Vibration Control of Wood-Polymeric Composite Structures using Piezoelectric Transducers

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Abstract: Vibration control has been a subject of engineering research for the past few decades. Recently, the use of smart materials-related components for vibration control has become an alternative to traditional vibration control techniques. Vibration control with smart materials have many advantages such as lighter weight, smaller size, flexibility in the structure and lower cost. They are especially suitable where the traditional techniques cannot be applied due to weight and size restriction. The scope of the study is to understand the nature of piezoelectric materials for converting mechanical energy to electrical energy and vice versa. Physical properties of piezoelectric materials for vibration sensing, actuation and dissipation were evaluated. An analytical study of the resistor-inductor (R-L) passive piezoelectric vibration shunt control of a cantilever beam was undertaken. The modal and strain analyses were performed by varying the material properties and geometric configurations of the piezoelectric transducer in relation to the structure in order to maximize the mechanical strain produced in the piezoelectric transducer. Numerical modelling of structures was performed and field-coupled with the passive piezoelectric vibration shunt control circuitry. The Finite Element Analysis (FEA) was used for the analysis and optimal design. Experiments with the passive piezoelectric vibration shunt control of beams were also carried out and the results compared to the analytical and computational work described above.

Keywords: vibration control, piezoelectric, energy, beam, shunt circuit, field coupled analyses.

1. Introduction

The use of smart materials-related technology in vibration control, in recent years, has become a viable alternative to traditional vibration control techniques. In the application described here, it is required that the transient vibration on a beam-like structure is reduced. This beam structure is essentially a wood-carbon fibre polymeric structure. The vibration reduction is to be facilitated by using a piezoelectric transducer attached to an electrical shunt network. The Lead Zirconium-Titanate (PZT), a piezoelectric ceramic material, can convert mechanical energy into electrical energy and vice versa. Hagood and von Flotow (1991) in [1], demonstrated that structural vibrations can be damped with piezoelectric materials and a passive electrical (shunt) network. The piezoelectric materials convert vibration energy into electrical energy and then dissipate the electrical energy in the form of joule heat through the electrical shunt network. This is also known as passive vibration damping. The analytical work presented here elucidates on the work by Cao *et al* [2], [3] where field-coupled analyses of strain and piezoelectric actuation are introduced.

A R-L electrical (shunt) network consists mainly of resistors and inductors connected across a piezoelectric transducer. This can be modeled as a strain controlled voltage source in series with an inherent capacitance, or a strain controlled current source in parallel with the inherent capacitance. The piezoelectric transducer can alter the stiffness and loss factor of the system attached, acting as a tuned mass damper (TMD) or tuned vibration absorber (TVA). Edberg *et al* [4] discuss the theoretical underpinnings of this procedure on structural truss members. Hollkamp [5] introduces the procedure to address multimodal vibration suppression using the R-L shunt circuit. Wu [6] does the same with parallel R-L shunt circuits. Wang *et al* [7], describes an energetic-based approach to the modeling of the vibration reduction protocol using semi-active piezoelectric networks. In the work by Fleming *et al* [8], they investigate the inductance requirements for effective vibration control of mechanical systems.

Strain and stress distributions in the composite beams are determined by the geometry size, material properties and beam structure. Understanding the strain/stress distribution in the composite beams is important for effectively attenuating vibration-based energy from the structure using the passive piezoelectric vibration control technique. In this paper, the influence of the neutral surface location on strain distribution in the composite beam will be discussed. The equations of neutral surface location, strain and stress distribution of a two-layer composite beam made of the PZT and a uniform beam are derived respectively. Simulations of the derived equations with different thickness ratios and Young's modulus ratios are presented. Some Finite Element Analysis (FEA) simulations results of the piezoelectric vibration shunt control for the beams made of various materials are presented. The experimental results are presented by Cao *et al* in [2] and [3] and Cao [11] and are for brevity purposes, not presented here.

2. Equation of Motion of a cantilever beam attached with a PZT transducer

Figure 1 shows a composite beam structure that is a cantilever beam with a PZT patch attached.

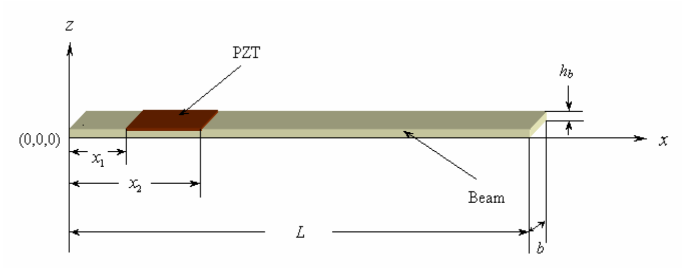


Figure 1 PZT attached composite beam

For the simplicity, only bending deformation of the beam is considered in this study. Shear and rotation deformations are ignored. The potential energy of beam due to bending is:

$$U_b = \frac{1}{2}\int_{V_b} \sigma_{11}\varepsilon_{11} dV_b = \frac{1}{2}\int_0^L E_b I_b \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{1}$$

The potential energy of the PZT due to bending is:

$$U_p = \frac{1}{2}\int_{V_p} (\sigma_{11}\varepsilon_{11} + D_3 E_3) dV_p = \frac{1}{2}\int_0^L \left[E_p I_p \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{1}{2} h_{31} A_p (h_b + h_p) D_3 \frac{\partial^2 w}{\partial x^2} + A_p \beta_{33} D_3^2 \right] [H(x-x_1) - H(x-x_2)] dx \tag{2}$$

The total potential energy $U = U_b + U_p$.

Similarly, the kinetic energy of beam plus PZT due to bending is

$$T = T_b + T_p = \frac{1}{2}\int_0^L \rho_b A_b \left(\frac{\partial w}{\partial t}\right)^2 dx + \frac{1}{2}\int_0^L \rho_p A_p \left(\frac{\partial w}{\partial t}\right)^2 [H(x-x_1) - H(x-x_2)] dx \tag{3}$$

where ,

$w(x,t)$ is the transverse displacement of composite beam; ε_{11} and σ_{11} are the axial strain and stress in x direction; I_b and I_p are the area moment of inertias of beam and PZT about neutral axis; L, $(x_2 - x_1)$, b, h_b, h_p, A_b, A_p, V_b and V_p are the lengths, widths, thickness, cross-sectional areas and volumes of beam and PZT respectively; E_b and E_p are the elastic modulus of beam and PZT; ρ_b and ρ_p are the densities of beam and PZT; h_{31} is the electrical displacement-stress coefficient of

PZT in "3-1" direction; b_{33} is the dielectric impermeability of PZT in "3-3" direction; D_3 and E_3 are the electrical displacement and electrical field of PZT in "3" direction, $E_3 = -h_{31}\mathrm{e}_{11} + \mathrm{b}_{33}D_3$; $H(x)$ is the Heaviside function.

According to the Hamilton's principle,

$$\mathrm{d}\int_{t_1}^{t_2}(T-U+W)\,dt = \int_{t_1}^{t_2}(\mathrm{d}T - \mathrm{d}U + \mathrm{d}W_f + \mathrm{d}W_p)dt = 0 \tag{4}$$

where $\mathrm{d}W = \mathrm{d}W_p + \mathrm{d}W_f$ represents virtual work, $\mathrm{d}W_p = \int_{V_p} E_3 \mathrm{d}D_3 dV_p$ is the virtual work due to electrical displacement, $\mathrm{d}W_f = \mathrm{d}\int_0^L f(x,t)w(x,t)dx$ is the virtual work due to applied force.

The motion equation of the composite beam therefore can be derived as

$$E_b I_b \frac{\partial^4 w}{\partial x^4} + E_p I_p \frac{\partial^4 w}{\partial x^4}[H(x-x_1) - H(x-x_2)] + 2E_p I_p \frac{\partial^3 w}{\partial x^3}[\mathrm{d}(x-x_1) - \mathrm{d}(x-x_2)] + E_p I_p \frac{\partial^2 w}{\partial x^2}[\mathrm{d}'(x-x_1) - \mathrm{d}'(x-x_2)] +$$

$$\mathrm{r}_b A_b \frac{\partial^2 w}{\partial t^2} + \mathrm{r}_p A_p \frac{\partial^2 w}{\partial t^2}[H(x-x_1) - H(x-x_2)] - h_{31} A_p (h_b + \frac{h_p}{2}) D_3 [\mathrm{d}'(x-x_1) - \mathrm{d}'(x-x_2)] = f(x,t) \tag{5}$$

with boundary condition:

$$\begin{cases} E_b I_b \dfrac{\partial^2 w}{\partial x^2}\mathrm{d}\left(\dfrac{\partial w}{\partial x}\right)\Bigg|_0^L = 0 \Rightarrow \dfrac{\partial^2 w}{\partial x^2} = 0 \text{ or } \mathrm{d}\left(\dfrac{\partial w}{\partial x}\right) = 0 \\ E_b I_b \dfrac{\partial^3 w}{\partial x^3}\mathrm{d}w\Bigg|_0^L = 0 \Rightarrow \dfrac{\partial^3 w}{\partial x^3} = 0 \text{ or } \mathrm{d}w = 0 \end{cases}$$

According to mode separation method $w(x,t) = \sum_{r=1}^{\infty} \mathrm{f}_r(x) q_r(t) \approx \sum_{r=1}^{N} \mathrm{f}_r(x) q_r(t)$, where mode function $\mathrm{f}_r(x)$ satisfies all the boundary condition. Substitute $w(x,t)$ into equation (5), using Galerkin's method and minimizing e by,

$$\int_0^L \mathrm{e}(x,t)\mathrm{f}_s(x)\,dx = 0 \quad (s = 1, 2, \ldots, N)$$

The partial differential equation (PDE) of equation (5) can be discretized into an ordinary differential equation (ODE) as shown below:

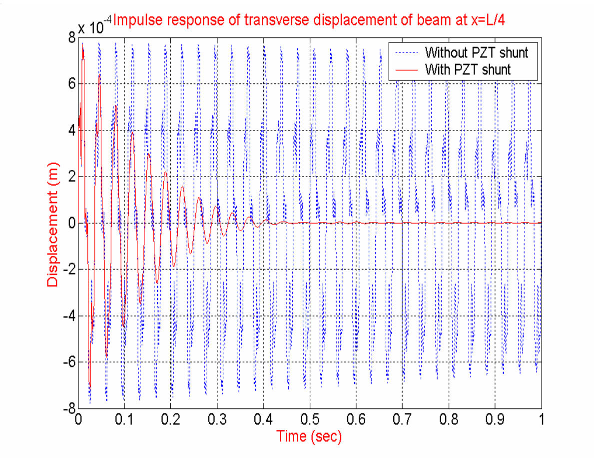
$$\sum_{r=1}^{N}\left(\int_0^L \left[\mathrm{r}_b A_b + \mathrm{r}_p A_p [H(x-x_1) - H(x-x_2)]\right]\mathrm{f}_r(x)\mathrm{f}_s(x)\,dx\right)\ddot{q}_r(t) + \sum_{r=1}^{N}\left(\int_0^L \left[E_b I_b + E_p I_p [H(x-x_1) - H(x-x_2)]\right]\mathrm{f}_r''(x)\mathrm{f}_s''(x)\,dx\right)q_r(t)$$

$$-h_{31} A_p (h_b + \frac{h_p}{2}) D_3 \int_{x_1}^{x_2} \mathrm{f}_s''(x)dx = \int_0^L f(x,t)\mathrm{f}_s(x)dx \tag{6}$$

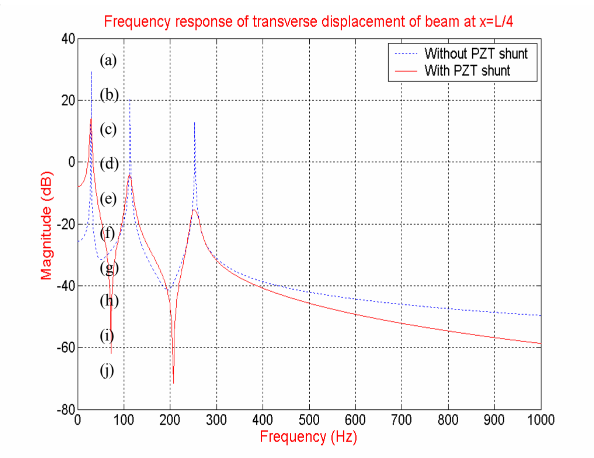
where,

$$D_3 = \frac{1}{\mathrm{b}_{33}}(E_3 + h_{31}\mathrm{e}_{11}) = \frac{1}{\mathrm{b}_{33}}\left(\frac{v(t)}{h_p} + h_{31}\mathrm{e}_{11}\right) \tag{7}$$

In [11] Equation (6) was converted to its equivalent state space form and solved with a numerical example. The beam response with the shunt circuit tuned to that of the beam's fundamental natural frequency. The resultant beam response is shown in Figure 2.



(a) Impulse response



(b) Frequency response

Figure 2 The tuned response for a Composite Wood-Carbon Fibre Composite Beam.

Figure 2 shows that the vibration amplitude of the beam is significantly reduced when the inductor of shunt circuit is tuned to the frequency (f_i) to be controlled, that is:

$$L_i = \frac{1}{(2\text{p}f_i)^2 C_p} \quad (8)$$

where C_p is the inherent static capacitance of the PZT transducer.

3. Experimental Results

The piezoelectric vibration shunt experiments were carried on aluminum, wood and carbon-fibre composite beams respectively. Figures 3 & 4 show the time and spectrum results of before and after shunting with the OROS spectrum analyzer. The beam lengths are about 500 mm, widths are between 25 ~ 40 mm, and thicknesses are between 2 ~ 3 mm. The boundary conditions are free-clamped. The PZT used is PSI-5A4E piezoelectric sheet (T107-A4E-602) from Piezo Systems. 8% for wooden beam (Figure 3), and 80% for carbon-fibre composite beam (Figure 4). The experimental results were predicted by simulation. By using carbon-fibre composite beam, the vibration reduction efficiency can be greatly improved when compared with the wooden beam since the Young's modulus of carbon-fibre composite beam can be chosen to be much closer to that of the PZT.

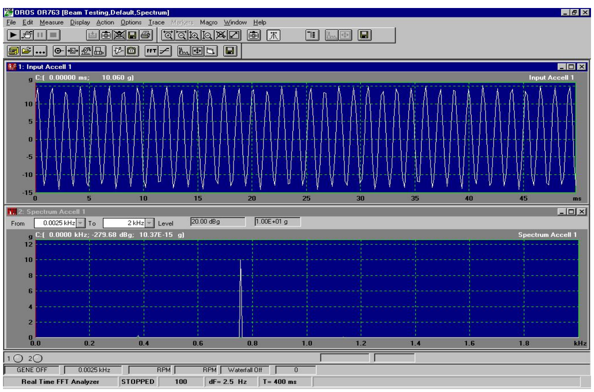


Figure 3 a Wooden beam before s hunting

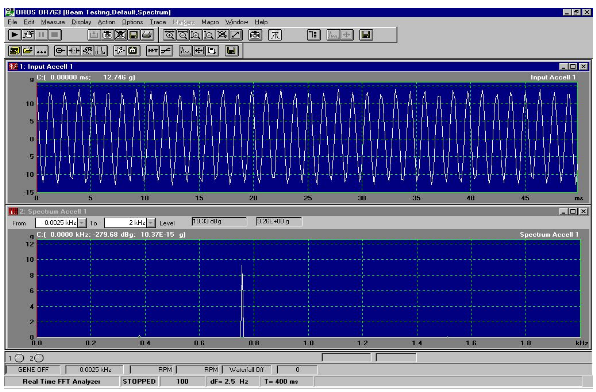
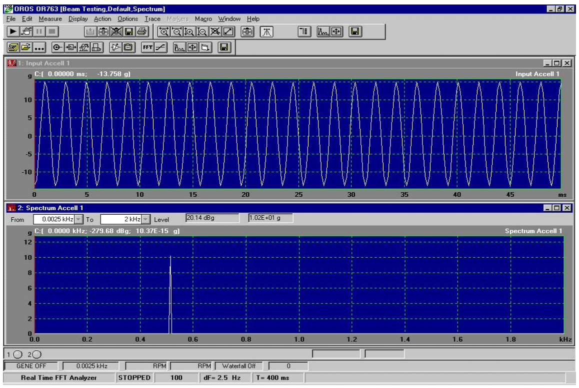


Figure 3b Wooden beam after shunting



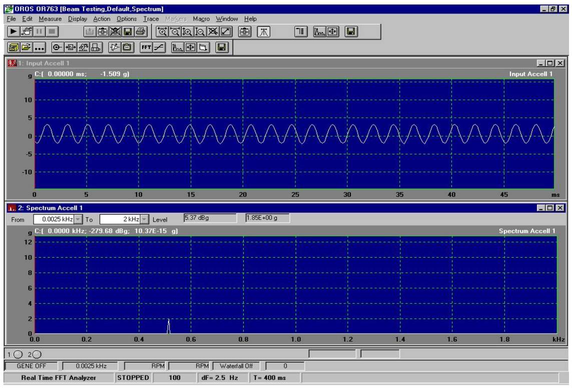


Figure 4a Carbon-fibre composite beam before shunting

Figure 4b Carbon-fibre composite beam after shunting

4. Field-Coupled FEA Analysis of a Composite Wood-Carbon-Fibre Composite Beam with an attached PZT Transducer

In order to improve the efficiency of vibration shunting for a wooden beam, a built-up beam was constructed as shown in Figure 5. The left section is wooden material, right section is a composite material with Young's modulus (E_p) equal to 66 GPa , and density (ρ_b) equal to 1500 kg/m^3. The PZT patch was attached on the top of composite material beam. Figure 6 are the simulation results of the built-up beam. It shows that the deductions of vibration amplitude at natural frequencies are markedly improved. To illustrate this finding, the after shunt circuit activation for the 3rd fundamental mode (Figure 6b) of vibration is compared to that before shunt activation (Figure 6a). Here 80% reduction in the amplitude of vibration is observed.

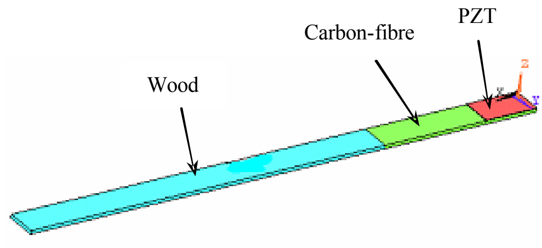
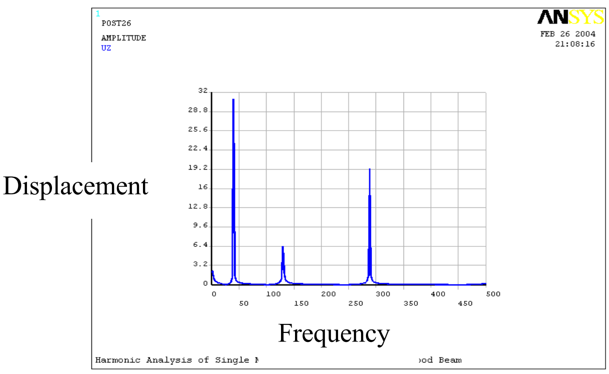
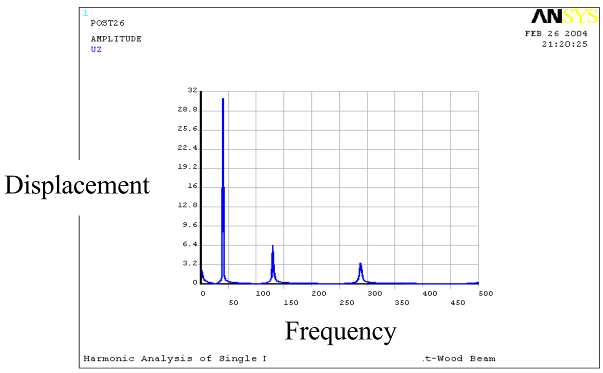


Figure 5 Built-up beam with PZT patch attached





(a) Before Shunt

(b) Shunt 3rd mode

Figure 6 Harmonic analysis of the built-up beam vibration

4. Conclusions

The use of piezoelectric materials in conjunction with resistor-inductor shunt circuits can effectively reduce the vibration amplitudes of structures. An analytical study on the parallel resistor-inductor piezoelectric shunt control using the Hamilton's principle and the Galerkin's method has been presented. On the other hand, the experiments are costly and time consuming. It is thus desirable to have a system to be properly simulated before a final design can be made. The paper has shown a numerical method of designing a piezoelectric vibration shunt control system by using Finite Element Analyses. With the help of such numerical methods, designers can have much more flexibility in designing an optimal control system. Simulation and experimental results have shown the impact of material property variations. Vibration control on the carbon-fibre beam when adhesively bonded to wood, in particular, was shown to be very effective using this technique. This enables the vibration of wooden structures to be controlled with the introduction of a polymeric carbon-fibre interface between the wooden substrate and the piezoelectric transducer connected to a shunt circuit. The results are corroborated from analytical, experimental and computational analyses.

5. Acknowledgements

This work is co-supported by an Australian Research Council (ARC) grant (Grant number LP 0222482).

6. References

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