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Real-time identification method of a heat transfer coefficient

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Abstract: The demands on machining accuracy have been increasing lately and therefore research of thermal behaviour of machine tool structures is crucial for successful manufacturing. Generated heat diffuses into the structure of the machine tool components, this process is affected by heat sinks such as heat transfer on the surfaces and cooling systems. Meanwhile the heat warms up the structure of the machine tool and thermal dilatation deforms the structure, which subsequently affects machining accuracy in a negative way. Different systems are used to eliminate the thermal error, but their efficiency corresponds to the quality of the thermal machine tool model. The key problems of machine tool thermal error reduction are not in the thermal model itself, but in the fast, or even real-time, identification of the heat sources and the heat transfer coefficients (HTC) on the surfaces. This paper brings up a new identification method of the HTC, which is based on an analytical description.

Keywords: heat transfer coefficient, machine tool, modelling, real-time identification, thermal error.

1 Introduction

Thermal deformation of machine tool structures can be reduced with a thermally stable construction, by cooling the structure to desired temperature or by implementing a correction algorithm in the driving system. The success of all three cases depends on a high quality thermal deformation model. The thermal deformation model can be based on empiricism, the finite element method or an analytical solution.

Empirical methods are grounded in finding the compensation function, which relates the relative thermal displacement to the temperature rise at some points on the structure. Regression techniques are used to express the compensation function in the form of polynomial equations of different orders or as a series of exponential terms with different time constants. Recently neural method techniques were also used. Empirical methods doesn't include the real base of the heat transfer problem, therefore the compensation function is satisfactory only for service conditions used for calibration of the empirical model.

Finite element method (FEM) is very a powerful tool in heat transfer analyses and it respects the real essence of the problem. On the other hand FEM is too robust and the solving is time demanding. Thus it isn't suitable for real-time calculations.

Recently new methods of heat transfer modeling appeared based on the analytical solution of a heat conduction equation (Eq. 1) [1, 2, 3]. These methods contain the nature of the heat transfer principles, thus the calibration of the empirical parameters is simpler and the model is in addition more reliable with untested inputs, because the data is forced to conform to the same principles as the real process. At the same time the results are obtained by two orders of magnitude faster [1]. These parameters make methods based on the analytical solution very suitable for fast estimation of the heat transfer coefficient.

2 Analytical description

In order to identify the heat transfer coefficient the transfer function between heat source and its temperature response must be found. Such a complex process, as heat transfer in a machine tool structure, can be approximated by the analytical solution to a similar, but simpler, phenomenon which is physically related [1]. In this case the phenomenon, called fundamental generalized problem (FGP), is heat conduction in a thin infinite plate with a convective boundary condition on the face and a radially symmetric central ring of heat generation. The heat transfer process is governed by the following differential equation:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r} - \eta \cdot T + \frac{q(r,t)}{\lambda} = \frac{1}{a} \cdot \frac{\partial T}{\partial t} \qquad (1)$$

where $\eta = \alpha/(\lambda \cdot h)$, α is the heat transfer coefficient on the plate surface /W.m^{-2}.K^{-1}/, λ is the material thermal conductivity /W.m^{-1}.K^{-1}/, h is the plate thickness /m/, a is thermal diffusivity /m^{2}.s^{-1}/ and q is the heat released per unit time and volume /W.m^{-3}/. The initial plate temperature is zero, therefore $T(r,0) = 0$.

2.1 General solution

Firstly the term "$\eta \cdot T$" is removed by implementing new variable in Eq. (1).

$$T(r,t) = \theta(r,t) \cdot e^{-\eta a t} \tag{2}$$

This leads to new formulation of Eq. (1):

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \theta}{\partial r} + \frac{q(r,t)}{\lambda} \cdot e^{\eta a t} = \frac{1}{a} \cdot \frac{\partial \theta}{\partial t} \tag{3}$$

This equation can be simplified by the Hankel transformation, where $J_0(Br)$ is a zero order Bessel function of the first kind:

$$\Theta(B,t) = \int_0^\infty r \cdot J_0(Br) \cdot \theta(r,t) \cdot dr \tag{4}$$

The Eq. (3) is multiplied by $r \cdot J_0(Br)$ and integrated with respect to *r*. The integral of the first two terms can be found in [4]:

$$\int_0^\infty \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \theta}{\partial r} \right) \cdot r \cdot J_0(Br) \cdot dr = -B^2 \cdot \Theta \tag{5}$$

If the heat source function q is zero everywhere in the plate except for the thin ring at $r = r_0$, the function can be defined by the space impulse function δ as follows:

$$q(r,t) = \frac{q_S(t)}{2\pi \cdot r_0 \cdot h} \cdot \delta(r - r_0) \tag{6}$$

Then the third term in Eq. (3) is transformed after the substitution of Eq. (6) into Eq. (3).

$$\frac{q_S(t) \cdot e^{\eta a t}}{2\pi \cdot r_0 \cdot h \cdot \lambda} \cdot \int_0^\infty \delta(r - r_0) \cdot r \cdot J_0(Br) \cdot dr = \frac{q_S(t) \cdot e^{\eta a t}}{2\pi \cdot h \cdot \lambda} \cdot J_0(Br_0) \tag{7}$$

The final form of the transformed Eq. (3) is a first order linear differential equation:

$$\frac{\partial \Theta}{\partial t} + aB^2 \cdot \Theta = \frac{q_S(t) \cdot a \cdot e^{\eta a t}}{2\pi \cdot h \cdot \lambda} \cdot J_0(Br_0) \tag{8}$$

Homogenous solution, Eq. (9), of this equation is found and substituted into it:

$$\Theta(B,t) = C(B,t) \cdot e^{-aB^2 t} \tag{9}$$

$$\frac{\partial C}{\partial t} \cdot e^{-aB^2 t} = \frac{q_S(t) \cdot a \cdot e^{\eta a t}}{2\pi \cdot h \cdot \lambda} \cdot J_0(Br_0) = s(t) \tag{10}$$

$$C(B,t) = \frac{a \cdot J_0(Br_0)}{2\pi \cdot h \cdot \lambda} \cdot \int_0^t q_S(t) \cdot e^{(\eta + B^2) a t} \cdot dt + C_B(B) \tag{11}$$

It is clear that $C_B = 0$, when the initial condition is considered. The final solution, Eq. (12), of Eq. (8) is transformed by the inverse Hankel transformation, Eq. (13), back to the space domain and finally substituted by the Eq. (2), Eq. (14).

$$\Theta(B,t) = \frac{a \cdot J_0(Br_0) \cdot e^{-aB^2 t}}{2\pi \cdot h \cdot \lambda} \cdot \int_0^t q_S(\tau) \cdot e^{(\eta + B^2) a \tau} \cdot d\tau \tag{12}$$

$$\theta(r,t) = \frac{a}{2\pi \cdot h \cdot \lambda} \cdot \int_0^\infty B \cdot J_0(Br) \cdot J_0(Br_0) \cdot e^{-aB^2 t} \cdot \left(\int_0^t q_S(\tau) \cdot e^{(\eta + B^2) a\tau} \cdot d\tau \right) \cdot dB \tag{13}$$

$$T(r,t) = \frac{a}{2\pi \cdot h \cdot \lambda} \cdot \int_0^\infty B \cdot J_0(Br) \cdot J_0(Br_0) \cdot e^{-(\eta + B^2) \cdot at} \cdot \left(\int_0^t q_S(\tau) \cdot e^{(\eta + B^2) a\tau} \cdot d\tau \right) \cdot dB \tag{14}$$

Equation (14) represents general analytical solution of heat conduction in a thin infinite plate.

2.2 Solution for constant heat source

The heat source q_s is considered to be constant during the identification process, $q_s = Q$. Then the general solution is expressed:

$$T(r,t) = \frac{Q}{2\pi \cdot h \cdot \lambda} \cdot \int_0^\infty \frac{B}{\eta + B^2} \cdot J_0(Br) \cdot J_0(Br_0) \cdot \left(1 - e^{-(\eta + B^2) \cdot at} \right) \cdot dB \tag{15}$$

$$T(r,t) = \frac{Q}{2\pi \cdot h \cdot \lambda} \cdot \left(F(r,0) - F(r,t) \right) \tag{16}$$

Where

$$F(r,t) = \int_0^\infty \frac{B}{\eta + B^2} \cdot J_0(Br) \cdot J_0(Br_0) \cdot e^{-a(\eta + B^2)t} \cdot dB \tag{17}$$

3 Identification method

This method is based on the analytical solution, Eq. (16) and allows identify heat transfer coefficient on the surface of a thin plate from temperature response in the vicinity of the defined heat source.

Let's assume the plate is heated with known and constant heat source Q. Then the temperature response in the distance $r=R$, as shown in Eq. (16), is a function of time and η, which linearly depends on the heat transfer coefficient.

$$T(R,t,\eta) = C \cdot \left(F(R,0,\eta) - F(R,t,\eta) \right) \tag{18}$$

Where

$$F(R,t,\eta) = \int_0^\infty \frac{B}{\eta + B^2} \cdot J_0(BR) \cdot J_0(Br_0) \cdot e^{-a(\eta + B^2)t} \cdot dB \tag{19}$$

This leads to an inverse problem, where η must be expressed from Eq. (19) analytically and then calculated for each pair of measured temperature and time $[T(R,t_i), t_i]$ so:

$$\eta(t_i) = \left(F_A(R, t_i, T_i / C) \right)_{INV} \tag{20}$$

Unfortunately the function F is very complex (Figure 1) and it's very difficult or nearly impossible to obtain its analytical solution. Therefore the function $F(\eta,t)$ is approximated for $r = R = 2r_0$ by the function $F_N(\eta,t)$, which can be the problem solve after. The approximation starts with substituting the functions F(1,t), identical to $F(\eta_{ref},t)$, and $F(\eta^*,T_0)$, where $\eta^* = \eta/\eta_{ref}$ (Figure 2):

$$F(1,t) \approx U(t) = \sum_{j=1}^{n} \frac{u_j}{1 + v_j \cdot t} \qquad n \cong 3 \tag{21}$$

$$F(\eta^*, T_0) \approx s(\eta^*) = \sum_{j=0}^{m} w_j \cdot (\eta^*)^j \qquad m \cong 3 \tag{22}$$

Then:

$$S(\eta^*, t_i) = F(\eta^*, t_i) - F(1, t_i) - \left[F(\eta^*, t_0) - F(1, t_0) \right] \tag{23}$$

can be very well substituted by (Figure 3):

$$S(\eta^*, t_i) = p(t_i) \cdot (\eta^* - 1) + q(t_i) \cdot (\eta^* - 1)^2 \quad (24)$$

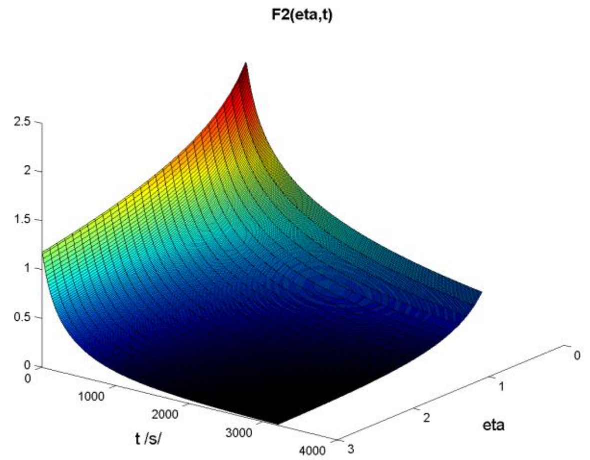

Figure 1. Numerical solution of F(t,η)

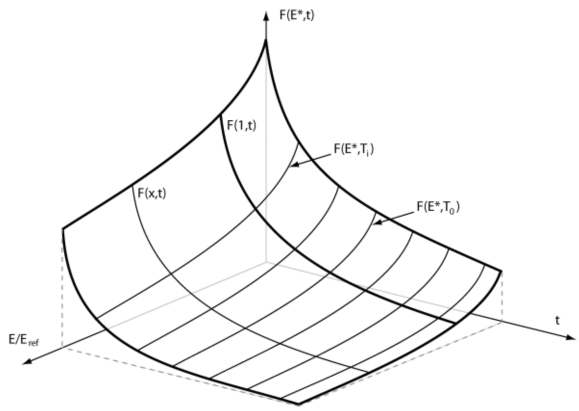

Figure 2. Approximation of F(t,E*), E*=η/ η_{ref}

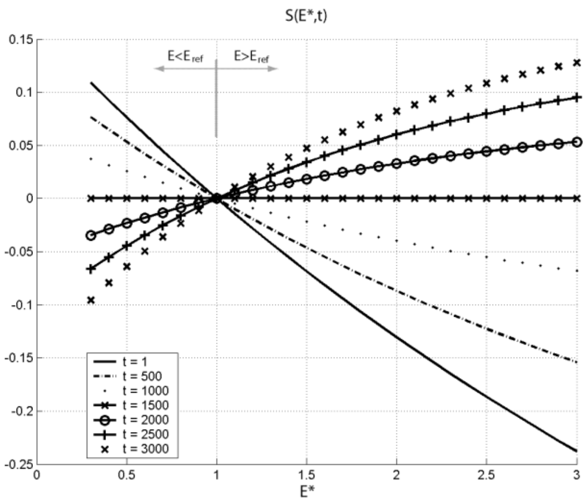

Figure 3. Function S(E*,t)

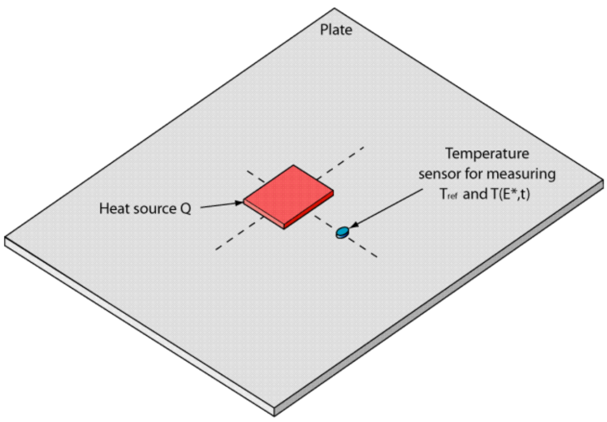

Figure 4. Scheme of the identification method

Substituting Eq. (24), (21) and (22) into (23) gives:

$$F(\eta^*, t_i) = S(\eta^*, t_i) - U(t_0) + s(\eta^*) + U(t_i) \quad (25)$$

The approximation becomes simpler and more applicable if the reference temperature is known.

$$T_{ref}(t) = C \cdot \left(F(R, 0, \eta_{ref}) - F(R, t, \eta_{ref})\right) \quad (26)$$

This temperature is measured at calibration conditions. Even if these conditions are unknown, i.e. η_{ref} is unknown, the resultant η^* shows the change of unknown η_{ref}, respectively α_{ref}, linearly which is sufficient information for the general temperature model.

Then the temperature difference between reference T_{Ref} and measured temperature T_M at unknown η^* is:

$$\frac{T_{ref} - T_M}{C} = \left(F(0, \eta_{ref}) - F(t, \eta_{ref})\right) - \left(F(0, \eta^*) - F(t, \eta^*)\right) \quad (27)$$

Substituting Eq. (25) into (27) results in:

$$\frac{\Delta T}{C} = S(\eta^*, t) - S(\eta^*, 0) \qquad \rightarrow \qquad \frac{\Delta T}{C} = P(t)\cdot(\eta^* - 1) + Q(t)\cdot(\eta^* - 1)^2 \tag{28}$$

Which leads to the quadratic equation:

$$A(t)\cdot(\eta^*)^2 + B(t)\cdot\eta^* + C(t) = 0 \tag{29}$$

The solution of Eq. (29) for each $t=t_i$ gives the resultant value of the unknown heat transfer coefficient on the surface of the plate at the time t_i.

4 Numerical experiments

The presented method was tested on a FEM model, created in ANSYS (Figure 4).

4.1 Plate model

The plate (*1600x1600x10 mm)* model represents surface of a machine tool with thickness *h=10 mm.* The heat transfer coefficient on the plate surface is $\alpha_{ref} = 10\ W.m^{-2}.K^{-1}$, the thermal conductivity $\lambda = 43\ W.m^{-1}.K^{-1}$,the thermal diffusivity $a=11.9081x10^{-6} m^2.s^{-1}$ and the square heat source, with side length 50 mm, $r_0=25\ mm$, $Q=10\ W$ is placed in the center.

The reference temperature was derived 45 mm from the center of the plane or the heat source respectively with the heat transfer coefficient on the surface α_{ref}. The function F was approximated for the following range: $[\eta^*, t] \in \langle 1 - 3 \rangle \times \langle 0 - 3200 \rangle$. Variables *P(t)* and *Q(t)* in the Eq. (28) were calculated for the range of time from the Eq. (19).

4.2 Results

Different heat transfer coefficients were applied on the surface and then the temperature responses were derived from the model, see Figure 5. Unknown values *η** were calculated from Eq. (29), see Figure 6 and Table 1.

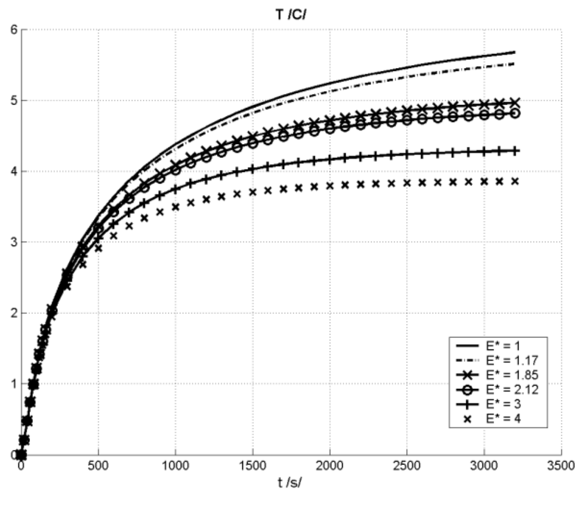

Figure 5. Temperature rise for various η* - plate

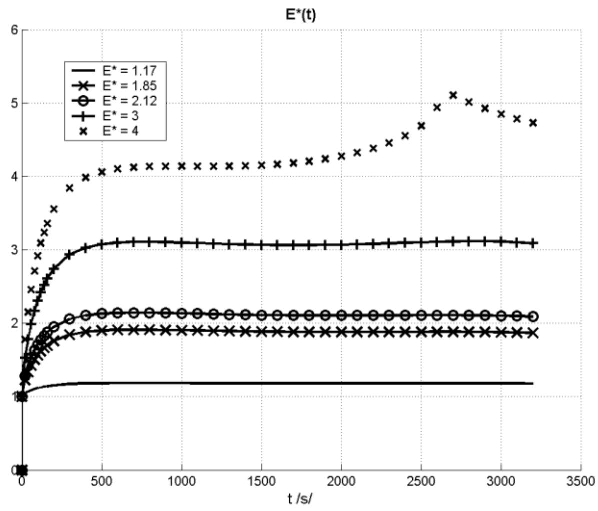

Figure 6. Identified values η*(t) - plate

The table below concludes the results averaged for the time spam (500 - 3200) s.

Table 1. Results

Applied HTC /$Wm^{-2}K^{-1}$/	Calculated HTC /$Wm^{-2}K^{-1}$/	Relative error /%/
11.7	11.806	0.91
18.5	18.88	2.05
21.2	21.142	-2.74
30	30.894	2.98
40*	44.011*	10.03*

*The method was tested for value outside the region of the approximated function F.

5 Conclusion

Presented method of heat transfer coefficient identification was tested on a numerical model and the applicability was proved, even for the square heat source. The relative error of the method is less than 3 % and the time needed for the identification is approximately 10 min. If the plate, which can be implemented in the structure of the machine tool instead of the real surface, is 1 mm thin its temperature response will be one minute, which is sufficient enough for real-time calculations of machine tool behaviour.

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