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Real-time hybrid testing in structural dynamics

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Abstract: Real-time hybrid testing is a method of simulating dynamic structural response by splitting the system being emulated into one or more physical test specimens of key parts, and a numerical model of the remainder. The simulation is achieved by passing data between the physical and numerical parts in real time as the test proceeds. The method has the potential to offer significant improvements in the realism of laboratory simulation of dynamic structural response. This paper gives an overview of the development of hybrid testing within the field of earthquake engineering, and discusses some of the main technical issues such as actuator delay compensation and fast numerical model solution. Some other applications and possible future developments are also briefly discussed.

Keywords: hybrid testing; structural dynamics; substructuring.

1 Introduction

Despite advances in modelling techniques, there remains a strong need for experimental assessment of dynamic performance of novel systems across numerous fields of mechanical and civil engineering. In the field of earthquake engineering, dynamic testing of structures has concentrated on two approaches – shaking tables and pseudo-dynamic (PsD) testing. In a shaking table test a (usually) small-scale model of a structure is mounted on the table and subjected to a prescribed base motion by servo-hydraulic actuators. While useful data have been obtained, severe problems have been encountered with control of tests on large, inelastic structures, and with scaling of results from model to prototype. In a PsD test, only the stiffness part of the structure under test is modelled physically. Inertia and damping forces are calculated numerically and applied slowly to the test specimen. Using this approach it is reasonably economical to test at large scale, eliminating the scaling difficulties of shaking tables. However, the method is obviously poorly suited to tests involving unpredictable rate-dependent behaviour, which cannot be captured over the expanded timescale of a PsD test.

These issues have provided the motivation for the development of a new family of test methods, referred to here as real-time hybrid testing, though other terminology is often used, such as real-time substructuring, or hardware-in-the-loop. While this paper concentrates mainly on the development of the method within earthquake engineering, parallel efforts are in progress in many other fields, such as automotive and aerospace engineering; these will be briefly discussed.

The paper will briefly summarise the history of hybrid testing in earthquake engineering, then discuss some of the main technical issues such as the overall control strategy, actuator delay compensation and fast numerical model solution. Some possible applications of the method will be presented. Finally, we will look at the future of the technique, including the development of distributed hybrid testing.

2 The real-time hybrid test method

The aim of the new generation of real-time test methods is to enable economical, large-scale testing of components at realistic loading rates. To achieve this, most methods make use of the concept of substructuring, based on the observation that non-linear, rate-dependent or otherwise unpredictable behaviour in structures tends to be quite localised. It is only these localised parts that really need to be tested in the laboratory; the majority of the structure behaves quite predictably, and so can be simulated numerically. In a substructure test, we aim to impose on the test specimen the forces and deformations that it would experience when part of a larger structure subjected to dynamic loads.

Figure 1 illustrates the basic principle. In the example shown, the system to be emulated comprises a braced frame subjected to seismic ground shaking, fitted with an energy dissipator in the bottom storey, whose properties are poorly understood, necessitating physical testing. The dissipator and the adjoining braces form the *physical substructure*, which is tested at large scale and in real time. The remainder of the structure is modelled using finite elements (the *numerical substructure*). At each timestep displacements at the interface points between the two substructures are computed by the finite element model and applied to the test specimen by hydraulic actuators, and the resulting forces are measured and fed back to the numerical model as part of the input to the next analysis step.

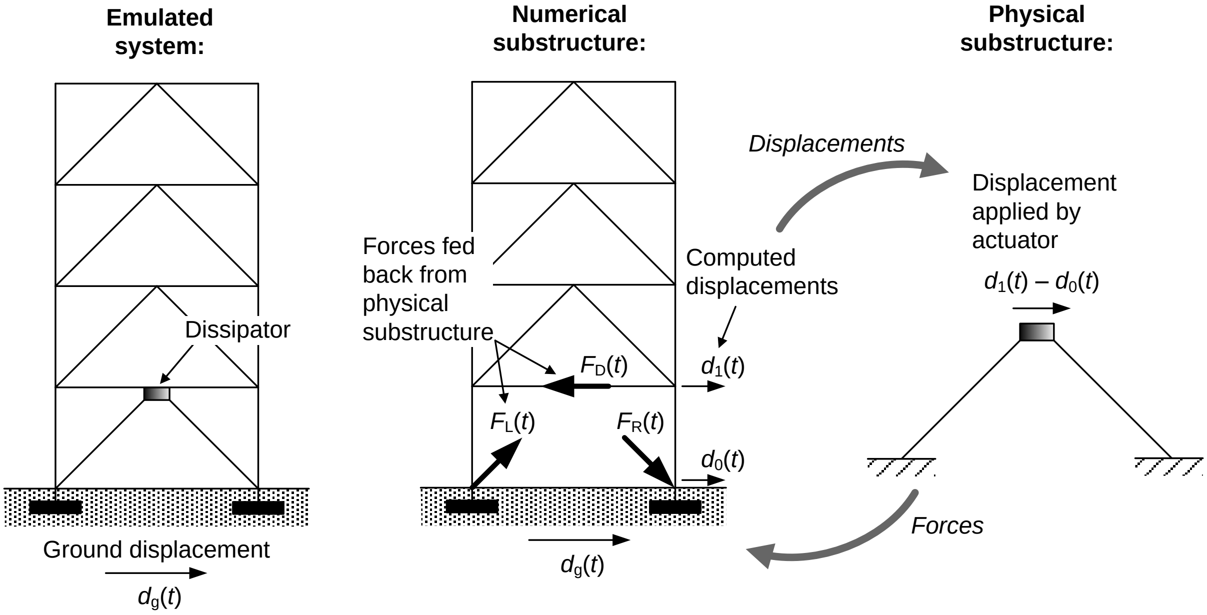

Figure 1. Real-time hybrid test on a frame with an energy dissipator

Figure 2 shows a schematic of the control loop required to achieve a real-time hybrid test. Typically, external excitation is applied to the numerical substructure. The model then outputs displacements at the interface with the physical substructure – these are the desired displacements, d_{des}, which we wish to apply instantaneously to the test specimen. The displacements are applied by actuators operating under a proprietary controller, usually of the PID type. We refer to the actuator-controller combination as the *transfer system.* The dynamics of the transfer system are imperfect, introducing both timing and amplitude errors, so that the actual displacement applied, d_{act}, is unlikely to be exactly equal to the desired value. Some form of compensation is therefore applied to the desired displacement and a modified command displacement d_{com} is sent to the inner loop controller, with the aim of minimising the discrepancy between d_{des} and d_{act}. Finally, the resistance forces generated by the application of d_{act} are fed back to the numerical substructure as part of the input to the next timestep.

The required speed of execution of this control loop will depend on the dynamics of the system under test, the frequency content of the input loads and the complexity of the numerical model which needs to be solved at each timestep. To simulate accurately the dynamic response of a structure to an earthquake ground motion is likely to require loop closure in around 10 ms or less.

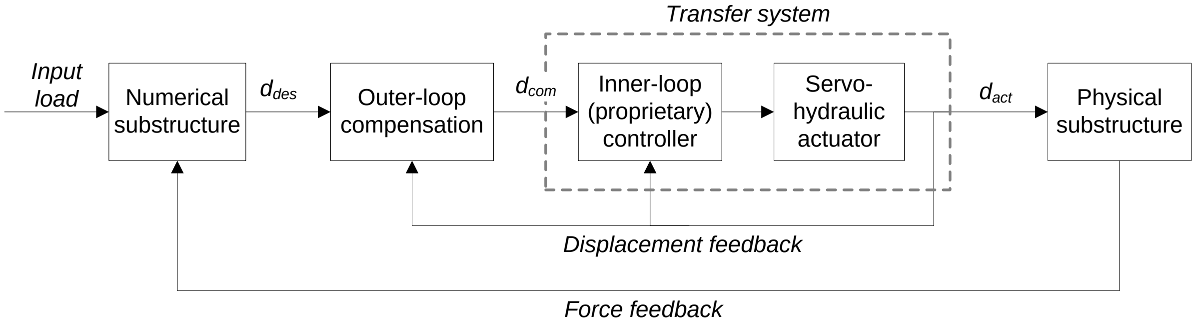

Figure 2. Control loop for a real-time hybrid test

The idea of hybrid testing is not new, but the necessary computational speeds and control algorithms have only become available quite recently. In earthquake engineering, the first reported real-time hybrid test was performed in Japan in 1992 by Nakashima et al. [1], who tested a viscous damper located at the base of a multi-storey building, which was a modelled as a single-degree-of-freedom (SDOF) system. In the UK in 1999, Darby et al. [2] performed real-time tests on a variety of physical systems, again coupled to a SDOF numerical model. Subsequently Horiuchi et al. [3], Nakashima and Masaoka [4] and Darby et al. [5] used more complex numerical substructures, though still linear. Blakeborough et al. [6] reported the first real-time test in which non-linear behaviour was permitted in both the numerical and physical parts. In the last few years, a broader range of researchers have become involved in hybrid testing, rapid advances have been made in numerical methods and control strategies, and the breadth of applications has increased. These developments are discussed in the remainder of this paper.

3 Overall testing strategy and equipment

To perform a real-time hybrid test requires very fast solution of numerical models, quick and accurate application of loads to test specimens, and rapid, robust control and and communications. The hardware and overall strategy to deliver these requirements are discussed here, with an emphasis on the system implemented at Oxford University.

3.1 Hardware

Large dynamic loads are generally applied by servo-hydraulic actuators, whose action is governed by flow of oil through a servo-valve, controlled by an electrical signal from a proprietary controller. In addition to this inner loop control, a second, outer loop is required in order to perform a hybrid test. In our system this is implemented using a dSpace digital signal processing board as shown in Figure 3. The board communicates with the host PC using a proprietary software package called ControlDesk, and is fully compatible with Matlab and Simulink.

Prior to a test, the numerical substructure model is compiled in Simulink and dowloaded to the board. The dSpace board then carries out the real-time execution, including fast communication with the inner loop controller. It is possible to monitor and adjust the processes running on the board via ControlDesk, without affecting the speed of execution. Separate monitoring of the actuators and inner-loop controller is performed on another PC, via a GPIB interface, which delivers data in packets at a slower rate than needed for real-time control.

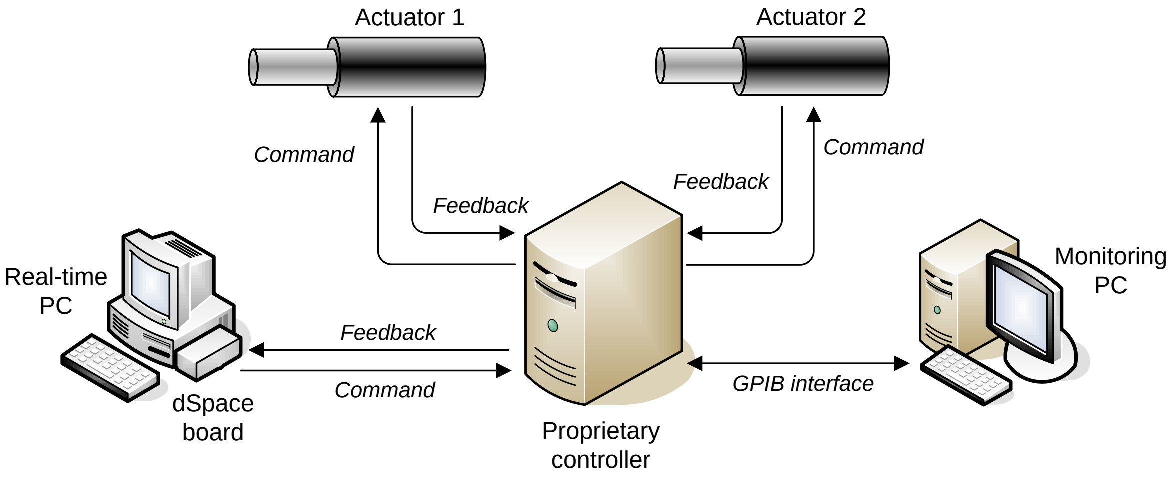

Figure 3. Simple hybrid test hardware setup

3.2 Testing procedure

The various processes involved in a real-time hybrid test are required to run synchronously, but may require different timesteps. The numerical model is likely to be reasonably complex and posibly non-linear, and is executed at relatively large time steps, referred to as *main steps*, to enable the solution to be obtained in real time. On the other hand, actuator control requires short time steps, referred to as *sub-steps*, so as to ensure smooth actuator motion and to achieve accurate velocity and acceleration profiles when only actuator displacements are being controlled. In a typical setup the main step is in the range 5-10 ms and the sub-step is around 0.2 ms.

A further issue relates to time delays within the system. These arise principally from the imperfect dynamics of the transfer system, which may take several milliseconds to impose the displacement output by the numerical model. The effect of this delay can be complex. For simple systems Horiuchi et al. [3] have shown that it is equivalent to introducing negative damping into the system, leading to instability; in more complex systems its effect can be more unpredictable. In any case, failure to correct for the delay will result in an inaccurate and possibly unstable simulation.

An appropriate test procedure to deal with these issues is as follows:

1. The numerical substructure is solved by an appropriate algorithm to give the desired actuator displacement at the next main step, d_{des}^{n+1}.
2. Some form of curve fit is performed, usually based on the current and the past few displacement points.
3. The curve fit is used to extrapolate forward by a time equal to the estimated actuator delay, to give the command displacement, d_{com}^{n+1}.
4. The same curve fit is then used for interpolation purposes, to calculate the d_{com} values at sub-steps. These are then sent to the inner loop controller, together with the current actuator position d_{act}.
5. Step 4 is repeated at sub-steps, until the next main step. A multi-tasking strategy is implemented on the board, with the sub-step control task given priority, and the main step tasks executing in the free time between sub-step tasks.

The following sections consider in more detail the two processes which are key to the performance of a stable, accurate real-time test: the choice of solution method for the numerical substructure, and the strategy for compensating for the transfer system dynamics.

4 Numerical integration schemes for real-time testing

The numerical scheme used to integrate the equations of motion needs to be sufficiently fast to enable real-time execution, while maintaining stability and reasonable accuracy. It is advantageous if it can handle non-linear behaviour (which is usually restricted to the stiffness term). An iterative approach of the type used by some researchers in PsD testing, in conjunction with an implicit time integration scheme, is unsuitable for real-time simulation since iterations would disrupt the physical substructure dynamics. Therefore, the available strategies can be classified in three types: fully explicit numerical schemes; implementation of an implicit scheme through an explicit predictor target; and implementation of an implicit scheme through a direct sub-step feedback.

Examples of fully explicit schemes are the central difference method, which was used in the earliest real-time hybrid tests [1, 2] and Newmark's method [7]. The latter is preferable as it has more favourable error-propagation characteristics. Its formulation is:

$$\mathbf{d}_{n+1} = \mathbf{d}_n + T\mathbf{v}_n + \frac{T^2}{2}\mathbf{a}_n \tag{1}$$

$$\mathbf{v}_{n+1} = \mathbf{v}_n + \frac{T}{2}\left(\mathbf{a}_n + \mathbf{a}_{n+1}\right) \tag{2}$$

$$\mathbf{Ma}_{n+1} + \mathbf{Cv}_{n+1} + \mathbf{R}_{n+1} = \mathbf{f}_{n+1} + \mathbf{F}_{n+1} \tag{3}$$

where where **d**, **v** and **a** are vectors of nodal displacement, velocity and acceleration, **M** and **C** are the mass and damping matrices, **R** is the vector of restoring forces due to the considered displacement (= **Kd** for a linear system), **F** is the vector of applied forces and **f** is the vector of forces fed back from the physical substructure.

The drawback of explicit approaches is the stability limit, proportional to the smallest natural period of the numerical substructure. For complex MDOF systems, the required time-step may be too small for the computing hardware to solve the model in real-time. Chang [8] devised a modification to the standard Newmark scheme, in which weighting matrices are applied to the velocity and acceleration terms in (1). The scheme has excellent accuracy characteristics and is unconditionally stable for linear systems.

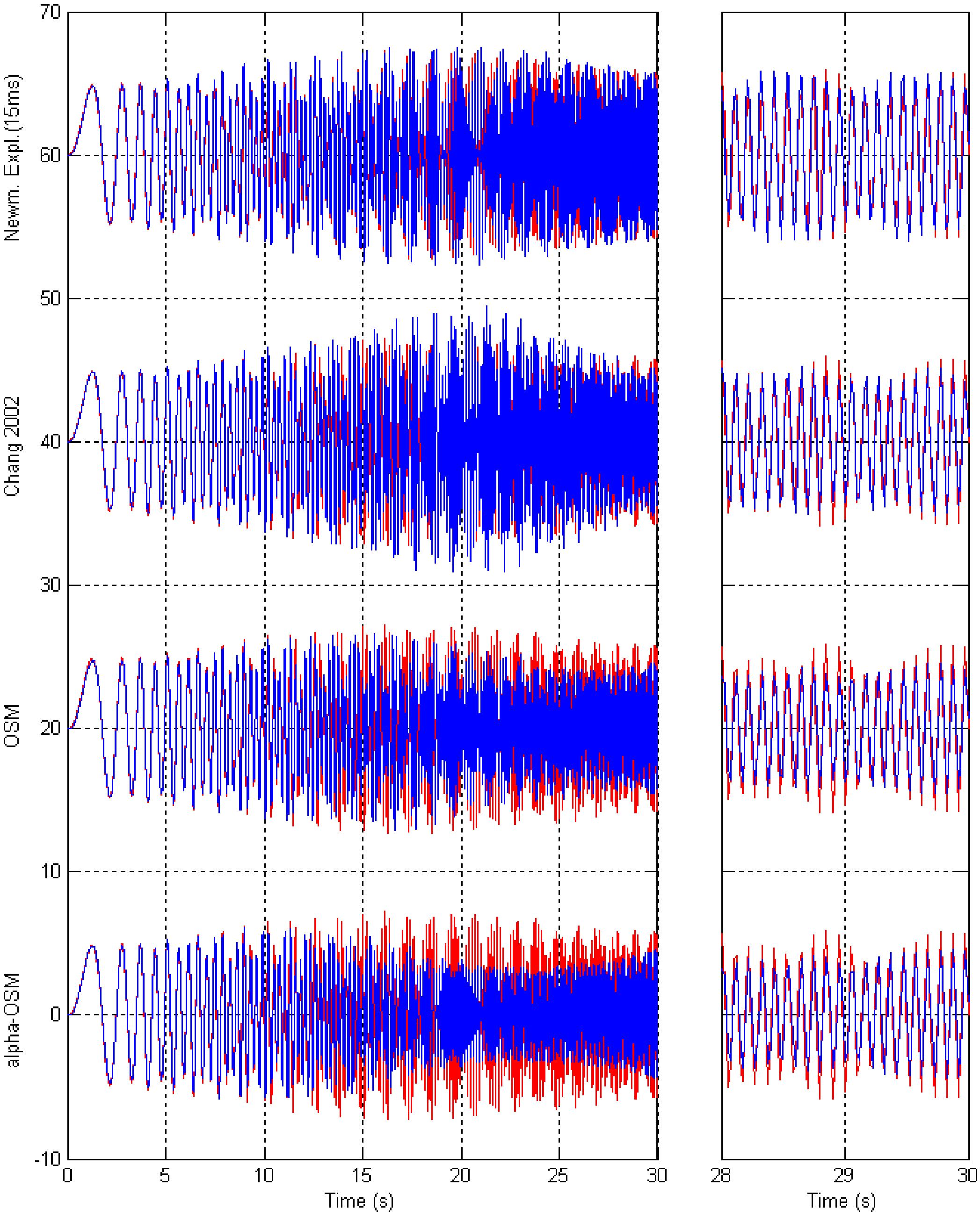

Figure 4. Time history plots of numerical substructure interface response (mm) for four schemes (blue) and numerical simulation result (red) for the 50 DOF test with 25 ms main steps (15 ms for Newmark)

Turning to implicit schemes, the operator-splitting method is an example of a scheme in which the displacement calculation is split into an explicit predictor and an implicit corrector step. The method can be combined with an α-shifted equilibrium equation of motion in order to introduce numerical damping into the algorithm [9]. In the operator-splitting approach, it is the explicit predictor displacement which is applied to the test specimen, with corrected value only being used for the next calculation step. To compensate for this, an approximate correction can be applied to the fed-back force vector to acount for the difference between the predicted and corrected displacements. Wu et al. [10] have proposed an alternative formulation of the operator-splitting approach which gives an explicit estimate of the velocity as well as the displacement.

An alternative to the predictor-corrector approach for implementing an implicit integration scheme is to take advantage of the existence of sub-steps. At each sub-step, a displacement increment is calculated based on the known parameters from the previous main step and fed-back forces from the previous sub-step. These increments effectively act like the iterations in a conventional implicit scheme, but they are weighted to ensure overshoot does not occur, and they are limited to the number of sub-steps in a main step, so that a correction may be required at the end of each main step. This method has been used in conjunction with the constant average acceleration method (CAAM) by Bayer et al. [11] and with the α-method by Jung and Shing [12].

All of the schemes described can be used successfully for real-time hybrid testing with a simple numerical substructure and a realtively short main step. As the complexity of the numerical model increases, the implicit methods with sub-step feedback are unable complete the necessary computations over the very short duration of each sub-step, so that real-time testing cannot be achieved. (For our system, the limiting model size is around 10 DOF, with a sub-step of 0.2 ms.) With large numerical models requiring a longer main step, the Newmark method may encounter stability problems.

Figure 4 shows results for a test comprising a 50 DOF numerical model linked to a SDOF physical system. The main step is 25 ms for all schemes except Newmark, which required a 15 ms step for stability. The sub-step is 0.2 ms, matching the inner loop controller timestep. The system is loaded by a base motion comprising a swept sine wave between 0 and 10 Hz. It can be seen that the Newmark scheme gives the best agreement with a numerical simulation. Chang's method slightly amplifies the response at the higher frequencies, while both operator-splitting methods tend to damp the higher-frequency response.

The results suggest a preference for the Newmark method if stability is not an issue, Chang's method if the model size requires a longer timestep, and an operator splitting method if numerical damping of higher modes is desired (this can be help reduce the amplification of control errors).

5 Compensation for transfer system dynamics

As already mentioned, the imperfect dynamics of the transfer system can introduce both timing and amplitude errors into the simulation, causing inaccuracy and possibly instability. To overcome this, some form of outer loop compensation is required. A compensator can be considered to have two components: a *forward prediction scheme*, whichs aim to achieve accuracy through extrapolation, assuming the delay and the amplitude error are known and accurate; and a *performance estimation scheme*, in which the delay and amplitude error are estimated more precisely as the test proceeds, starting from rough estimates provided by the user. In many systems this second component has been omitted, but it has been shown that the delay can vary significantly during a test, and that failure to take this into account can lead to instability [13].

Forward prediction schemes include:

- Exact polynomial – in which, for example, a cubic is fitted exactly through the last four main step displacement points and extrapolated forward by the required delay [3].
- Least squares – a line of best fit is obtained using a larger number of data points, e.g. a fourth order polynomial fitted through twelve points. This approach is useful for eliminating the effects of noise in systems whose dynamics do not change rapidly [14].
- Linear acceleration – extrapolation is base on only the previous two acceleration values and has been shown to have superior stability properties to the exact polynomial approach [15].

- Laguerre sub-step extrapolator – in which the sub-step diaplacements within the current main step are used as input, and weighted by the Laguerre polynomials [16].

Figure 5 illustrates the effectiveness of a delay compensation scheme. In this case an exact third-order polynomial extrapolation has been used to compensate the delay in a hybrid test on a 2DOF system which has been split into SDOF physical and numerical substructures. The first plot shows the actuator output against the actuator command, with the delay causing the elliptic shape. The centre plot shows the command against the desired output, with a similar elliptic shape caused this time by the forward prediction. On the right these two effects are combined to give an almost linear relationship between actual and desired displacement, showing that the delay has been mostly corrected.

Performance estimation aims to achieve real-time delay estimates, either by attempting to measure the delay directly, or by inferring a delay from the difference between the desired and actual displacements. Bonnet et al [16] used the latter approach to generate updates to the delay estimate δ according to:

$$\delta^{i+1} = \delta^{i} + \left(\frac{d_{des}^{i}}{A_{fac}} - d_{act}^{i} \right) \left(\frac{A_{fac}.T}{d_{des}^{i+1} - d_{des}^{i}} \right) \tanh \left(C_{v} \frac{\mathrm{abs}\,(d_{des}^{i+1} - d_{des}^{i})}{T} \right) \tag{4}$$

where where A_{fac} is the amplitude correction factor and C_{v} is a user-defined gain. This is based on treating the displacement error (the first bracketed term) as the product of the delay and the actuator velocity (the second bracketed term). The tanh term effectively switches off the correction at times of zero velocity. A typical variation in delay estimates produced by (4) during a twin-actuator test on a very stiff test specimen is shown in Figure 6.

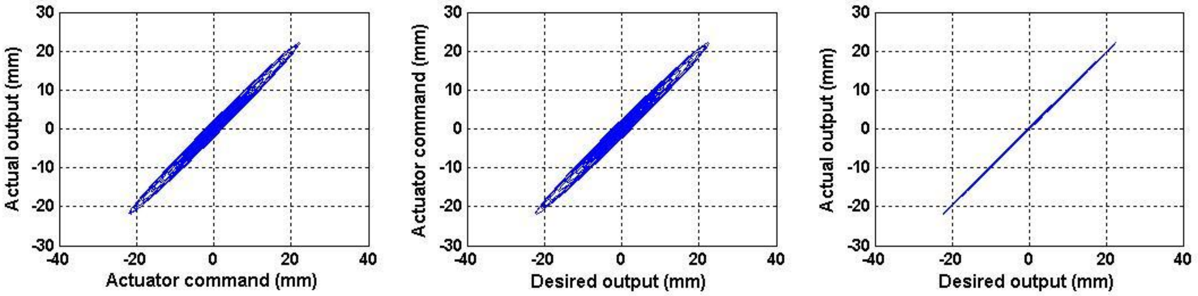

Figure 5. Synchronization plots showing (from left to right), effect of actuator delay, compensation of the delay, and the resulting agreement between actual and desired output

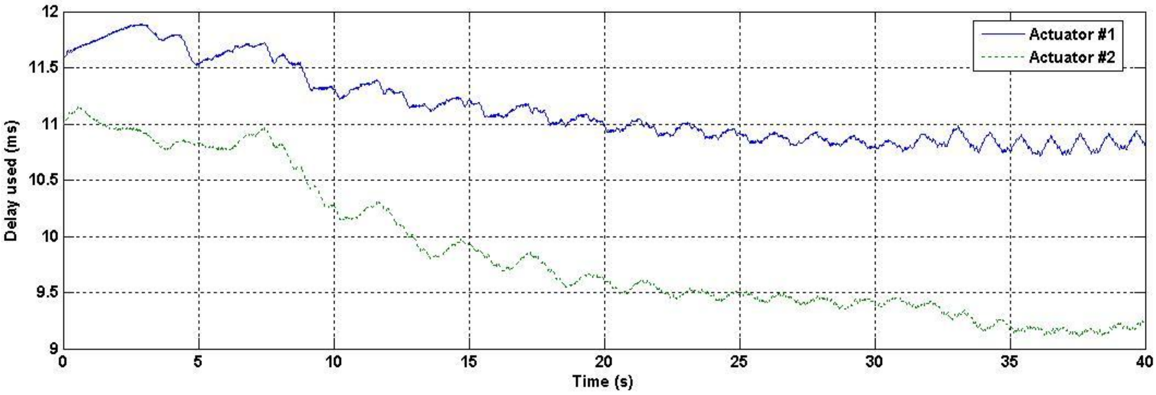

Figure 6. Actuator delay estimates produced by error-based updating scheme

Other researchers have attempted to deal with the transfer system dynamics through a fundamentally different approach, in which an outer-loop, adaptive controller is used in place of a delay estimation scheme. The most promising such approach appears to be the minimal control synthesis algorithm with modified demand (MCSmd) [17, 18].

6 Applications

As hybrid testing technology matures, it is finding applications in numerous branches of civil and mechanical engineering, aside from earthquake engineering. Examples include:

- Crowd-structure interaction in cantilever grandstands – tests underway at Oxford make use of a full-scale section of raked grandstand supporting up to fifteen people. This is embedded in a hybrid test loop so as to impose on the test subjects the realistic dynamic response of a full-scale grandstand, as illustrated in Figure 7.
- Automotive component testing – hybrid simulation can be used to test components such as suspension units or tyres, coupled to a numerical chassis model.
- Aerospace – for example, physical testing of lag dampers used to prevent instabilities in helicopter rotor assemblies, coupled to numerical models of the main system.
- Formula One – Figure 8 shows an example in which aerodynamic down forces due to forward motion are simulated numerically, while other ride parameters such as road roughness are applied to a prototype vehicle at full scale.

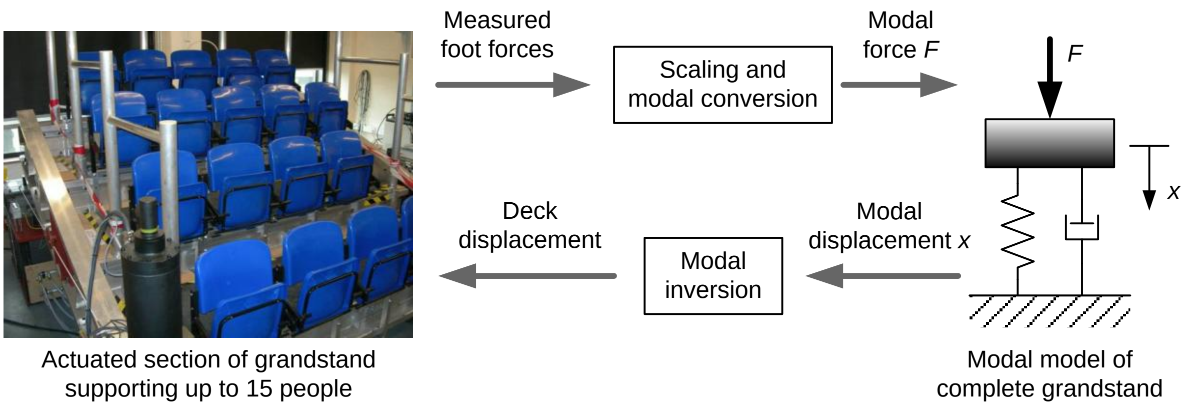

Figure 7. Hybrid test of crowd loading of a cantilever grandstand

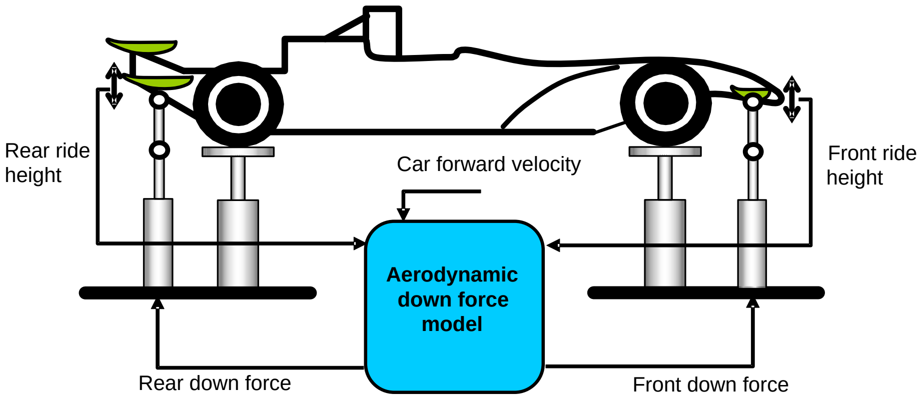

Figure 8. Hybrid test of a Formula One car (*courtesy of A. Plummer, University of Bath*)

7 Future Developments

Hybrid testing is now close to becoming a mature, reliable technology, and its use likely to spread to new application areas. Continuing increases in computer power are adding to the size and complexity of numerical substructures that can be used. There is scope for improving the performance of the transfer system and the control/compensation techniques used to correct for this.

An interesting extension of hybrid testing is the idea of linking several geographically separate laboratories to perform a *distributed hybrid test*. In this approach, a very large and/or complex structure is split into several physical substructures, tested simultaneously in different laboratories around the world and linked together by grid computer resources. A schematic of a possible pilot test, on an idealised bridge structure, is shown in Figure 9. In the first instance, such a test would have to run at an expanded timescale, but the development of faster internet systems may make real-time testing possible in the future.

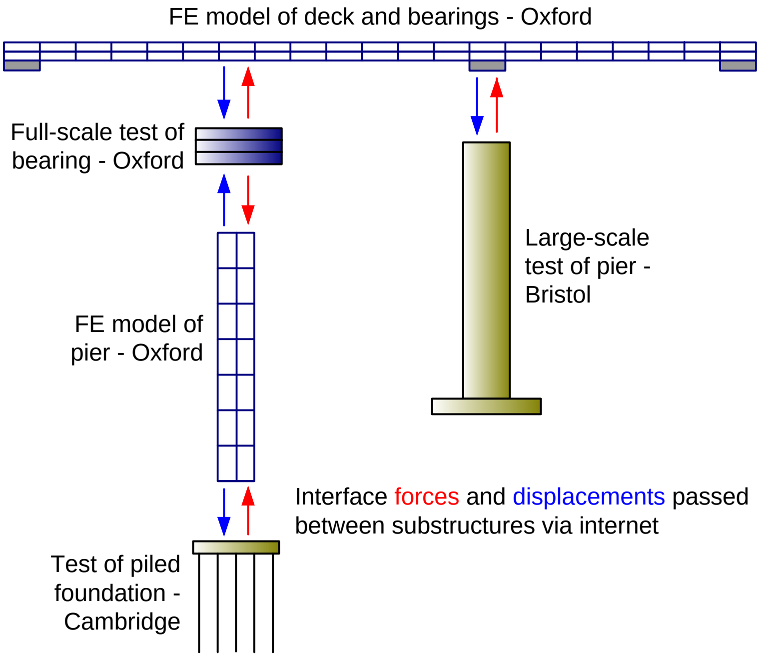

Figure 9. Hybrid test of a bridge structure, distributed between three laboratories

8 Conclusions

Hybrid testing has the potential to offer significant improvements in the realism of laboratory simulation of dynamic structural response. While this paper has focused on the method's development in the context of seismic testing, parallel efforts are underway in many branches of civil and mechanical engineering. Implementation of the method has required significant development of numerical algorithms and techniques for compensating for the transfer system dynamics. Some further improvements are needed to develop hybrid testing into a robust, reliable tool.

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References

[1] Nakashima M., Kato H., Takaoka E. (1992) Deelopment of real-time pseudodynamic testing. *Earthquake Engng Struct. Dyn.*, **21**, 79-92.

[2] Darby A.P., Blakeborough A., Williams M.S. (1999) Real-time substructure tests using hydraulic actuator. *J. Engng Mech.*, **125**, 1133-1139.

[3] Horiuchi T., Inoue M., Konno T., Namita Y. (1999) Real-time hybrid experimental system with actuator delay compensation and its application to a piping system with energy absorber. *Earthquake Engng Struct. Dyn.*, **28**, 1121-1141.

[4] Nakashima M., Masaoka N. (1999) Real-time on-line test for MDOF systems. *Earthquake Engng Struct. Dyn.*, **28**, 393-420.

[5] Darby A.P., Blakeborough A., Williams M.S. (2000) Improved control algorithm for real-time substructure testing. *Earthquake Engng Struct. Dyn.*, **30**, 431-448.

[6] Blakeborough A., Williams M.S., Darby A.P., Williams D.M. (2001) The development of real-time sub-structure testing. *Phil. Trans. R. Soc. Lond.*, A**359**, 1869-1891.

[7] Newmark N.M. (1959) A method of computation for structural dynamics. *J. Engng Mech.*, **85**, 67-94.

[8] Chang S.Y. (2002) Explicit pseudodynamic algorithm with unconditional stability. *J. Engng Mech.*, **128**, 935-947.

[9] Combescure D., Pegon P. (1997) α-Operator splitting time integration technique for pseudodynamic testing: error propagation analysis. *Soil Dyn. Earthquake Engng*, **16**, 427-443.

[10] Wu B., Xu G., Wang Q., Williams M.S. (2006) Operator-splitting method for real-time substructure testing. *Earthquake Engng Struct. Dyn.*, **35**, 293-314.

[11] Bayer V., Dorka U.E., Füllerkrug U. (2000) A new algorithm for real-time sub-structure pseudo-dynamic tests. *Proc. 12th World Conf. Earthquake Eng.*, Auckland, New Zealand, Paper 0401.

[12] Jung R.Y., Shing P.B. (2006) Performance evaluation of a real-time pseudodynamic test system. *Earthquake Engng Struct. Dyn.*, **35**, 789-810.

[13] Darby A.P., Williams M.S., Blakeborough A. (2002) Stability and delay compensation for real-time substructure testing. *J. Engng Mech.*, **128**, 1276-1284.

[14] Wallace M.I., Sieber J., Neild S.A., Wagg D.J., Krauskopf B. (2005) Stability analysis of real-time dynamic substructuring using delay differential equation models. *Earthquake Engng Struct. Dyn.*, **34**, 1817-1832.

[15] Horiuchi T., Konno T. (2001) A new method for compensating actuator delay in real-time hybrid experiments. *Phil. Trans. R. Soc. Lond.*, A**359**, 1893-1909.

[16] Bonnet P.A., Williams M.S., Blakeborough A. (2007) Compensation of actuator dynamics in real-time hybrid tests. *Proc. Instn Mech. Engrs, Part I: J. Systems & Control Engng*, **221**, 251-264.

[17] Lim C.N., Neild S.A., Stoten D.P., Drury D. (2005) Real-time dynamic substructure testing via an adaptive control strategy. *Proc. 1st Int. Conf. on Advances in Exp. Struct. Engng*, Nagoya, Japan.

[18] Bonnet P.A., Lim C.N., Williams M.S., Blakeborough A., Neild S.A., Stoten D.P., Taylor C.A. (2007) Real-time hybrid experiments with Newmark integration, MCSmd outer-loop control and multi-tasking strategies. *Earthquake Engng Struct. Dyn.*, **36**, 119-141.