predicts stable buckles for widths of the order of 100 mm. Buckles with widths of 30 mm, as seen in defects, should only occur only for abnormally high tensions or friction conditions.

Extra strip tension occurs in the buckle region as it is wrapped around the roll, as the radius of the buckle region around the roll centre will be the roll radius plus the buckle height, causing the strip buckle region to stretch for the same roll-strip contact angle. The extra tension stress in the buckle region is then:

$$\sigma_x = E\frac{w}{R}$$

where w is the buckle height. For a roll radius of 200 mm, only a buckle height of 0.5 mm is necessary to produce yielding of strip with a yield stress of 500 MPa, being consistent with the buckle heights measured after ironing-in. Lower buckle heights will be sufficient for the ironing-in of the buckle for softer strip.

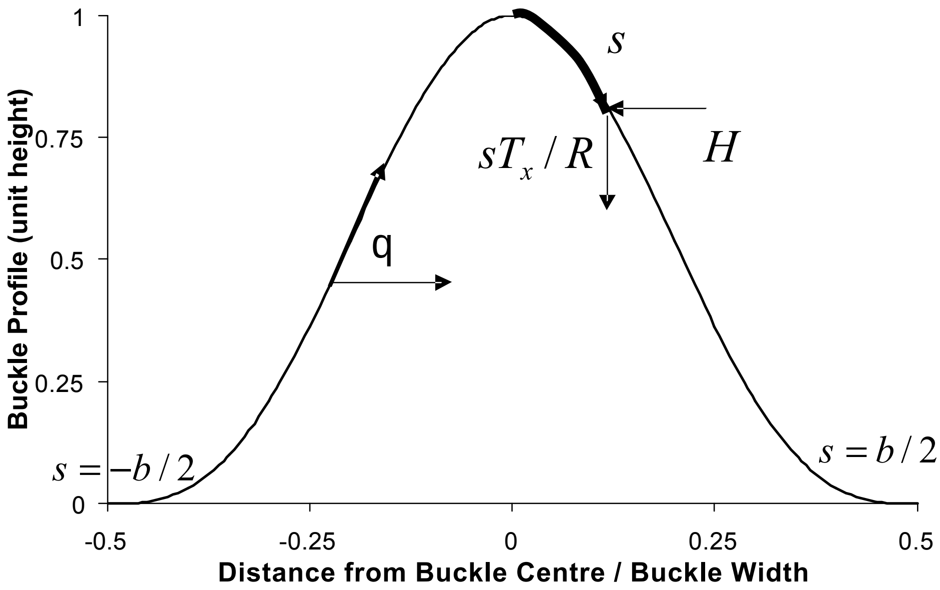


Figure 4. Co-ordinate system (s,q) for Elastica Equation under compressive horizontal force H and vertical force sT_x / R.

4 Conclusions

A mathematical model to predict line conditions for the occurrence of a strip defect known as "tension crease" has been developed. Results of this model are consistent with observations where it is predicted that tension crease may be caused by strip camber and is more likely under conditions of thin wide strip under high friction conditions, according to the simple formula given by Eqn.(15). It is also predicted that if the tension is sufficiently high such that upstream buckles do not occur then the crease may be avoided, as is shown in Figure 3.

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