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Title – Enriched Finite Element and Meshfree Methods for Dynamic Crack Propagation Problems

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Abstract: Extended meshfree methods for dynamic crack propagation are reviewed. One class of methods enforces crack path continuity while the other class of method treats the crack as set of cracked particles. In the first method, crack path continuity is enforced either by use of a crack tip enrichment or by use of Lagrange multipliers. All methods are implemented in two and three dimensions and are compared to experimental results.

Keywords: meshfree, fracture, material instability, enrichment, localization.

1 Introduction

Recently, the local partition of unity is applied to meshfree methods for cohesive fracture problems, [1,2,3,4,5,6,7,8,9], in which the solution space is enriched by a priori knowledge about the behaviour of the solution near the discontinuity. Since only nodes close to the crack are enriched, the number of additional degrees of freedom for the enrichment is minimized.

Two classes of methods are proposed to describe the crack. The first class ensures crack path continuity and the second class does not. Methods that do enforce crack path continuity are generally more accurate but difficult to implement and their applicability is limited to problems with few cracks. Methods that do not enforce crack path continuity need a higher resolution in order to obtain adequate results but are simple to implement and especially useful for problems with many cracks and applications with complicated crack patterns such as crack branching and joining cracks.

2 Meshfree Methods

The meshfree approximation can be written similarly to finite elements as:

$$\boldsymbol{u}^h(\boldsymbol{X},t) = \sum_I N_I(\boldsymbol{X})\,\boldsymbol{u}_I(t)$$

where $N_I(X)$ are the shape functions that are computed in the element-free Galerkin (EFG) method, the meshfree method that we use in our computations, by

$$N_I(\boldsymbol{X}) = \boldsymbol{p}(\boldsymbol{X})^T\,\boldsymbol{A}(\boldsymbol{X})^{-1}\boldsymbol{D}(\boldsymbol{X}_J)$$
$$\boldsymbol{A}(\boldsymbol{X}) = \sum_J \boldsymbol{p}(\boldsymbol{X}_J)\,\boldsymbol{p}^T(\boldsymbol{X}_J)W(\boldsymbol{X}-\boldsymbol{X}_J,h)$$
$$\boldsymbol{D}(\boldsymbol{X}_J) = \boldsymbol{p}(\boldsymbol{X}_J)W(\boldsymbol{X}-\boldsymbol{X}_J,h)$$

Where $\boldsymbol{p}$ is the polynomial basis, W is a weight function and h is the size of the domain of influence. For most problems, we employed linear basis polynomials, i.e. $\boldsymbol{p}(\boldsymbol{X}) = [1 \quad X \quad Y]$ in two dimensions and $\boldsymbol{p}(\boldsymbol{X}) = [1 \quad X \quad Y \quad Z]$ in three dimensions. For more details, see e.g. [10].

3 Treatment of Discontinuities

3.1 Methods that enforce crack path continuity

If there is a crack in the domain of interest, the displacement approximation $\boldsymbol{u}^h(\boldsymbol{X},t)=\boldsymbol{u}^0(\boldsymbol{X},t)+\boldsymbol{u}^e(\boldsymbol{X},t)$ is decomposed into a standard part $\boldsymbol{u}^0(\boldsymbol{X},t)$ (as described in section 2) and into an enriched part $\boldsymbol{u}^e(\boldsymbol{X},t)$ that describes the crack kinematics. The enriched part is given by

$$\boldsymbol{u}^e(\boldsymbol{X},t)=\sum_{J\in S}\boldsymbol{u}^{e,J}(\boldsymbol{X},t) \quad \text{and} \quad \boldsymbol{u}^{e,J}(\boldsymbol{X},t)=\sum_{I\in W_J}N_I(\boldsymbol{X})\psi_I^J(\boldsymbol{X})\boldsymbol{a}_I^J(t)+\sum_{I\in R_J}N_I(\boldsymbol{X})B\;(\boldsymbol{X})\boldsymbol{b}_{kI}(t)$$

where *S* is the set of all the cracks in the domain, W_J is the set of particles whose domain of influence is cut by the crack *J*, ψ_I^J is the enrichment function for particle *I* whose domain of influence is cut completely by the crack *J*, figure 1, $\boldsymbol{a}$ and $\boldsymbol{b}$ are additional degrees of freedom, R_J is the set of particles whose domain of influence are cut partially by the crack *J* and $B(\boldsymbol{X})=r^m\sin\theta/2$ is the crack tip enrichment for cohesive cracks where *r* is the closest distance of a material point to the crack tip and *θ* is the angle between the crack line (at its crack tip) and the line that connects a material point with the crack tip. The step function S is chosen as enrichment function. The jump of the displacement field is governed only by the enrichment. The same structure applies to the test functions.

The crack tip enrichment guarantees that the crack is closed at the crack tip and simultaneously maintains the discontinuous character of the crack kinematics, [1,3]. Another opportunity to close the crack at the crack tip without the tip enrichment is the domain-decrease method [2]. In the domain decrease method, the domain of influence is reduced around the crack tip such that only completely cut domains of influence exist. However, this method is only applicable in two dimensions. An alternative to the domain-decrease method is the use of Lagrange multipliers in order to ensure crack closure of the crack tip as is shown in figure 1 exemplarily for the two-dimensional case. For more details, see [2,4].

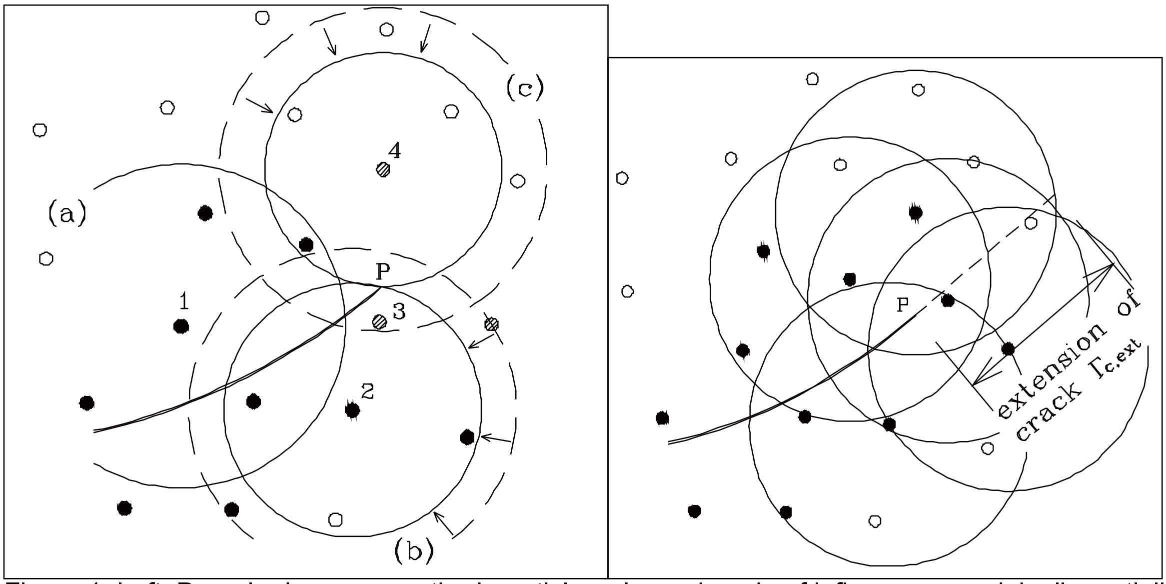


Figure 1: Left: Domain decrease method; particles whose domain of influence are originally partially cut by the crack are shown dashed; right: Lagrange multiplier method shows the domain of influence of several particles and the crack plus its virtual extension.

3.2 Methods that do not enforce crack path continuity

The cracking particle method [5,6] describes the crack as set of cracked particles as shown in figure 2. The basic idea is also based on the decomposition of the displacement field into a continuous and discontinuous part. However, the crack is required to pass through a particle. Only particles that are cracked are enriched:

$$\boldsymbol{u}^h(\boldsymbol{X},t) = \sum_{I \in S} N_I(\boldsymbol{X})\, \boldsymbol{u}_I(t) + \sum_{I \in W} N_I(\boldsymbol{X}) S(\boldsymbol{X}) \boldsymbol{a}_I(t)$$

where *W* denotes the set of particles that are cracked.

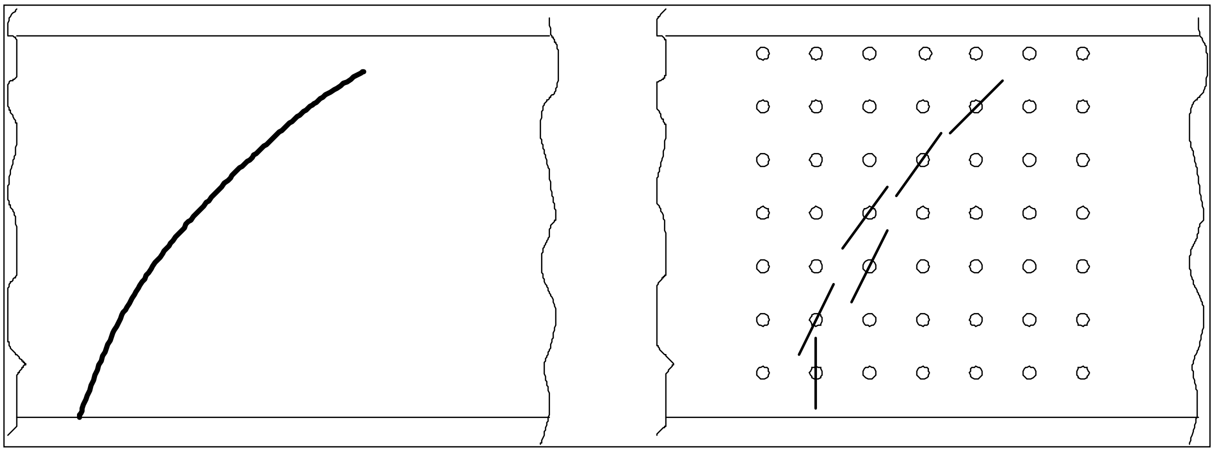


Figure 2: Crack representation in the cracking particle method

4 Governing equations

The strong form of the momentum equation in the total Lagrangian description is given by

$$\rho_0\, \ddot{\boldsymbol{u}} = \nabla_0 \cdot \boldsymbol{P} + \rho_0\, \boldsymbol{b} \quad \text{in} \quad \Omega \setminus \Gamma_0^c$$

with boundary conditions

$$\boldsymbol{u} = \bar{\boldsymbol{u}} \quad \text{on} \quad \Gamma_0^u$$

$$\boldsymbol{t} = \bar{\boldsymbol{t}} \quad \text{on} \quad \Gamma_0^t$$

$$\boldsymbol{n}_0 \cdot \boldsymbol{P}^- = \boldsymbol{n}_0 \cdot \boldsymbol{P}^+ = \boldsymbol{t}_{c0} \quad \text{on} \quad \Gamma_0^c$$

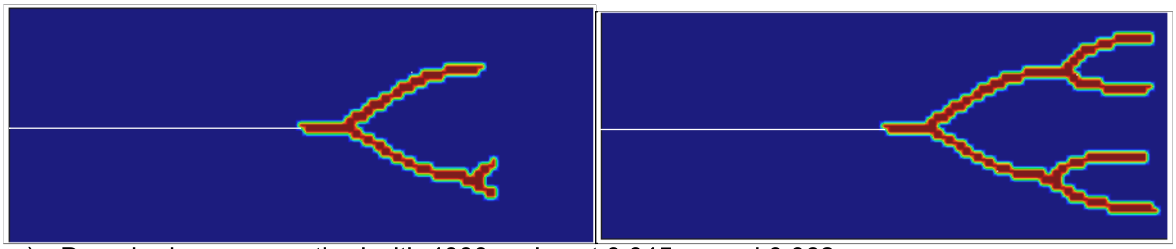
$$\boldsymbol{t}_{c0} = \boldsymbol{t}_{c0}([[\boldsymbol{u}]]) \quad \text{on} \quad \Gamma_0^c$$

where ρ_0 is the initial mass density, $\ddot{\boldsymbol{u}}$ is the acceleration, $\boldsymbol{P}$ denotes the nominal stress tensor, $\boldsymbol{b}$ designates the body force, $\bar{\boldsymbol{u}}$ and $\bar{\boldsymbol{t}}$ are the prescribed displacement and traction, respectively $\boldsymbol{n}_0$ is the outward normal to the domain and $\Gamma_0^u \cup \Gamma_0^t \cup \Gamma_0^c = \Gamma_0, \left(\Gamma_0^u \cap \Gamma_0^t\right) \cup \left(\Gamma_0^u \cap \Gamma_0^c\right) \cup \left(\Gamma_0^c \cap \Gamma_0^t\right) = 0$. The cohesive traction on the crack face is a function of the crack opening displacement. Loss of material stability or the Rankine criterion is used for both crack initiation and crack propagation, see e.g. [1-6].

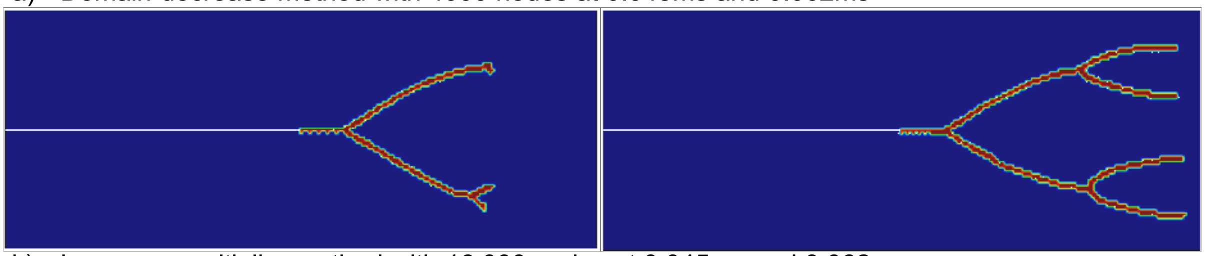
5 Results

5.1 Crack branching in two dimensions

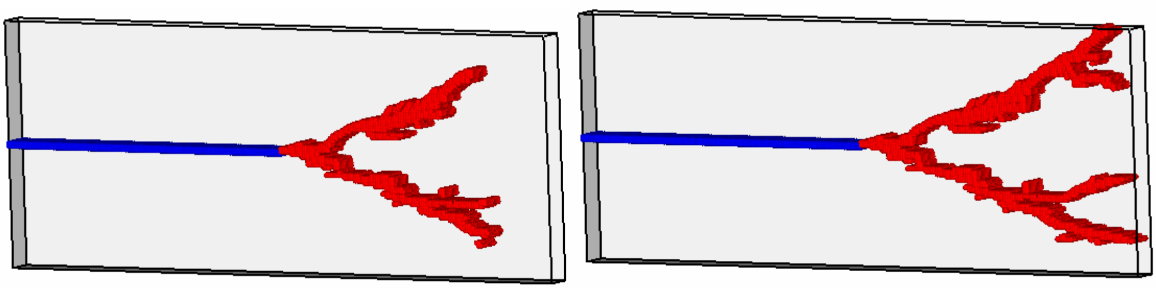
We consider a rectangular prenotched specimen as shown in figure 3. the length of the rectangle was 0.1m and the width 0.04m. Plane strain conditions were assumed. Initially, there was a horizontal notch from the left edge to the center of the plate. A tensile traction of 1 MPA was applied on the top and bottom edges. The constitutive model is described in [1]. The crack pattern at different stages is shown in figure 3a,b for methods that do enforce crack path continuity. Figure 3c shows the crack pattern for the cracking particle method. Note that a three-dimensional analysis for carried out in the latter case



a) Domain-decrease method with 4000 nodes at 0.045ms and 0.062ms



b) Lagrange multiplier method with 16,000 nodes at 0.045ms and 0.062ms



c) Cracking particle method at 0.046ms and 0.06ms; 3D simulation with approximately 370,000 nodes

Figure 3: Cracking patterns of the plate with an edge crack at different time steps for different methods

5.2 Concrete under explosive loading

This is an example that shows the limitation of methods that enforce crack path continuity. Here, we show results obtained by the cracking particle method. In several experiments [11], concrete slabs with different thicknesses and strengths were subjected to explosive loading (TNT). We show results of a concrete slab that was completely perforated. The deformation of the concrete slab and the explosive is shown in figure 4. Figure 5 compares the final crack pattern of the simulation with the one of the experiment, figure 6. Figure 7 compares a thicker concrete slab that was charged with less TNT. This slab was damaged mainly at the top surface.

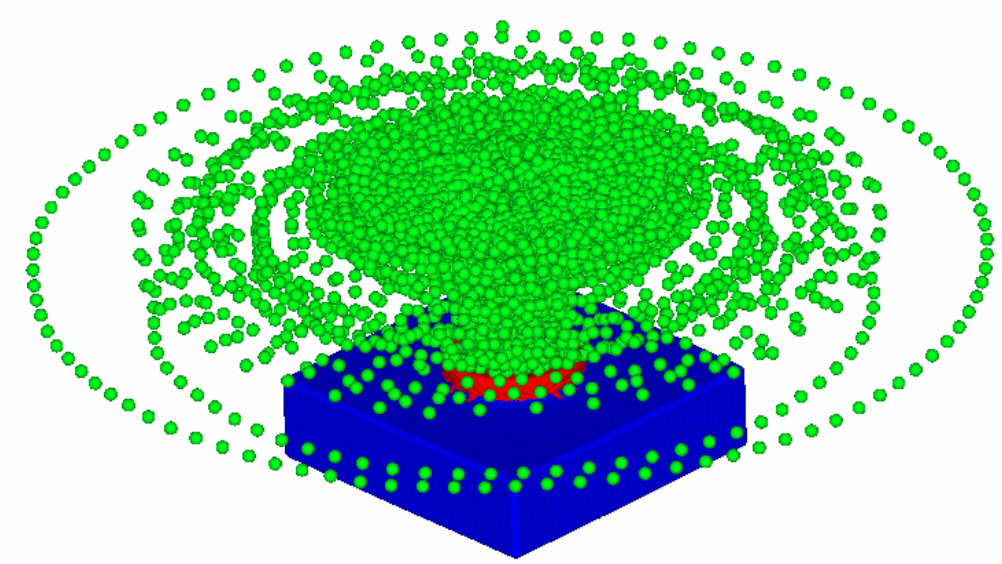


Figure 4: Concrete under explosive loading, green: explosive, red and blue: concrete, red particles are cracked while blue particles are uncracked

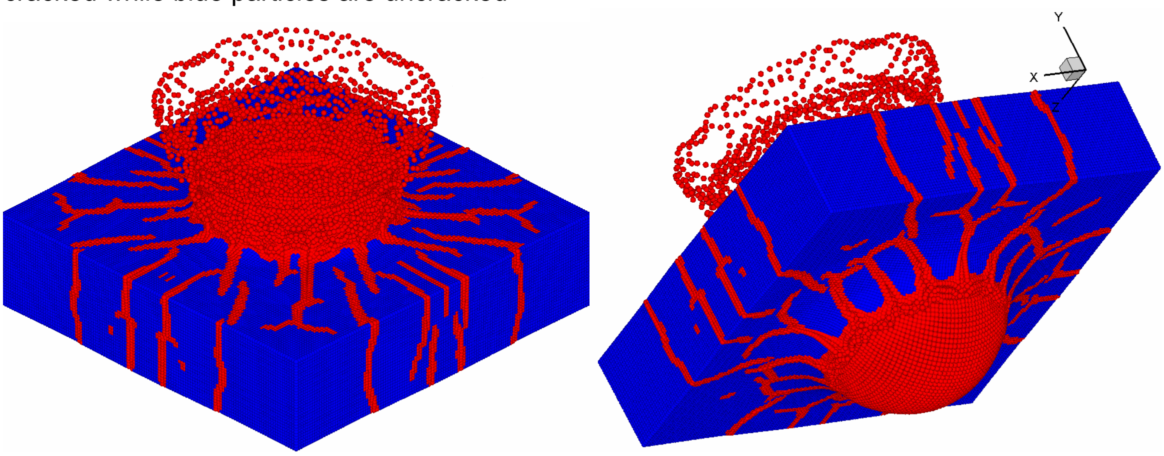


Figure 5: Concrete under explosive loading after cracking is completed.

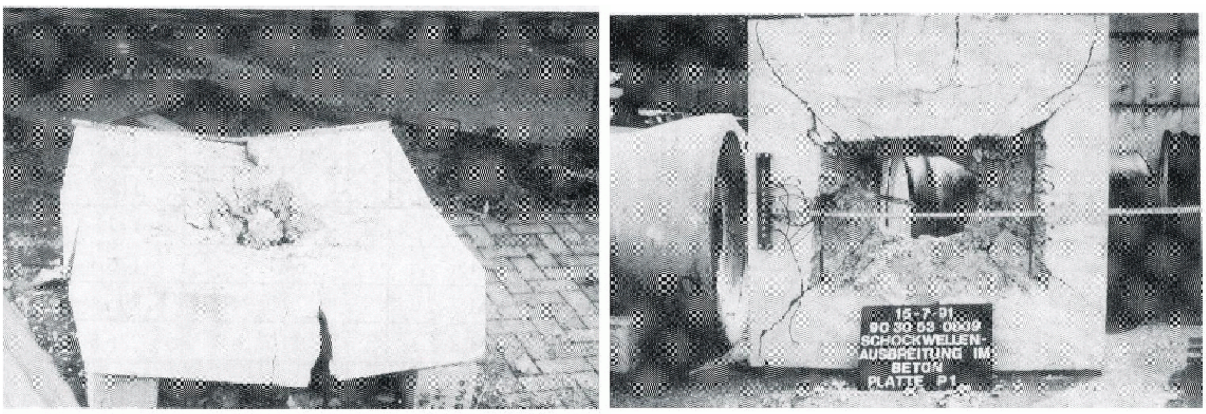


Figure 6: Concrete slab under explosive loading after the experiment, [11]

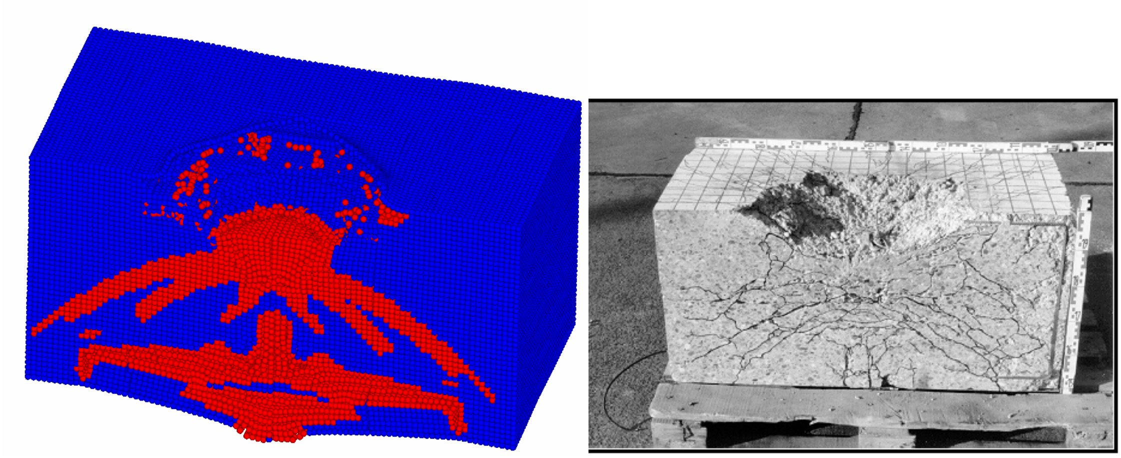


Figure 7: Concrete slab of the simulation compared to the experimental result

6 Conclusions

We have presented enriched meshfree methods based on the local partition of unity concept. One class treats the crack as continuous crack surface while the other class describes the crack as set of cracked particles. The first class is more accurate but difficult to implement and hence applicable to problems with only a few number of cracks while the latter method is applicable to many cracks though a higher number of particles are needed in order to obtain sufficient results. We have demonstrated the capability of these methods for several dynamic problems.

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