Applying Psychological Theory to Helping Students Overcome Learned Difficulties in Mathematics

An Alternative Approach to Intervention

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ABSTRACT The appearance of systematic errors in computation suggests relatively unlinked computational knowledge to conceptual knowledge, and hence difficulties with forward learning of mathematics. The provision of programs of good teaching, where concrete materials are used to exemplify and thus legitimize algorithmic processes, frequently are not effective for use with upper primary students: systematic errors often resurface. A novel and quite alternate approach to intervention is the Old Way/New Way (O/N) strategy (Lyndon, 1989) based on psychological principles of memory, forgetting and interference. In this article, issues associated with intervention, systematic errors and upper primary students are addressed through a discussion of results of previous research into seventh graders' subtraction knowledge development by overcoming error patterns in subtraction computation. By comparing re-teaching strategies and O/N, it is proposed that both good teaching and effective intervention strategies should be integral to the craft of teaching, particulary in the middle school.

Background

Mathematical knowledge

In the past, basic mathematical knowledge was viewed as a student's proficiency in arithmetical calculation (Putnam et al., 1990), and thus traditionally, the teaching of computational skill was dominant in mathematics instruction (Lampert, 1986). From this perspective, it

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School Psychology International Copyright © 2003 SAGE Publications (London, Thousand Oaks, CA and New Delhi), Vol. 24(1): 95–114. [0143–0343 (200302) 24:1; 95–114; 032582]

could be argued that a student who exhibited errors in calculation would be regarded as having little or poor mathematical knowledge. However, definitions of mathematics knowledge and understanding state that computational performance is only one element encompassed within a definition of knowing. For example, Resnick (1982) suggested that mathematical knowledge is both *syntactic* and *semantic*, where syntactic knowledge is correct performance of mathematical procedures, and semantic knowledge is the understanding of the meaning of those procedures. Leinhardt (1988) suggested that mathematics knowing derives from four knowledge types: *intuitive*, *concrete*, *computational*, and *principled-conceptual*, where, intuitive knowledge is 'everyday' or real world application knowledge which is normally acquired before formal instruction; concrete knowledge is knowledge associated with representation by appropriate concrete materials during instruction; computational knowledge is algorithmic performance; and principled-conceptual knowledge is the '. . . underlying knowledge of mathematics from which the constraints can be deduced.' (p. 122). It is generally accepted that the development of mathematical knowledge can compartmentalize, and hence knowledge types can grow in isolation. For example, a child who is proficient in computational skill may have limited understanding of the mathematical principles of the computation; a child who can perform calculations mentally in real-life situations may not be able to perform the same calculation in the school setting. Computational proficiency therefore cannot be regarded as a true indicator of knowledge, just as poor computational skill does not necessarily mean the student possesses little principled-conceptual knowledge. Comprehensive mathematics knowledge, then, is where knowledge is linked; the knower has developed various internalized representations of related mathematical ideas, and easily moves between each representation (Putnam et al., 1990).

From descriptions of mathematical knowledge, it can be seen that for a child to know subtraction means that he/she is not simply proficient in computation. Subtraction knowledge is the melding of principled-conceptual and intuitive knowledge from which computational knowledge is derived (Leinhardt, 1988); it is intuitive knowledge, a child's ability to solve subtraction problems as experienced in real life, providing the basis for formal knowledge growth and the meaningful application of the standard subtraction algorithm (Ginsburg 1977); it is the link of the syntax (the procedures used in the subtraction algorithm) with the semantics (the meaning of the procedures) in the mind of the learner (Resnick, 1982). Computational errors in subtraction therefore could be regarded as indicative of a student's underdeveloped computational knowledge, or the absence of linkage between computational subtraction knowledge and principled-conceptual knowledge, particularly if the error is a consistent automated response.

Error patterns in computation

Traditionally, students who made errors in their work were regarded as suffering from some learning disability (Kephart, 1960). Errors were indicative of a lack of knowledge. Such a deficit model of error production assumed that minimal learning occurred as a result of the original teaching effort, and so slow and progressive re-teaching was required. Error pattern research has provided an alternative to this perspective. Analysis of errors in computation has revealed that many student errors are not careless or random, but occur regularly and consistently (Brumfield and Moore; 1985, Cox, 1975). Consistency in error production indicates that the student is in fact capable of learning, but has somehow acquired a *learned disability* rather than a *learning disability* (Ashlock, 1994). Consistent errors are active knowledge constructions (Confrey, 1990a; Resnick et al., 1989) and therefore indicate the presence rather than the absence of learning. What has been learned are merely incorrect ways of doing things.

The development of consistent errors in computation, according to Resnick et al. (1989), occurs as children attempt to integrate new knowledge with established knowledge when they are confronted with mathematical examples which extend beyond their current knowledge base. Children apply known mathematical procedures to unfamiliar activities. This description of the development of consistent errors is consistent with Brown and Van Lehn's (1982) *Repair Theory*. Repair Theory suggests that when learners are confronted with tasks which they are unsure of how to perform (on which they have become 'stuck'), they use a simple 'repair' tactic which enables them to produce a solution and become 'unstuck'. In this way, repairs occur as learners choose alternative solution paths in order to produce answers. However, if the repair is erroneous and is left unchecked, the incorrect repair will become a habit through repetition and practice. The repair then becomes a consistent error.

The development of error consistency can also be seen to be similar to the development of skill automaticity (Anderson, 1985). According to Anderson, there are three stages to skill learning: (1) a cognitive stage; (2) an associative stage and (3) an autonomous stage. Anderson suggested that the cognitive stage of skill learning demands application of much cognitive effort to learn a skill or procedure. At the associative stage the procedure is practised and more efficient means of performing the task are discovered. As a result of practise the procedure enters the third stage, the autonomous stage, and is performed with little conscious thought. Consistent errors in computation, then can be seen to be procedures practised to automaticity which are performed with little cognitive effort by the students in response to appropriate stimuli. Through repetition, the errors become learned habits. They are produced automatically in response to a stimulus, and in contrast to random, careless errors, are not self-detected nor self-corrected. They are conceptual and learned (Ashlock, 1994).

Overcoming errors in computation

Once acquired, students' errors and misconceptions are extremely difficult to overcome (Confrey, 1990b; Fischbein, 1987; Graeber and Baker, 1991), and without appropriate instructional intervention, systematic, learned errors persist for long periods of time (Cox, 1975). For the remediation of systematic errors in computation, approaches incorporating the close linkage of the written representation with the concrete/pictorial representation have been suggested (e.g. Ashlock et al., 1983; Booker et al., 1980; Resnick, 1982). Such approaches are characterized by the use of various materials to promote concept development, and typically incorporate the manipulation of specific materials to demonstrate procedures within written algorithms. For promoting subtraction knowledge, and hence overcoming systematic errors in subtraction computation, Booker et al. (1980) described a sequence of lessons that begin with activities for bundling and unbundling popsicle sticks in groups of tens and ones, leading to activities involving grouping, regrouping and trading of base 10 blocks, to finally symbolically recording the algorithm whilst simultaneously manipulating base 10 blocks. Studies on teaching subtraction using similar approaches have been reported in the literature. For example, a teaching sequence for subtraction was trialled with first and second grade children by Fuson and Briars (1990) where subtraction algorithms were explored in concrete, verbal and symbolic modes. Concrete materials were used until children reached the point at which the blocks became unnecessary and were dispensed with. One of the aims of this sequence was to ensure that error patterns in computation did not develop. Fuson and Briars stated that at every point in the teaching sequence, students' performance was carefully monitored so that any errors made were not practised. In another study, Leinhardt (1988), described a similar teaching sequence that began with the concrete representation of a subtraction problem, progressed to an expanded decomposition algorithm and finally moved to the simple decomposition algorithm. A variety of manipulatives were also incorporated into this sequence. Drawing from her definition of knowing, Leinhardt's sequence was aimed at guiding the development of students' principled-conceptual subtraction knowledge through building knowledge of the following principle:

The place value system can be partitioned and is conveniently arbitrary in its notation. 26 = 20 + 6 can be expressed as 10 + 16 or 26 ones if value is maintained as equal. Using this notational rearrangement column subtraction can be applied. (Leinhardt, 1988: p. 24)

The teaching sequences described by Leinhardt (1988) and Fuson and Briars (1990) were reported as successful in promoting the subtraction knowledge of the first and second grade students in these studies. However, using a similar teaching sequence with students who had developed consistent errors in subtraction computation, Resnick (1982), found quite contrasting results. In her study, Resnick found that the students who received intensive instruction using concrete materials and place-value games performed only marginally better than students in the control group. Of this study, Resnick (1992) stated:

> Despite the intensive personal instruction, only half the children taught learnt the underlying semantics well enough to construct an explanation of why the algorithm worked and what the marks represented. More surprisingly, even children who did give evidence of good understanding of the semantics often reverted to their buggy calculation procedures once the instructional sessions were over (p. 394).

Resnick's study, using good re-teaching approaches, provides evidence that students can develop and hold appropriate concepts without giving up their prior, inappropriate concept or error pattern. This phenomenon has been described as knowledge compartmentalization, where a learner holds two pieces of knowledge that are in conflict with each other as separate entities in the mind (Posner et al., 1982; Vinner, 1990).

The recurrence of systematic errors and misconceptions despite the intensity and quality of re-teaching programs has shown the limitations of good re-teaching for assisting students with learned mathematical difficulties. This is one of the major factors affecting programs of intervention. It is well documented that good re-teaching does not always result in the complete eradication of errors and misconceptions (e.g. Bourke, 1980; Connell and Peck, 1993; Resnick, 1992). It is extremely important to assist students overcome their mathematical errors and misconceptions as they 'act as barriers to the acquisition of new conceptual knowledge' (Mansfield and Happs, 1992: p. 453). Prior knowledge has an interfering effect on the development of new knowledge. In a study by Connell and Peck (1993) where concrete materials and good re-teaching sequences were used with students who had developed mathematical errors and misconceptions, this factor became apparent. As they stated:

> When clearly identifiable student conceptual change occurred, it had limited effect due to interference from previously acquired mental structures. Newly acquired information appeared to serve in a superordinate capacity with previously learned procedures or concepts being automatically applied (p. 329).

The reported lack of sustainability of conceptual change away from the intensive remedial setting is also a factor affecting intervention programs. It is well documented that some students appear to make satisfactory progress under closely supervised and individualized instruction, but these gains do not transfer to the regular classroom. Although improvement may occur in the short term, these gains appear to fade over time (Read, 1987).

A further factor that plays a significant role in intervention programs is the affective domain of the learner. Typical responses from students in intervention situations include slowness to respond, apathetic attitude to task, frustration and task avoidance. Often, negative responses to mathematical situations are in the form of avoidance behaviours, where a student shows extreme reluctance to perform any mathematical task, particularly those tasks with which the student has had a long history of failure. Of avoidance behaviour, the American psychologist W. James stated over 100 years ago that avoidance behaviours are exhibited as a response to avoiding the situation: the best way to avoid failure is never to try anything new, because if there is not attempt there can be no failure and with no failure there can be no humiliation (James, 1890). Fernald (1971) suggested that students who have experienced repeated failure in performing mathematical tasks will only approach anything mathematical with a degree of fear and loathing, and that such students need 'emotional reconditioning'. This is achieved by ensuring that students experience success on the first day of the remedial session by organizing instruction for 'ensuring early success, and avoiding situations that lead to failure' (Cole and Chan, 1990; p. 14).

For students who exhibit consistent errors in mathematics, the provision of successful intervention programs is not a simple task. Many issues associated with mathematics intervention are apparent. Some of these issues are summarized in the following list:

- Errors are knowledge;
- Consistent errors are learned and habitual;
- Computational errors indicate the absence of linkage to principled-conceptual knowledge;
- Errors/misconceptions interfere with development of new knowledge and forward learning;
- Re-teaching programs are often ineffective in eradicating consistent errors;
- Recurrence of error patterns after intense intervention influences the affective domain of the learner;

- Learners in intervention situations frequently exhibit avoidance behaviours.

New alternatives for intervention

Re-teaching approaches do not overtly focus on the specific errors/misconceptions held by the child. Re-teaching approaches appear to align the approach to intervention programs as suggested by Gagne (1983):

> The effects of incorrect rules of computation, as exhibited by faulty performance, can most readily be overcome by deliberate teaching of correct rules. . . This means that teachers would best ignore the incorrect performances and set about as directly as possible teaching the rules for correct ones. . . To make students fully aware of the nature of their incorrect rules before going on to teach correct ones . . . seems to me . . . is very likely a waste of time (p. 15).

Gagne proposed that to overcome errors is to aim for 'extinction' (in psychological terms) of that error by ignoring this inappropriate knowledge. An alternative view is that errors/misconceptions must be the focus of intervention, and intervention must assist unlearning of inappropriate knowledge. This view is in agreement with Ausubel (1968) who stated:

> The role of preconceptions in determining the longevity and qualitative content of what is learned and remembered is crucial. . . [The] unlearning of preconceptions may well prove to be the most determinative factor in the acquisition and retention of . . . knowledge (p. 135).

Alternative intervention programs have been described in the literature which appear to take an 'unlearning' perspective on intervention. Rather than being programs of re-teaching these methods begin with the students' errors/misconceptions, or incorporate procedures to enable students' errors/misconceptions to surface. Errors/misconceptions are used in varying degrees to accelerate conceptual change. Such methods include error pattern analysis and intervention (e.g. Ashlock, 1994; Gable et al., 1991); cognitive conflict and conflict teaching (Bell, 1986–87; Swedosh, 1999; Swedosh and Clark, 1998); using errors as springboards for enquiry (Borassi, 1994); belief-based teaching (Rauff, 1994) and teaching by analogy (e.g. Tirosh, 1990). Rather than taking a linear approach, these methods can be regarded as splintering from the error in a number of pathways.

Another program which provides a further dimension to intervention is provided by the Conceptual Mediation (CM) program (Lyndon, 1995) incorporating the Old Way/New Way (O/N) strategy (Lyndon, 1989). CM is similar to intervention programs which actively focus on students' errors and misconceptions. CM differs in that its theoretical background provides persuasive answers to questions that have plagued remedial teachers and researchers such as those asked by Erlwanger (1975):

> Why is it that remedial children often display patterns of errors, hold tenaciously to their own procedures, appear to become confused and emotionally disturbed during remedial work and to require prolonged individual assistance and guidance? (p. 171).

Conceptual Mediation

The Conceptual Mediation program is an integrated approach to intervention that incorporates a particular strategy for assisting students overcome systematic errors and misconceptions. The strategy is called Old Way/New Way (O/N), and by using this strategy, the student and teacher engage in a two-way dialogic process where the child's misconception or error is explored and compared and contrasted to information presented by the teacher.

The fundamental principle upon which the O/N strategy rests, and which makes it different from other 'unlearning' strategies, is that knowledge is held within the mind and protected from change by a mental mechanism, referred to in psychological terms as proactive inhibition (PI). Acknowledging this brain mechanism and its influence on conceptual change provides a new perspective to learned mathematical difficulties and programs of intervention.

Proactive inhibition

As a psychological concept, PI has been widely researched over many years (e.g. Dyer, 1973; Melton and Von Lockum, 1941; Postman and Underwood, 1973; Stroop, 1935; Underwood, 1948; 1957; 1966; Underwood and Postman, 1969). PI is a knowledge protection mechanism that acts to protect first learned knowledge by preventing the association of conflicting ideas (Baddeley, 1990; Houston, 1991; Underwood, 1966). The PI mechanism in action can be clearly seen through the following example provided by Underwood (1966):

> If we are told that: 2 x 2 now is 11
>
> 8 – 4 now is 1
>
> 3 + 3 now is 27
>
> we can imagine the difficulty we would have in remembering and applying this new information. Interference, indeed frustration might well occur. (p. 516)

The following example provided by Baddeley (1990) provides a further description of proactive inhibition: 'Being taught that C means 'caldo' which means hot, but none the less 'forgetting' and turning the wrong tap would be an instance of proactive interference' (p. 40). The psychology literature provides evidence that the PI mechanism is an involuntary mechanism over which the individual has little or no control (e.g. Baddeley, 1990; Houston, 1991), but that considerable variation exists within the population in the level of PI one inherits (Stroop, 1935).

Proactive inhibition and intervention

Proactive inhibition as an innate universal brain mechanism common to everyone has significant implications for education and programs of intervention. Re-visiting errors in light of PI, it can be seen that errors as knowledge will be protected from change by PI. PI does not prevent learning from occurring, it merely prevents the association of conflicting ideas. In the remediation setting, then, for students who have developed consistent errors or misconceptions, the teacher is most likely presenting information that is in conflict with their own knowledge. The PI mechanism thus will ensure that the student's erroneous knowledge is protected. In such a situation, the brain will experience confusion, a state of disbelief, and the new information will be rejected or distorted so as to come closer into agreement with what has been learned before.

According to Lyndon (1989), many factors affecting intervention can be attributable to the action of PI. As discussed above, the recurrence of errors/misconceptions despite the intensity of the intervention is due to that particular program's inability to overcome the influence of PI. With application of much cognitive effort, students will appear to make progress whilst closely monitored during intervention, but these gains will fade when the student is working alone. The student's correct performance is cue-dependent; that is, dependent upon the presence of the remediator. In the absence of the remediator, PI will cause accelerated forgetting of the new knowledge and this new knowledge will be dismissed due to the cognitive effort required by the individual to overcome the power of PI. The recurrence of errors will reinforce in the students their feelings of failure, and thus students will exhibit avoidance behaviour in remedial situations.

The Old Way/New Way strategy

For effective remediation, which contrasts programs of re-teaching, Lyndon (1989), stated that the influence of PI as an inhibitor of knowledge change must be acknowledged. Further, Lyndon stated that '. . . the inhibitory effects of PI may be reduced by the use of the O/N method; and the use of O/N may lead to the retroactive inhibition (i.e. forgetting) of the "old knowledge"'.

The O/N procedure is based upon bringing the learner's 'old way' to a conscious level and exchanging it for a 'new way' by means of discrimination learning, followed by practise with the correct 'new way'. There are four steps in the procedure, as follows:

1. Reactivation of the error memory;
2. Labelling and offering an alternative;
3. Discrimination; and
4. Generalization.

The application of O/N to a student's systematic error in the subtraction algorithm (306 – 149) would proceed in the following manner. In step 1, *reactivation of the error memory*, the student would be presented with the subtraction exercise 306 – 149 and asked to complete that calculation in his/her usual way. Prior to this step, it would have been determined that the student produces consistent errors in such exercises. For step 2, *labelling and offering an alternative*, the student would be asked if his/her particular method of performing that computation could be called the 'old way'. After gaining consent, the student would then be asked if a 'new way' for computing 306 – 149 could be shown. Using carefully selected language, the remediator would then perform the algorithm the standard way. The difference between the two algorithms would then be carefully pointed out, with both the student and the remediator suggesting differences. In step 3, *discrimination*, the student would then be asked to perform the computation his/her old way again, then to perform it the new way, and then asked to contrast the two ways. This discrimination step, using the same problem (306 – 149) is repeated five times, in the sequence: old way, new way, difference; old way, new way, difference. For step 4, *generalization*, the student would be provided with six subtraction exercises and asked to complete them using the new way. The sequence of four steps in O/N is called a learning trial, and takes approximately 10 minutes.

The application of O/N

The application of O/N to errors and misconceptions has been explored in mathematics (e.g. Dole, 1990, 1993, 1995, 1999) and science (Lyndon and Dawson, 1995; Rowell et al., 1990). In a particular research study conducted by Dole (1993), two programs of intervention were utilized in an attempt to overcome Grade 7 students' (age 12–13 years) error patterns in subtraction computation, and to promote links between computational, concrete and principled-conceptual subtraction knowledge. One program of intervention was based on a conventional good re-teaching approach; the other program utilized O/N. The specific purposes of the study were four-fold, as follows:

1. To compare the effectiveness of two intervention programs upon changing Year 7 students' erroneous computational subtraction knowledge and their potential for linking computational knowledge with concrete and principled-conceptual subtraction knowledge;
2. To document subjects' responses to the two methods;
3. To search for evidence of PI affecting conceptual change;
4. To hypothesize an effective program of intervention for upper primary students.

In this study, 16 students from a pool of 60 Grade 7 students demonstrated systematic errors in subtraction computation, and were randomly allocated to one of two treatment groups. Identification of error consistency was via a diagnostic error analysis test that consisted of five types of subtraction exercises classified according to skill level (see Table 1). The test consisted of five subtraction exercises from each level of computational skill (25 items), presented in random order. Test performance was scored by examining errors for the existence of a pattern. For any given skill level, a systematic error was defined as one which occurred three or more times out of five attempts (Cox, 1975).

This test not only assisted selection of subjects, but also highlighted the fact that the students' computational knowledge was inappropriate. The students were also interviewed prior to treatment to ascertain the appropriateness of their intuitive, concrete and principled-conceptual subtraction knowledge. For concrete knowledge, subjects were required to demonstrate the procedures in a two-digit subtraction algorithm (with regrouping) using base 10 blocks. For intuitive/real life knowledge,

Table 1 *Five skill levels of subtraction algorithms*

Skill level	*Skill*	*Example*
Level A	Subtracting a two-digit from a two-digit number with regrouping	53 –14
Level B	Subtracting a two-digit number from a three-digit number with renaming in ones and tens place	523 –78
Level C	Subtracting a three-digit from a three-digit number containing zero in the ones place with regrouping in the tens place	260 –156
Level D	Subtracting a three-digit from a three-digit number containing zero in the tens place with renaming in the hundreds place	608 –134
Level E	Subtracting a three-digit from a three-digit number containing zero in the tens place with renaming across the tens to the hundreds place	302 –158

subjects had to solve a real world subtraction problem, and also create a real world subtraction problem. For principled-conceptual knowledge, subjects had to demonstrate the following understandings:

1. That by increasing/decreasing the minuend or subtrahend in a subtraction exercise, the solution alters accordingly;
2. Addition is the inverse of subtraction, by stating that subtraction computation could be checked through addition;
3. Regrouping/renaming is a legitimate process used for decomposition subtraction algorithms, by being able to explain, for example, that 28 ones can be regrouped as 1 ten and 18 ones;
4. Place value, by organizing expanded numbers into appropriate columns to perform subtraction computation.

Interview data gathered prior to treatment revealed that students in both groups had inappropriate computational knowledge, but that their intuitive, concrete and principled-conceptual knowledge showed variation. For all students in this study, therefore, the subtraction algorithm existed in relative isolation and was unlinked to other subtraction knowledge types. Also, it was found that two students performed subtraction computations in the same erroneous manner, yet of these two students, one demonstrated appropriate principled-conceptual knowledge in three of the four subcategories of this knowledge type, whilst the other failed to demonstrate any such knowledge. It can be seen that the students' errors in subtraction computation effectively masked deeper subtraction knowledge.

Intervention commenced after all interviews had been conducted. For this study, a two-week period was set aside for intervention, and students in each treatment group were withdrawn from their regular classroom at the same time each day over this period of ten consecutive days. Conventional intervention consisted of a series of lessons which centred upon using appropriate materials to demonstrate the legality of the subtraction process within the base ten numeration system. Students in this group experienced 10 lessons of approximately 25 minutes duration beginning with regrouping activities using bundling sticks and base 10 blocks for the subtraction of single digit numbers from two digit numbers, and progressing through the five skill levels of algorithms used on the diagnostic error analysis test.

The lessons followed a sequential framework, but planning of each lesson was also contingent upon progress in the previous lesson. The O/N intervention consisted of an O/N trial performed individually with each student on the first day of intervention. Subtraction exercises selected for intervention using O/N were determined through analysis of individual students' error patterns. For example, a skill Level E algorithm was used with a student who demonstrated an error pattern in skill Level E algorithms. For students who demonstrated error patterns at several skill levels, skill Level B algorithms were selected for O/N as Level B algorithms were the algorithms that students had the most difficulty with second to Level E algorithms.

Each O/N trial took approximately 10 minutes. On the second day of O/N intervention, these subjects met as a whole group, and were presented with five skill level B and five skill level E exercises. Subjects were expected to complete the exercises relating only to the skill level that had been targeted using O/N. From this information, the experimenter could ascertain which subjects required further O/N trials for other skill level algorithms. (Further O/N trials were not required, however).

During the two-week treatment time allocation, Group 2 met together on one other occasion for a skill maintenance check, where ten subtraction exercises were presented for computation. Comprehensive field notes were gathered during the intervention sessions, particularly students' responses and attitudes to the intervention approaches. Second interviews were conducted after treatment, and responses on interview items were compared to data collected during the first interview.

After treatment, students in both groups demonstrated change in subtraction knowledge across particular knowledge categories. The O/N was successful in changing computational knowledge for all students, and also surprisingly appeared to have some influence on concrete and principled-conceptual knowledge development, though the extent of the development of these two knowledge categories was much less than the development of computational knowledge. The conventional approach, which was more time intensive compared to O/N, was less successful in improving computational knowledge and only marginally better in building concrete and principled-conceptual knowledge.

Subjects' attitudes to the two intervention methods also showed stark contrast. When O/N was used individually with students, initially presentation of a subtraction exercise caused feelings of anxiety, both about mathematics in general, or the subtraction exercise in particular. Such student comments were: 'I can't do these, I always get them wrong.', 'I'm no good at Maths.', 'Oh, I hate these . . . with the zero. I always get them wrong.' The students also expressed task avoidance (slowness to take up the pencil, leaning back on the chair, distancing body from the table). After the O/N trial, students expressed the following comments: 'Oh, that's good. Now I know how to do it. Good.', 'Oh yeah. I just used to forget about the zero.', 'Oh this is easy. I know how to do this now.' After one O/N trial, one subject asked whether he could take his work home to show his mother! With these students, there was a change of attitude from negative to positive.

Table 2 *Number of Group 1 and Group 2 subjects demonstrating appropriate computational, intuitive, concrete and principled-conceptual subtraction knowledge – interview 1 and interview 2*

	Group 1		*Group 2*	
Subtraction knowledge	*Interview 1*	*Interview 2*	*Interview 1*	*Interview 2*
1. Computational	0	4	0	8
2. Concrete	0	5	2	5
3. Intuitive				
(a) Solving	2	4	5	7
(b) Creating	5	8	7	8
4. Principled-conceptual				
(a) Subtraction operation	2	4	4	6
(b) Inverse	3	4	6	7
(c) Renaming	1	2	5	6
(d) Place-value	4	6	6	7

For the students in the conventional intervention group, however, the reverse occurred. As the group came together for treatment on the first day, the mood of the Group appeared positive, and the students appeared excited. Comments noted included: 'Are we going to have some fun?', 'What are we going to do? Make something?' The students indicated that they had never used materials such as bundling sticks or base ten blocks for subtraction, and they readily engaged in bundling and unbundling groups of ten sticks, and exploring the value of the various base ten blocks. When students were asked to demonstrate subtraction exercises with base ten blocks whilst simultaneously recording the algorithm, the students became very reluctant to perform. The students were continually informed that the demonstration of the process itself was more important than attainment of the correct solution. This resulted in one student physically distancing himself from the main group, the slow and deliberate pounding of the blocks onto the place value chart by another student, and the construction of towers with the blocks by another student. Comments from other students were: 'This is babyish.', 'I feel a bit daggy doing this.', 'Oh, this is so cinch.', 'Oh, why do we have to do it with blocks?', 'This is so basic.', 'I don't need to use blocks. I know how to do this.' Many subjects disregarded the concrete model and, when not monitored, calculated the solution using their own (upon inspection, erroneous) computational methods. From comments and observed body language it was apparent that enthusiasm for these sessions had waned for some students in the group, despite the fact that errors were still being made.

From this study, limitations of conventional approaches to intervention with upper primary students became apparent. Specifically, the conventional intervention sequence failed to:

- Cater to the ability levels of the subjects;
- Control the desire of all students to produce answers rather than analyse the procedure used to gain the answer;
- Maintain motivation to enable all tasks to be completed;
- Promote the translation of the concrete process to cognitive structure, and to the pen and paper procedure, evidenced by the discarding of materials at the earliest convenience; and
- Confront subjects' existing knowledge, as evidenced by the recurrence of systematic errors.

This method was also time and energy intensive for all parties. In contrast, the O/N intervention, was seen to be very appealing for use with Year 7 students. The strengths of O/N lay in the short amount of time and effort required for implementation, its power to motivate students and its ability to confront the effect of PI as recurrence of erroneous computation procedures was not evident.

Good teaching and effective intervention

The two intervention approaches explored in this study attacked subtraction computation from entirely different angles. O/N looked first at the error, and was concerned with replacing the erroneous procedure with a correct procedure. The conventional approach aimed to use concrete materials to legitimize the steps in the algorithm. From this study, O/N appeared to provide an excellent starting point for remediation with students of Grade 7. Students displayed confidence in their ability, and a perceptible sense of relief at finally being shown the correct way. Once the subtraction algorithm was correctly performed, these students appeared ready to engage in activities designed to develop other aspects of subtraction knowledge. Aspects of conventional intervention, such as exemplifying the legitimacy of the subtraction algorithm with concrete materials, exploring the use of addition to check subtraction calculations, linking subtraction to the real world and developing estimation skills to approximate answers appeared to be the next logical steps in the intervention process, in an effort to promote and link computational subtraction knowledge to concrete and principled-conceptual knowledge.

This study highlighted the value of using a simple process to break through the barrier of erroneous knowledge, and enable forward knowledge growth and development. In light of this study, an effective method of intervention for use with upper primary students is proposed as one which would consist of the following steps:

1. Identification of systematic errors;
2. Structured interview to establish depth of intuitive, concrete and principled-conceptual knowledge;

3. Application of O/N upon systematic computational errors;
4. Use of good re-teaching methods to link computational knowledge to concrete knowledge and principled-conceptual knowledge.

Implications for practicing school psychologists

The recidivist nature of errors is frequently observed by those working with learners in educational contexts. For practitioners trained in remedial techniques, methods and strategies to assist students with learned difficulties can be regarded as good teaching techniques. A common assumption appears to be that students who are struggling in academic situations require good teaching at a slower pace, encapsulated by the following quote taken from the Bullock Committee (HMSO, 1975):

> There is no mystique about remedial education, not are its methods intrinsically different from those employed by successful teachers anywhere. The essence of remedial work is that the teacher is able to give additional time and resources to adapting these methods to the individual child's needs and difficulties.

Similarly, Adelman and Taylor (1986) have echoed similar thoughts: '. . . remedial strategies involve no new principles of instruction. . .' (p. 167) and '. . . conventional remedial practice is not distinguishable from good teaching practise. . .' (p. 176).

In this study, Lyndon's O/N strategy was an effective and efficient means for overcoming systematic computational errors. The theoretical background of O/N offers a framework for interpreting error recidivism. The O/N method itself offers a prescriptive approach to attacking consistent errors, which was readily accepted by students in this study. Exploration of O/N in this study has enabled the difference between good teaching strategies and remedial teaching strategies to be enunciated. There is a clear difference between good teaching and intervention teaching. Good teaching strategies fall into a category distinct from true remediation or intervention strategies. Only strategies that attack, rather than build on prior knowledge, and that are effective in overcoming the power of PI, can be classed as mediation or remediation strategies. Practitioners in school settings therefore, need to make conscious decisions about intervention methods when they work with students. The decision is whether to apply good re-teaching strategies or to use methods for 'un'-teaching, and this is determined through diagnosis or assessment of students' knowledge. Planning for instruction is thus a two-step process consisting of (1) diagnosis of students' knowledge of the topic, including identification of systematic errors and/or misconceptions and (2) planning instruction for either (i) teaching (and/or re-teaching) or (ii) un-teaching. Teaching and re-teaching should occur via implementation of good teaching strategies for building students' knowledge of the topic. For un-teaching, intervention strategies, such as O/N should be utilized.

Concluding comments

Current trends in mathematics intervention research have come to focus on the centrality of considering students' errors and misconceptions. The need to assist students unlearn has given rise to a variety of methods and strategies. The O/N strategy as described in this article, is a prescriptive method based on the psychological concept of PI as a knowledge protection mechanism. Acceptance of PI in this manner reflects the absolute necessity for both good teaching strategies and effective intervention (un-learning) strategies as vital for effective instruction.

References

Adelman, H. S. and Taylor, L. (1986) *An Introduction to Learning Disabilities*. Glenview, IL: Scott Foresman.

Anderson, J. R. (1985) *Cognitive Psychology and its Implications*. New York: W. H. Freeman and Company.

Ashlock, R. B. (1994) *Error Patterns in Computation – A Semi-Programmed Approach* (6th edn) New York: Macmillan.

Ashlock, R. B., Johnson, M. L., Wilson, J. W. and Jones, W. L. (1983) *Guiding each Child's Learning of Mathematics: A Diagnostic Approach to Instruction*. Columbus, OH: Charles E. Merrill.

Ausubel, D. (1968) *Educational Psychology: A Cognitive View*. New York: Holt, Rinehart and Winston.

Baddeley, A. (1990) *Human Memory: Theory and Practice*. Hillsdale, NJ: Lawrence Erlbaum.

Baxter, E. P. and Dole, S. (1990) 'Working with the Brain, not Against It: Correction of Systematic Errors in Subtraction', *British Journal of Special Education* 17(1): 19–22.

Bell, A. (1986–87) 'Diagnostic Teaching 1–3', *Mathematics Teaching*, Nos. 115, 116, 118.

Booker, G., Irons, C. and Jones, G. (1980) *Fostering Arithmetic in the Primary School*. Canberra: Curriculum Development Centre.

Borassi, R. (1994) 'Capitalising on Errors as "Springboards for Inquiry": A Teaching Experiment', *Journal for Research in Mathematics Education* 25(2): 166–208.

Bourke, S. F. (1980) *Numeracy in School: An Integrated Approach to Community Expectation, Student Assessment, and Error Analysis*. Unpublished master's thesis, Monash University, Melbourne.

Brown, J. S. and Van Lehn, K. (1982) 'Towards a Generative Theory of "Bugs"', in T. P. Carpenter, J. M. Moser and T. A. Romberg (eds), *Addition and Subtraction – A Cognitive Perspective*, pp. 117–35. Hillsdale, NJ: Lawrence Erlbaum Associates.

Brumfield, R. and Moore, B. (1985) 'Problems with the Basic Facts may not be the Problem', *Arithmetic Teacher* 33(3): 17–18.

Cole, P. and Chan, L. (1990) *Methods and Strategies for Special Education*. New York: Prentice Hall.

Confrey, J. (1990a) 'What Constructivism Implies for Teaching', in R. B. Davis, C. A. Maher and N. Noddings (eds) *Constructivist Views on Teaching and Learning of Mathematics*, pp. 107–22. Reston, VA: NCTM.

Confrey, J. (1990b) 'Inconsistencies and Cognitive Conflict: A Constructivist's View', *Focus on Learning Problems in Mathematics* 12(3 & 4): 99–109.

Connell, M. and Peck, D. (1993) 'Report of a Conceptual Change Intervention in Elementary Mathematics', *Journal of Mathematical Behaviour* 12: 329–50.

Cox, L. S. (1975) 'Diagnosing and Remediating Systematic Errors in Addition and Subtraction Computations', *Arithmetic Teacher* 22(4): 202–20.

Dole, S. (1999) *Percent Knowledge: Effective Teaching for Learning, Relearning and Unlearning*. Unpublished doctoral dissertation, Queensland University of Technology, Australia.

Dole, S. (1995) 'Gaining Access to Mathematical Literacy: Application of Old Way/New Way to Mathematics Instruction for Adult Learners', *Proceedings of the 3rd Annual International Conference on Post-Compulsory Education and Training*, Brisbane.

Dole, S. (1993) 'Error Patterns and Subtraction Knowledge Development – A Comparison of Methods', in J. Novak (ed.) *Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics*. Ithaca, NY: Correll University. (Electronic publishing of seminar proceedings)

Cole, S. and Cooper, T. (1995) 'Errors as Springboards for Remediation of Year 7 Students' Subtraction Knowledge', in B. Atweh and S. Flavell (eds) *Eighteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)*, pp. 235–40, Northern Territory University.

Dyer, F. (1973) 'The Stroop Phenomenon and its Use in the Study of Percentual, Congitive and Response Processes', *Memory and Cognition* 1(2): 106–20.

Erlwanger, S. H. (1975) 'Case Studies of Children's Conceptions of Mathematics', *Journal of Mathematical Behaviour* 1(3): 157–283.

Fernald, G. M. (1971) *Remedial Techniques and Basic School Subjects (Edited by Lorna Idol)* Austin, TX: ProEd.

Fischbein, E. (1987) *Intuition in Science and Mathematics: An Educational Approach*. AH Dordrecht, Holland: D. Reidel.

Fuson, K. and Briars, D. J. (1990) 'Using a Base-Ten Blocks Learning/Teaching Approach for First and Second Grade Place Value and Multidigit Addition and Subtraction', *Journal for Research in Mathematics Education* 21(3): 180–206.

Gable, R. A., Enright, B. E. and Hendrickson, J. M. (1991) 'A Practical Model for Curriculum-Based Assessment and Instruction in Arithmetic', *Teaching Exceptional Children* 24(1): 6–9.

Gagne, R. (1983) 'Some Issues in the Psychology of Mathematics Instruction', *Journal for Research in Mathematics Education* 14(1): 7–18.

Ginsburg, H. (1977) *Children's Arithmetic: The Learning Process*. New York: Van Nostrond.

Graeber, A. and Baker, K. M. (1991) 'Curriculum Materials and Misconceptions Concerning Multiplication and Division', *Focus on Learning Problems in Mathematics* 13(3): 25–38.

HMSO (1975) *A Language for Life*. Report of the Committee of Inquiry Appointed by the Secretary of State for Education and Science under the Chairmanship of Sir Alan Bullock.

Houston, J. P. (1991) *Fundamentals of Learning and Memory*, 4th edn. Orlando, FL: Harcourt Brace Jovanovich.

James, W. (1890) *Principles of Psychology*. New York: Holt, Rinehart and Winston.

Kephart, N. C. (1960) *The Slow Learner in the Classroom*. Columbus, OH: Charles E. Merrill.

Lampert, M. (1986) 'Knowing, Doing and Teaching Multiplication', *Cognition and Instruction* 3(4): 305–42.

Leinhardt, G. (1988) 'Getting to Know: Tracing Student's Mathematical Knowledge from Intuition to Competence', *Educational Psychologist* 23(2):119–44.

Lyndon, H. (1995) 'Conceptual Mediation – A New Approach to an Old Problem', *Voice – A Journal for Educators* 3(2):00–00

Lyndon, H. (1989) 'I Did it My Way! An Introduction to the Old Way/New Way Methodology', *Australasian Journal of Special Education* 3(1): 32–37.

Lyndon, E. H. and Dawson, C. J. (1995) *The Conceptual Mediation Program: A Practical Outcome from a Novel Perspective on Conceptual Change*, paper presented at the Australian Association of Science Education Annual Conference, Adelaide, July 1995.

Mansfield, H. and Happs, J. (1992) 'Using Grade-Eight Students Existing Knowledge to Teach about Parallel Lines', *School Science and Mathematics* 92(8): 450–54.

Melton, A. and Von Lockum, W. (1941) 'Retroactive and Proactive Inhibition in Retention: Evidence for a Two-Factor Theory of Retroactive Inhibition', *American Journal of Psychology* 54: 157–73.

Posner, G., Strike, K., Hewson, P. and Gertzog, W. (1982) 'Accommodation of a Scientific Computation: Toward a Theory of Conceptual Change', *Science Education* 66: 211–27.

Postman, L. and Underwood, B. (1973) 'Critical Issues in Interference Issues', *Memory and Cognition* 1: 19–40.

Putnam, R. T., Lampert, M. and Peterson, P. C. (1990) 'Alternative Perspectives on Knowing Mathematics in Elementary Schools', *Review of Research in Education* 16: 57–149.

Rauff, J. V. (1994) 'Constructivism, Factoring and Beliefs', *School Science and Mathematics* 94(8): 421–26.

Read, G. (1987) 'Following the Progress of Children who have Received "Remedial" Tuition', *Support for Learning* 2(1): 52–9.

Resnick, L. B. (1992) 'From Protoquantities to Operators: Building Mathematical Competence on a Foundation of Everyday Knowledge', in G. Leinhardt, R. Putnam and R. A. Hattrup (eds) *Analysis of Arithmetic for Mathematics Teaching*, pp. 373–429. Hillsdale, NJ: Lawrence Erlbaum.

Resnick, L. B. (1982) 'Syntax and Semantics in Learning to Subtract', in T. P. Carpenter, J. M. Moser and T. A. Romberg (eds) *Addition and Subtraction – A Cognitive Perspective*, pp. 136–55. Hillsdale, NJ: Lawrence Erlbaum Associates.

Resnick, L., Nesher, P., Leonard, F., Magone, M., Omanson, S. and Peled, R. (1989) 'Conceptual Bases of Arithmetic Errors: The Case of Decimal Fractions', *Journal for Research in Mathematics Education* 20(1): 8–27.

Rowell, J. A., Dawson, C. J. and Lyndon, E. H. (1990) 'Changing Misconceptions: A Challenge to Science Education', *International Journal of Science Education* 12(2): 167–75.

Stroop, J. R. (1935) 'Studies of Interference in Serial Verbal Reactives', *Journal of Experimental Psychology* 18: 643–62.

Swedosh, P. (1990) 'Reducing the Incidence of Mathematical Misconceptions in "Middle Band" Students', in J. Truran and K. Truran (eds) *Making the Difference: Proceedings of the Twenty-Second Annual Conference of the Mathematics Education Research Group of Australasia*. Adelaide: MERGA.

Swedosh, P. and Clark, J. (1998) 'Mathematical Misconceptions – We Have an Effective Method for Reducing their Incidence but will the Improvement Persist?', in C. Kanes, M. Goos. E. Warren (eds) *Teaching Mathematics in New Times: Proceedings of the Twenty-First Annual Conference of the Mathematics Education Research Group of Australasia (2)*, pp. 588–95. Broadbeach: MERGA.

Tirosh, D. (1990) 'Inconsistencies in Students' Mathematical Constructs', *Focus on Learning Problems in Mathematics* 12 (3 & 4): 111–29.

Underwood, B. J. and Postman, L. (1969) 'Extra-Experimental Sources of Interference in Forgetting', *Psychological Review* 67(2): 73–95.

Underwood, B. J. (1966) *Experimental Psychology*, 2nd edn. New York: Appleton-Century Crofts.

Underwood, B. J. (1957) 'Interference and Forgetting', *Psychological Review* 64(1): 49–60.

Underwood, B. J. (1948) 'Retroactive and Proactive Inhibition after Five and Forty-Eight Hours', *Journal of Experimental Psychology* 38: 29–38.

Vinner, S. (1990) 'Inconsistencies: Their Causes and Function in Learning Mathematics', *Focus on Learning Problems in Mathematics* 2(3 & 4): 85–99.