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Bone functional remodeling under multi-field loadings

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Abstract: Bone remodeling process is investigated theoretically and numerically within the framework of the extended adaptive elasticity. The coupling between the internal and surface remodeling is considered for the solution of the bone remodeling process. A semi-analytical solution based on the state-space method was used to analyze remodeling process of inhomogeneous bone materials subjected to multi-field loadings. The results show that, in contrast to the former works without considering the coupling between the internal and the surface remodeling, the magnitude of the electrical field changes is obviously lower and the remodeling time is shorten significantly. This implies that the coupling between the internal and the surface remodeling plays an important role in the bone remodeling process.

Keywords: Coupling of internal remodeling and surface remodeling, Bone remodelling, Electromagnetic field, Biomechanics, inhomogeneous bone materials.

1 Introduction

The process of bone functional remodeling is defined as bone formation and resorption to adapt its historical architecture to changes in long term loadings. It can be classified as two categories: internal bone remodeling and surface bone remodeling [1]. Internal remodeling describes the process of the resorption of a bone tissue by decreasing or increasing the bulk density of the bone, while the surface remodeling describes the deposition of bone material on the external surface of the bone [2]. During the past decades, many theoretical models have been proposed for the study of bone remodeling [3-11]. Among all the theories, Cowin's adaptive elasticity is one of the most popular ones due to its explicitness and simplicity. In former works, however, the internal bone remodeling and the surface bone remodeling are considered separately by ignoring the coupling phenomena between them. As a matter of fact, bone's internal remodeling and surface remodeling are coupled each other and the remodeling is an interaction of the two kinds of processes. Therefore it is necessary to investigate bone remodeling process by considering the internal and surface remodeling simultaneously. On the other hand, extremely low frequency pulsed electromagnetic field was widely applied in the treatment of skeletal disease such as osteoporosis, tendonitis, osteonecrosis, fracture and nonunion in recent years. And much research has been carried out on the response of bone tissue under an extremely low frequency electromagnetic field [12-14]. All these results have shown that a pulsed electromagnetic field with extremely low frequency can trigger bone tissue to remodel itself. But the mechanism of their interaction is still under investigation.

In this paper, an improved theoretical model was proposed to investigate the internal and the surface remodeling which are coupled each other based on the extended theory of adaptive elasticity. In particular, we are interested in exploring the effect of the coupling on the bone remodeling process. A semi-analytical solution based on the state-space method was obtained to describe the strain field, electrical field and the magnetic field of inhomogeneous bone materials. The influence of all the external loadings, such as transverse pressure, electrical field and magnetic filed on the bone remodeling process are discussed in detail.

2 Basic equations for bone remodeling

In this section, the theory of extended adaptive elasticity [2, 15] used to analyzing the internal and the surface bone remodeling process is re-extended to include magnetic effect. The bone material used in this work is assumed to be a hollow circular cylinder composed of linearly magnetoelectroelastic bone materials subjected to axisymmetric multi-field loadings. Since this problem is axisymmetric about the axis of the cylinder, a cylindrical coordinate system is used in the analysis. The axial, circumferential and normal to the middle-surface co-ordinate length parameters are denoted by z, θ and r, respectively.

2.1 Equations for internal remodeling

To investigate the influence of magnetic field on bone remodeling process, we extend the theory of adaptive elasticity [2] to the new regulation equation:

$$\begin{aligned}\dot{e} = A^{*}(e) + A_{r}^{E}(e)E_{r} + A_{z}^{E}(e)E_{z} + F_{r}^{E}(e)H_{r} + F_{z}^{E}(e)H_{z} \\ + A_{rr}^{s}(e)(s_{rr} + s_{\theta\theta}) + A_{zz}^{s}(e)s_{zz} + A_{rz}^{s}(e)s_{rz}\end{aligned} \tag{1}$$

where e is a change in the volume fraction of bone matrix material from its reference value, say ξ_0, s_{ij}, E_i and H_i are components of strain, electric field and magnetic field respectively, and $A^{*}(e)$, $A_i^E(e)$, $F_i^E(e)$ and $A_{ij}^s(e)$ are material coefficients dependent upon the volume fraction change e and can be approximated by the following equations for small value of *e* [16].

2.2 Equations for surface bone remodeling

The surface bone remodeling equations for the theory of extended adaptive elasticity to consider the magnetic field can be shown as follows [15]:

$$\begin{cases} -\dfrac{\partial a}{\partial t} = C_{rr}^{e}s_{rr}^{e} + C_{\theta\theta}^{e}s_{\theta\theta}^{e} + C_{zz}^{e}s_{zz}^{e} + C_{rz}^{e}s_{rz}^{e} + C_{r}^{e}E_{r}^{e} + C_{z}^{e}E_{z}^{e} + G_{r}^{e}H_{r}^{e} + G_{z}^{e}H_{z}^{e} - C_{0}^{e} \\ \dfrac{\partial b}{\partial t} = C_{rr}^{p}s_{rr}^{p} + C_{\theta\theta}^{p}s_{\theta\theta}^{p} + C_{zz}^{p}s_{zz}^{p} + C_{rz}^{p}s_{rz}^{p} + C_{r}^{p}E_{r}^{p} + C_{z}^{p}E_{z}^{p} + G_{r}^{p}H_{r}^{p} + G_{z}^{p}H_{z}^{p} - C_{0}^{p} \end{cases} \tag{2}$$

where *t* is the time, $a(t,e,b)$ is the radius of the inner surface and $b(t,e,a)$ denotes the radius of the outer surface, and C_{ij}, C_i and G_i are surface remodeling coefficients. The superscript e and p denote the endosteum and the periosteum.

2.3 Solutions to regulating equations for bone remodeling

We consider a piezoelectromagnetic bone cylinder subjected to a quasi-static axial load P, an external pressure *p*, an electric load φ_a (and/or) φ_b and a magnetic load ψ_a (or/and ψ_b). The constitutive equations of a piezoelectromagnetic solid in cylindrical coordinate system can be given by [17]:

$$\begin{aligned}
\sigma_{rr} &= c_{11}s_{rr} + c_{12}s_{\theta\theta} + c_{13}s_{zz} - e_{31}E_{z} - \alpha_{31}H_{z} \\
\sigma_{\theta\theta} &= c_{12}s_{rr} + c_{11}s_{\theta\theta} + c_{13}s_{zz} - e_{31}E_{z} - \alpha_{31}H_{z} \\
\sigma_{zz} &= c_{13}s_{rr} + c_{13}s_{\theta\theta} + c_{33}s_{zz} - e_{33}E_{z} - \alpha_{33}H_{z} \\
\sigma_{zr} &= c_{44}s_{zr} - e_{15}E_{r} - \alpha_{15}H_{r} \\
D_{r} &= e_{15}s_{zr} + \kappa_{1}E_{r} + d_{1}H_{r} \\
D_{z} &= e_{31}(s_{rr} + s_{\theta\theta}) + e_{33}s_{zz} + \kappa_{3}E_{z} + d_{3}H_{z} \\
B_{r} &= \alpha_{15}s_{zr} + d_{1}E_{r} + \mu_{1}H_{r} \\
B_{z} &= \alpha_{31}(s_{rr} + s_{\theta\theta}) + \alpha_{33}s_{zz} + d_{3}E_{z} + \mu_{3}H_{z}
\end{aligned} \tag{3}$$

where σ_{ij}, D_i and B_i are components of stress, electrical displacement and magnetic induction, respectively, c_{ij}, e_{ij} and α_{ij} are elastic stiffness, piezoelectric constants, and piezomagnetic constants, and κ_i, d_i and μ_i are dielectric permitivities, magnetoelectric constants and magnetic permeabilities.

It can be seen from Eqs. (1) and (2) that both internal and surface modeling equations are written in terms of strain field, electrical field and magnetic field. Once all these external loads are determined, the solutions to $e(t,a,b)$, $a(t,e,b)$ and $b(t,e,a)$ can be obtained from Eqs. (1) and (2). In the case of homogeneous bone material, analytical solutions to the bone remodeling process can be obtained [2, 15]. In the case of inhomogeneous bone material, however, the analytical solutions cannot be obtained due to the fact that the analytical solution to the strain field is not available for the structures of inhomogeneous material, and thus we will use the semi-analytical methods to investigate the interaction process of the bone remodeling. The solution procedure for the strain field, electrical field and magnetic field is given in the work by Qin *et al*. [2, 15] which made an attempt to investigate the inhomogeneous internal or surface bone remodeling based on the state-space method. They considered an initially inhomogeneous bone structure and examined the effect of inhomogeneity on bone remodeling. This paper applies the same method to investigate the influence of transverse loadings on an initially homogeneous bone structure by considering the coupled internal and surface bone remodeling process.

Once these fields are determined, the volume fraction change *e* and the surface radii *a* and *b* of a bone can be calculated, and the interaction between internal and surface bone remodeling can be investigated. In the calculation, the variables e, a and b are evaluated by using the forward Euler scheme:

$$e^{t+\Delta t} = e^{t} + \frac{\partial e}{\partial t}\times\Delta t\,,\quad a^{t+\Delta t} = a^{t} + \frac{\partial a}{\partial t}\times\Delta t\,,\quad b^{t+\Delta t} = b^{t} + \frac{\partial b}{\partial t}\times\Delta t \qquad (4)$$

For convenience, the following non-dimensional parameters are introduced for the variation rate of the radii of inner and outer surfaces.

$$\varepsilon = \frac{a}{a_0} - 1\,,\qquad \eta = \frac{b}{b_0} - 1 \qquad (5)$$

3 Results and discussions

In this section, three loading cases will be distinguished to examine the influence of transverse pressure, the electrical field and the magnetic field on the bone remodeling. The results can be seen as follow.

Case 1: The effect of electric field on the surface bone remodeling

First, we examine the effect of electric field on the surface bone remodeling: (1) considering the coupling of the internal and the surface remodeling and (2) ignoring the internal remodeling. The axial load $P = 1500\,\mathrm{KN}$ and the electric load $\varphi = \varphi_b - \varphi_a = -2, -1, 1, 2\ \mathrm{V}$ are applied.

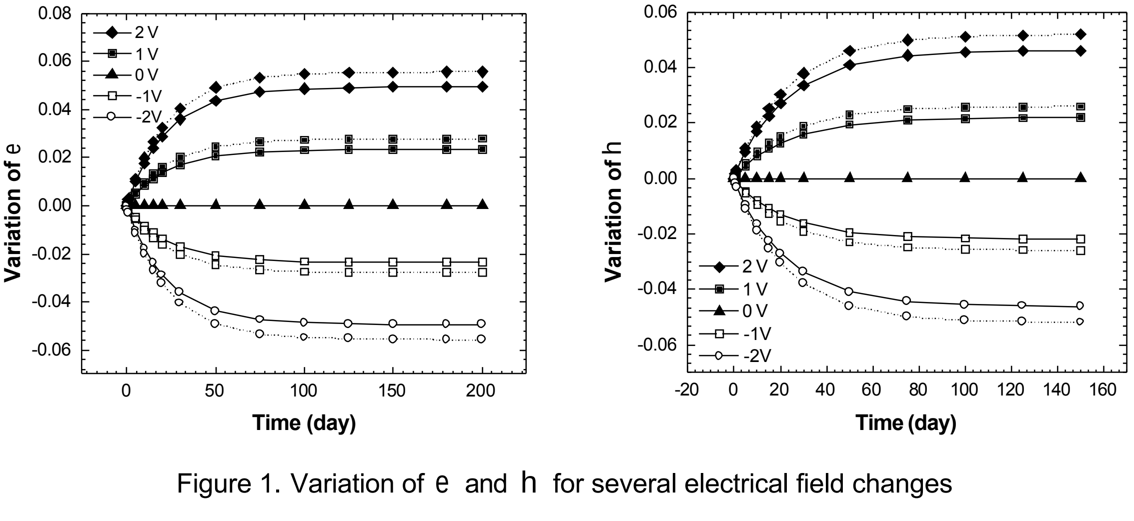

Figure 1. Variation of ε and η for several electrical field changes

Fig. 1 shows the variation rates of inner and outer surfaces versus time *t*. The solid line curves are obtained by considering both the bone internal remodeling and the surface remodeling, and the dash line ones are obtained by ignoring the internal bone remodeling. Similar to the results obtained by ignoring the internal bone remodeling, the solid curves by considering both the bone internal remodeling and the surface remodeling indicate that positive electrical field can increase both the inner and the outer radii of the bone's surface. And as the time increase, the variation rate will approach to a constant. The ultimate change rates of the two surfaces are almost the same and the area of bone's cross section increases, which means that the bone structure becomes thicker and the strains of the bone materials decrease. However, the negative one can just do the opposite. It also can be seen that a more intense field results in more changes in bone's inner and outer surface radii. At the same time a faster remodeling process can also observed when the intensity of the electrical field increase.

As can be seen from Fig. 1 the results obtained by considering the coupling are much different from those by ignoring the internal remodeling in spite of the similar changes of bone remodeling in both cases. Both the change rate and the remodeling rate obtained by considering the coupling are less than those obtained by ignoring the internal remodeling. But the time cost by bone remodeling in the coupling case is shorter than that by ignoring the internal remodeling. The reason can be illustrated as follow. The uncoupling method only considers the bone internal remodeling and surface remodeling respectively. When the internal bone remodeling is investigated, the influence of the surface remodeling is omitted and vice versa. But actually, they can interact and both of them contribute to the bone structure changes. If we omit one of them, the task to be done by another one is heavier, which enhance the values of the results. It's obvious that the cooperation of the two processes can remodel the bone materials more accurately. So the improved method is better for the simulation of bone remodeling process.

Case 2: The effect of magnetic field on bone internal remodeling

To investigate the effect of magnetic field, we simulate the internal bone remodeling under magnetic field with considering the coupling process. The axial load *P* = 1500 KN and the magnetic loads $y = y_b - y_a = -2, -1, 1, 2\ \mathrm{A}$ are applied. The influences of magnetic field and days on the volume fraction change are shown in Figs. 2a and 2b, respectively.

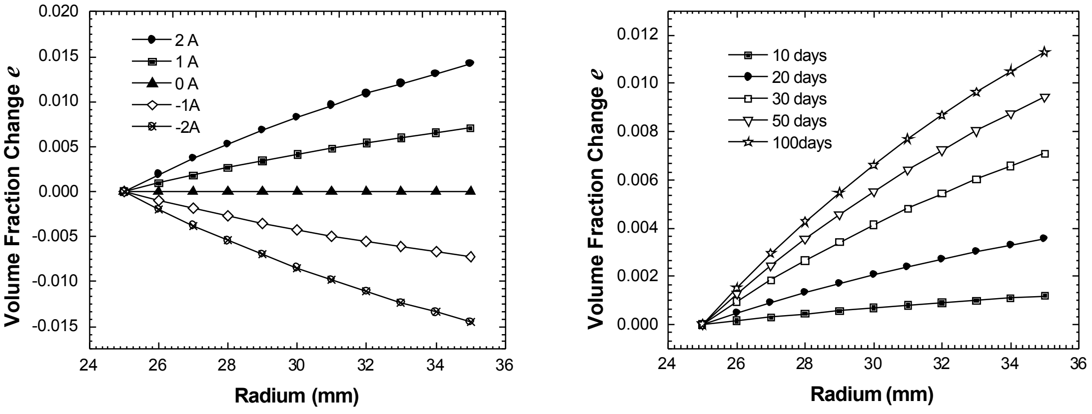

Figure 2. Bone internal remodeling for several electrical field changes

Fig. 2 shows that the volume fraction change e increases along the radial direction with the smallest in the inner surface and the largest in the outer surface. It indicates that the transverse magnetic loadings can remodel an initially homogenous bone structure to an inhomogeneous one. This can be illustrated by the theory of adaptive elasticity: An inhomogeneous stress field is generated when the transverse magnetic field is loaded, and the stress of the inner surface is smaller than that of the outer one. As the bone remodeling process is ongoing, the strain field is becoming homogeneous. To achieve this, the bone tissue must change to a state with more porous endosteum and less porous periosteum, which results in an inhomogeneous bone structure and the inhomogeneity is dependent on the intense of magnetic field and time-related. It is seen from Fig. 2 that transverse magnetic loads can indeed change the bone structure although the value of e is very small. With the increase of the intense of magnetic field or time, the volume fraction change e increases, which means the bone structure becomes more inhomogeneous.

In addition, Fig. 2 also shows that positive magnetic field can result in a more density bone tissue by adding new bone materials and increase the porosity of the bone, which can decrease the strain of the structure. The more intense the positive magnetic field is, the more density bone structure we can obtain. Also, it can be seen in Fig. 2 that as the time passes, the changes of volume fraction becomes less and less. This indicates an equilibrium state of the bone remodeling will be finally attained and after that the volume fraction doesn't change any more.

Case 3: The effect of transverse pressure on the surface and internal bone remodeling

To illustrate the influence of the transverse pressure on the surface bone remodeling, the variation rate of the radii of the inner surface is calculated, as shown in Fig. 3a. The applied loading are p = 0.1, 0.2, 0.3, 0.4, 0.5 MPa and P = 1500 KN.

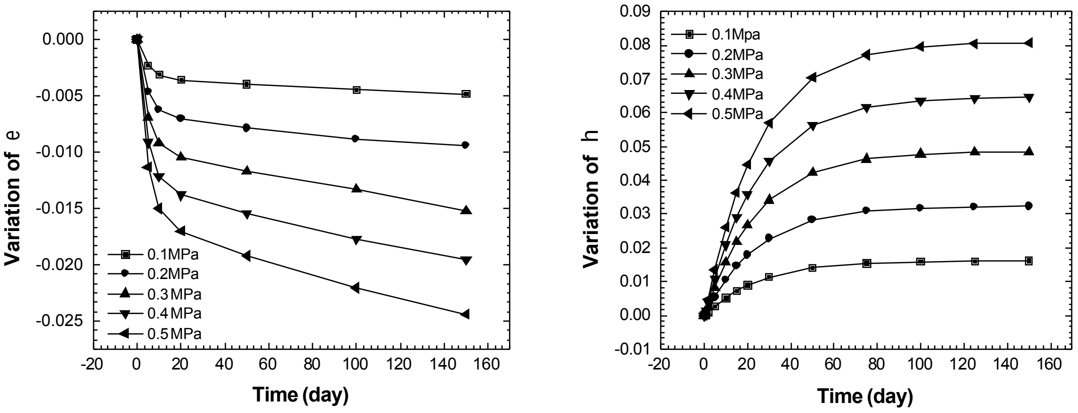

Figure 3. Bone surface remodeling under transverse pressure

The inner and outer surface bone remodeling under transverse pressures are presented in Fig. 3. In contrast to the effect of the electric field on the bone surface remodeling where the inner surface of the bone becomes bigger when the intense of the positive electrical field increases and smaller when the intense of the positive electrical field decreases, the inner surface of the bone becomes smaller when the transverse pressure increases and bigger when the transverse pressure decreases. On the other hand, the outer surface becomes bigger when the transverse pressure increases and smaller when the transverse pressure decreases, just the same as its changes with the positive electrical field, as shown in Fig. 1. The effect of the transverse pressure on the inner and outer surfaces indicates that the transverse pressure can result in an increase in the bone's cross sectional area and, consequently, a thicker and stronger bone which can decrease the strains of the bone structure. The larger the pressure, the more the surfaces of the bone change, and a larger pressure can also increase the velocity of bone surface remodeling and, thus, accelerate the recovery of the injured bone.

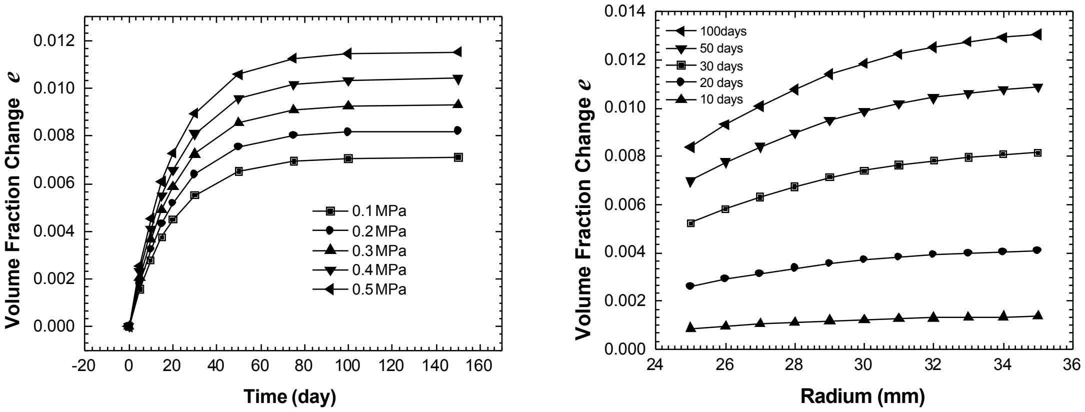

Figure 4. Bone internal remodelling under transverse pressure

To show the influence of the transverse pressure on the internal bone remodeling, the bone internal remodeling is simulated and shown in Fig. 4. It can be seen from Fig. 4 that the transverse pressure can also inhomogenize the initial homogeneous bone material as the magnetic field does. And the inhomogeneous degree is also related to the time and the magnitude of the transverse pressure. The same reason why the magnetic field can inhomogenize the bone structure can explain this phenomenon. And transverse pressure can also strengthen the bone tissues by depositing new bone material and reducing porosity. Then the density and the elastic modular increase as well, which result in the decrease of the strain of the bone structure.

4 Conclusions

An efficient methodology has been established for the coupled internal and surface remodeling of a bone structure with inhomogeneous material within the framework of extended adaptive elasticity. The state-space method is introduced to solve the regulating functions of bone remodeling, and a semi-analytical solution is obtained to simulate the bone remodeling process. In particular, the coupling analysis of the internal and surface remodeling of the bone structure under the magnetic field is reported for the first time in this paper.

Using the modified theoretical model, numerical simulations have been carried out to examine the coupling effect on the internal and surface bone remodeling process. Results show that the coupling between the internal and the surface remodeling plays an important role in the bone remodeling process, and thus the improved method is more accurate than the previous theoretical model which ignores the coupling of the internal and the surface remodeling for the simulation of bone remodeling process. Also the numerical results indicate that the transverse loadings, including transverse pressure, electric load and magnetic load, can strengthen or weaken bone structures and make them inhomogeneous. This feature can be considered and applied in the clinical practice after the experimental validation.

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