

16<sup>th</sup> Australasian Fluid Mechanics Conference  
Crown Plaza, Gold Coast, Australia  
2-7 December 2007

## A Comparative Study on High Reynolds Number Turbulent Boundary Layers

S. Hafez [shafaz@kau.edu.sa]

Department of Aeronautical Engineering  
King Abdul Aziz University, Jeddah, 21589 Saudi Arabia

### Abstract

An increasing number of high quality measurements of turbulent boundary layers at high Reynolds number have been reported in the literature. These measurements come from flows that were established and developed employing different approaches and facilities. It is interesting to assess how the choice of an experimental set-up and the employment of different types of tripping device influence the state of development for such layers. The present study aims to establish, qualitatively, a relationship between mean flow parameters and higher order statistics of the flow, namely, the streamwise turbulence intensity. Detailed mean flow parameters and turbulence intensity profiles for normally tripped layers, in the sense of Coles [4], do not show dependence on the type of tripping device at high Reynolds number. However, there is some evidence indicating that naturally transition flows and artificially tripped layers that are highly perturbed during initial stages of development will not follow similar behaviour. It appears that flows with high Coles' profile parameter,  $\Pi$ , will exhibit high levels of turbulence intensity in the outer flow region.

### Introduction

Turbulent boundary layers encountered in practical applications are complex in nature with various parameters that may affect their development. The most extensively studied class of flow is the two-dimensional turbulent boundary layer developing over a smooth surface without the influence of other parameters such as pressure gradient or surface quality to name a few. This type of flow will be referred hereafter as a turbulent boundary layer, TBL. Despite extensive efforts and recourses dedicated to study such flow, we are still facing outstanding fundamental questions such as the proper scaling of the mean-flow velocity profiles. Recent reviews regrading our state of knowledge and relevant issues including this type of flow are addressed by Buschmann & Gad-el-Hak [1], Fernholz & Finley [6] and Gad-el-Hak & Bandyopadhyay [8].

Buschmann & Gad-el-Hak [1] ruled out neither logarithmic nor power law behaviour for the mean flow velocity profile of turbulent wall-bounded flows with the possibility of Reynolds number dependence. It is evident that there is a lack of detailed and reliable measurements at high Reynolds number,  $R_\theta$ . Where Reynolds number is based on the freestream velocity,  $U_1$ , momentum thickness,  $\theta$ , and the kinematic viscosity,  $\nu$ .

Recent publications and research work aimed at providing new information at high  $R_\theta$  yielded different outcomes, e.g. Hites [11], Österlund [17] and DeGraaff & Eaton [5]. The main objective of previous studies, and the present work as well, is to collect and present highly resolved experimental data that closely represents this class of flow at the highest possible  $R_\theta$ . However, observed differences in the measured quantities, even derived from mean flow measurements show large variations, which exceed experimental uncertainty, but may be attributed to the way these layers have been established and subsequently developed. Some of the major factors that may have large impact on the quality of the results are the choice of tripping device and the spatial resolution of the flow.

Nickels *et al.* [17] addressed the effect of spatial resolution on the streamwise turbulence intensity profiles and its spectra at high  $R_\theta$ . It was found that lack of spatial resolution may lead to false conclusions such as the appearance of a second peak for the turbulence intensity profiles away from the viscous dominated wall region. Also, spectra, within the turbulent wall region, will be affected as well over a wide range of the spectrum resulting in lack of collapse of these profiles as anticipated by physical model for the structure of turbulent boundary layer.

The use of tripping devices in boundary layers research was initially intended to enhance spatial resolution of the flow by promoting much thicker layers, compared with free transition layers at the same Reynolds number. Klebanoff & Diehl [13] and Coles [4] demonstrated experimentally that some types of tripping devices can produce long lasting effects on boundary layers, eventually yielding a distorted layer. Calculation of tripping device size can be guided by criterion developed by different researchers, e.g. Tani [23], and these criterions should be consulted only to provide a preliminary estimate of the size of the tripping device. Detailed measurements should follow to optimize the size of the tripping device to match the incoming flow and it should have a diminishing effect on boundary layer development leading to high  $R_\theta$ . Support for this argument is evident from Coles [4] analysis of normally developing TBL's at low  $R_\theta$  and culminating in Coles' empirical correlation between profile parameter,  $\Pi$ , and  $R_\theta$ .

Boundary layers studies at high  $R_\theta$  are commonly carried out by examining boundary layers developed over the floor or sidewalls of wind tunnels. Given the relatively large size of these facilities, the TBL may begin to develop inside the contraction due to surface irregularities or other external factors. The work of Fernholz *et al.* [7] and the NDF floor boundary layer by Hites [11] are typical examples of this configuration. Measured flow statistics for this type of flow, uncontrolled transition, may depend on the test facility. Another approach is the use of mounted models within the wind tunnel. This provides a better defined origin of the boundary layer, e.g. the work of Österlund [18], Knobloch & Fernholz [14] and the smooth cylinder model of Hites [11].

The range of  $R_\theta$  that can be achieved in the laboratory is low to moderate, compared with most practical engineering applications. This has led to the development of similarity scaling laws for turbulence quantities to predict flow parameters at high  $R_\theta$ . The work of Marusic *et al.* [16] and the recent extension by Marusic & Kunkel [15] is a typical example of such an approach which is supported by quality data.

Castillo *et al.* [2] proposed an outer layer scaling law for turbulence quantities as a function of the type and location of the tripping device in addition to the evolution of the TBL or "flow history". According to these arguments, for a typical tripping device, distributions of turbulence quantities are related to  $U_1$  irrespective of  $R_\theta$ . This contradicts the classical scaling arguments and implies that there is no unique asymptotic state at high  $R_\theta$ . The present work aims to assess some of these arguments based on recent measurements by the author and other researchers over a large range of freestream velocities and Reynolds numbers.

### Apparatus and Measuring Techniques

Measurements reported in the current study were carried out at the high Reynolds number turbulent boundary layer wind tunnel at the University of Melbourne. The test facility is an open return blower wind tunnel with a unique 27m working length and a 2m x 1m cross section. Measurements were carried out for boundary layers developing on the tunnel's floor, which is covered by aluminium plates of 6m x 2m and 6mm in thickness. A schematic view of the test facility is shown in figure 1.

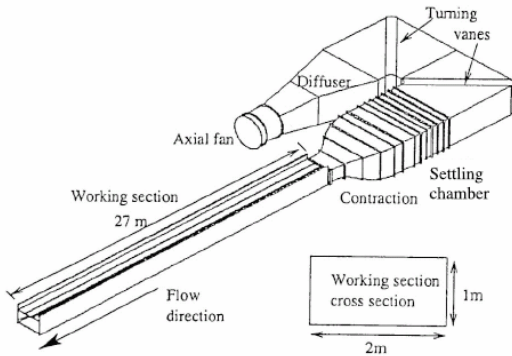


Figure 1: Isometric view of wind tunnel.

Boundary layers developing along the inner surface of the contraction were tripped using sandpaper sheets, grade 40 of 115mm width, placed 750mm upstream from the exit. A nominally zero pressure gradient was maintained along the working section using a series of 1m width adjustable ceiling panels. Air bleeding and adjusting the height of these panels were the two mechanisms used to control the streamwise pressure variation.

The Clauser chart technique was used employing traditional constants,  $\kappa = 0.41$  and  $A = 5.0$ , to deduce local skin friction coefficient,  $C_f = 2 \tau_w / \rho U_I^2$ . Here  $\tau_w$  is the wall shear stress,  $\rho$  is the fluid density.

The normal single sensor (DANTEC 55P05) is used with a constant temperature anemometer system (AN-1003 from AA lab systems) operating at overheat ratio of 1.8. The frequency response of the system to a 2kHz internal pulse, was more than 200kHz. Wollaston wires were soldered to the probe and etched to give a platinum filament with core diameters of 5.0 $\mu$ m and 2.5 $\mu$ m, with active length of approximately 0.9mm and 0.5-0.6mm respectively.

A static calibration techniques with a third order polynomial curve fit was used to convert the measured anemometer output voltage into velocity. The normal hot-wire was statically calibrated against a Pitot-static tube pair, located within the mid-height of the tunnel and about 5cm apart. Hot-wire signals were sampled on-line with an IBM compatible personal computer, using Microstar 16 bit data acquisition board model DAP3000A/21. Turbulence intensity measurements were taken in bursts of 8000 samples and four bursts were sufficient to obtain converged results to within 1%. The signals were sampled at 200Hz and filtered at 20kHz using a Frequency Devices filter model LP00. The hot-wire probe was attached to streamlined sting, protruding from the tunnel's floor, and driven using a computer controlled stepping motor. This arrangement significantly reduced the aerodynamic loading on the probe body and hence the pitch angle of the sting and also eliminates the need for any yaw correction.

### Results and Discussion

Detailed measurements comprising Pitot-static tube and normal hot-wire measurements were reported by Hafez *et al.* [10,11].

The turbulent boundary layer results are analysed according to the classical two-layers model. The friction velocity  $U_\tau = \sqrt{\tau_w / \rho}$ , is used to scale turbulence quantities for inner and outer flow scaling. The viscous length,  $\nu/U_\tau$ , is used to scale the wall-normal distance,  $z$ , with inner flow scaling. The outer flow length scale is the boundary layer thickness, as defined by Perry *et al.* [20] i.e.

$$\delta = \delta^* S / C_I \quad (1)$$

where  $\delta^*$  is the displacement thickness of the boundary layer,  $S = U_I / U_\tau$  and  $C_I$  is a constant found by integrating the velocity defect profile, i.e.

$$C_I = \int_0^1 \left( \frac{U_I - U}{U_\tau} \right) d\eta \quad (2)$$

The non-dimensional wall-normal distance with inner and outer flow scaling is  $z^+ = zU_\tau/\nu$  and  $\eta = z/\delta$  respectively.

Measurements for the main study, developing flows, were limited to three reference unit Reynolds numbers  $U_\infty/\nu = 6.48 \times 10^5$ ,  $1.33 \times 10^6$  and  $1.94 \times 10^6/m$ . They are corresponding to nominal reference freestream velocity,  $U_\infty$ , of 10m/s, 20m/s and 30m/s respectively. Measurements for the matched Reynolds number profiles, same  $R_\theta$ , are carried out at three stations along the plane of symmetry of the tunnel. The Reynolds number, for the most downstream station at the lowest reference velocity is approximately  $R_\theta = 20,000$ . This value is kept constant to within  $\pm 0.7\%$  of the other two runs. The measuring stations were located at 21.7m, 13.7m and 8.7m from the tripping device with the corresponding nominal  $U_\infty$  of 10m/s, 16m/s and 24m/s respectively. Mean-flow velocity profiles, scaled and plotted with inner flow scaling are shown in figure 2 and complete collapse throughout the whole layer is evident. The agreement with the DNS data of Spalart [22],  $R_\theta = 1410$ , for  $z^+ < 100$  is excellent.

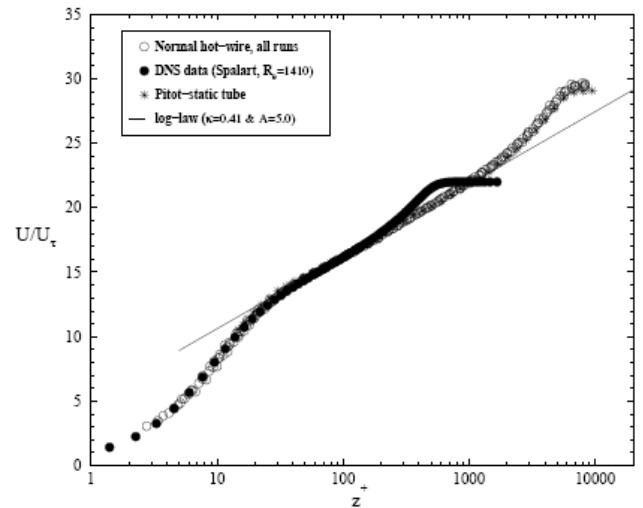


Figure 2: Mean-flow velocity profiles,  $R_\theta = 20,000$ , with inner flow scaling.

If the matched mean-flow velocity profiles, shown in figure 2, are to be plotted in the velocity defect form it will show a similar degree of collapse throughout the whole boundary layer. The present data in the classical velocity defect form,  $(U_I - U)/U_\tau$  is directly related to the Castillo *et al.* [2] outer flow scaling formulation,  $(U_I - U)/U_I$ , as

$$(U_1 - U)/U_\tau = S (U_1 - U)/U_1 \quad (3)$$

The excellent collapse of the mean-flow velocity profiles invalidate the velocity defect similarity arguments as proposed by Castillo *et al.* [2].

The effect of  $R_\theta$  is investigated by considering data at low and very high  $R_\theta$ . The relevant mean-flow velocity defect profiles with outer flow scaling are shown in figure 3, from measurements that were carried out at two stations. The first measuring station is located at  $x = 1.6\text{m}$  and  $U_\infty = 10\text{m/s}$  ( $R_\theta = 2580$ ). The second station is located at  $x = 21.6\text{m}$  with  $U_\infty = 30\text{m/s}$  ( $R_\theta = 52,100$ ) and  $10\text{m/s}$  ( $R_\theta = 20,000$ ).

Figure 3 shows that there is no Reynolds number effect outside the turbulent wall region, TWR, i.e.  $z^+ > 100$  &  $\eta < 0.15$ . However, within the TWR there is a lack of collapse between measurements at  $R_\theta = 20,000$  and  $52,100$  which can be attributed to boundary layer development.

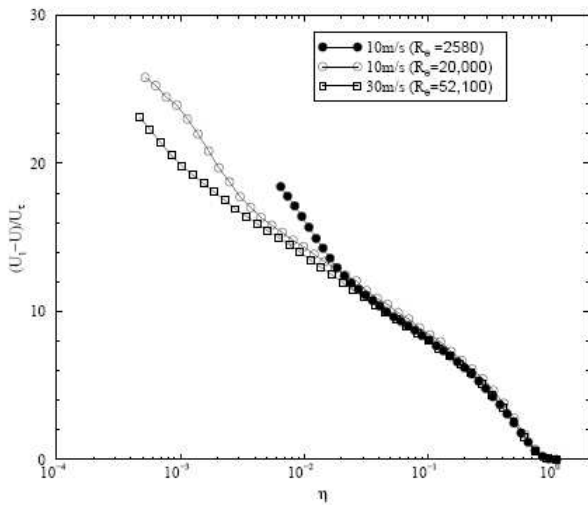


Figure 3: Mean-flow velocity defect profiles with outer flow scaling.

This behaviour can be explained by inspecting figure 4 that shows the variation of Coles' wake parameter,  $\Delta U^+ = \Delta U/U_\tau$  where  $\Delta U$  is the maximum deviation from the log-law of the wall, with  $R_\theta$ . Coles [4] showed that  $\Delta U^+$  is highly dependent on Reynolds number, for  $R_\theta < 6000$  and reach a "state of equilibrium" for much higher Reynolds numbers with a value of  $\approx 2.93$ . The data of Smith & Walker [21], as compiled by Coles, showed a consistent drop of  $\Delta U^+$  for  $R_\theta > 1.5 \times 10^4$  with an asymptotic value of 2.05.

Coles' empirical correlation between  $\Delta U^+$  and  $R_\theta$  for low Reynolds number has been employed in our study to justify the selection of the tripping device and to validate its performance under different freestream conditions.

A typical functional form for this correlation is  $\Pi = \Pi_{max}(1 - \exp(-0.243a^{0.5} - 0.298a))$ , where  $a = ((R_\theta/425) - 1)$ , Cebeci & Smith [3]. The minimum  $R_\theta$  at which  $\Pi$  can exist is  $\approx 425$  and  $\Pi_{max}$  or asymptotic value, according to Coles, can take a value between 0.55 to 0.60. This corresponds to  $\Delta U^+$  between 2.68-2.93.

Figure 4 shows that the three main flow cases are properly tripped as compared with Coles' criterion at low  $R_\theta$ . The data show a peak at  $R_\theta \approx 7000$  with  $\Delta U^+ \approx 3.0$ , followed by a consistent decrease with increasing  $R_\theta$ .

Rotta [20] identified different boundary layers where an "equilibrium or self-similar layers" can be established. According to Rotta's criterion and Perry *et al.* [19], the zero pressure gradient turbulent boundary layer will reach an equilibrium at very high Reynolds number,  $R_\theta \rightarrow \infty$ . The present study suggests that such state can practically be represented by data from  $R_\theta >$

$2.0 \times 10^4$ ,  $\Delta U^+ \approx 2.3$ . The data of DeGraaff & Eaton [5], Smith & Walker [21] and our present study give strong support for the Reynolds number limits but not the terminal value. These data were analysed using similar techniques and employing the same traditional constants. However, different techniques were employed to deduce the skin friction velocity and this could be the reason for reaching different asymptotic values.

Fernholz *et al.* [7] have found that their data showed the wake component has not reached an asymptotic state till  $R_\theta \approx 6 \times 10^4$ . The reason for this difference is not clear and it could be due to uncontrolled transition in their experiments. Further work is needed to refine  $R_\theta$  limit and resolve the differences in the asymptotic values.

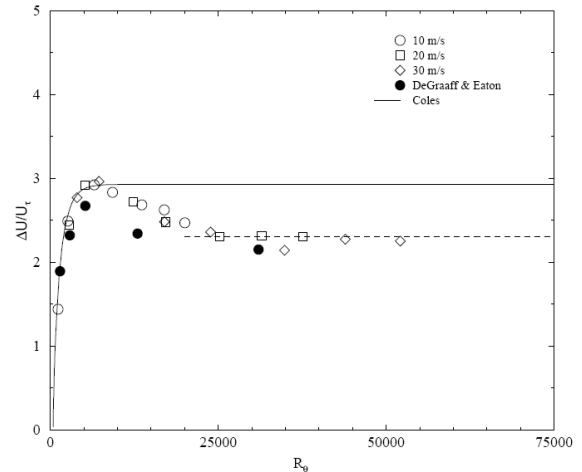


Figure 4: variation of Coles' wake parameter with  $R_\theta$ .

Streamwise turbulence intensity profiles with outer flow scaling are shown in figure 5. The trend shown for  $z^+ > 100$  is consistent with Townsend's attached eddy hypothesis, its extensions by the University of Melbourne fluids group and the recent work of Marusic & Kunkel [15]. The high  $R_\theta$  data show an extensive outer flow overlap region with peeling off following the increase of  $R_\theta$  of the flow. The case of  $R_\theta = 2580$  can marginally be considered to follow the same trend.

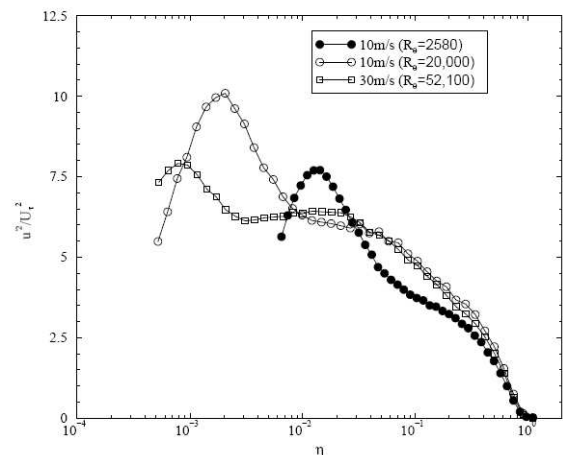


Figure 5: Streamwise turbulence intensity profiles with outer flow scaling.

The effect of employing different approaches and facilities to reach high  $R_\theta$  flows is shown in figure 6. Data comprise of measurements at two typical Reynolds numbers of  $R_\theta \approx 20,000$  and  $39,000$ , and measurements are presented with inner flow scaling.

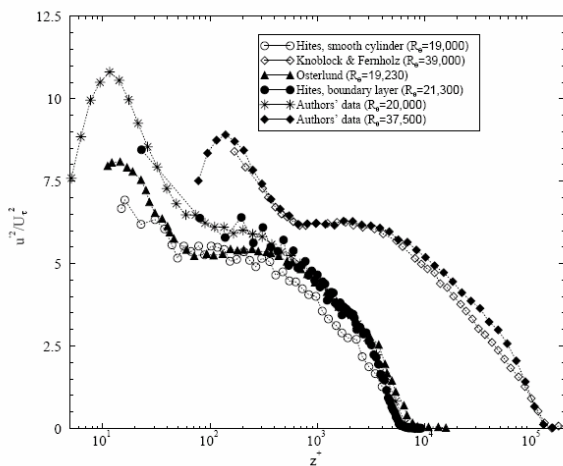


Figure 6: Streamwise turbulence intensity profiles with inner flow scaling. Note shift in abscissa.

Close to the wall, the measurements of Österlund [18], the smooth cylinder model of Hites [11] and the author's data are affected by spatial resolution. Away from the wall, the smooth cylinder model of Hites showed an attenuated profile throughout most of the boundary layer. This behaviour could be a result of lateral straining of the TBL during its course of longitudinal development over an external curved surface. Mean-flow velocity profiles with inner flow scaling showed differences in terms of the Coles' profile parameter,  $\Pi$ . Observed differences between streamwise turbulence profiles with outer flow scaling, not shown here, appears to be linked to  $\Pi$ . Profiles with high levels of turbulence intensity are directly related to mean-flow velocity profiles with high value of  $\Pi$ . This result is consistent with Coles' interpretation of  $\Pi$  as a measure of energy of the large eddies, on the average.

### Conclusions

Normal hot-wire measurements at a range of  $R_\theta$  2580 to 52,100 showed that mean flow velocity profiles and streamwise turbulence intensity profiles can be represented by classical scaling laws either for the inner or outer flow region. Mean-flow velocity defect may approach self similarity at very high  $R_\theta$  and self similar profiles based on the arguments by Castillo *et al.* (2004)<sup>[1]</sup> is not supported by the present measurements. The effect of initial conditions was assessed by carrying out comparative studies at  $R_\theta \approx 20,000$  and 39,000. The near-wall region showed very good agreement between different data sets provided that the flow was adequately resolved. This behaviour was expected since this region will not be influenced by the initial conditions especially at high  $R_\theta$ . The outer flow region was found to be influenced by initial conditions. Mean-flow velocity profiles with high values of  $\Pi$  are found to exhibit high levels of turbulence intensity in the outer flow region. Further work is needed in order to confirm these findings.

### Acknowledgement

The author wish to acknowledge the financial assistance of the Australian Research Council Discovery Grant.

### References

[1] BUSCHMANN, M.H. & GAD-EL-HAK, M. Recent developments in scaling of wall-bounded flows. *Prog. Aerospace Sci.*, **42**, 2007, 419-467.  
 [2] CASTILLO, L., SEO, J., HANGAN, H. & JOHANSSON, T.G. Smooth and rough turbulent boundary layers at high Reynolds number. *Exp. Fluids*, **36**, 2004, 759-774.

[3] CEBECI, T. & SMITH, A. M.O. Analysis of turbulent boundary layers. *Academic press, New York*, 1974.  
 [4] COLES, D.E. The turbulent boundary layer in a compressible fluid. Rand Rep. R-403-PR, Appendix A: A manual of experimental boundary layer practice for low-speed flow, 1962.  
 [5] DEGRAAF, D.B. & EATON, J.K. Reynolds-number scaling of the flat plate turbulent boundary layer. *J. Fluid Mech.*, **422**, 2000, 319-346.  
 [6] FERNHOLZ, H.H. & FINLEY, P.J. The incompressible zero-pressure-gradient turbulent boundary layer: An assessment of the data. *Prog. Aerospace Sci.*, **32**, 1996, 245-311  
 [7] FERNHOLZ, H.H., KRAUSE, K., NOCKEMANN, M. & SCHOBER, M. Comparative measurements in the canonical boundary layers at  $Re_{\delta_2} \leq 6 \times 10^4$  on the wall of the German-Dutch windtunnel. *Phys. Fluids* **7**, 1995, 1275-1281.  
 [8] GAD-EL-HAK, M. & BANDYOPADHYAY, P.R. Reynolds number effects in wall bounded turbulent flows. *Appl. Mech. Rev.*, **47**, 1994, 307-365.  
 [10] HAFEZ, S., JONES, M.B. & CHONG, M.S. The zero pressure gradient turbulent boundary layer and its approach to equilibrium. *Proc. Of the 10<sup>th</sup> ACFM*, 2004, Peradeniy, Sri Lanka.  
 [9] HAFEZ, S., CHONG, M.S., JONES, M.B. & MARUSIC, I. Observations on high Reynolds number turbulent boundary layer. *Proc. Of the 15<sup>th</sup> AFMC*, 2004, Sydney, Australia.  
 [11] HITES, M.H. Scaling of High-Reynolds Number Turbulent Boundary Layers in the National Diagnostic Facility. PhD thesis, 1997, Illinois Institute of Technology.  
 [12] HITES, et al xxxM. H. 1997 Scaling of High-Reynolds Number Turbulent Boundary Layers in the National Diagnostic Facility. PhD thesis, Illinois Institute of Technology.  
 [13] KLEBANOFF, P.S. & DIEHL, Z.W. Some Features of Artificially Thickened Fully Developed Turbulent Boundary layers with zero pressure gradient. NACA Rep. No. **1110**, 1952.  
 [14] KNOBLOCH, K. & FERNHOLZ, H.H. Statistics, correlations and scaling in a turbulent boundary layer at  $Re_\delta \leq 1.15 \times 10^5$ . *IUTAM Symposium on Reynolds number Scaling in Turbulent Flow. Sep. 2002, Princeton. Kluwer Academic Pub.*  
 [15] MARUSIC, I. & KUNKEL, G.I. Turbulence intensity laws for turbulent boundary layers. *IUTAM Symposium on Reynolds number Scaling in Turbulent Flow. Sep. 2002, Princeton. Kluwer Academic Pub.*  
 [16] MARUSIC, I, UDDIN, A.K.M. & PERRY, A.E. Similarity law for the streamwise turbulence intensity in zero-pressure-gradient turbulent boundary layers. *Phys Fluids* **9**(12): 1997, 3718-3726.  
 [17] NICKLES, T.B., MARUSIC, I., HAFEZ, S., HUTCHINS, N. & CHONG, M.S. Some predictions of the attached eddy model for a high Reynolds number boundary layer. *Phil. Trans. R. Soc. A*, **365**, 2007, 807-822.  
 [18] ÖSTERLUND, J.M. Experimental studies of Zero Pressure Gradient Turbulent Boundary-Layer Flow. PhD thesis, 1999, Royal Institute of Technology, Stockholm, KTH, Sweden.  
 [19] PERRY, A.E., MARUSIC, I. & JONES, M.B. Similarity law for the streamwise turbulence intensity in zero-pressure-gradient turbulent boundary layers. *J. Fluid Mech.* **461**, 2002, 61-91.  
 [20] ROTTA, J.C. Turbulent boundary layers in incompressible flow. *Prog. Aeronaut. Sci.*, **2**, 1962, 1-220.  
 [21] SMITH, D.W. & WALKER, J.H. Skin friction measurements in incompressible flow. NACA Report No. R-26, 1959.  
 [22] SPALRAT, P.R. Direct simulation of turbulent boundary layer up to  $R_\theta = 1410$ . *J. Fluid Mech.* **187**, 1988, 61-98.  
 [23] TANI, I. Boundary-Layer transition. *Ann. Review Fluid Mech.* **1**, 1969, 33.