The second fundamental law of gearing and contact stress calculation of high order contact gearing

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Abstract: After an extensive research on the fundamental theory, the theoretical basis for the Logix gearing will be presented in this paper. This includes the theory for gear meshing with high degree of contact. In comparison to previous studies, the theory of this paper is more restricted going beyond third order parameters into the fourth order, the teeth profile had one order of contact higher than that of the Logix gearing. So that further improved the contact strength of tooth profile. A large category of gearings with high order of contact was presented, while the Logix gear is only one of them, or the special example in realization of my theory. In the Logix gearing, the zigzag curvature center curve of media rack does not always lay about the pitch line and does not extend along the pitch line continuously and steadily. It goes forward and returns. So that, in terms of properties of the transverse engagement, the Logix gearing has not much improvement over the traditional Novikov gearing (in which the transverse engagement is temporally). In my gearing the curvature center of basic rack extend along the pitch line continuously and steadily. The transverse engagement factor is larger than that of Logix gearing.

Keywords: gear law, meshing, basic rack

1 Introduction

Professor Novikov of former Soviet Union presented the point contact circular gearing, so started the history of "Concave-convex" contact transmission. Professor Neumann of Germany presented the Neumann worm transmission [1]. Such researches improved the contact strength of tooth surface in some degree. But circular gearing can only mesh temporally in the same transverse section. The contact of the profile of Neumann worm transmission was still the "second degree "contact. In the early 1990s, the Japanese scholar Komori T *et al.* presented a new gear profile having zero relative curvature at many contact points improved the contact strength of tooth profile [2-3]. However, after the presentation of this new gear, so called "Logix" gear, there were not many significant researches on this field. In recent years, some institutes in China had been attracted by this gearing approach and started to follow the research of this kind of gearing. But most researches concentrated on the topics such as the foot-cut and interference, which are applications of the ready-made principles [4-5]. After an extensive research on the fundamental theory, the theoretical basis for the Logix gearing will be presented in this paper. In comparison to previous studies, the theory of this paper is more restricted going beyond third order parameters into the fourth order, the teeth profile had one order of contact higher than that of the Logix gearing. So that further improved the contact strength of tooth profile. In the gearing proposed in this work the curvature centre of basic rack extends along the pitch line continuously and steadily. The transverse engagement factor is larger than that of Logix gearing. From the viewpoint of derivative geometry, the most simple and strict way to judge the closeness between two bodies is the concept of "contact".
In the end of last century, The NASA of United State, (National Air and Space Agency) had carried out a program of ART (Advanced Rotary Transmission). In which, they presented a new kind of profile, so called NIF/HRC (Nun-involutes / High Rate of Contact) profile. So that increased the contact and bending strength of gearing dramatically. And therefore deceased the total weight of the whole engine. But the detail is very hard to get.

2 The Three definitions of gear law:

(1) First definition: 1886, Oliver present first way to conform the conjugate gear surfaces, and then present second way to conform the conjugate gear surfaces: sometimes called Oliver first law of gear meshing, Oliver second law of gear meshing.[7]

(2) Second definition: the first law of gearing defines the instantaneous relationship between infinitesimal displacements of an output body to an infinitesimal angular displacement of an input body for a specified tooth contact normal. The second law of gearing establishes a relation between the instantaneous gear ratio. The third law of gearing establishes instantaneous relationship for the relative curvature of two conjugate surfaces [8,9].

(3) The third definition:

The first law of gearing: The transmission ratio of two meshing gears is inversely proportional to the ratio of two line segments cut from the center line by the common normal of tooth profiles through the contact point. Wherever the teeth contact, the common normal of tooth profile through the contact point must intersect the centerline at fixed point: $x + yy' = 0$

The two surfaces would conjugate with each other; there are no additional requirements for the profile of media rack. The gap between tooth surfaces at adjacent area of mesh point is an infinite small of second order. The parameter of second order can be expressed by $(x + yy')' = 0$

The second law of gearing: If $(x + yy')' = 0$, the curvature centre of media rack must lay on the pitch line, the gap between tooth surfaces at adjacent area of mesh point is an infinite small of third order.

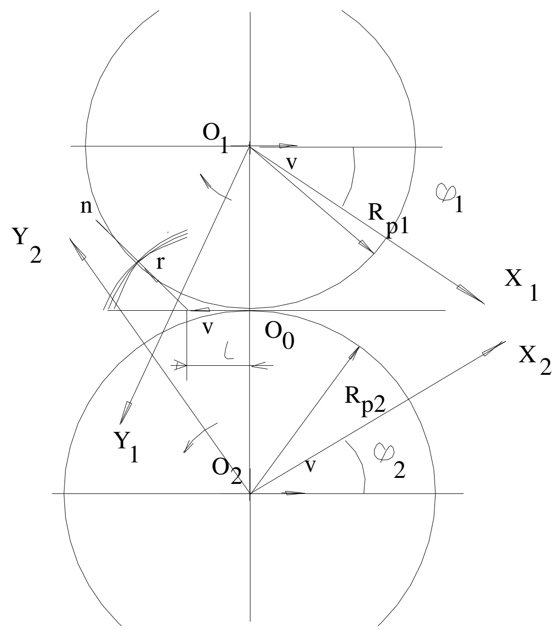

Fig.2 Basic rack and the generation of the conjugate gears

Illustrations of the notations in this figure:

x , y ------the coordinates of the basic rack

n_x ,n_y ------the normal vector of the basic rack

X,Y ------the coordinates of the gear

N_X,N_Y ------the normal vector of the gear

ϕ ------the angular displacement of the gear

l ----- the transverse displacement

Rp -----the radius of the pitch circle

3 The condition for the gap between tooth surfaces to be an infinite small of 3rd order

According to the theories of gear meshing [6], supposing that the profile of basic rack is: $y = y(x)$, the corresponding profile and normal vector of the gear would be (see Fig.2)

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} -R_p\phi \\ R_p \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} N_x \\ N_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} -y' \\ 1 \end{bmatrix} / \sqrt{y'^2+1} \tag{2}$$

According to the theory of gear meshing, the transverse displacement l of rack and the angular displacement of the gear ϕ must satisfy the following equation:

$$\phi = \frac{l}{R_p} = \frac{x + yy'}{R_p} \tag{3}$$

The form of Matrix can express Equ.3:

$$\{R\} = M\{r\} + M\{\Phi\} \tag{4}$$

Where: $\{R\} = \begin{bmatrix} X \\ Y \end{bmatrix}$, $M = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$, $\{r\} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\{\Phi\} = \begin{bmatrix} -R_p\phi \\ R_p \end{bmatrix}$

Differentiate $\{R\}$ form 1st and 2nd order□

$$\{R\}' = M'\{r\} + M\{r\}' + M'\{\Phi\} + M\{\Phi\}' \tag{5}$$

$$\{R\}'' = M''\{r\} + 2M'\{r\}' + M\{r\}'' + M''\{\Phi\} + 2M'\{\Phi\}' + M\{\Phi\}'' \tag{6}$$

In the above equation, the differentials of $\{r\}$ and $\{\Phi\}$ are:

$$\{r\}' = \begin{bmatrix} 1 \\ y' \end{bmatrix}, \{r\}'' = \begin{bmatrix} 0 \\ y'' \end{bmatrix}, \tag{7}$$

$$\{\Phi\} = R_p \begin{bmatrix} -\phi \\ 1 \end{bmatrix}, \{\Phi\}' = R_p \begin{bmatrix} -\phi' \\ 1 \end{bmatrix}, \{\Phi\}'' = R_p \begin{bmatrix} -\phi'' \\ 1 \end{bmatrix}, \tag{8}$$

Since that the original point of coordinate is the intersect point of normal vector and the pitch line of rack. Where:

$$\phi = \frac{l}{R_p} = \frac{x + yy'}{R_p} = 0□ \tag{9}$$

So that, we have:

$$M\big|_{\phi=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{10}$$

$$M'\big|_{\phi=0} = \begin{bmatrix} -\phi'\sin\phi & \phi'\cos\phi \\ -\phi'\cos\phi & -\phi'\sin\phi \end{bmatrix}_{\phi=0} = \begin{bmatrix} 0 & \phi' \\ -\phi' & 0 \end{bmatrix} \tag{11}$$

$$M''\big|_{\phi=0} = \begin{bmatrix} -\phi''\sin\phi - \phi'^2\cos\phi & \phi''\cos\phi - \phi'^2\sin\phi \\ -\phi''\cos\phi + \phi'^2\sin\phi & -\phi''\sin\phi - \phi'^2\cos\phi \end{bmatrix}_{\phi=0} = \begin{bmatrix} 0 & \phi'' \\ -\phi'' & 0 \end{bmatrix} \tag{12}$$

$$\{R\}'' = \phi'' \begin{bmatrix} y \\ -x \end{bmatrix} + 2\phi' \begin{bmatrix} y' \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ y'' \end{bmatrix} + R_p \begin{bmatrix} 0 \\ 2\phi'^2 \end{bmatrix} \tag{13}$$

The normal vector of the gear is determined by the normal vector of the rack, so that we have:

$$\begin{bmatrix} N_x \\ N_y \end{bmatrix}_{\phi=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -y' \\ 1 \end{bmatrix} / \sqrt{y'^2+1} \quad = \begin{bmatrix} -y' \\ 1 \end{bmatrix} / \sqrt{y'^2+1} \tag{14}$$

The second parameter of the Taylor series of the tooth profile: $n \cdot \dfrac{d^2\mathbf{r}}{du^2}$ is:

$$\{N\}^T\{R\}'' = \phi''\begin{bmatrix} -y' & 1 \end{bmatrix}\begin{bmatrix} y \\ -x \end{bmatrix} + 2\phi''\begin{bmatrix} -y' & 1 \end{bmatrix}\begin{bmatrix} y' \\ -1 \end{bmatrix} + \begin{bmatrix} -y' & 1 \end{bmatrix}\begin{bmatrix} 0 \\ y''' \end{bmatrix} + R_p\begin{bmatrix} -y' & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2\phi'^2 \end{bmatrix}$$

$$\phi''(-y'y - x) + 2\phi'(-y'^2 - 1) + y'' + 2R_p\phi'^2 \quad = -2\phi'(y'^2 + 1 + 2R_p\phi') + y'' \tag{15}$$

($\because \phi = 0, l = x + yy' = 0$)

For gear 1 and gear 2 we have the expressions as following respectively:

$$\{N_1\}^T\{R_1\}'' = -2\phi_1'(y'^2 + 1 + 2R_{p1}\phi_1') + y'', \phi_1 = \frac{l}{R_{p1}} = \frac{x + yy'}{R_{p1}}$$

$$\{N_2\}^T\{R_2\}'' = -2\phi_2'(y'^2 + 1 + 2R_{p2}\phi_2') + y'', \phi_2 = \frac{x + yy'}{R_{p2}}$$

Let the second parameter equal zero:

$$\{N_1\}^T\{R_1\}'' - \{N_2\}^T\{R_2\}'' = 0$$

We have:

$$\phi_1' = \frac{l'}{R_{p1}} = 0,\ \phi_2' = \frac{l'}{R_{p2}} = 0,\ l' = 0 \qquad (x + yy')' = 0,\ 1 + y'^2 + y'' = 0, \tag{16}$$

The coordinate of the curvature center:

$$y_c = y + \frac{1 + y'^2}{y''} = 0$$

This means the curvature centre of basic rack must lie on the pitch line.

4 the configuration of the rack profile.

1. First, choose the appropriate value: y_0, x_0, y_0', y_0''

2. Create a shot curve passing through the point (x_0, y_0):

$$y = y_0 + y_0'(x - x_0) + \frac{y_0''}{2}(x - x_0)^2 \tag{17}$$

$$y' = y_0' + y_0''(x - x_0) \tag{18}$$

$$y'' = y_0'' \tag{19}$$

3. Find the curvature centre of the curve:

$$y_c = y + \frac{1 + y'^2}{y''} \tag{20}$$

$$x_c = x - \frac{y'\left(1 + y'^2\right)}{y''} \tag{21}$$

4. Choose a point at the end of the curve. Connect this point and its curvature centre. Find out the intersect point of this line and the *x* axes□

$$x_c' = x - \frac{y\left(x - x_c\right)}{y - y_c} \tag{22}$$

5. Let the new point to be the centre of curvature, revise y'' accordingly. Repeat the 3rd step again.

6. The original data are as following:

$y_0 = 10 \qquad x_0 = 3.64 \qquad y_0' = 2.74748 \qquad y_0'' = -0.8585$

7. Calculate processes are as following:

$y_0 = 10 \qquad x_0 = 3.64 \qquad y_0' = 2.74748 \qquad y_0'' = -0.8585$

□ $\Delta x = -1$

$$y = y_0 + y_0'\left(x - x_0\right) + \frac{y_0''}{2}\left(x - x_0\right)^2 = 10 + 2.74748 \times (-1) + \frac{-0.8585}{2}(-1)^2 = 6.825$$

$$x = 3.64 - 1 = 2.64$$

$$y' = y_0' + y_0''\left(x - x_0\right) = 2.74748 - 0.8585(-1)^2 = 3.602$$

$$y'' = -0.8585$$

$$y_c = y + \frac{1 + y'^2}{y''} = 6.825 + \frac{1 + 3.602^2}{-0.8585} = -9.45$$

$$x_c = x - \frac{y'\left(1 + y'^2\right)}{y''} = 2.64 - \frac{3.602(1 + 3.602)^2}{-0.8585} = 61.27$$

$$x_c' = x - \frac{y\left(x - x_c\right)}{y - y_c} = 2.64 - \frac{6.825(2.64 - 61.27)}{6.825 + 9.45} = 27.23$$

□ $\Delta x = -1$

$y = 6.825 \qquad x = 2.64 - 1 = 1.64 \qquad y' = 3.602$

$$y'' = -\frac{1 + y'^2}{y} = \frac{1 + 3.602^2}{6.825} = -2.05$$

$$y = y_0 + y_0'\left(x - x_0\right) + \frac{y_0''}{2}\left(x - x_0\right)^2 = 6.825 + 3.602 \times (-1) - \frac{2.05}{2}(-1)^2 = 2.2$$

$$y' = y_0' + y_0''\left(x - x_0\right) = 3.602 + 2.05 = 5.652$$

$$y'' = -2.05$$

$$y_c = y + \frac{1 + y'^2}{y''} = 2.2 + \frac{1 + 5.652^2}{-2.05} = -13.87$$

$$x_c = x - \frac{y'\left(1 + y'^2\right)}{y''} = 1.64 - \frac{5.652\left(1 + 5.652^2\right)}{-2.05} = 92.47$$

$$x_c' = x - \frac{y(x-x_c)}{y-y_c} = 1.64 - \frac{2.2(1.64-92.47)}{2.2+13.87} = 14.07$$

□ $\Delta x = -0.1$

$$y = 2.2 \qquad x = 1.54 \qquad y' = 5.652$$

$$y'' = -\frac{1+y'^2}{y} = -\frac{1+5.652^2}{2.2} = -14.98$$

$$y = y_0 + y_0'(x-x_0) + \frac{y_0''}{2}(x-x_0)^2 = 2.2 + 5.652(-0.1) - \frac{14.98}{2}(-0.1)^2 = 1.56$$

$$y' = y_0' + y_0''(x-x_0) = 5.652 + 1.498 = 7.15$$

$$y'' = -14.98$$

$$y_c = y + \frac{1+y'^2}{y''} = 1.56 + \frac{1+(7.15)^2}{-14.98} = -1.92$$

$$x_c = x - \frac{y'(1+y'^2)}{y''} = 1.54 - \frac{7.15(1+7.15^2)}{-14.98} = 26.42$$

$$x_c' = x - \frac{y(x-x_c)}{y-y_c} = 1.54 - \frac{2.2(1.54-26.42)}{2.2+1.92} = 14.81$$

5 Contact stress

According to the research of G.L.Gladwell and Popov, if the deformation could be expressed by Qibchev polynomial, the contact stress can be expressed by Qibchev polynomial as well.

$$w = -d + \sum_{n=0}^{N} b_n T_n(\xi)$$

Then, the contact stress:

$$p(t) = (1-t^2)\ \sum_{n=0}^{N} a_n T_n(t)$$

In the equation□ $a_n = \vartheta^{-1} n b_n$ The equation can be rewritten to the form:

$$(1-t^2)^{-1/2} p(t) = \sum a_n T_n(t)$$

$$(1-t^2)^{-1/2} p(t) = \sum a_n T_n(t) - \sum a_n T_n(1)$$

$$= a_0[T_0(t) - T_0(1)] + a_2[2t^2 - 1 - 1] + a_4[8t^4 - 8t^2 + 1 - 1]$$

$$= 2a_2(t^2-1) + a_4 8t^2(1-t^2)^{1/2}$$

$$p(t) = 2a_2(1-t^2)^{1/2} + 8a_4 t^2(1-t^2)^{1/2} = \vartheta^{-1}[4b_2(1-t^2)^{1/2} + 32b_4 t^2(1-t^2)^{1/2} \quad \text{□57□}$$

6 Conclusion

Some essential findings are:

(1) If $x + yy' = 0$, the two surfaces would conjugate with each other, there are no additional requirements for the profile of basic rack. The gap between tooth surfaces at adjacent area of mesh point is an infinite small of the second order. The parameter of the second order can be expressed by: $(x + yy')'$.

(2) If $(x + yy')' = 0$, the curvature centre of basic rack must lay on the pitch line, the gap between tooth surfaces at adjacent area of mesh point is an infinite small of the third order. The parameter of the third order can be expressed by: $(x + yy')''$.

(3)The contact stress of high order contact gearing can not be calculated by Hertz equation, it can be expressed by Qibchev polynomial.

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Fig.3 The gear

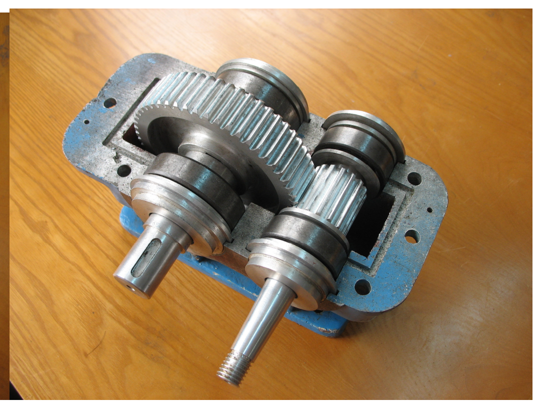

Fig.4 The reducer