5th Australasian Congress on Applied Mechanics, ACAM 2007
10-12 December 2007, Brisbane, Australia

Determination of Modal Parameters of a Half-Car Fitted with a Hydraulically Interconnected Suspension from Simulated Free decay Responses

Nong Zhang[1], Wade Smith[1], Jeku Jeyakumaran[1] and William Hu[1]

[1]*Faculty of Engineering, University of Technology, Sydney*

Abstract: This paper presents an alternative approach for determining the vibration modal parameters, in terms of natural frequencies, damping ratios and modal shapes of a roll-plane half-car fitted with a general hydraulically interconnected suspension system. The dynamic model of the system, which consists of the sprung mass and the HIS and the wheels, is formulated using the state space representation approach. The state variables describing rigid body motions of the sprung and unsprung masses are heavily coupled with those describing the dynamics of HIS fluid circuits. A numerical simulation scheme is developed to obtain the transient and free decay responses of the half car vehicle using specific initial conditions or road inputs. The obtained results are compared with those determined from the free vibration analysis of the system using the transfer matrix method. Discussions on the advantages and limitations of the presented method are also provided.

Keywords: Hydraulically interconnected suspensions, multi-body systems, dynamics of fluid circuits

1 Introduction

Fatal crashes due to vehicle rollovers have been frequently reported in recent years around the world. Four Wheel Drive vehicles (4WDs), typically having higher mass centres, are particularly vulnerable to this type of accident, with over one third of 4WD fatalities involving rollover [1]. The events leading to vehicle rollovers are complex with many factors influencing the vehicle motion. However, good suspensions can greatly reduce vehicular rollover propensity during extreme manoeuvres.

Recently, research efforts have been focused on advanced suspension systems, with a view to overcoming the inevitable compromise between ride and handling performance encountered in conventional suspensions. These advanced suspensions generally include variable or adjustable stiffness or damping parameters, most often achieved through active or semi-active means [2, 3]. However, completely passive hydraulically interconnected suspension (HIS) is also becoming increasingly popular for passenger vehicles. In addition to having the normal functionality of a conventional suspension, a HIS has advantages in providing additional stiffness in roll, bounce, pitching, articulation, or their combinations, depending on the system configuration. A typical HIS often contains several fluid circuits that link double-acting cylinders, damper valves and accumulators through pipe, hose or curved fitting elements. Kinetic H2 suspension is one example [4]. A HIS behaves nonlinearly in relation to external disturbances. Therefore, unlike a conventional suspension, a lumped mass assumption does not apply to a vehicle with a HIS because the highly pressurised fluid within the circuits is a distributed mass system. The vibration modes of bounce, roll, pitching, articulation and wheel hopping of the vehicle system can not be easily determined using the conventional multiple lumped mass, spring and damping approach. Although rich practical knowledge on HIS systems has been gained from experimental studies and applications in both racing and passenger vehicles [4-6], only recently a systematic theory and solution procedure for determining the dynamic characteristics of HIS vehicles has been presented by Smith, et al [7] and Zhang, et al [8].

This paper presents an alternative approach for determining the vibration modal parameters, in terms of natural frequencies, damping ratios and modal shapes of a roll-plane half-car fitted with a general hydraulically interconnected suspension system. The sprung mass, suspension springs, wheels and tires are modelled using the free body diagram method. The individual fluid elements, such as lines, valves and accumulators of the HIS fluid circuits are modelled using their own dynamic models. A state vector for the vehicle dynamic system is so defined that it contains all the independent displacements and velocities of the sprung mass and wheels, and the pressure and flow at all nodes (between which are various hydraulic elements) of the fluid circuits. The dynamic model of the system, which couples the sprung mass and the HIS and the wheels, is then formulated using the state space representation approach (see the details in reference [8]). The state variables describing rigid body motion are heavily coupled with those describing the dynamics of the HIS fluid circuits. The dynamic model consists of time-variant coefficient matrices, which accommodate the nonlinearities in the fluids circuits, and can be used to obtain the transient responses induced by external disturbances.

A numerical simulation scheme is developed to obtain the transient and free decay responses of the half car vehicle using specific initial conditions or road inputs. The state variable method is applied to identify the modal parameters of the half car system from the simulated time domain responses. The obtained results are compared with those determined from the free vibration analysis of the system using the transfer matrix method. Discussions on the advantages and limitations of the presented approach and the free vibration analysis method are provided.

2 A Half-Car with a Hydraulically Interconnected Suspension (HIS) System

In an effort to retain simplicity, whilst still accounting for fluid interconnections between wheel stations, a lumped-mass four-degree-of-freedom half-car model, as shown in Fig. 1, is used in this investigation. The system consists of linear tire damping and springing, linear conventional suspension springing, and a typical HIS system.

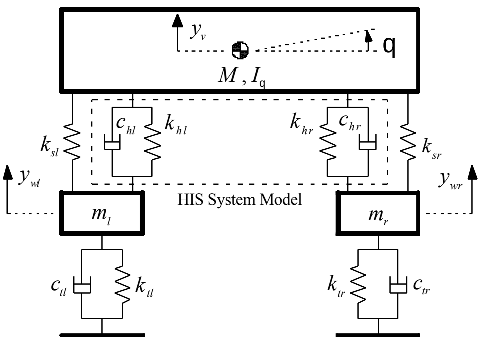

Fig. 1 Schematic of a half-car with an HIS

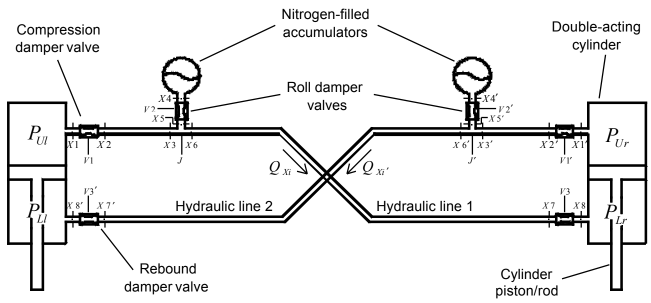

Fig. 2 Schematic of a typical HIS for half-car applications

From Fig. 1, it can be seen that the shock absorbers of conventional independent suspensions between the sprung and unsprung masses are replaced by a HIS system. A typical roll-resistant HIS arrangement for this application consists of two identical hydraulic circuits as shown in Fig. 2.

The two hydraulic circuits are coupled with each other kinetically via two identical double acting piston-cylinder absorbers. Each of the hydraulic circuits comprises three damper valves, a nitrogen-filled diaphragm accumulator, and a hydraulic pipeline. HIS circuits might typically include additional elements, such as hydraulic fittings and flexible hoses, but they fall beyond the consideration of this investigation. Within the two hydraulic circuits, the cylinders are mounted on the car body and the piston rods are fixed on the wheel stations. The dynamic interaction between the hydraulic system and the sprung and unsprung masses can be described as such: relative velocities in the suspension struts cause fluid flows in both circuits and accompanying pressure changes in the cylinder chambers, which leads to new suspension strut forces being applied to the sprung and unsprung masses. As a result, vehicle body and wheel motions occur, which, in turn, affect the hydraulic system. This interaction will continue until the system reaches a new equilibrium. HIS systems can provide greater freedom to independently specify modal stiffness and damping characteristics. Ideally, HIS system function is characterised entirely by mode, though factors such as fluid compressibility and frequency-dependent hydraulic circuit impedance cause imperfect system function. The working mechanisms and features of the half-car HIS system can be found from [8].

3 Modelling of the Half-Car with a HIS System

The details of the derivation of the equations of motion of the half-car model for free vibration analysis using transfer matrix method are presented in [8]. Here a brief description of the dynamic model, i.e., the state space representation of the integrated half-car system for the analysis of the transient responses caused by road inputs is provided.

For the integrated half-car system, the system state vector is defined as

$$\mathbf{X} = [y_{wl}, y_{wr}, y_v, \theta, \dot{y}_{wl}, \dot{y}_{wr}, \dot{y}_v, \dot{\theta}, P_{X8}, P_{X1}, P_{X8'}, P_{X1'}, \ldots,]^T \quad (1)$$

which includes the system displacement vector $\mathbf{Y} = [y_{wl}, y_{wr}, y_v, \theta]^T$, velocity vector $\dot{\mathbf{Y}}$, and pressure vector $\mathbf{P}$, which includes the end nodes of the fluid circuits and several internal nodes depending on the desired solution accuracy.

Applying Newton's second law, the equation of motion of the system is:

$$\mathbf{M}\ddot{\mathbf{Y}} + \mathbf{C}\dot{\mathbf{Y}} + \mathbf{K}\mathbf{Y} = \mathbf{F} = \mathbf{F}_{ext} + \mathbf{F}_h \quad (2)$$

Concentrating only on transient responses due to external forces, $\mathbf{F_{ext}}$, is caused by the road input from the two tires and $\mathbf{F_h}$ is the forces applied to the sprung and unsprung masses from the suspension fluid circuits resulted from the relative motion between sprung and unsprung masses. The equation of motion for the integrated system can then be determined in the form:

$$\dot{\mathbf{X}} = \mathbf{J}\mathbf{X} + \tilde{\mathbf{F}}_{ext} \quad (3)$$

in which the system coefficient matrix is

$$\mathbf{J} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{-M^{-1}K} & \mathbf{-M^{-1}C} & \mathbf{M^{-1}D} \\ \mathbf{0} & \mathbf{-V^{-1}A} & \mathbf{G} \end{bmatrix} \quad (4)$$

where **M**, **K**, **C** are the mass, stiffness and damping coefficient matrices respectively, see Eq. (1); **D**, **V**, **A** are the coefficient matrices coupling the motions and dynamics of the rigid body system and the suspension fluid circuits; **G** is the coefficient matrix determining the dynamics of the fluid circuits. We should note that coefficient matrices **V** and **G** are time variant as their values depend on the system state and the external force vector $\left[\tilde{\mathbf{F}}_{ext}\right]_{12\times1} = \left[\mathbf{0}, \mathbf{M}^{-1}\mathbf{F}_{ext}, \mathbf{0}\right]^T$ depends on the road input only.

4 Modelling of the HIS fluid circuits

To obtain the solution of equation (3), the dynamic state of the rigid body system and that of the fluid circuits can be determined simultaneously using the first 8 equations for the rigid body system and the last *n* equations for the fluid circuits respectively. This section provides a brief description of the modelling and the numerical solution scheme of the fluid circuit dynamics.

A HIS fluid circuit, as shown in Figure 2, consists of key elements such as the cylinder-piston unit, damper valves, straight and curved pipes or fittings and an accumulator. These elements need to be properly modelled so their dynamic effects on the integrated system are taken into account. Due to space limitations, only the differential equations describing the key hydraulic elements are provided.

Damper valves - The damper valves play an important role in the HIS system performance. The relationship between flow rate and pressure drop of the dampers is usually nonlinear. For obtaining the system modal parameters, it is common to assume that the vibration of the system involves small oscillations about the equilibrium position. For simplicity, therefore, the flow rates of the damper valves are modelled as linear with respect to the pressure drop between their inlets and outlets, as stated mathematically below.

$$Q_{dv} = C_{dv}(p_{in} - p_{out}) \quad (5)$$

where Q_{dv} and C_{dv} stand for flow rate and pressure loss coefficient of the damper valve respectively.

Pipelines - The vehicle layout and packaging constraints of the suspension system requires relatively long flexible pipelines. A lumped parameter model is developed by dividing the fluid pipelines into several elements. Each fluid pipeline of constant diameter is handled as one element. The mean pressure and mean flow in each element is assumed as an arithmetic mean of the pressure and flow rate at both ends of the pipe. The fluid flow in the pipe is assumed as one-dimensional compressible flow to accommodate the water hammer phenomenon. Assuming the pressure losses due to viscosity are proportional to the mean flow rate, and the magnitude of losses is the same as the inertia and pressure forces, the momentum equation can be written as

$$\frac{\rho l_i}{A_i}\dot{Q}_i = (p_{i1} - p_{i2}) - k_i l_i Q_i \tag{6}$$

where the viscous loss $k_i=8\pi\rho/A^2_i$, ρ is the fluid density, l_i the pipe length, and A_i the pipe section area. The continuity equation for the pipeline is written in terms of the mean pressure and flow difference between the ends of the pipe element as

$$\dot{p}_i = \frac{\beta(p)}{V_i}(Q_{i1} - Q_{i2}) \tag{7}$$

where $\beta(p)$, the bulk modulus, is expressed as a function of mean pressure p. The mean pressure and mean flow rate of the pipe element are given as the arithmetic mean at both ends of the element, i.e.,

$$p_i = \frac{1}{2}(p_{i1} + p_{i2}) \tag{8}$$

$$Q_i = \frac{1}{2}(Q_{i1} + Q_{i2}) \tag{9}$$

It is noted that Eqs. (6-9) apply to each pipe element. Substituting Eq. (8) and (9) into Eq. (6) results in one coupled first order differential equation governing the pressures at defined nodal points of the suspension fluid circuits. Flow rates and their first order derivatives at nodes can be determined in terms of the nodal pressures, pressure change rates and/or boundary conditions.

Accumulator - As shown in Figure 2, the HIS system features gas-pressurized hydraulic accumulators to reduce shock pressure loading due to system inputs. The accumulator consists of a pressure housing divided into two chambers by an elastomeric diaphragm. One chamber is filled with gas and the other with hydraulic fluid. The compressibility of the oil in the accumulator is neglected, as the oil stiffness is much greater than that of the nitrogen contained in the bladder. A drop in the system pressure is accompanied by flow from the accumulator and therefore the accumulator needs to be sufficiently large to meet the peak flow demands without appreciable drop in the system pressure. The accumulator is modelled by assuming an adiabatic process.

$$pV^{\gamma} = p_0 V_0^{\gamma} = const. \tag{10}$$

The adiabatic gas law is used to model the accumulator pressure as a function of gas volume at a precharged pressure. Taking the partial time derivative of Eq. (10) noting that the flow into the accumulator, Q, is given by $Q=-\partial V/\partial t$, the pressure gradient of the accumulator can be written as a nonlinear function of pressure p, i.e.,

$$\dot{p} = \frac{\gamma Q p}{V_0}\left(\frac{p}{p_0}\right)^{1/\gamma} \tag{11}$$

Finite element method based numerical solution scheme – In order to obtain the transient state of the suspension fluid circuits, a finite element method based numerical scheme is developed. The long pipe lines are meshed into several smaller elements and at each node of each straight internal pipe, the out flow rate value of the upstream element is equal to that of the in flow rate of the neighbouring down steam pipe element. Note that at a junction node, net in flow is equal to net out flow, and the pressures (for different fluid elements) are the same. Because of these physical constraints, the flow rates at internal nodes can be eliminated, or represented by nodal pressures and those at boundary nodes can be determined from the relative motion of the rigid bodies. Through mathematical manipulation, a set of n first order differential equations, which describes the dynamics of the suspension fluid circuits, are obtained. These equations are then combined into the state space equation for the integrated system, i.e., Eq. (3).

As this investigation primarily focuses on the determination of the modal parameters of the multi-body system, we try to obtain the free vibrations of the system dominated by single vibration modes by using specific road bump inputs. At time 0, the system is at its static equilibrium state and the initial state can be determined from the static force balancing equations. At the next instant, the disturbances from specified road inputs are applied to the system, so the dynamic state of the integrated system is determined from the state space system model combining the dynamics of the rigid body system and the suspension fluid circuits using the Runge-Kutta numerical integration method.

5 Results and Discussion

In this section, the developed half-car model of the HIS system is simulated for specified road inputs in order to determine the frequencies, damping ratios and modal shapes of vibration modes of bounce, roll and wheel hop. The mechanical and hydraulic system parameters used for the simulation are presented in Table 2 of the Appendix [8]. The simulation is performed with the accumulator pre-charge pressure of 0.5 MPa and system pressure of 2.0 MPa.

Bounce mode – To excite the response of the half-car dominated by the bounce mode, a step bump input, with amplitude of 0.05m, was applied to the two tires simultaneously. The obtained transient responses for y_{wl}, y_{wr}, y_v, and θ are plotted in Fig. 3. The results show that the sprung mass has a decayed response, and its roll is almost zero. The wheels have identical vertical displacement. The response is comprised mainly of the bounce mode and in-phase wheel hop mode.

Roll mode – To excite the response of the half-car dominated by the roll mode, two specific step bump inputs, with amplitude of ±0.05m, were applied to the two tires simultaneously. The obtained transient responses for y_{wl}, y_{wr}, y_v, and θ are plotted in Fig. 4. The results show that the sprung mass has a clean decayed response in roll and its bounce is almost zero. The wheels vibrate with the same amplitudes but out of phase, and they decay very rapidly.

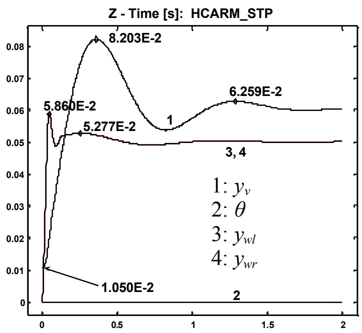

Fig. 3 Bounce and wheel hoop responses

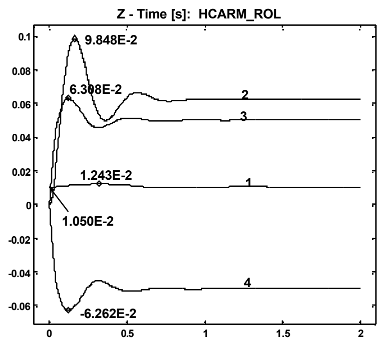

Fig. 4 Roll and wheel hoop responses

Table 1 Four system natural modes dominated by the half-car multi-body motion

Mode & frequency (Hz)	**Bounce** (1.05)[1]	**Bounce** (1.08)[2]	**Roll** (2.60)[1]	**Roll** (2.43)[2]
State variable	-2.875 ± 6.598i	-2.478 ±6.786i	-6.568 ±14.987i	-5.591 ±15.28i
Damping ratio	0.399	0.344	0.402	0.349
Left wheel (y_{wl})	0.098, 47.1°	0.099, 39.9°	0.279, 48.2°	0.291, 42.3°
Right wheel (y_{wr})	0.098, 47.1°	0.099, 39.9°	0.279, -48.2°	0.291, -42.3°
Centre of gravity (y_v)	1.000	1.000	0.000	0.000
Roll angle (θ)	0.00	0.00	1.000	1.000
Mode & frequency (Hz)	**Hop 1** (10.8)[1]	**Hop 1** (10.8)[2]	**Hop 2** (10.8)[1]	**Hop 2** (12.6)[2]
State variable	-30.61 ±60.54i	-30.61 ±60.54i	-62.62 ±25.52i	-78.54 ±11.94i
Damping ratio	0.451	0.451	0.926	0.989
Left wheel (y_{wl})	1.000	1.000	1.000	1.000
Right wheel (y_{wr})	1.000	1.000	-1.000	-1.000
Centre of gravity (y_v)	0.085, 77.1°	0.074, 62.5	0	0
Roll angle (θ)	0	0	0.343, 24.8°	0.303, 7.5°

[1] – The results of free vibration analysis [8]; [2] – the results of the presented transient analysis.

Modal parameters of the multi-body system – From the simulated transient responses, the modal parameters, in terms of natural frequencies, damping ratios and modal coefficients, of all four vibration modes of the multi-body system are determined using the State Variable Method [9]. All of these results and those obtained in the free vibration analysis method [8] are listed in Table 1 for comparison. From the first upper-left three columns in Table 1, for the bounce mode, it is seen that the damped natural frequency obtained using the presented method is about 3% higher than that obtained from the free vibration analysis. The newly obtained damping ratio is about 14% less and the modal coefficients for the wheels have almost the same magnitude but 15% difference in their phase referred to the sprung mass. From the first two upper-right columns, for the roll mode, the relative errors between the modal parameters obtained by the methods are all less than 10%. From the lower part of Table 1, for wheel hop mode 1, it is seen that the damped natural frequencies and damping ratios are the same for both methods. The modal coefficients for the sprung mass have 13% difference in magnitude and 19% difference in their phase referred to the wheels. For wheel hoop mode 2, the relative errors between the obtained results are even larger than those for the other modes because this mode is heavily damped.

Discussion – From the results listed in Table 1, although large relative errors occurred for the state variables where the absolute coefficients are very small (less than 10% of those of the reference state variables), most of the modal coefficients obtained with the two methods compare favourably. Therefore it is reasonable to state that, in general, the modal parameters obtained from the presented method agree well with those obtained from the free vibration analysis method. Therefore the transient analysis of the half-car system can provide the free decay type of system responses, from which the modal parameters of the system can be determined with adequate accuracy.

In this investigation, for simplicity, the half-car system is almost symmetric from the left wheel to the right wheel. So the bounce mode and wheel hoop mode 1 can be easily excited by applying the same step inputs to the tires (see Figure 3) and the roll mode and wheel hoop mode 2 can be easily excited by applying two opposite step inputs to the tires (see Figure 4). But it should be noted that the fluid circuits are not exactly left-right symmetric due to the location of the accumulators. So the simulated roll response is not zero in Figure 3 and the sprung mass vertical response is not zero in Figure 4. In real applications, the left and right sides of the half car are not symmetric and therefore all of the four modes will contribute to the response when step inputs are applied to the tires. This will be left for further investigation due to the limited space.

The most obvious limitation of the free vibration method is its linearity assumption, the details of which are discussed in [8]. For the presented method, for simplicity, all of the hydraulic elements except the accumulators are also assumed as linear. The friction effects, such as those in the piston rod seals and those between the piston and cylinder are ignored in the simulation. However, the presented method can be easily modified to include these frictional effects if so required. Furthermore, nonlinearities such as the chamber volume coupling nonlinearity, and nonlinear damper valve characteristics, can also be considered in the transient analysis.

6 Conclusions

This paper has presented an alternative approach for determining the modal parameters of a vehicle fitted with a general hydraulically interconnected suspension system. A simplified linear roll-plane half-car model is used as an example to illustrate the application of the proposed methodology. The individual fluid elements, such as lines, valves and accumulators of the HIS fluid circuits are modelled using a first principles approach. The dynamic model of the system, which couples the sprung mass and the HIS and the wheels, is then formulated using the state space representation approach. The state variables describing rigid body motion are heavily coupled with those describing the dynamics of the HIS fluid circuits. The dynamic model consists of time-variant coefficient matrices, which accommodate the nonlinearities in the fluids circuits, and can be used to obtain the transient responses induced by external disturbances.

A numerical simulation scheme is developed to obtain the transient responses of the half car vehicle using specific road inputs. The simulated free decay responses are then used to determine the modal parameters of the half-car system. The obtained modal parameters are compared with those determined from the free vibration analysis of the system using the transfer matrix method. It has been found that, in general, the results obtained from the presented method agree well with those obtained from the free vibration analysis method. Thus the presented transient analysis method can obtain the free decay type of system responses, from which the modal parameters of the system can be determined with adequate accuracy.

References

[1] National Highway Traffic Safety Administration (2004), Traffic Safety Facts.

[2] Sims, N.D. and Stanway, R. (2003), Semi-active vehicle suspension using smart fluid dampers: a modelling and control study. *International Journal of Vehicle Design*, **33** (1-3) pp. 76-102.

[3] Milanese, M., Novara, C., Gabrielli, P. and Tenneriello, L. (2004), Experimental Modelling of Vertical Dynamics of Vehicles With Controlled Suspensions. *SAE Paper Series*, 2004-01-1546.

[4] Wilde, J.R., Heydinger, G.J., Guenther, D.A., Mallin, T. and Devenish, A.M. (2005), Experimental Evaluation of Fishhook Maneuver Performance of a Kinetic Suspension System. *SAE Technical Paper Series*, 2005-01-0392

[5] Bhave, S.Y. (1992), Effect of connecting the front and rear air suspensions of a vehicle on the transmissibility of road undulation inputs. *Vehicle System Dynamics*, 21 (4) pp. 225-245.

[6] Fontdecaba, J. (2002), Integral Suspension System for Motor Vehicles Based on Passive Components. *SAE Technical Paper Series*, 2002-01-3105

[7] Smith, M.C. and Walker, G.W. (2005), Interconnected Vehicle Suspension. *Journal of Automobile Engineering*, 219 (3) pp. 295-307.

[8] Zhang, N, Smith, W., Jeyakumaran, J., Free vibration of a half-car fitted with a hydraulically interconnected suspension system, under review.

[9] Zhang, N. and Hayama, S. (1990), Identification of structural system parameters from time domain data (Identification of global modal parameters of structural system by improved state variable method), *Intl. Journal of JSME, Series 3*, Vol. 33, No.2, pp. 168-175.

Appendix

The parameter values used in this study were assumed based on available data in the public domain and, to a lesser extent, on realistic estimates made by the authors. Cylinder and roll damper loss coefficients were chosen to achieve bounce and roll damping ratios of approximately 40%. Since no mechanical friction or hysteretic damping has been considered, the damper loss coefficients selected here would be larger than those required in practice. Some parameters of the half-car system are shown in Table 2. The mean system operating pressure $\bar{P}$ of the fluid system is 2 MPa. The parameter quantities used in the study other than those listed in the table can be found in [8].

Table 2 Properties of the half-car multi-body system

Symbol	Value	Units	Description
M, m	750, 35	kg	Sprung and unsprung mass
I_q	320	kg/m^2	Sprung mass moment of inertia about roll axis
k_s, k_t	20, 200	kN/m	Mechanical suspension spring and tire spring stiffness
c_{tj}	300	N.s/m	Tire damping
μ	0.05	$N.s/m^2$	Hydraulic oil viscosity
b_{oil}	1400	MPa	Hydraulic oil bulk modulus
l_{23}, l_{67}	0.5, 1.5	m	Length of pipe from X_2 to X_3 and from X_6 to X_7
d	7.07	mm	Internal pipe diameter
V_p	200	ml	Accumulator pre-charge gas volume
P_p	5	bar	Accumulator pre-charge pressure
A_U, A_L	506.7, 412.4	mm^2	Upper and lower piston areas
R_V	5e+09, 3.2e+09	$kg/s.m^4$	Loss coefficient for cylinder valves and accumulator valves