

Photon frequency-mode matching using acousto-optic frequency beam splitters

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(Received 26 September 2005; published 17 March 2006)

It is a difficult engineering task to create distinct solid state single photon sources which nonetheless emit photons at the same frequency. It is also hard to create entangled photon pairs from quantum dots. In the spirit of quantum engineering we propose a simple optical circuit which can, in the right circumstances, make frequency distinguishable photons frequency indistinguishable. Our circuit can supply a downstream solution to both problems, opening up a large window of allowed frequency mismatches between physical mechanisms. The only components used are spectrum analysers or prisms and an acousto-optic modulator. We also note that an acousto-optic modulator can be used to obtain Hong-Ou-Mandel two photon interference effects from the frequency distinguishable photons generated by distinct sources.

DOI: [10.1103/PhysRevA.73.033813](https://doi.org/10.1103/PhysRevA.73.033813)

PACS number(s): 42.50.Dv, 42.65.Lm, 03.67.Lx

I. INTRODUCTION

Frequency indistinguishable photons are very useful in protocols for quantum computing and communication that exploit polarization, space or time degrees of freedom. Unfortunately achieving this indistinguishability can be practically very challenging, especially when the photons are produced by solid state devices. Two photons can be considered to be frequency distinguishable when their bandwidths are much smaller than the difference in their average frequencies. The practical challenges of tuning two distinct physical processes so that they generate photons which are frequency indistinguishable can sometimes be avoided. Instead, pairs of photons can be rendered frequency indistinguishable *after* they have been emitted. Variants of the circuit we propose can render photons, which initially differ in frequencies by tens of megahertz up to a few gigahertz, frequency indistinguishable and we hope that it will have broad uses across quantum optics.

Our circuit is composed of only a pair of prisms and an acousto-optic modulator (AOM) which acts as a form of frequency beam splitter [1–3]. In [1] the authors describe a radio frequency half-wave plate. They use an highly asymmetric Mach-Zehnder interferometer combined with a double pass of an AOM and obtain a circuit which is very similar in effect to the one described below (see also [4,5] for further work). Their aim was to demonstrate that qubits can be defined and manipulated using sideband modes, whereas this paper uses a frequency beam splitter (FBS) to allow greater experimental flexibility by allowing imperfect frequency mode matching to be avoided.

Having introduced the AOM and our circuit in Sec. II, Sec. III shows that it can be useful in realizing entanglement between separated sources (this is important for the family of entangling schemes recently proposed for cluster state computation [6]). In Sec. IV we show how to use our circuit to create entangled photon pairs from asymmetric single quantum dots (such a resource has broad applications, from uses in quantum cryptography [7] to quantum computation [8]).

II. ACOUSTO-OPTIC MODULATORS AS FREQUENCY SHIFTERS

In this section we review how AOMs are used as frequency shifters and show how they can be used as the frequency analog of a beam splitter, i.e., a frequency beam splitter (FBS). The AOM can be viewed as a coupler of two electric fields via a phonon field [9]. Acoustic waves are used to generate periodic, propagating, inhomogeneities in a crystal's refractive index and these act as a moving diffraction grating, scattering incoming light.

The classical relationship between the fields at the input, $[E_i(0), E_d(0)]$, and output, $[E_i(l), E_d(l)]$, if both beams travel a distance l through the crystal is

$$E_i(l) = E_i(0)\cos(\eta l) + iE_d(0)\sin(\eta l), \quad (1)$$

$$E_d(l) = E_d(0)\cos(\eta l) + iE_i(0)\sin(\eta l), \quad (2)$$

The constant η has the following form: $\eta = (\omega/2\sqrt{2}c)\sqrt{[(n^6 p^2/\rho v_s^3)I_{\text{acoustic}}]}$, here I_{acoustic} is the acoustic intensity of the sound waves applied to the crystal, n is its refractive index, v_s its speed of sound, p its photoelastic constant, and ρ its density [10]. Taking ω_i as the frequency of $E_i(0)$ and ω_d that of $E_d(0)$ and assuming that $\omega_i - \omega_d \ll \omega_d$ we then define the ω in the expression for η as $\omega = \omega_i \approx \omega_d$, see Fig. 1. Considering only the forward propagating field components and quantizing these, the input and output modes are coupled as below

$$b_i = a_i \cos(\eta l) + i a_d \sin(\eta l), \quad (3)$$

$$b_d = a_d \cos(\eta l) + i a_i \sin(\eta l), \quad (4)$$

where a_i is the annihilation operator for modes at the input frequency ω_i and a_d for those at frequency ω_d and the frequency of the modulation of the crystal is $\omega_i - \omega_d$.

Consider the circuit in Fig. 2. The first prism splits the state by its frequency, the AOM rotates between the two frequencies and the second prism recombines the state. It effects the map

$$\alpha|\omega_i\rangle + \beta|\omega_d\rangle \rightarrow [\alpha \cos(\eta l) + i\beta \sin(\eta l)]|\omega_i\rangle + [i\alpha \sin(\eta l) + \beta \cos(\eta l)]|\omega_d\rangle. \quad (5)$$

Consider modulating the AOM at $\omega_i - \omega_d$ and arranging

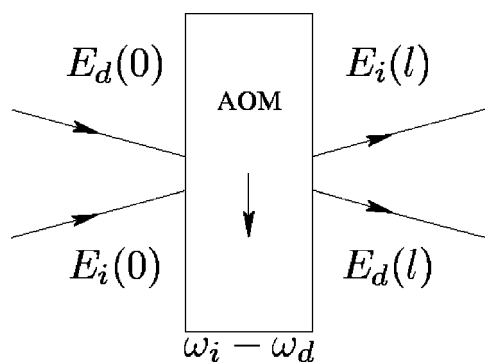


FIG. 1. The classical model of the AOM as a coupler of two fields. $E_d(0), E_i(0)$ are the field components in the incident direction at the crystal input (at frequencies ω_d and ω_i , respectively) and $E_d(l), E_i(l)$ the fields at the output. The arrow inside the box indicates the direction in which the acoustic wave is traveling and the crystal is modulated at the frequency difference between the two input fields, $\omega_i - \omega_d$.

the apparatus so that $\eta l = \pi/4$ (giving a 50% conversion efficiency). In this case the circuit performs the maps $|\omega_i\rangle \rightarrow (1/\sqrt{2})|\omega_i\rangle + i(1/\sqrt{2})|\omega_d\rangle$ and $|\omega_d\rangle \rightarrow i(1/\sqrt{2})|\omega_i\rangle + (1/\sqrt{2})|\omega_d\rangle$. This is similar to a spatial beam splitter in its action. It should be noted that η depends on the frequency of the incident light and thus bandwidth effects could make the action of the above FBS more complicated. In Sec. IV we argue that this will not prove a practical issue for relevant photon bandwidths. Also note that AOM's can only be modulated at frequencies up to a few GHz [11]. In practice, the prisms we describe would need to be very good spectrometers. Devices which can resolve frequency differences, $\omega_i - \omega_d$, of less than a GHz are just on the edge of what is currently experimentally achievable. By using the FBS in slightly modified ways the schemes presented below avoid the need for such high resolution spectrometers.

In what follows we provide two applications for variants of the above circuit. The first is in entangling emitters placed in cavities, the second is in entangling correlated photon pairs from quantum dots.

III. SINGLE PHOTON USE: ENTANGLING IONS IN CAVITIES

A number of schemes exist for entangling pairs of atoms or dots which have been placed in spatially separated cavities or traps [12–18]. To be concrete we will consider the protocol of Barrett and Kok [12]; they have a technique for generating the highly entangled cluster states needed in one-way

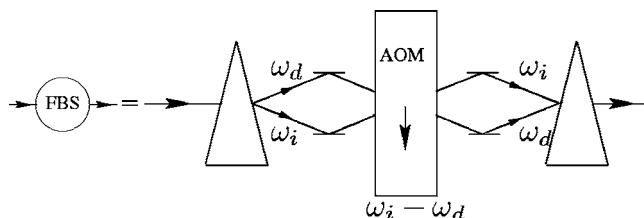


FIG. 2. A frequency beam splitter, FBS. The AOM is modulated at the difference of the two frequencies ω_i and ω_d .

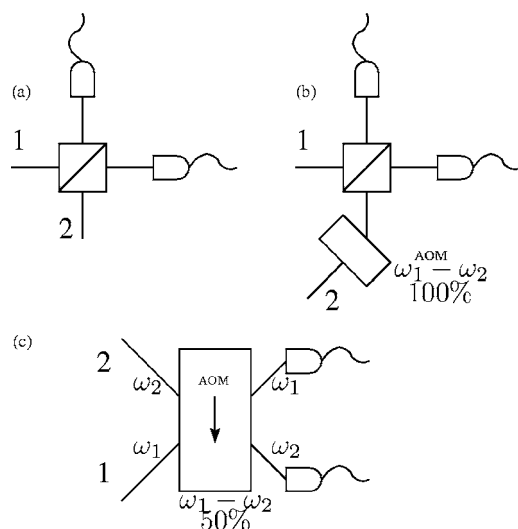


FIG. 3. (a) Photons either from cavity 1 at frequency ω_1 or cavity 2 at frequency ω_2 are incident on nonpolarizing beam splitters. (b) Photons from cavity 2 are frequency up shifted with ideally 100% efficiency so that they have the same frequency as photons from cavity 1. (c) In this scheme the AOM need only frequency shift half the light, 50% efficiency.

quantum computation [6]. They suggest using the apparatus in Fig. 3(a) to entangle two photon sources. Either a single photon comes from source 1 or 2. Simultaneous emission events from 1 and 2 are discarded. If the photons from 1 and 2 are at the same frequency, the presence of the beam splitter ensures that, when one of the detectors clicks, the experimenter cannot determine whether the photon came from cavity 1 or 2. For solid state single photon sources [19] it is very difficult to fix the frequencies of the two sources to be the same, as is required in Fig. 3(a); the photons from directions 1 and 2 can often be frequency distinguished. Though the authors suggest filtering out unmatched photons, this leads to an appreciable reduction in the efficiency with which their scheme makes large cluster states.

A first solution is to use an AOM on one of the photons to frequency match the pair as in Fig. 3(b). It is, however, difficult to perform this frequency shift with 100% efficiency, normally some light is unshifted (highly efficient AOM's also absorb more light, so 100% efficient devices with low absorptivity are difficult to fabricate). Figure 3(c) provides an alternative approach. Conceptually this apparatus moves the challenge of tuning distinct components to the easier task of tuning the apparatus of detection. The photon passes through a 50% efficient frequency beam splitter at modulation frequency $\omega_1 - \omega_2$. From a click at either detector, the experimenter will be unable to deduce the cavity from which the photon came. If the cavities differ in their frequency of emissions by up to a few gigahertz the circuit can still allow them to become entangled. We have increased the window of suitable emitters. Depending on the size of the frequency shift, an AOM with 50% diffraction efficiency should be feasible with current components [11]. Figure 3(c) performs the transformation [using Eqs. (3) and (4) when $\eta l = \pi/4$]

$$a_1^\dagger |G_1\rangle \langle E_1| + a_2^\dagger |G_2\rangle \langle E_2| \rightarrow b_1^\dagger (|G_1\rangle \langle E_1| + i |G_2\rangle \langle E_2|) + b_2^\dagger (i |G_1\rangle \langle E_1| + |G_2\rangle \langle E_2|). \quad (6)$$

Where if cavity i emits a photon then it is in state $|G_i\rangle$ and if it does not emit a photon it is in the state $|E_i\rangle$ and $a_i^\dagger|0\rangle$ indicates the presence of a photon at frequency ω_i on path i on the input to the AOM.

Though Fig. 3(c) was described in the context of single photons, it also has a two photon role. Experiments investigating the Hong-Ou-Mandel dip [20] for successive photons from the same single photon solid-state source have already been studied [21]. The scheme in Fig. 3(c) could be used to detect a dip between photons from *distinct* sources when the sources are not frequency matched. Using Eqs. (3) and (4) again when $\eta l = \pi/4$ we find that

$$a_1^\dagger a_2^\dagger \rightarrow \frac{1}{2}(b_1^\dagger + i b_2^\dagger)(i b_1^\dagger + b_2^\dagger) \rightarrow \frac{i}{2}(b_1^\dagger b_1^\dagger + b_2^\dagger b_2^\dagger). \quad (7)$$

Both photons leave the AOM in the same spatial mode and at the same frequency.

IV. TWO PHOTON USE: RECTIFYING POLARIZATION ENTANGLEMENT IN BIEXCITON EMISSION

In this section we provide a use for frequency beam splitters in the generation of entangled photon pairs. Quantum dots can emit correlated photon pairs by a cascade of photon emission from a biexciton via an exciton [22]. Asymmetries in the dot can prevent the photon pairs from being entangled. A symmetric dot might generate the two photon state

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|x, \omega_A; x, \omega_B\rangle + e^{i\nu}|y, \omega_A; y, \omega_B\rangle), \quad (8)$$

where $|x, \omega_A\rangle$ is a photon linearly polarized in the x direction at frequency ω_A (similarly for $|x, \omega_B\rangle$) and $|y, \omega_A\rangle$ is a y -polarized photon at frequency ω_A ($|y, \omega_B\rangle$ similarly) and ν is a phase [23]. In practice asymmetries are hard to avoid when constructing the dots [25] and the following state might be expected instead

$$|\phi'\rangle = \frac{1}{\sqrt{2}}(|x, \omega_1; x, \omega_2\rangle + e^{i\nu}|y, \omega_3; y, \omega_4\rangle), \quad (9)$$

where $\omega_3 > \omega_1 > \omega_2 > \omega_4$ and $\omega_3 = \omega_1 + \Delta = \omega_2 + \Delta + \xi = \omega_4 + 2\Delta + \xi$. We have $\Delta = (\omega_3 - \omega_1) = (\omega_2 - \omega_4)$ as the doublet splitting due to dot asymmetry and $\xi = \omega_1 - \omega_2$ as the biexciton shift. Since $|x, \omega_1\rangle$ and $|y, \omega_3\rangle$ are frequency distinguishable (and also $|x, \omega_2\rangle$ and $|y, \omega_4\rangle$) no polarization entanglement will be found [26]. Each photon will also have a certain narrow bandwidth rather than being strictly monochromatic. The lack of coherence detected in emissions from such dots [27,28] has been understood as indicating that there is no appreciable frequency overlap between each of the photons emitted at different frequencies (their bandwidth is less than the doublet splitting). Rather than tuning the dots to eliminate asymmetries or using cavities to control the emitted frequencies [26] our circuit offers a downstream solution [see Figs. 4(a) and 4(b)] with general applicability. In Fig.

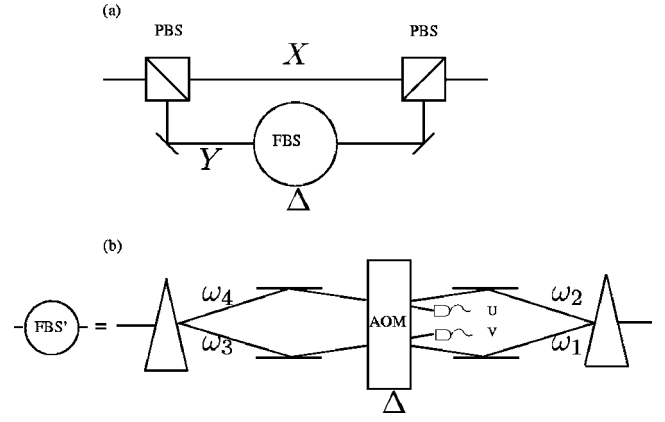


FIG. 4. (a) To demonstrate two-photon interference using the Mach-Zehnder interferometer, X polarized photons take the top path and Y polarized photons are reflected and pass through the FBS defined in Fig. 2 (modulated at the frequency Δ). Here PBS is a polarizing beam splitter. (b) Assuming that AOM's do not frequency shift all incident light we propose this circuit, FBS', as a substitute for the FBS in (a). Here photon detectors are placed at points U and V and these detect all frequency unshifted photons. Unlike the situation in Figs. 2 and 3(c), there is no longer a mixing of frequency shifted and unshifted photons; there is a sense in which the AOM is being used as two separate devices. The AOM is thus not modulated at the frequency difference of the input beams, as was the case in Figs. 2 and 3(c).

4(a) the AOM is modulated at $\omega_3 - \omega_1$ and performs the map

$$|\phi'\rangle \rightarrow \frac{1}{\sqrt{2}}\{[x, \omega_1; x, \omega_2] + e^{i\kappa}[i \sin(\eta l)|y, \omega_1\rangle + \cos(\eta l)|y, \omega_3\rangle] \times [\cos(\eta l)|y, \omega_4\rangle + i \sin(\eta l)|y, \omega_2\rangle]\} \quad (10)$$

If the AOM is 100% efficient at the desired frequency ($\eta l = \pi/2$) then the polarization entangled state $(1/\sqrt{2})(|x, \omega_1; x, \omega_2\rangle + e^{i(\kappa+\pi)}|y, \omega_1; y, \omega_2\rangle)$ is obtained [compare with Eq. (8)].

Depending on the dot's specific asymmetries a frequency difference Δ of the order of 800 MHz might be obtainable. There exist AOM's that operate at this frequency (shifting by around 5 μ eV), though they cannot shift all incident light to different frequencies (typically efficiencies are $< 80\%$ [11]). It is thus practically difficult to have $\eta l = \pi/2$. If one wants a maximally entangled state, $|\phi\rangle$, but cannot frequency shift all light or perform complicated multiple or single copy purifications we propose the substitution of the circuit FBS' [Fig. 4(b)] for the FBS in Fig. 4(a). In the FBS' the two photons are being addressed individually and do not cross in the AOM, so though the circuit looks like Fig. 2 it is actually slightly different. One could, alternatively, use two distinct modulators operating at the same frequency on each arm of the FBS'. Each frequency shift fails with probability $(1 - \alpha)$ and then the unshifted photon can be detected. For perfect detectors in FBS', if neither clicks the desired frequency shift has been obtained (this occurs with probability α^2) conversely any clicks herald a failure. In practice single photon detectors produce both false positives and negatives and are not essential here. Photon loss at U, V , will mean that any

two photon experiments will fail with probability $1 - \alpha^2$ but with current technology two photon events are rare so a factor of α^2 should not significantly change count rates. AOM's typically absorb $<0.05\%$ of incident radiation so the circuit will produce maximally polarization entangled photons with probability $\geq 0.95^2 \alpha^2$ ($\alpha < 0.8$).

The quantity η is frequency dependent. This means that photons at different frequencies are shifted in differing proportions. It is assumed in the derivation of Eqs. (1) and (2) that the two input frequencies are close (that the crystal modulation frequency is small compared to the light frequency) but this must be checked in this case. The biexciton shift, Δ , is $\sim 1 \times 10^9$ Hz (by contrast $\xi \sim 1 \times 10^{11}$ Hz) and typical optical photon frequencies are $\sim 1 \times 10^{15}$ Hz. The proportion of light of frequency ω which is frequency shifted is $\sin^2(\omega R)$ where R is a constant depending on the crystal properties and the intensity and frequency of modulation [10]. For GaP crystals of 1 mm thickness modulated with intensity 1×10^{-6} W/m², R is $\sim 1 \times 10^{-15}$ s. The ratio of frequency shifts for photons at frequencies ω and $\omega + \delta$, δ small, is $\sin^2[(\omega + \delta)R]/\sin^2(\omega R)$ which is $1 + 2\delta R \cos(\omega R)/\sin(\omega R)$ to first order in δR .

The value of δR here is $\sim 1 \times 10^{-6}$ so the assumption that the AOM has approximately the same effect on optical photons differing by Δ is reasonable, i.e., the error is negligible. One must further note that it has been assumed that all photons are monochromatic: in reality they have a bandwidth. The AOM has a frequency dependent action but providing that the bandwidth of each photon is much less than their mean frequency the above analysis can be

applied again. Since the bandwidth is less than the biexciton shift by at least an order of magnitude, this error is negligible.

V. CONCLUSION

In this paper we showed that the inevitable irregularities present in arrays of solid state cavities and the asymmetries of individual quantum dots need not proscribe their use as sources for entanglement generation or distribution. The frequency beam splitter described opens up a window for frequency errors which can be much larger than the bandwidth of individual photons. If physical differences between two mechanisms yield photons which differ in frequency greater than their individual bandwidth, this frequency which-way information can prevent the detection of entanglement in other degrees of freedom. We show how this which-way information can sometimes be removed. Our scheme has, we believe, the twin virtues of simplicity and general applicability and hope it will be useful in scenarios beyond the examples described here.

ACKNOWLEDGMENTS

The authors would like to thank Sean Barrett, John Rarity, Tony Short, and Andrew Shields. N.J. would like to thank the Royal Commission for the Exhibition of 1851, the EPSRC, and the Toshiba Research Europe Ltd. for support. T.M.S acknowledges the QIP IRC for financial support.

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