Propagative identification of SIMO systems using the Mean Differential Cepstrum (MDC)

W. L. Chia[1], R. B. Randall[1] and J. Antoni[2]

[1] *School of Mechanical and Manufacturing Engineering, University of New South Wales, Sydney 2052, Australia*

[2] *Laboratoire Roberval de Mécanique, Université de Technologie de Compiègne, 60200 Compiègne, France*

Abstract: An alternative SIMO system identification technique is presented in this paper, requiring only response measurements. The technique is based on the Mean Differential Cepstrum, but in a different format from the original, and has two solution formulations, which are differentiated by the use of a Taylor series approximation. The identification processes give both the magnitude and phase in a propagative manner, solving in the frequency domain from one frequency to the next. Initial values near zero frequency can be taken from the static stiffness properties (for fixed systems) or inertial properties (for free-free systems). The technique has the advantage of not requiring the assumption of minimum-phase properties for the system being identified, which is successfully demonstrated on simulated minimum and non-minimum phase systems. A discussion on the stability of the two solution formulations is given, together with the results from the application to measurements from an experimental test rig. Since the method is limited to transient inputs, the excitations used are both burst random and impulsive forces in each test scenario.

Keywords: single-input multiple-output (SIMO) system identification, mean differential cepstrum (MDC), Taylor series approximation, propagative, non-minimum phase systems.

1 Introduction

The single-input multiple-output (SIMO) system identification method presented in this paper forms part of the completed work [1] intended for multiple-input multiple-output (MIMO) systems. The objective is to address the common assumption of spectrally white input in prevailing operational modal analysis (OMA) methods [2][3]. In reality, input spectra are closer to being broadband than white, leading to incorrect scaling between different mode shapes in the absence of input information. Successful use of existing cepstral methods [4][5] in correcting these inaccuracies fuelled the pursuit of a cepstrum-based system identification methodology. The alternative Mean Differential Cepstrum (MDC) method proposed here differs from the original format [6] and has two solution formations, differentiated by the use of a Taylor series approximation. Each directly estimates both magnitude and phase of the frequency response function (FRF) in a propagative manner. It has the advantage of not requiring the assumption of minimum-phase characteristics for the system, hence suited for estimating non-minimum phase systems.

2 Background

The term cepstrum was first coined in 1963, and was initially developed for echo detection [7]. The homomorphic nature of the cepstrum gives an additive relation between the input and the system, as illustrated in (1), the cepstrum of the generic system equation, $Y(\omega) = H(\omega)\,X(\omega)$.

$$\begin{aligned} \mathbf{C_y}(\tau) = Y(\tau) &= \mathcal{F}^{-1}(ln\,Y(\omega)) \\ &= \mathcal{F}^{-1}(ln\,[H(\omega)\,X(\omega)]) \\ &= \mathcal{F}^{-1}(ln\,H(\omega) + ln\,X(\omega)) \quad = \quad H(\tau) + X(\tau) \end{aligned} \tag{1}$$

The Cepstrum is defined as the inverse Fourier transform of the logarithmic spectrum and the symbols X, Y, H, ω and τ are respectively the input, output, system, frequency and quefrency. Quefrency has the same units as time and corresponds to the cepstral domain as does frequency to the spectral domain. Examples of successful cepstrum application include separation of glottal excitation and vocal tract impulse response in speech analysis [8][9], and separation of excitation and structural responses in gearboxes [10].

The application of a derivative led to the Differential Cepstrum (DC) [11][12], which does not require phase unwrapping, unlike the Cepstrum. The generalisation of DC into the Mean Differential Cepstrum (MDC) [6][13] introduces an averaging aspect into its application while preserving the useful homomorphic property. The MDC is defined as the inverse Fourier transform of the spectral correlation density of an output signal, $Y(\omega)$ and the following shows the cepstral and spectral domain definition,

$$\mathbf{d_y}(\tau) \quad = \quad \mathcal{F}^{-1}\,(\ \mathbf{D_{yy}}(\omega)\ ) \tag{2}$$

$$\mathbf{D_{yy}}(\omega) \quad = \quad \triangle_{\alpha,\omega}\ \ ln\ \mathbf{C_{yy}}(\alpha,\omega)|_{\alpha=0} \ = \ \frac{\mathbf{E}\,(Y'(\omega)\,Y^*(\omega))}{\mathbf{E}\,(Y(\omega)\,Y^*(\omega))} \tag{3}$$

where partial derivative: $\triangle_{\alpha,\omega} = \frac{\partial}{2\partial\omega} - \frac{\partial}{\partial\alpha}$ is with respect to frequency, ω and cyclic frequency, α, and $Y'(\omega) = \frac{\partial}{\partial\omega}\,Y(\omega)$ for $\mathbf{C_{yy}}(0,\omega) \neq 0$. Earlier work has seen the successful use of the MDC for non-minimum phase system identification [6].

2 Theoretical development

The following SIMO system identification technique uses the same MDC definition presented in (3). It can be distinguished from earlier MDC literature [6][13] by its propagative solution sequence and the use of the Taylor series approximation in one of its two solution formulations. Assumptions on the input excitation signal are spectral whiteness and by convention it has unitary power (scale indeterminacy). The technique is non-iterative, non-parametric and estimates both magnitude and phase of the system in the frequency domain. While a parallel development for MIMO system identification involving the matrix version of the MDC exists [14], the development and results for the scalar MDC used here are presented for the first time.

2.1 Propagative solution sequence

Substituting $Y \ = \ H\,X$ into the scalar MDC definition in (3),

$$\begin{aligned}
\mathbf{D_{yy}} \quad &= \quad \frac{\mathbf{E}\left((HX)'\ (HX)^*\right)}{\mathbf{E}\left((HX)\ (HX)^*\right)} \\
&= \quad \frac{\mathbf{E}\left((H'\,X + H\,X')\,(\,X^*H^*)\right)}{\mathbf{E}\left((H\,X)\,(\,X^*H^*)\right)} \\
&= \quad \frac{\left(H'\,\underbrace{\mathbf{E}\,(\,X\,X^*)}_{\mathbf{S_{xx}}=\mathbf{1}}\,H^* + H\,\underbrace{\mathbf{E}\,(\,X'\,X^*)}_{jC}\,H^*\right)}{\left(H\,\underbrace{\mathbf{E}\,(\,X\,X^*)}_{\mathbf{S_{xx}}=\mathbf{1}}\,H^*\right)}
\end{aligned}$$

assumption of unitary input power (i.e. auto-spectrum $\mathbf{S_{xx}} = 1$) gives,

$$\begin{aligned}
\mathbf{D_{yy}}\ \ (H\,H^*) \quad &= \quad (H'\,H^* + H\ [jC]\ H^*) \\
\mathbf{D_{yy}}\ H \quad &= \quad H' \ + \ H\ [jC]
\end{aligned}$$

$[jC]$ is a purely imaginary constant, corresponding to a time displacement, which can be set to zero since there is no absolute time reference in response measurements.

$$H' \ - \ \mathbf{D_{yy}}\ H \quad = \quad 0 \tag{4}$$

Making the derivative term the subject and defining it as a backward difference,

$$
\begin{aligned}
H' &= \mathbf{D_{yy}}\ H \\
\frac{H(\omega_k) - H(\omega_{k-1})}{d\omega} &= \mathbf{D_{yy}}(\omega_\mathbf{k})\ H(\omega_k) \\
\Big(1 - d\omega\,.\,\mathbf{D_{yy}}(\omega_\mathbf{k})\Big) H(\omega_k) &= H(\omega_{k-1})
\end{aligned}
$$

$$
(\mathbf{H_{DirOrn}}) \qquad H(\omega_k) = \Big(1 - d\omega\,.\,\mathbf{D_{yy}}(\omega_\mathbf{k})\Big)^{-1} H(\omega_{k-1}) \tag{5}
$$

(5) solves for the system directly in a propagative manner, from one frequency to the next. This original formulation, denoted by $\mathbf{H_{DirOrn}}$, has a Taylor series approximated version, presented in the next section.

2.2 Taylor series approximation

The multiplicative factor relating one frequency value to the next in (5) can be approximated using the Taylor series approximation,

$$
\begin{aligned}
(\mathbf{H_{DirTay}}) \quad H(\omega_k) &= \Big(1 + \frac{(-d\omega\,.\,\mathbf{D_{yy}}(\omega_\mathbf{k}))}{1!} + \frac{(-d\omega\,.\,\mathbf{D_{yy}}(\omega_\mathbf{k}))^2}{2!} + \frac{(-d\omega\,.\,\mathbf{D_{yy}}(\omega_\mathbf{k}))^3}{3!} + \dots\Big)^{-1} H(\omega_{k-1}) \\
&= \Big(e^{-d\omega.\mathbf{D_{yy}}(\omega_k)}\Big)^{-1} H(\omega_{k-1})
\end{aligned} \tag{6}
$$

The intended use of the approximation was to preserve hermitian property of matrices in MIMO systems application [14]. Its extension to the current form in (6) has the advantage of a more stable identification outcome as will be evident in the results section.

3 Results

The SIMO identification method is tested on a simulated 5 DOF system (system parameters outlined in figure 1), the experimental set-up in figure 2 and a simulated non-minimum phase system. The transient input excitations used are impulsive or burst random signals. The identification outcome from $\mathbf{H_{DirOrn}}$ and $\mathbf{H_{DirTay}}$ are plotted in black and blue respectively, together with the reference FRF in red. The plots are intentionally displaced from each other for better visual presentation. Note that the constant amplification factor between the input and output is unknown in the absence of input information. The first frequency bin value used to initiate the solution propagation can be obtained from system properties (mass or stiffness) or a previously known FRF. The results generated are based on the latter.

3.1 Set-up

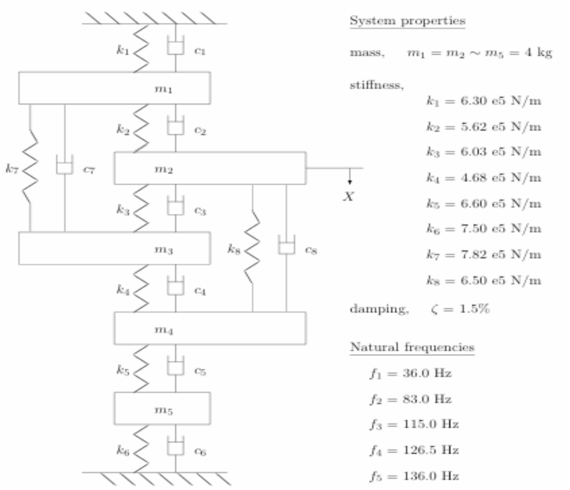

Figure 1: Simulation set-up

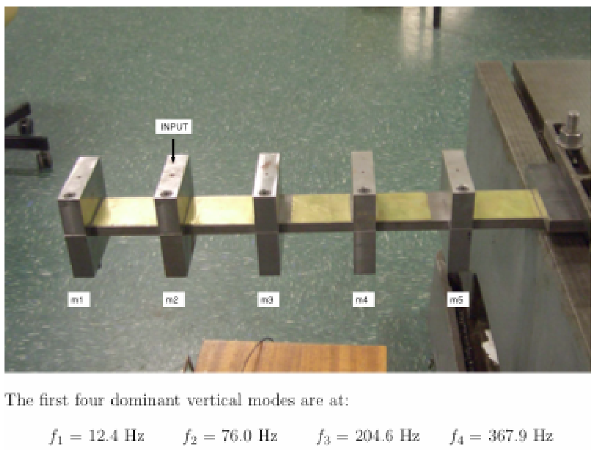

Figure 2: Experimental set-up

3.2 Simulation results

The magnitude and phase results in figures 3 to 6 are based on driving point measurements made at m2 for both simulation and experiment.

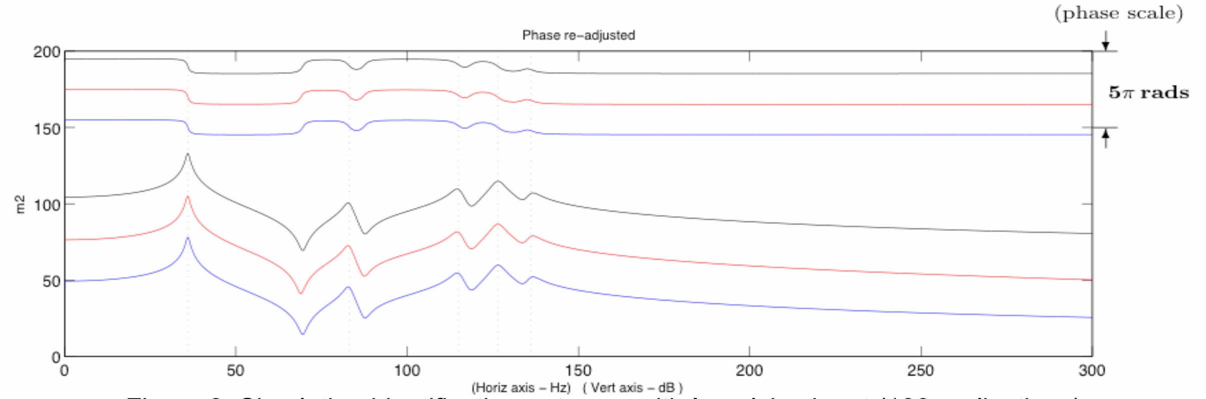

Figure 3: Simulation identification outcome with Impulsive input (100 realisations)

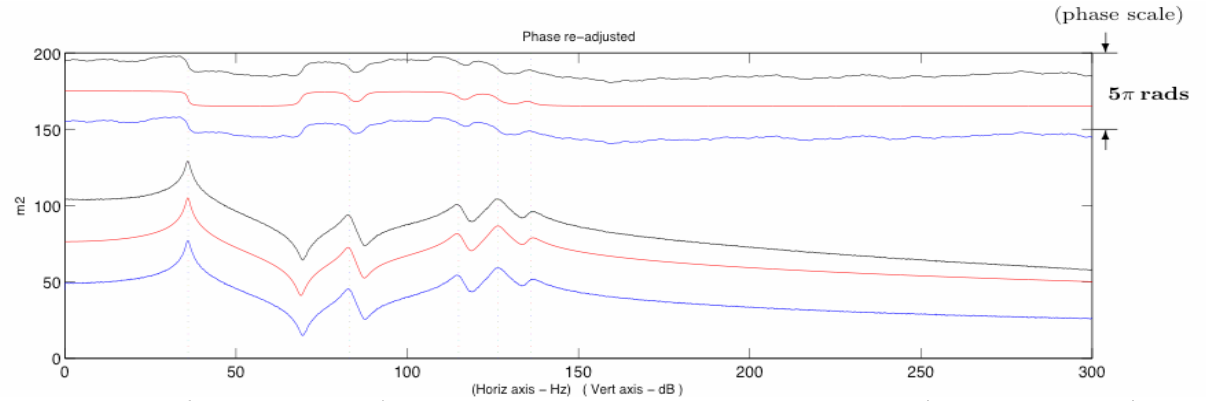

Figure 4: Simulation identification outcome with 4s Burst Random input (1000 realisations)

3.3 Experimental results

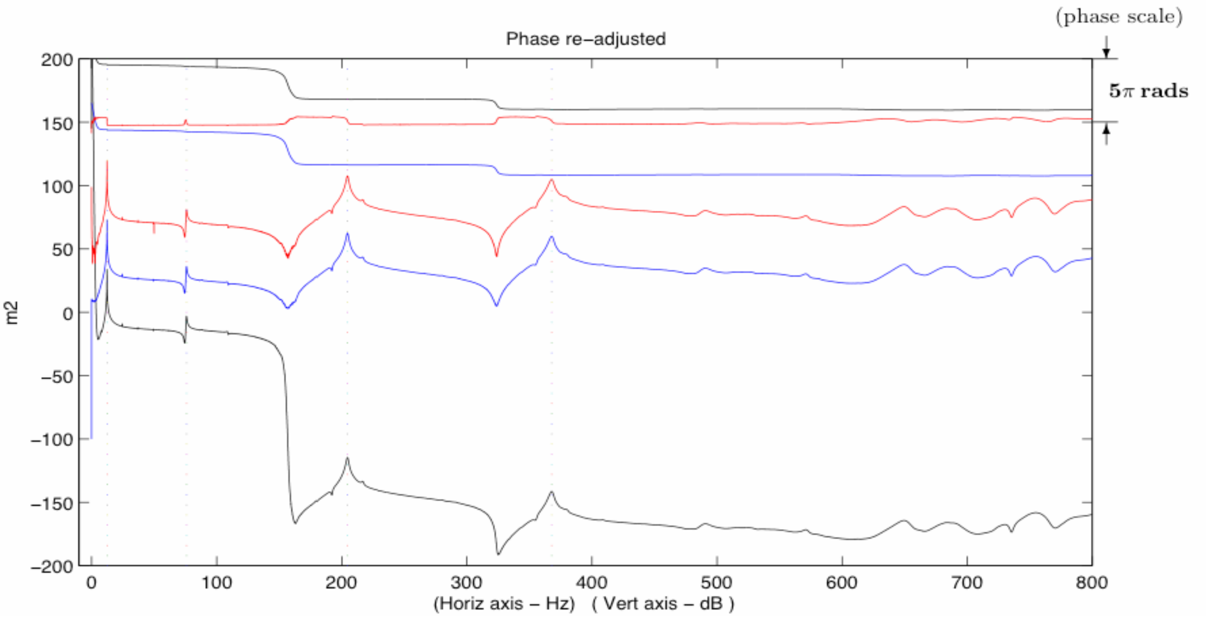

Figure 5: Experiment identification outcome with Hammer Impulsive input (100 realisations)

$(\mathbf{H}_{\mathbf{DirOrn}}$ ——, $\mathbf{H}_{\mathbf{DirTay}}$ ——, $\mathbf{H}_{\mathbf{ref}}$ ——$)$

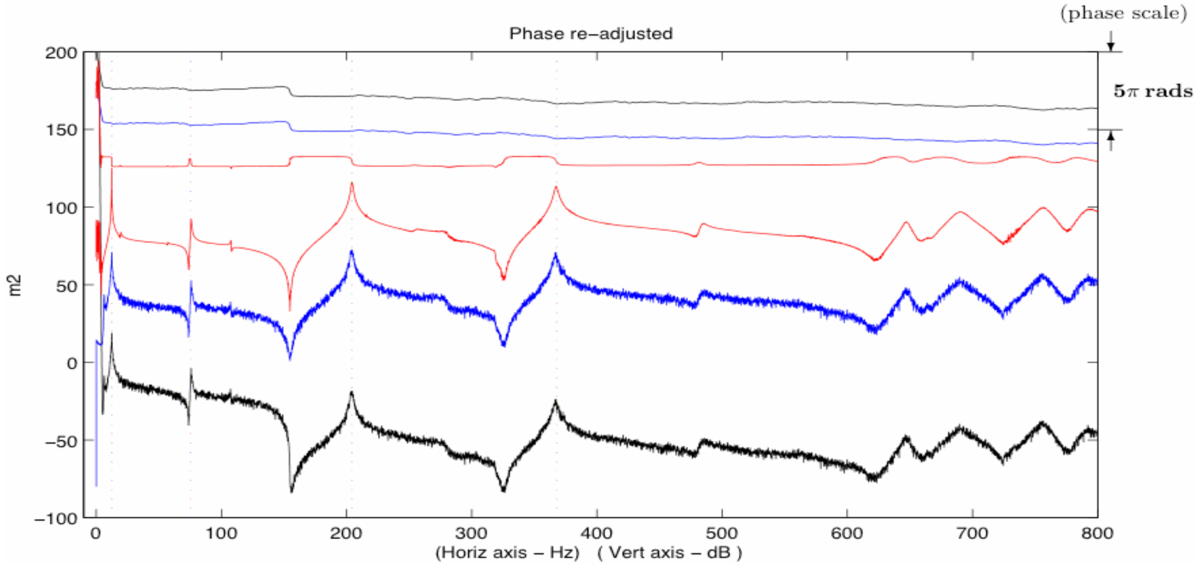

Figure 6: Experiment identification outcome with Shaker 4s Burst Random input (100 realisations)

Figures 3 to 6 show the identification outcome of varying quality from both formulations, $\mathbf{H_{DirOrn}}$ and $\mathbf{H_{DirTay}}$. The impulsive input simulation shows an excellent identification quality with accurate prediction of both poles (resonances) and zeros (anti-resonances) frequency location estimation (details are found in [1]), and phase prediction in Figure 3. For the burst random input simulation in figure 4, the estimated phases have some errors. However, they agree with the reference phase plot in general and better results are achievable with increased averages [1]. The slight negative slope distortion in the magnitude plot of $\mathbf{H_{DirOrn}}$ in Figure 4 indicate a lower quality solution outcome compared to $\mathbf{H_{DirTay}}$, which is more evident in figures 5 and 6.

In the experimental results, higher solution stability is observed from the estimated magnitude plots of $\mathbf{H_{DirTay}}$ in figures 5 and 6. Poor phase identification is observed in all experimental results despite reasonable estimation of pole and zero frequency locations [1]. Note that a time delay between the excitation time and the start of response time records manifests itself as slope distortions in the estimated phases. All estimated phase plots presented in this paper have undergone a slope adjustment at the post-processing stage to eliminate any slope based on the low frequency region below the first resonance.

3.4 Identification of simulated non-minimum phase system

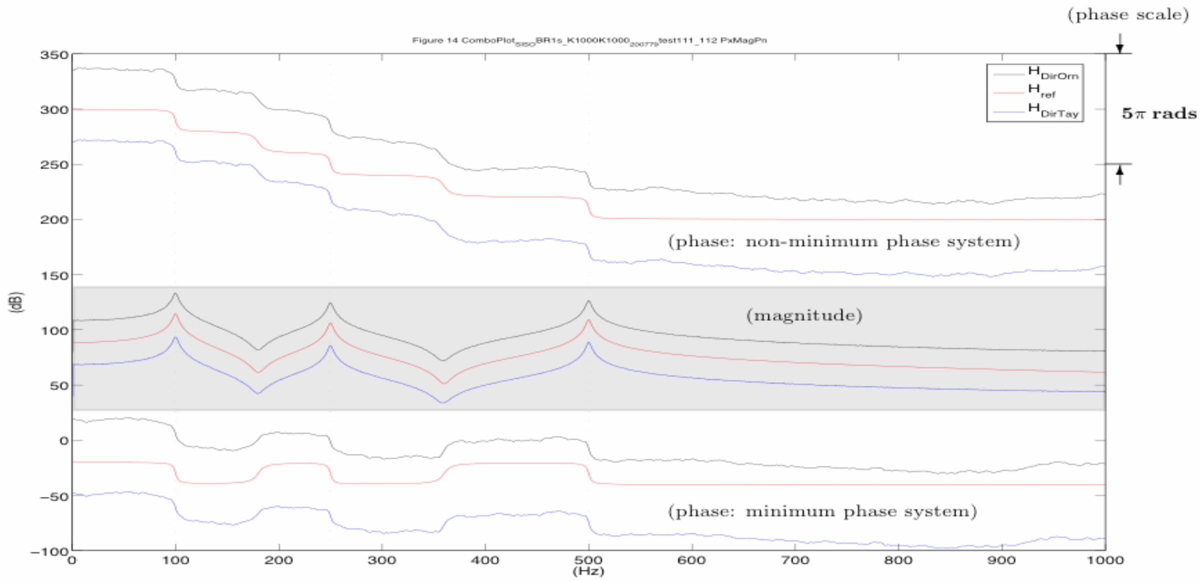

Figure 7: Simulated Non-Minimum phase system identification, Burst Random input(1000 realisations)

Figure 7 shows the identification outcome of a non-minimum phase system. The estimated phase plots from simulation of an equivalent minimum phase system (by having minimum phase rather than maximum phase zeros) are also given (bottom of the figure) for comparison. Both formulations gave very good magnitude estimation, as previously seen in figure 4. Note that the estimated magnitude plots for the minimum phase system, that is absent from the figure, is almost identical to the presented magnitude plot from the non-minimum phase system.

The differences between a non-minimum and a minimum phase system can be appreciated from the respective reference phase plots. At each zero (anti-resonance), the non-minimum phase system shows a decrease in phase by π radians, as opposed to an increase for the minimum phase system. The net effect is two very distinct sets of phase plots. All estimated phases have an error, as seen in figure 4. Improvement can be achieved with increased averages. The estimated phases generally agree with the reference phase and phase change at each pole or zero can be clearly identified despite the uncertainty.

4 Conclusions

Assessment based on both estimated magnitude and phase plots showed that the newly proposed MDC method is applicable for both minimum and non-minimum phase system identification. It worked well in the simulations and gave reasonable magnitude estimation in the experiments. Comparison between magnitude plots indicates that a better identification outcome can be obtained from $\mathbf{H}_{\mathbf{DirTay}}$ compared to $\mathbf{H}_{\mathbf{DirOrn}}$. This seems to reflect increased robustness against noise issues with the adoption of the Taylor series approximation. Future works on the method will include addressing the limitation of transient inputs requirement with the application of random decrement technique on a continuously excited response measurement.

5 References

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