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Dynamics of a Vapour Bubble inside a Vertical Rigid Cylinder

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Abstract

In this Paper dynamics of a vapour bubble generated due to a local energy input inside a vertical rigid cylinder and in the absence of buoyancy forces is investigated. Different ratios of the rigid cylinder to the maximum radius of the bubble are considered. The Boundary Integral Equation Method is employed for numerical simulation of the problem. Results show that during the collapse phase of the bubble inside a vertical rigid cylinder, two liquid micro jets are developed on the top and bottom sides of the vapour bubble and are directed inward. Results also show that existence of a deposit rib inside the vertical rigid cylinder slightly increases the life time of the bubble. It is found that by increasing the ratio of the cylinder diameter to the maximum radius of the bubble, the rate of the growth and collapse phases of the bubble increases and the life time of the bubble decreases.

Introduction

Dynamics of a vapour bubble generated due to a local energy input in the vicinity of different kinds of surfaces is of significant importance in medicine and industry. Numerical and experimental results have shown that a vapour bubble generated due to a local energy input in the vicinity of a rigid boundary is attracted by the rigid surface. In this case during the collapse phase of the bubble a liquid micro jet is developed on the far side of the bubble from the rigid boundary and is directed towards it [2,4 & 10]. Numerical results have also shown that during the growth and collapse of a vapour bubble generated due to a local energy input beneath a free surface, the vapour bubble have a different behaviour. In this case the vapour bubble is repelled by the free surface. During the collapse phase a liquid micro jet is developed on the closest side of the bubble to the free surface and is directed away from it [3,5 & 8]. In the case of the pulsation of a vapour bubble near a rigid surface, the impingement of the liquid micro jet to the rigid surface is an important cause of mechanical erosion. Experimental investigations also show that at the end of the collapse phase of the vapour bubble and just before its rebound a shock wave is emitted in the liquid domain. The emission of the shock wave inside the liquid domain is also cause of the rigid surface destruction [6]. In this paper dynamics of a vapour bubble inside a vertical rigid cylinder with and without a deposit rib generated due to a high local energy input is numerically investigated by employing the Boundary Integral Equation Method. Different ratios of the rigid cylinder diameter to the maximum radius of the bubble are considered. Numerical study on the behaviour of a vapour bubble inside a vertical rigid cylinder is of great importance in medicine and industry.

Geometrical definition

The vapour bubble is generated due to a local energy input inside a vertical rigid cylinder with and without a deposit rib. The problem is axisymmetric. The vertical and radial axes are shown in figure 1. The vertical rigid cylinder is assumed as a long pipe.

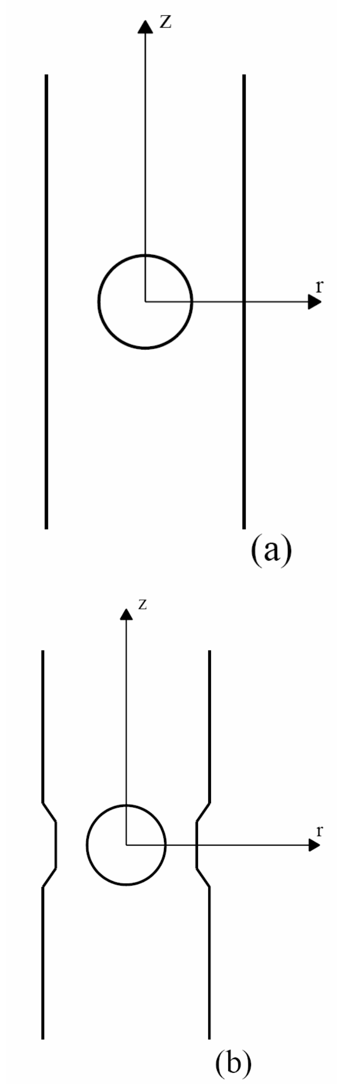

Figure 1. Schematic representation of the vapour bubble inside a vertical rigid cylinder (a)without a deposit rib, (b) with a deposit rib.

Hydrodynamic equation

The liquid flow around the vapour bubble is assumed to be inviscid, irrotational and incompressible and the surface tension is neglected. Therefore the liquid flow around the vapour bubble is a potential flow and the Green's integral formula governs the hydrodynamic behaviour of the problem.

$$C(p)f(p)+\int_S f(q)\frac{\partial}{\partial n}\left[\frac{1}{|p-q|}\right]dS=\int_S \frac{\partial}{\partial n}[f(q)]\left(\frac{1}{|p-q|}\right)dS \tag{1}$$

where f is velocity potential and S is the boundary of the liquid domain which includes the bubble boundary and the internal surface of the vertical rigid cylinder. p is any point on the boundary or in the liquid domain and q is any point on the boundary. $C(p)$ is $2p$ when p is on the boundary and is $4p$ when p is inside the liquid domain.

The unsteady Bernoulli equation in its Lagrangian form is used for calculating the velocity potential at the successive time steps and is given as

$$\frac{Df}{Dt} = \frac{P_\infty - P_b}{r} + \frac{1}{2}\left|\nabla f\right|^2 \tag{2}$$

where P_∞ is pressure in the far field, P_b is pressure inside the vapour bubble, r is density and t is time.

Discretization of the boundaries

The boundary of the vapour bubble is discretized into M cubic spline elements and the internal surface of the vertical rigid cylinder is divided into $2N$ linear segments. The origin of the vertical and radial axes is located at the centre of the initial spherical minimum vapour bubble. The vertical rigid cylinder in both upward and downward of the radial axis is discretized up to physical infinity, where the growth and collapse phases of the vapour bubble have not considerable effects on the fluid flow. Figure 2 illustrates the discretized representation of the vapour bubble generated due to local energy input inside a vertical rigid cylinder without and with a deposit rib.

Discretization of the equations

Equation (3) is a system of linear equations which represents the discretized form of equation (1)

$$2pf(p_i) + \sum_{j=1}^{M+2N}\left\{f(q_j)\int_{S_j}\frac{\partial}{\partial n}\left[\frac{1}{\left|p_i - q_j\right|}\right]dS\right\} = \sum_{j=1}^{M+2N}\left\{\frac{\partial}{\partial n}\left[f(q_j)\right]\int_{S_j}\left(\frac{1}{\left|p_i - q_j\right|}\right)dS\right\} \tag{3}$$

Whereas equation (4) represents the discretized form of unsteady Bernoulli equation and allows the velocity potential to be time marched over a time increment of Δt.

$$\left[f_i\right]_{t+\Delta t} = \left[f_i\right]_t + \Delta t\left\{\frac{P_\infty - P_b}{r} + \frac{1}{2}\left|\nabla f\right|^2\right\} \tag{4}$$

Evolution of the vapour bubble

A variable time increment is defined as

$$\Delta t = \min\left|\frac{\Delta f}{\frac{P_\infty - P_c}{r} + \frac{1}{2}\left(y^2 + h^2\right)}\right| \tag{5}$$

where Δf is some constant and represents the maximum increment of the velocity potential between two successive time steps. Also P_c is saturated vapour pressure, y is normal velocity on the boundary of the liquid domain and h is tangential velocity.

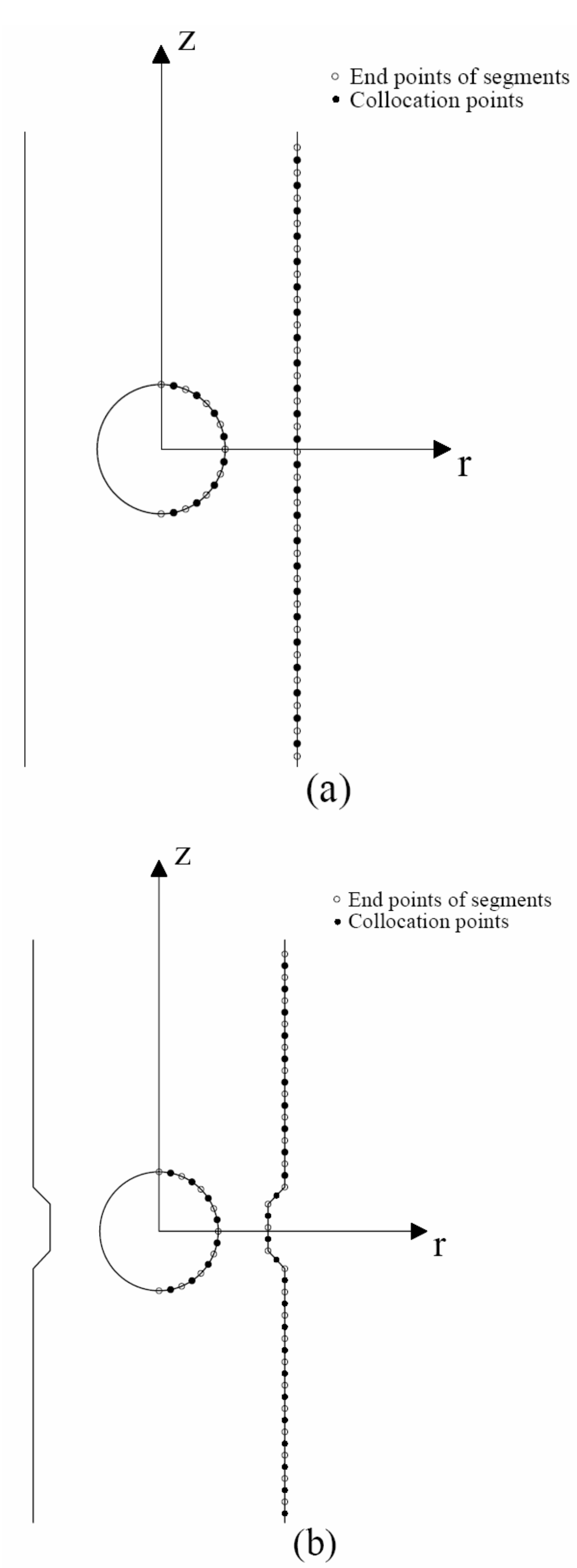

Figure 2. Discretized representation of the vapour bubble generated due to a local energy input inside a vertical rigid cylinder in the cases of (a) without a deposit rib, (b) with a deposit rib.

At the beginning of calculations the vapour bubble has a minimum spherical volume with a very high pressure in its inside. Mathematical model for predicting the initial minimum radius of the bubble and the corresponding pressure in its inside is based on the Rayleigh equation [7] which is developed by Best [1]. Details of the calculations for specifying initial values of the bubble volume and its inside pressure is given by Shervani-Tabar et. al. [9]. Normal derivative of the velocity potential on the internal surface of the vertical rigid cylinder and the velocity potential on the bubble boundary at the beginning of the calculations are known and are equal to zero. Therefore the linear system of equation (3) is solved by employing Gauss elimination method. Consequently the normal velocity on the bubble boundary and the velocity potential on the internal surface of the vertical rigid cylinder are found. By having distribution of the velocity potential on the bubble boundary, the tangential velocity on the bubble boundary is obtained by differentiating the velocity poptential on the bubble surface with respect to its arc length. By having normal and tangential velocity on the bubble boundary, radial and vertical components of the velocity on the bubble boundary are obtained. Since the radial and vertical components of the velocity on the bubble boundary are known, then by employing a second order Runge-Kutta scheme, the evolution of the vapour bubble over a time increment of Δt is obtained. Also by using the discretized form of Bernoulli equation the distribution of the velocity potential over the bubble surface at the next time step is calculated.

Non-dimensional parameters
The problem under investigation is non-dimensionalized by employing maximum radius of the bubble, R_m, diameter of the vertical rigid cylinder, D, liquid density, r , pressure in the far field, P_∞ and saturated vapour pressure P_c. The non-dimensional parameters which are used in this paper are defined as:

$$l = \frac{D}{R_m},$$

$$t = \frac{t}{R_m}\left(\frac{P_\infty - P_c}{r}\right)^{\frac{1}{2}},$$

$$\Psi = y\left(\frac{r}{P_\infty - P_c}\right)^{\frac{1}{2}}.$$

Numerical results and discussion
Numerical results are obtained in two parts. In part 1 the vapour bubble is generated due to a local energy input inside a vertical rigid cylinder without a deposit rib. While in part 2 the vapour bubble is generated due to a local energy input inside a vertical rigid cylinder with a deposit rib.

Part 1. A vapour bubble inside a vertical rigid cylinder without a deposit rib
Figure 3 illustrates the growth and collapse phases of a vapour bubble generated due to a local energy input inside a vertical rigid cylinder with $l = 3$. Figure 3a shows that the vapour bubble during its growth phase elongates in the vertical direction. During the collapse phase of the bubble two broad liquid micro jets are developed on the top and bottom sides of the bubble and are directed inward.

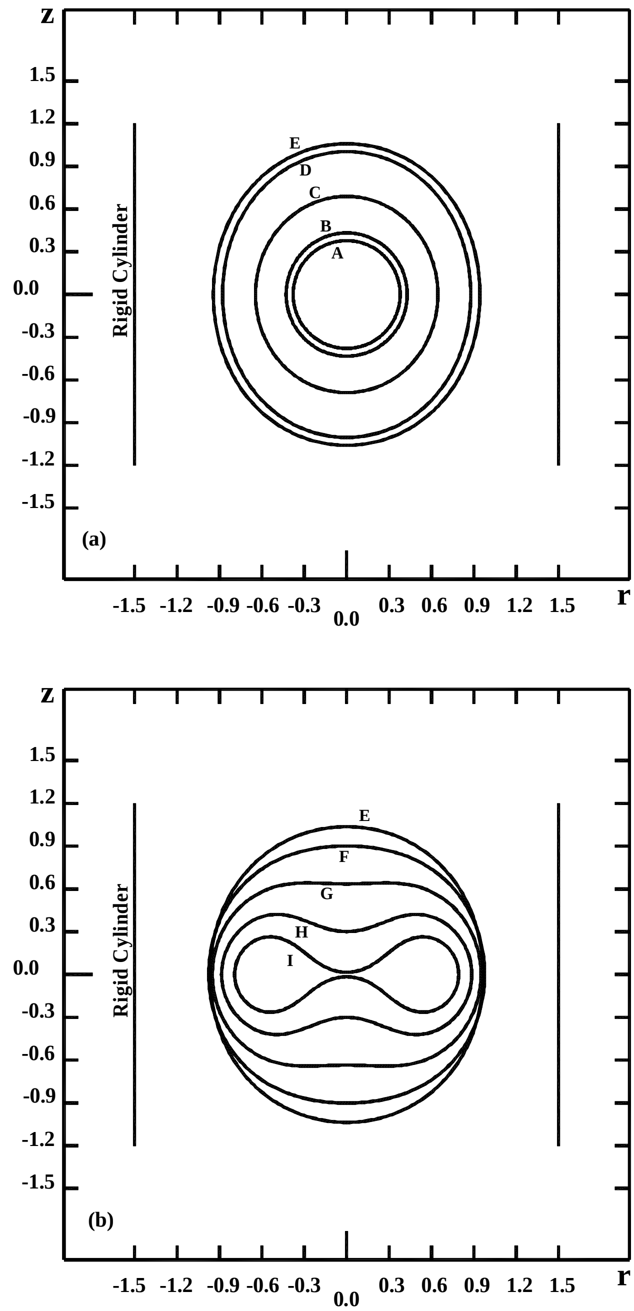

Figure 3. Growth and collapse of a single vapour bubble inside a rigid vertical cylinder in the absence of buoyancy forces with $l = 3$. The non-dimensional times corresponding to successive profiles are
(a) Growth phase: A) 0.00364 , B) 0.12644 , C) 0.50345 , D) 1.42588, E) 2.46031,
(b) Collapse phase: E) 2.46031, F) 3.09105, G) 3.68906, H) 4.17126, I) 4.51301.

Figure 4 shows that by decreasing the ratio of the vertical rigid cylinder diameter to the maximum radius of the bubble, l , elongation of the vapour bubble during its growth phase in the vertical direction is more appreciable.
A comparison between figure 3 and figure 4 shows that the liquid micro jets in the case of figure 3 are much broader. Thus it should be noted that by increasing l the liquid micro jets which are developed on the top and bottom sides of the bubble becomes broader.

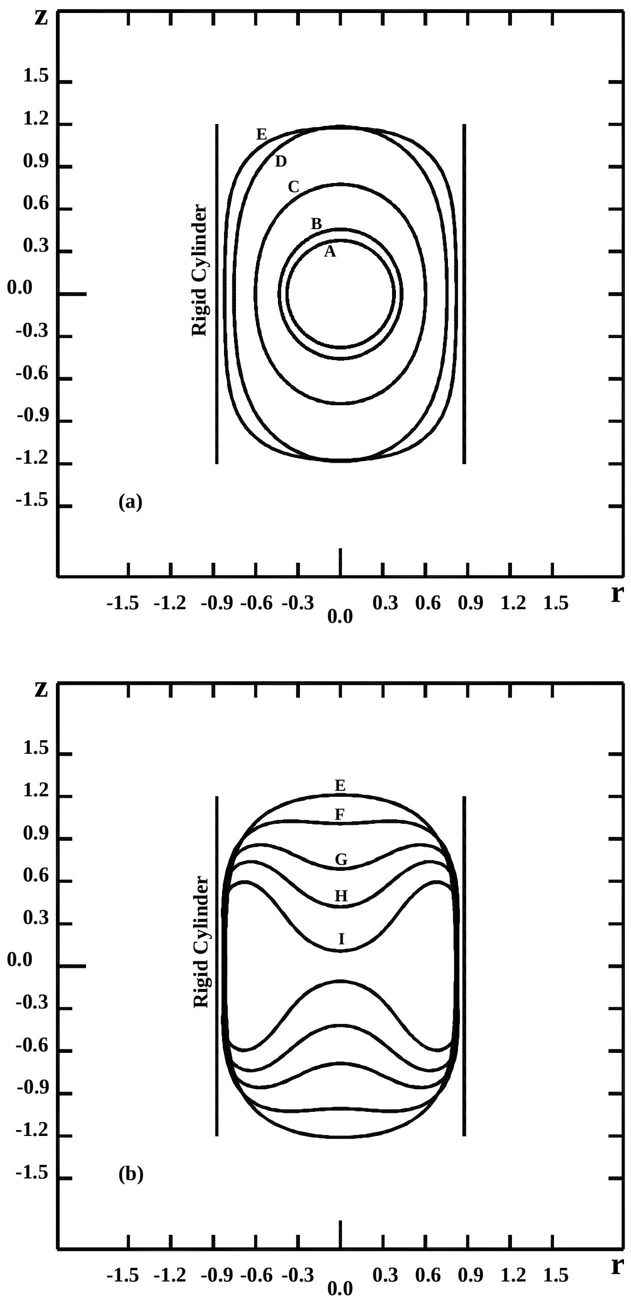

Figure 4. Growth and collapse of a single vapour bubble inside a rigid vertical cylinder in the absence of buoyancy forces with $l = 1.75$. The non-dimensional times corresponding to successive profiles are

(a) Growth phase: A) 0.00364, B) 0.17474, C) 0.62814, D) 1.65911, E) 2.97233,

(b) Collapse phase: E) 2.97233, F) 3.60799, G) 4.18045, H) 4.57527, I) 5.005.

Figure 5 shows that by increasing l the rate of the growth and collapse of a vapour bubble inside a vertical rigid cylinder becomes higher and the life time of the bubble becomes shorter.

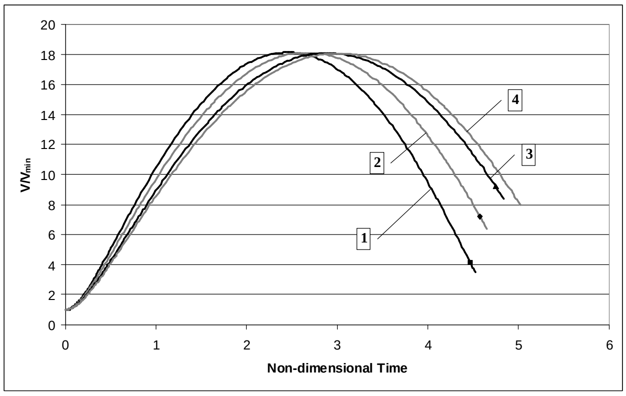

Figure 5. Variation of rational volume of the bubble in the absence of buoyancy forces against non-dimensional time in the cases of
(1) l =3, (2) l =2.5, (3) l =2, (4) l =1.75

Figure 6 shows that by increasing l the velocity of the liquid micro jets at the latest stages of the collapse phase become higher.

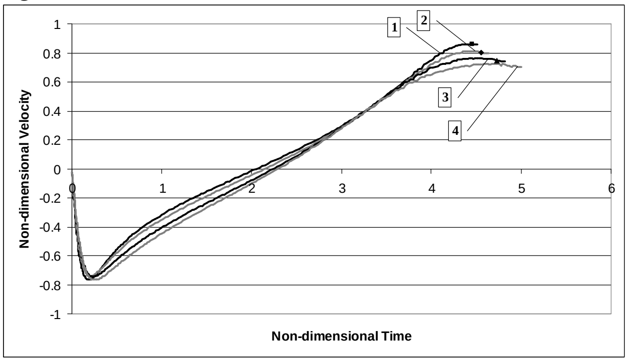

Figure 6. Variation of non-dimensional velocity of liquid jets in the absence of buoyancy forces against non-dimensional time in the cases of
(1) l =3, (2) l =2.5, (3) l =2, (4) l =1.75

Part 2. A vapour bubble inside a vertical rigid cylinder with a deposit rib

In this part a deposit rib have been considered inside the vertical rigid cylinder with the geometrical characteristics accoarding to Figure 7.

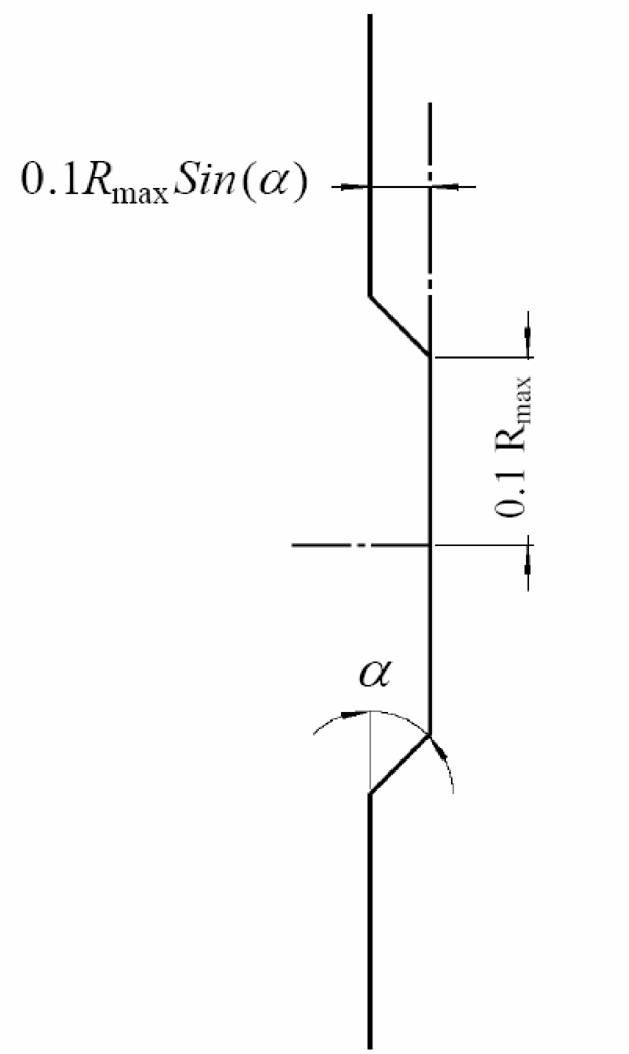

Figure 7. Geometrical characteristics of the deposit rib inside the vertical rigid cylinder

Figure 8 illustrates the growth and collapse of a vapour bubble generated due to a local energy input inside a vertical rigid cylinder with a deposit rib. As it is shown in figure 8(a) the vapour bubble during its growth phase elongates in the vertical direction. During the collapse phase two broad liquid micro jets are developed on top and bottom sides of the vapour bubble and directed inward. A comparison between figure 3 and figure 8 shows that existence of a deposit rib decreases the life time of the bubble very slightly.

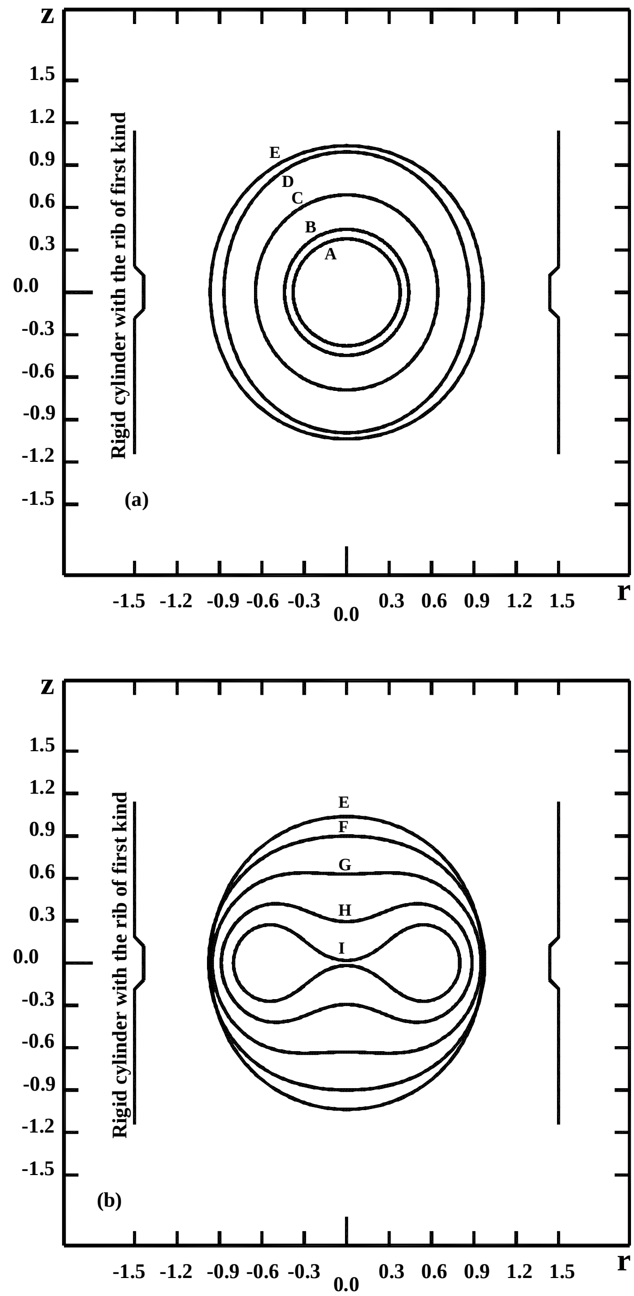

Figure 8. Growth and collapse of a single vapour bubble inside a rigid vertical cylinder with a deposit rib in the absence of buoyancy forces with $l = 3$. The non-dimensional times corresponding to successive profiles are

(a) Growth phase: A) 0.00364, B) 0.14384, C) 0.50346, D) 1.36254, E) 2.46015,

(b) Collapse phase: E) 2.46015, F) 3.09044, G) 3.68707, H) 4.16801, I) 4.49676.

Figures 8 and 9 in comparison with figures 3 and 4 show that the life time of the bubble inside a vertical rigid cylinder in the presence of a deposit rib very slightly becomes shorter.

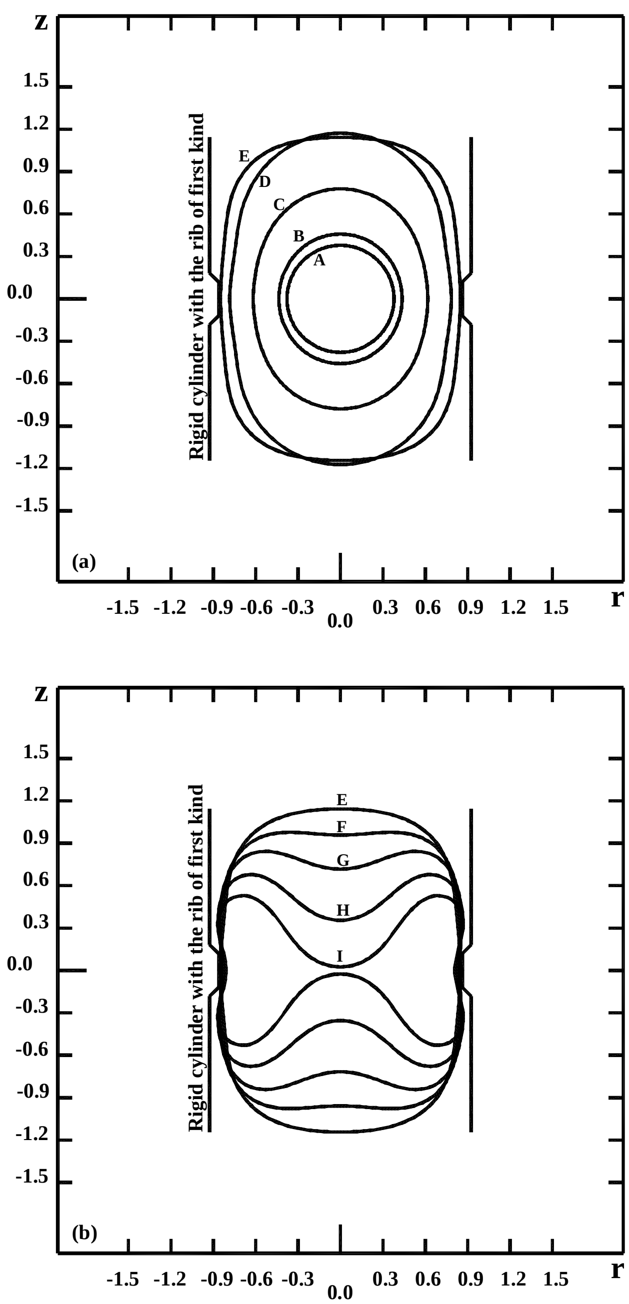

Figure 9. Growth and collapse of a single vapour bubble inside a rigid vertical cylinder with a deposit rib in the absence of buoyancy forces with $l = 1.85$. The non-dimensional times corresponding to successive profiles are

(a) Growth phase: A) 0.00364, B) 0.17577, C) 0.64217, D) 1.69388, E) 2.97972,

(b) Collapse phase: E) 2.97972, F) 3.49183, G) 3.9324, H) 4.46728, I) 4.92459.

Figure 10 shows the variation of the vapour bubble volume with respect to non-dimensional time when the bubble is inside a vertical rigid cylinder with different values of l and in the presence of a deposit rib.

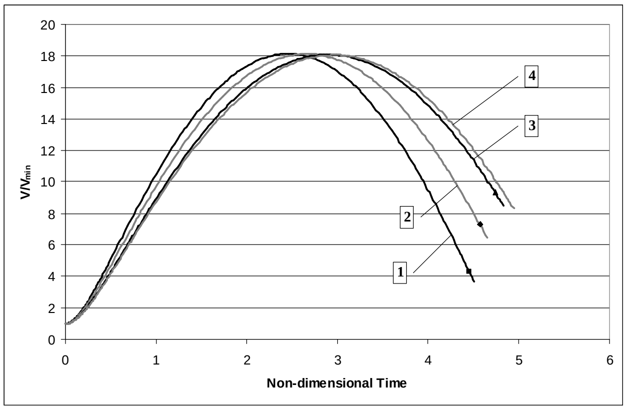

Figure 10. Variation of rational volume of the bubble inside a cylinder with a deposit rib in the absence of buoyancy forces against non-dimensional time in the cases of

(1) l =3, (2) l =2.5, (3) l =2, (4) l =1.85

Figure 11 shows the variation of the non-dimensional velocity of liquid micro jets with respect to non-dimensional time when the vapour bubble is inside a vertical rigid cylinder and with different values of l. A comparison between figure 6 and figure 11 illustrates that presence of a deposit rib has not considerable effect on the non-dimensional velocity of the liquid micro jets.

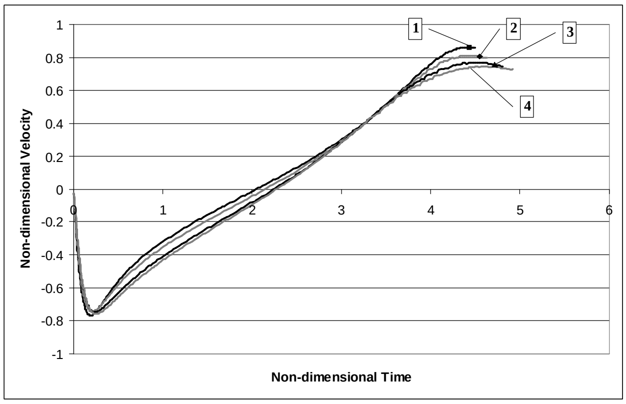

Figure 11. Variation of non-dimensional velocity of liquid jets on top and bottom sides of the bubble inside a vertical rigid cylinder with a deposit rib in the absence of buoyancy forces against non-dimensional time and in the cases of

(1) l =3, (2) l =2.5, (3) l =2, (4) l =1.85

Conclusions

In this paper dynamic behaviour of a vapour bubble generated due to a local energy input in the absence of buoyancy forces is numerically investigated by employing a Boundary Integral Equation Method.

Numerical results show that the vapour bubble during its growth phase elongates in the vertical direction. During the collapse phase of the bubble two broad liquid micro jets are developed on the top and bottom sides of the bubble and are directed inward.

Results also show that by increasing l, the ratio of the cylinder diameter to the maximum radius of the bubble, the rate of the growth and collapse of the vapour bubble inside a vertical rigid cylinder becomes higher and the life time of the bubble becomes shorter.

It is found that existence of a deposit rib inside the vertical rigid cylinder slightly decreases the life time of the bubble.

It is also found that the presence of a deposit rib on the internal surface of the vertical rigid cylinder has not considerable effect on the non-dimensional velocity of the liquid micro jets.

References

[1] Best, J. P., The dynamics of underwater explosions, PhD Thesis, University of Wollongong, Wollongong, Australia., 1991

[2] Blake, J. R., Taib, B. B. and Doherty, G., Transient cavities near boundaries; Part 1. Rigid boundary, J. Fluid Mech., 1986, Vol. 170, pp. 479-497.

[3] Blake, J. R., Taib, B. B. and Doherty, G., Transient cavities near boundaries; Part 2. Free surface, J. Fluid Mech., 1986, Vol. 181, pp. 197-212.

[4] Blake, J. R. and Gibson, D. C., Cavitation bubbles near boundaries, Ann. Rev. Fluid Mech., 1987, Vol. 19, pp. 99-123.

[5] Blake, J. R. and Gibson, D. C., Growth and collapse of a vapour cavity bubble near a free surface, J. Fluid Mech., 1981, Vol. 111, pp. 123-140.

[6] Lauterborn, W., Cavitation and coherent optics, Cavitation and Inhomogenities in Underwater Acoustics, Proceedings of the First International Conference, *Göttingen* , Fed. Rep. of Germany, Lauterborn (Ed.), Springer-Verlag, 1980, pp. 3-12.

[7] Rayleigh, Lord, On the pressure developed in a liquid during collapse of a spherical void, Phil. Mag., 1917, Vol. 34, pp. 94-98.

[8] Shervani-Tbar, M. T., Computer study of a cavity bubble near a rigid boundary, a free surface, and a compliant wall, PhD Thesis, University of Wollongong, Wollongong, Australia., 1995

[9] Shervani-Tabar, M. T., Abdullah, A. and Shabgard, M. R., Numerical study on the dynamics of an electrical discharge generated bubble in EDM, Engineering Analysis with Boundary Elements, 2006, Vol. 30, pp. 503-514.

[10] Soh, W. K. and Shervani-Tabar, M. T., Computer model for a pulsating bubble near a rigid surface, Computational Fluid Dynamics JOURNAL, 1994, Vol. 3, No. 1, pp. 223-236.