Two-spin measurements in exchange interaction quantum computers

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We propose and analyze a method for single shot measurement of the total spin of a two electron system in a coupled quantum dot or donor impurity structure, which can be used for readout in a quantum computer. The spin can be inferred by observing spin-dependent fluctuations of charge between the two sites, via a nearby electrometer. Realistic experimental parameters indicate that the fidelity of the measurement can be larger than 0.999 with existing or near-future technology. We also describe how our scheme can be used to implement various one- and two-qubit measurements, and hence to implement universal quantum computation.

Semiconductor technology is rapidly reaching the level where quantum systems comprising of one or two electron spins can be confined and coherently manipulated within a single nanostructure.[1–4] These experiments are important from a fundamental point of view, and will also pave the way to new applications in quantum information processing (QIP).[5–7] A key requirement for these QIP schemes is the realization of a single shot readout technique, whereby the spin state of a one- or two-electron system can be determined in a single measurement run. Such measurements are important both for the ongoing experimental development of spin qubit systems (e.g., for characterizing qubit parameters, and studying the physics of decoherence processes) and, ultimately, for developing scalable QIP architectures.

Recently, elegant experiments have demonstrated single shot readout of single electron spins, via optical techniques in a defect center in diamond,[8] and using an all-electrical technique involving spin to charge conversion in a quantum dot, together with charge detection with a nearby quantum point contact (QPC).[2] A number of other methods based on spin-to-charge conversion have also been proposed.[5,6,9,10] Of particular interest are readout schemes in donor impurity implementations of QIP,[6,7] in which an electric field is applied to induce spin-dependent polarization of a two-electron double-donor system. It is believed that these proposals may fail due to the short lifetime of the quasibound two electron system under the large E field required to observe a significant polarization.[11,12] In this paper we discuss an alternative scheme, whereby the total spin of a two electron system can be inferred by observing spin-dependent *fluctuations* of charge between two tunnel coupled quantum dots (CQD),[30] even in the absence of an external electric field. Our scheme therefore avoids the lifetime issue in donor impurity systems, and furthermore may be easily integrated into proposed QIP architectures, as it does not require magnetic tunnel barriers, tunnel coupling to electron reservoirs, or external rf fields.

Spin-dependent charge fluctuations can be observed by continuously measuring the CQD system with an electrometer adjacent to one of the dots (see Fig. 1). For the purposes of this work, we model the electrometer as a QPC, although it might equally be a single electron transistor. Note that the QPC has a dual role in this scheme: it acts both as a noise source which can induce inelastic transitions in the CQD system, and as a detector to observe these transitions.

In what follows, we first describe a model for the CQD-QPC system, and subsequently derive master equations for both the unconditional and conditional dynamics of the CQD system. We use these results to simulate individual runs of the measurement scheme, and to characterize the detector output for the different measurement outcomes. We show that observing the detector output leads to a quantum measurement in the singlet-triplet basis: for a singlet state (total spin $S=0$) one observes fluctuations in the output current, while for a triplet state ($S=1$), these fluctuations are energetically suppressed, owing to the Pauli principle. We then determine the single-shot singlet-triplet measurement time and fidelity for realistic experimental parameters. We conclude by briefly describing how the readout technique can be used to implement various one- and two-qubit measurements, and hence also to implement universal quantum computation.

Model. A number of authors have considered the problem of a continuously observed *single* charge in a coupled dot system.[9,13–16] Measurement of two charge qubits simultaneously coupled to a detector has also been considered.[17] Here, we adopt the quantum trajectories approach of Ref. 15 and Ref. 16, which take into account the role of energy exchange mechanisms between the observed system and a QPC detector at finite voltage bias. We describe the two-electron coupled-dot system by a two site Hubbard Hamiltonian of the form
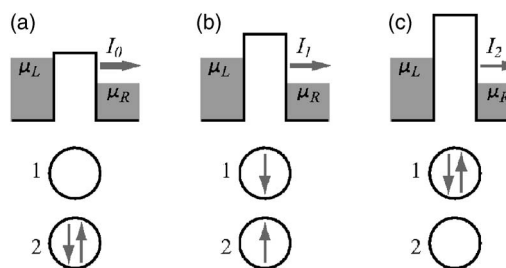

FIG. 1. A schematic of the singlet-triplet readout scheme. Configurations (a) and (c) are energetically forbidden for triplet states, and thus monitoring fluctuations in the QPC current allows a measurement in the singlet-triplet basis.

$$H = \hbar\Delta(t)\sum_{\sigma=\uparrow,\downarrow}(a_{1\sigma}^\dagger a_{2\sigma} + a_{2\sigma}^\dagger a_{1\sigma}) + \hbar U \sum_{i=1,2} n_{i\downarrow}n_{i\uparrow}, \quad (1)$$

where $a_{i\sigma}^{(\dagger)}$ is the fermionic annihilation (creation) operator for an electron on site i with spin σ, $n_{i\sigma}=a_{i\sigma}^\dagger a_{i\sigma}$, $\Delta(t)$ is the tunneling amplitude between sites, and U is the on-site Coulomb energy. We allow $\Delta(t)$ to be time dependent, since switching between different constant values of $\Delta(t)$ can lead to shorter measurement times (see below), although in what follows, we frequently suppress this time dependence for clarity. H is spanned by the four singly occupied states $|\sigma_1\sigma_2\rangle$ (where $\sigma_i=\uparrow,\downarrow$ denotes the spin on dot i) and two doubly occupied states $|d_i\rangle = a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger|0\rangle$ ($i=1,2$). The eigenstates of H are as follows. The singly occupied triplet states $|t_+\rangle=|\uparrow\uparrow\rangle$, $|t_0\rangle=2^{-1/2}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)$ and $|t_-\rangle=|\downarrow\downarrow\rangle$ form a degenerate subspace with eigenvalue 0. The remaining states (with total spin $S=0$) are $|s_0\rangle=2^{-1/2}\cos(\theta/2)(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)$ $-2^{-1/2}\sin(\theta/2)(|d_1\rangle+|d_2\rangle)$, $|s_1\rangle=2^{-1/2}\sin(\theta/2)(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)$ $+2^{-1/2}\cos(\theta/2)(|d_1\rangle+|d_2\rangle)$, and $|s_2\rangle=2^{-1/2}(|d_1\rangle-|d_2\rangle)$, where $\theta=\tan^{-1}(4\Delta/U)$. These states have eigenvalues $-J$, $U+J$, and U, respectively, where $J(t)=(\sqrt{U^2+16\Delta^2(t)}-U)/2$ $\approx 4\Delta^2(t)/U$ is the exchange splitting between the lowest energy singlet and triplet states. $J(t)$ can be varied over many orders of magnitude by varying a surface gate voltage.[5,18]

The entire system (i.e., CQD coupled to a QPC) has a Hamiltonian $H_{tot}=H+H_{tun}+H_{leads}$ where

$$H_{tun} = \hbar \sum_{k,q,\sigma}(T_{kq}+\chi_{kq}n_1)a_{Lk\sigma}^\dagger a_{Rq\sigma} + \text{H.c.}, \quad (2)$$

$$H_{leads} = \hbar\sum_{k,\sigma}\omega_{Lk}a_{Lk\sigma}^\dagger a_{Lk\sigma} + \hbar\sum_{q,\sigma}\omega_{Rq}a_{Rq\sigma}^\dagger a_{Rq\sigma}. \quad (3)$$

Here, $a_{Lk\sigma}$ ($a_{Rq\sigma}$) are fermionic annihilation operators for electrons in the kth (qth) mode on the left (right) side of the barrier with spin σ and angular frequency ω_{Lk} (ω_{Rq}). Tunneling between the left and right baths is described by the factor $T_{kq}+\chi_{kq}n_1$, where $n_1=n_{1\uparrow}+n_{1\downarrow}$ is the total occupancy of dot "1." $T_{kq}\approx T_{av}$ and $\chi_{kq}\approx\chi_{av}$ are assumed to vary slowly over the energy range where tunneling is allowed. We neglect coupling to other environmental degrees of freedom such as phonons.[19] For the parameters we give below, the coupling to the QPC dominates over such effects.

Unconditional master equation. We now derive an unconditional master equation (UME) for the evolution of the reduced density matrix of the CQD system, $\rho(t)$. The UME is obtained by first transforming to an interaction picture in which the dynamics are governed by $H_I(t)=U^\dagger(t)H_{tun}U(t)$, with $U(t)=e^{-i(H+H_{leads})t}$. In the Born and Markov approximations,[20] the resulting dynamics for the CQD system is given by

$$\dot\rho_I = \text{tr}_{leads}\left\{-\frac{1}{\hbar^2}\int_{-\infty}^{t}dt'[H_I(t),[H_I(t'),\rho_I(t)\otimes\rho_L\otimes\rho_R]]\right\}. \quad (4)$$

Here, $\rho_I(t)=U^\dagger(t)\rho(t)U(t)$, and ρ_L and ρ_R are the lead density matrices, given by Fermi-Dirac distributions with chemical potentials μ_L and μ_R. The interaction picture tunneling

Hamiltonian may be written explicitly as $H_I(t)=\Sigma_{k,q,\sigma}[T_{kq}$ $+\chi_{kq}n_1(t)]e^{i(\omega_{Lk\sigma}-\omega_{Rq\sigma})t}a_{Lk\sigma}^\dagger a_{Rq\sigma}+\text{H.c.}$, where the time dependence of $n_1(t)$ is given by $n_1(t)=\frac{1}{2}-e^{-i(J+U)t}\sin(\theta/2)|s_0\rangle\langle s_2|$ $+e^{iJt}\cos(\theta/2)|s_1\rangle\langle s_2|+\text{H.c.}$. Note that Eq. (4) corresponds exactly to the first nonvanishing term in a perturbative expansion of $\rho_I(t)$ in powers of $H_I(t)$.[20]

We now make two further controlled approximations. First, we make a rotating wave approximation, setting very rapidly oscillating factors $e^{i\alpha_i t}\to 0$, where $\alpha_i=\{U,J+U\}$. This is valid for $\alpha_i\gg\nu^2 V$, where $\nu=\sqrt{4\pi g_L g_R}\chi_{av}$ is a dimensionless coupling constant, g_i is the density of states in lead i, and $V=(\mu_L-\mu_R)/\hbar$. Second, we neglect small terms of order $\nu^2\sqrt{JV}$, which is valid when $J\ll V$. At low temperatures ($k_B T\ll\hbar V$), and taking $V\gg J+U$, the UME is

$$\dot\rho(t) = -\frac{i}{\hbar}[H,\rho(t)] + \sum_{j=1}^{3}\mathcal{D}[c_j]\rho(t) \equiv \mathcal{L}\rho(t), \quad (5)$$

where $\mathcal{D}[c]\rho=\mathcal{J}[c]\rho-\mathcal{A}[c]\rho$, $\mathcal{J}[c]\rho=c\rho c^\dagger$, $\mathcal{A}[c]\rho=(c^\dagger c\rho+\rho c^\dagger c)/2$. The Lindblad operators are given by

$$c_1 = \nu\sqrt{V-(J+U)}\sin\frac{\theta}{2}|s_2\rangle\langle s_0|, \quad (6)$$

$$c_2 = \nu\sqrt{V+(J+U)}\sin\frac{\theta}{2}|s_0\rangle\langle s_2|, \quad (7)$$

$$c_3 = \sqrt{V}\left[\mathcal{T}+\nu-\nu\cos\frac{\theta}{2}(|s_1\rangle\langle s_2|+\text{H.c.})\right], \quad (8)$$

where $\mathcal{T}=\sqrt{4\pi g_L g_R}T_{av}$. The c_i's can be associated with different types of tunneling process in the QPC. $c_{1,2}$ correspond to inelastic transitions, in which electrons tunneling through the QPC exchange energy with the CQD system, and are accompanied by transitions between the low energy, singly occupied state $|s_0\rangle$, and the high energy, doubly occupied state $|s_2\rangle$. This can be understood by noting that the corresponding rates for these transitions are proportional to the factors $V\mp(J+U)$, which correspond to the window of allowed QPC tunneling processes, when the Pauli exclusion principle and total energy conservation are taken into account. c_3 corresponds to a quasielastic transition, in which electrons tunnel through the QPC without changing energy.

Conditional dynamics. Individual measurement runs can be simulated using a conditional master equation (CME), which describes the evolution of the CQD system conditioned by the observed detector output, and also permits calculation of the current power spectrum. To derive the CME, we use an explicit model of the measurement process in terms of projective measurements of the number of electrons that have tunneled through the QPC, similar to that presented in Refs. 16 and 20. The CME is found to be
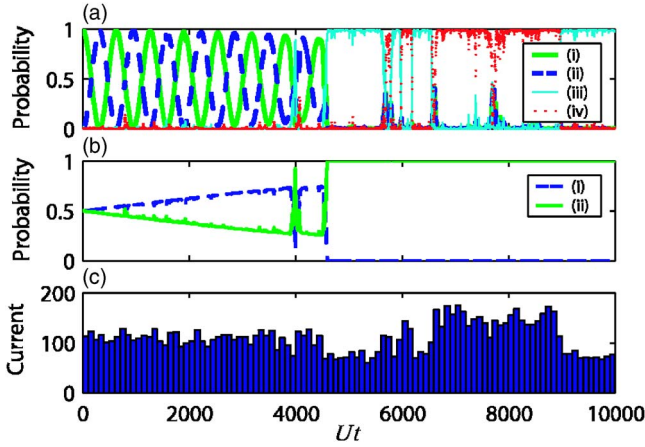
FIG. 2. (Color online) A sample evolution of the conditional density matrix, ρ_c, during a single measurement run. (a) Curves i = {(i)···(iv)} denote the probabilities $P(i,t)=\text{tr}[|\phi_i\rangle\langle\phi_i|\rho_c(t)]$ for the system to be in the state $|\phi_i\rangle\langle\phi_i|$ at time t, where $|\phi_i\rangle$ = {$|\uparrow\downarrow\rangle,|\downarrow\uparrow\rangle,|d_1\rangle,|d_2\rangle$}. (b) Corresponding probabilities for the system to be in a triplet state [curve (i)] or singlet state [curve (ii)]. (c) Corresponding QPC current (in arbitrary units) as a histogram of tunneling events. The parameters used in this simulation are Δ = $0.05U$ [we keep $\Delta(t)$ constant for this simulation], $V=5U$, T = $\sqrt{0.3}\approx 0.54$, $\nu=-0.15T\approx -0.082$. The initial state is taken to be $\rho_c=|\uparrow\downarrow\rangle\langle\uparrow\downarrow|$. In this measurement run, the final state of the system is a singlet state.

$$d\rho_c(t) = dN_c(t)\left[\frac{\sum_{n=1}^{3}\mathcal{J}[c_n]}{P_{1c}(t)} - 1\right]\rho_c(t)$$
$$+ dt\left\{-\sum_n \mathcal{A}[c_n]\rho_c(t) + P_{1c}(t) + i[H,\rho_c(t)]\right\}. \quad (9)$$

Here, $dN_c(t)$ is a stochastic point process, corresponding to the number of electrons (0 or 1) which tunnel through the QPC in an interval dt, $P_{1c}(t)dt \equiv \text{tr}\{\sum_n \mathcal{J}[c_n]\rho_c(t)\}dt$ is the corresponding probability of observing such a tunneling event. Note that, in deriving Eqs. (5) and (9), we have used a coarse graining in time, which means that these equations are valid for time scales longer than U^{-1}. In practice $U \gtrsim 10^{12}$ s^{-1}, and so this is not a significant restriction.

Two sample solutions of Eq. (9) are shown in Figs. 2 and 3. In Fig. 2(a) the system initially undergoes almost coherent oscillations between the singly occupied states $|\uparrow\downarrow\rangle\langle\uparrow\downarrow|$ and $|\downarrow\uparrow\rangle\langle\downarrow\uparrow|$ at the exchange frequency J. After a certain time the dynamics undergoes a qualitative change: the system suddenly jumps into the doubly occupied state $|d_1\rangle\langle d_1|$, and thereafter undergoes stochastic jumps between states with distinct charge configurations. This behavior is reflected in the QPC output [Fig. 2(c)]: when both electrons are localized on dot "1" ("2"), the current is smaller (larger) than when the system is in a singly occupied state. This is reminiscent of a random telegraph switching (RTS) signal. At the first jump, the system becomes localized in the singlet subspace [Fig. 2(b)]. This is because the doubly occupied state $|d_1\rangle\langle d_1|$ is a
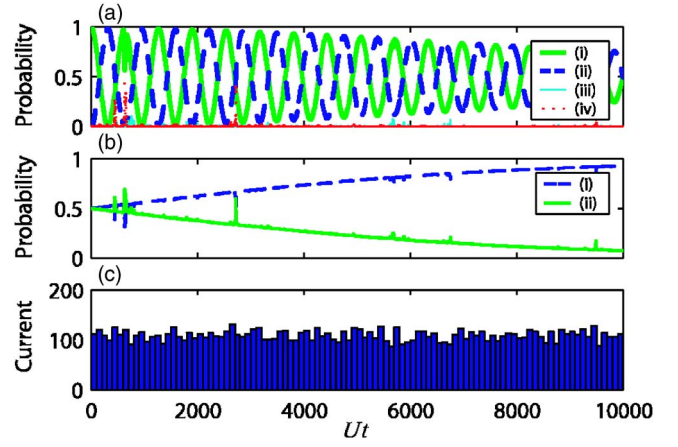
FIG. 3. (Color online) Another sample evolution. In this measurement run, the state of the system becomes localized in the triplet subspace. Parameters as in Fig. 2.

spin-singlet state. Doubly occupied triplet states are energetically forbidden, and therefore when a transition into the doubly occupied subspace is observed, one infers that the system is in a spin-singlet state. In Fig. 3, no jump into the doubly occupied subspace is observed, and the detector output current remains constant (up to shot noise). This leads to a gradual increase in the observer's confidence that the CQD system is in a spin-triplet state [Fig. 3(b)]. In both cases, the final state of the system is localized in either the singlet or triplet subspace, and a strong measurement of the total spin of the system in the singlet-triplet basis is obtained.

Detector power spectra. The possible detector outputs can be characterized by the power spectrum of the current, $I(t)$, through the QPC. The power spectrum is given by $S(\omega)$ $=2\int_{-\infty}^{\infty}d\tau G(\tau)e^{-i\omega t}$, where $G(\tau)=E[I(t+\tau)I(t)]-E[I(t+\tau)]$ $\times E[I(t)]$, and $E[\cdots]$ denotes the classical expectation. Following Refs. 14–16, we have $G(\tau)=e^2[\text{tr}\{\sum_{n,n'}\mathcal{J}[c_n]e^{\mathcal{L}\tau}$ $\times\mathcal{J}[c'_n]\rho_\infty\}-\text{tr}\{\sum_n\mathcal{J}[c_n]\rho_\infty\}^2+\text{tr}\{\sum_n\mathcal{J}[c_n]\rho_\infty\}\delta(\tau)]$, where ρ_∞ is a steady state solution of the UME. Spectra corresponding to the different measurement outcomes can be found by evaluating $G(\tau)$ for different ρ_∞, corresponding to steady states localized in the singlet or triplet subspaces.

For a singlet state outcome, the steady state is $\rho_{S,\infty}=[(V+U+J)|s_0\rangle\langle s_0|+(V-U-J)(|s_1\rangle\langle s_1|+|s_2\rangle\langle s_2|)]/(3V-U-J)$. In the parameter regime of interest (ω, J, $\nu^2 V\ll J+U\lesssim V$), the power spectrum is well approximated by

$$S_S(\omega) = 2e\bar{I} + \frac{16\delta I^2 J^2\nu^2 V\left(\frac{V-U-J}{3V-U-J}\right)}{(\omega^2 - J^2)^2 + 4\omega^2(\nu^2 V)^2}, \quad (10)$$

where $\bar{I}=e^2(\mathcal{T}+\nu)^2 V$ is the average current through the detector, and $\delta I=e^2(2\mathcal{T}\nu)V$. In the limit $J\ll\nu^2 V$, Eq. (10) corresponds to a RTS process with switching rate t_{RTS}^{-1} $=J^2/(4\nu^2 V)$ which is the rate at which the system jumps between the states $|d_1\rangle$ and $|d_2\rangle$. For a triplet state outcome, the steady state is an arbitrary state in the (singly occupied) triplet subspace. The power spectrum only contains a shot noise component, and is given by $S_T(\omega)=2e\bar{I}$. Power spectra
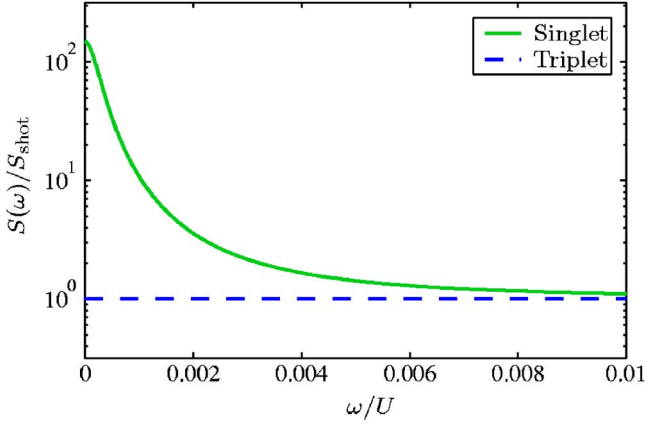
FIG. 4. (Color online) Normalized steady state power spectra for the QPC detector current for both singlet and triplet steady states. Parameters are as in Fig. 2.

for both singlet and triplet outcomes are illustrated in Fig. 4.

Measurement time and fidelity. The total time required to perform a single run of the singlet-triplet measurement scheme, t_{meas} has two components, $t_{meas} = t_{rel} + t_{det}$, where t_{rel} is the the time required to incoherently drive a singlet state into the doubly occupied subspace, and $t_{det} = t_{RTS} \approx 4\nu^2 V/J^2$ is the time required to determine the presence or absence of the RTS signal. By analyzing Eq. (5), we find $t_{rel}^{-1} \approx \nu^2 J(V-U)/U$. t_{meas} can be minimized by varying $J(t)$ over the course of the measurement, such that $J(t)$ is large during the initial "relaxation" phase, and small during the subsequent "detection" phase. In order to minimize t_{rel}, J_{rel} should be as large as possible, but to obtain a strong RTS signal, we require $J_{det} \lesssim \nu^2 V$. We now evaluate t_{rel} and t_{det} for two possible implementations.[5,6] For P donor qubits in Si,[6] we take $U = 43.8$ meV, $V = 45.4$ meV,[31] $\nu^2 = 6 \times 10^{-5}$ (corresponding to a change in conductance of 5% for each electron added to dot 1, assuming $\mathcal{T} = 0.3$), $J_{rel} = 1.0$ meV and $J_{det} = 0.8\nu^2 V = 2.2$ μeV,[17] which gives $t_{rel} = 300$ ns and $t_{det} = 1.51$ ns. For quantum dot qubits in GaAs,[5] we take $U = 1$ meV, $V = 2$ meV, $\nu^2 = 6 \times 10^{-5}$, $J_{rel} = 0.1$ meV, and $J_{det} = 0.8\nu^2 V = 0.11$ μeV, giving $t_{rel} = 110$ ns and $t_{det} = 34.4$ ns. Note that t_{rel} could be further reduced by using an additional noise source to induce relaxation to the doubly occupied subspace.

The fidelity of the measurement is limited by unwanted transitions between the singlet and triplet subspaces, during the course of the measurement operation. Such transitions can be characterized by the *mixing rate*, t_{mix}^{-1}, for unwanted transitions between the singlet and triplet subspaces. Using a simple rate equation it is straightforward to show that the probability of such a transition occurring during a measurement is approximately t_{meas}/t_{mix}, provided $t_{meas} \ll t_{mix}$. The resulting fidelity is thus $F \sim 1 - t_{meas}/t_{mix}$. Owing to the form of the detector-dot interaction in Eq. (2), the detector back-action does not induce such mixing transitions, and therefore the dominant contribution to t_{mix}^{-1} is due to interactions with the environment. Such interactions were not included explicitly in H_{tot}, but appropriate values of t_{mix} can be extracted from recent experiments. For GaAs systems, a lower bound

(limited by the experimental technique) of $t_{mix} \gtrsim 70$ μs has been measured;[3] thus $F \gtrsim 1 - 10^{-3}$. For P donors in Si, no data exist for double donors. We therefore take $t_{mix} \sim \min[T_1, T_2]$, where T_1 and T_2 are the single spin relaxation and dephasing times, as both processes can contribute to t_{mix}: T_1 can be hours,[21] and $T_2 = 50$ ms was recently measured,[22] giving $F \sim 1 - 5 \times 10^{-6}$.

We briefly review the prospects for the experimental demonstration of this scheme in the near future. The experimental requirements can be summarized as follows: (i) a pair of well defined single-spin qubits in adjacent dots; (ii) the ability to achieve exchange couplings up to the values of J_{rel} given above; (iii) rapid gate voltage switching, such that different values of J can be accessed on a time scale shorter than t_{meas}, say 100 ns; (iv) an integrated detector, with sensitivity of around 5%; (v) a large detector bandwidth, such that the signal can be extracted on a time scale of order t_{meas}. In GaAs systems, requirements (i)–(iv) have been met in recent experiments,[3,4] while with regard to requirement (v), fast QPC detectors (with response times much less than 1 μs) are currently being developed.[23] While isolated donor-impurity spin qubits have yet to be demonstrated in Si, recent experiments[24] have demonstrated requirements (iii–v), and theoretical calculations[17] indicate that (ii) should be possible in this system.

This scheme, when augmented with unitary operations and/or ancilla electron spins, can be used to perform a variety of one- and two-qubit measurements. First, readout of a single spin qubit (which we denote by q) can be performed with the aid of a single ancilla spin, by (for example) first preparing the spins in the state $|q\downarrow\rangle_{12}$, applying the operations $\text{CNOT}_{12}H_1$, where H denotes the Hadamard operation, and finally measuring in the two-spin system in the singlet-triplet basis using our scheme. Second, as described in Ref. 25, a useful encoding for exchange interaction is to encode a single logical qubit in three electron spins. Such a qubit can be read out in the computational basis by directly performing our singlet-triplet measurement of two out of the three physical spins.[25] Third, for a pair of single spin qubits, a variety of two qubit measurements are possible. A full Bell basis measurement can be implemented by the sequence: (i) apply the singlet-triplet measurement (STM); (ii) apply the operation X_1; (iii) apply the STM; (iv) apply Z_1; (v) apply the STM. The procedure can be terminated if at any of the STM steps a singlet state outcome is obtained. From the sequence of measurement outcomes it is possible to infer the state of the two qubit system in the original Bell basis. For example the sequence of outcomes "triplet, triplet, singlet" imply that the measured state was $2^{-1/2}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$, whereas "triplet, triplet, triplet" implies that the state was $2^{-1/2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. A partial Bell basis measurement, which allows (nondeterministic) projection onto a two dimensional subspace, is possible by applying only stages (i–iii) of the above procedure. Finally, we note that the scheme can be used to perform universal quantum computation. For example, Ref. 26 demonstrates that this can be achieved using only singlet-triplet measurements of the type described here, together with an initial supply of highly mixed spins.

In conclusion, we have proposed and analyzed a scheme for single shot measurement of a two spin qubit system in the

singlet-triplet basis. The detector in this scheme has a dual role: it induces spin-dependent transitions in the system, while simultaneously observing these transitions. The achievable fidelities are particularly promising for the implementation of single shot measurements at the level required for fault tolerant quantum computation.[27,28]

Recently, another proposal for two-spin measurements in exchange interaction quantum computers was proposed.[29]

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[30] Note that our scheme is applicable to both quantum dot or donor impurity based implementations, although hereafter we will refer to the electron sites as "dots."

[31] In order to prevent direct ionization of the donors by the detector noise, we require $V \le E_{bind}$, where $E_{bind} = 45.5$ meV is the single electron binding energy for phosphorous donors in silicon.