16th Australasian Fluid Mechanics Conference
Crown Plaza, Gold Coast, Australia
2-7 December 2007

Comparison of added mass coefficients for a floating tanker evaluated by conformal mapping and boundary element methods

W. Hem Lata and K P Thiagarajan[1]
School of Mechanical Engineering
The University of Western Australia, Crawley, WA 6009, AUSTRALIA

Abstract

One of the important parameters needed to model ship motions in a seaway is the added mass matrix of the hull. Current state-of-the-art boundary element methods routinely evaluate the 6 x 6 added mass matrices as part of the radiation problem solution. These developments have largely superseded conventional approaches to sectional added mass evaluation using conformal mapping techniques. However, conformal mapping techniques are still attractive in terms of their mathematical explicitness and computational simplicity.

The recurrent form of Bieberbach Method of conformal mapping was developed for mapping the exterior of a closed curve i.e. the two-dimensional ship cross section and its mirror image, into the exterior of the circle oscillating vertically at free surface and to compute the added mass coefficients. By incorporating a strip theory approximation the added mass coefficients of a three dimensional structure can be estimated from its two-dimensional section coefficients at different drafts. In this paper we have applied this method to calculate the heave, pitch and heave induced pitch added mass coefficients of a tanker. The applicability of these conformal mapping techniques to floating platforms under consideration is discussed, by comparing the results with state-of-the-art industry standard boundary element methods, AQWA and SESAM.

Introduction

Added Mass is the pressure force per unit acceleration acting on an oscillating floating body, due to the acceleration field set up in the surrounding fluid. It is different in different degrees of motion and depends upon the geometry of the body. The governing equations of motion for a floating rigid body are given by:

$$\left(M_{ij} + a_{ij} \right) \ddot{x}_j + b_{ij} \dot{x}_j + k_{ij} x_j = F_i(t) \qquad (1)$$

where

M_{ij} = Oscillating mass/ moment of inertia
a_{ij} = Added mass induced in i due to unit acceleration in j
b_{ij} = Damping
k_{ij} = Restoring stiffness
Fi = Exciting forces in the ith direction
$x_j, \dot{x}_j, \ddot{x}_j$ = the displacement, velocity and acceleration of the vessel in the jth direction
i, j = 1, .. 6, denote the six degrees of freedom surge, sway, heave, roll, pitch and yaw respectively as shown in Figure 1.

An added mass coefficient C_{ij} is the ratio of the added mass to the mass of the body as given below
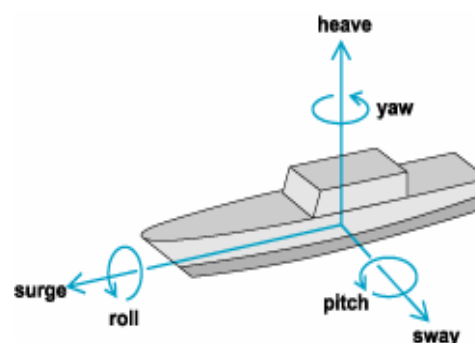
$$C_{ij} = \frac{a_{ij}}{M_{ij}} \qquad (2)$$

---

[1] Corresponding author, krish.thiagarajan@uwa.edu.au

Figure 1. 6 DOF of a rigid body[2]

Motion prediction of floating platforms is accomplished using one of the several boundary element packages available in the market. Out of these packages, AQWA [1], and SESAM [2] are used by offshore industry worldwide. The common procedure adopted in these methods is to discretize the underwater surface into discrete elements and solve for the incompressible, irrotational velocity potential of the flow around the body. The boundary value problem is formulated using Green's function from linear diffraction and radiation theory. One solution outcome is the 6x 6 added mass matrix as a function of the frequency of oscillation.

There are no known sources of experimental or 3-D computational results for radiation forces on systematic series of hull forms, which is perhaps not surprising due to the substantial effort involved in either case. On the other hand, such data has been available for 2-D forms for quite some time. Vugts [3] published a comprehensive set of experimental data, along with some theoretical results, for 2-D cylinders including semicircles, triangles, several ship-like sections, and rectangles at range of drafts.

In the '60s, Professor Landweber and M. Macagno from Iowa Institute of Hydraulic Research attempted a number of added mass coefficient calculations methods including two parameter, three parameter and conformal mapping developments to calculate added mass coefficients of two dimensional forms [4, 5, 6]. The method of conformal mapping was found comparatively better than the other two methods, and provides higher level of accuracy as compared to the other two methods [7]. The first two methods are based on the assumption that a ship section having the same principal geometric characteristics as a member of one

---

[2] (http://www.answers.com/topic/ship-powering-maneuvering-and-seakeeping?cat=technology)
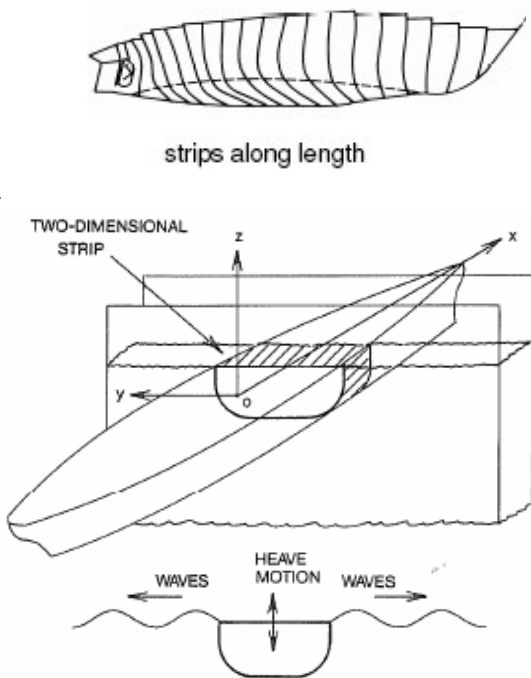
strips along length

Figure 2. Two dimensional strips in heave [6].

of the particular families, the Lewis forms or the three-parameters forms, will have the same added mass coefficient as that member. This assumption is not exact and no level of accuracy can be provided [6]. This method of conformal mapping thus has gone largely unnoticed by software developers.

Once coefficients of a two-dimensional section are known, the coefficients for the entire ship may be found by strip theory (see for e.g. [8] or [9]). This theory is applicable to slender bodies i.e. when the length of the body is much larger compared to the lateral dimension as shown in Figure 2. It is based on the assumption that the radiated wave lengths are of same order of magnitude as beam of the tanker and short as compared to the length of the tanker. Strip theory has the limitation of predicting transverse motions better than longitudinal motions.

We present here the basics of the theory of Landweber and Macagno [6] followed by its implementation into a strip theory formulation. Results for added mass coefficients from this theory are compared with boundary element packages for an offshore floating tanker vessel. It is to be noted that this method is basically applicable to the case of infinite frequency only.

Theory

The radiation problem comprises of a body shape oscillating at an angular frequency ω. The boundary condition on the underwater surface is basically satisfaction of no flow through the surface, and is given in terms of the velocity potential as

$$\omega^2 \phi = g \frac{\partial \phi}{\partial y} \tag{3}$$

The limiting condition when frequency $\omega \to \infty$ is $\Phi = 0$.

The conformal method is based on mapping the exterior of a ship-section and its mirror image about x = 0, in z-plane into the exterior of the circle in ζ-plane as shown in Figure 2. Here B and T are the beam and draft of the floating body respectively. Using
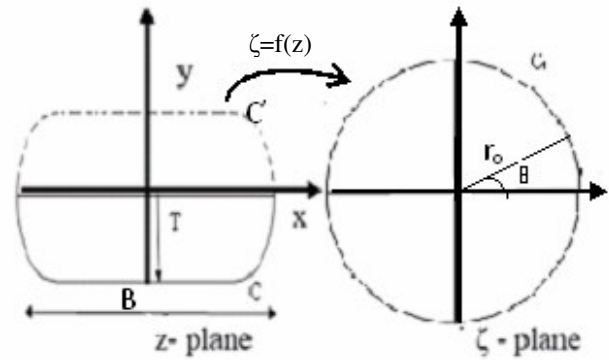

Figure 3 Mapping the z-plane one-to-one into ζ-plane.

a double-body contour eliminates free surface influence in the problem. The boundary condition is satisfied by supposing that the entire shape oscillates as a single shape with instantaneous velocity V which gives

$$\frac{\partial \phi}{\partial n} = V \frac{\partial y}{\partial n} \tag{4}$$

The above equation relates the velocity potential to the shape of the contour. At infinite frequency, the amplitude of oscillation diminishes infinitesimally, resulting in a finite velocity. The kinetic energy resident in the fluid due to this oscillation may be directly related to the added mass as

$$KE = \frac{1}{2} a_{33} V^2 = -\rho \oint \phi d\psi \tag{5}$$

where Ψ is the stream function. Solving for KE subject to the boundary conditions gives the added mass coefficient as

$$C_{33} = C_V = \frac{1}{b^2} \left[2 \left(r_o^2 + b_1 \right) - \frac{S}{\pi} \right] \tag{6}$$

Where

C_V = Added Mass Coefficient due to vertical (heave) oscillations

b = half beam of the ship section

r_o = Mean radius of the ship section.

S = Area of the ship section

b_1 = the first coefficient of mapping from z-plane to ζ-plane.

It remains to find the parameters b_1, r_0 and S. Let us consider a double ship like contour cc', symmetrical with respect to the x, y axes with the x-axis in the free surface as shown in Figure 3. The transformation as per the Bieberbach method of conformal mapping is given by [6]

$$z = \zeta + \frac{a_1}{\zeta} + \frac{a_2}{\zeta^3} + \frac{a_3}{\zeta^5} + \frac{a_4}{\zeta^7} + \ldots\ldots \tag{7}$$

Where $z = re^{i\theta}$ is the complex coordinate. Its inverse given by

$$\zeta = z + \frac{b_1}{z} + \frac{b_2}{z^3} + \frac{b_3}{z^5} + \frac{b_4}{z^7} + \ldots\ldots \tag{8}$$

is the transformation of mapping the exterior of the closed curve (Ship section and its mirror image) in the z-plane into the exterior of the closed circle in ζ-plane (Figure 3). The form of these mapping functions as expansions of odd powers of z and ζ with real coefficients is required in order to satisfy the condition that both the original section cc' and its transformation in ζ-plane be symmetrical with respect to their coordinate axes. Here a_i and b_i are the coefficients of the mapping.

The Bieberbach method is based on the property that among the closed curves obtained from the conformal mapping, by transformations of Eq. 8, of the exterior of the given closed curve bounding a simply connected region, the circle will bound the maximum area. Ritz procedure is applied to find the values of finite number of coefficients of mapping i.e. b_i's in Eq. 8 such that they yield maximum area subject to this restriction.

The area bounded by the curve CC' as shown in Figure 3 is given by

$$\oint_{cc'} \bar{z} dz = i \oint_{cc'} r^2 d\theta = 2iS \tag{9}$$

Thus we get the area as

$$S = \frac{1}{2} \oint_{cc'} r^2 d\theta \tag{10}$$

Let C_1 be the closed curve in ζ-plane obtained by mapping CC' one-to-one in z-plane. From Eq. 8, we obtain

$$\oint_{cc'} \bar{\zeta} d\zeta = 2iS \tag{11}$$

Substituting Eq. 8, we get

$$\oint_{cc'} \bar{\zeta} d\zeta = \oint_{cc'} \left(\bar{z} + \sum_{j=1}^{n} \frac{b_j}{\bar{z}^{2j-1}} \right) \left[1 - \sum_{k=1}^{n} \frac{(2k-1)b_k}{z^{2k}} \right] dz \tag{12}$$

Substituting values in Eq. 12 from Eqs. 9 and 11, the area, S' bounded by this curve is given by,

$$S' = S - \sum_{j=1}^{n} \sum_{k=1}^{n} \beta_{jk} b_j b_k + 2 \sum_{j=1}^{n} e_j b_j \tag{13}$$

Where

$$e_j = -\frac{i}{4} \oint_{cc'} \left[\frac{1}{\bar{z}^{2j-1}} - \frac{(2j-1)\bar{z}}{z^{2j}} \right] dz$$

$$\Rightarrow e_j = \begin{cases} \frac{2j-1}{2j-2} \int_0^{2\pi} \frac{\cos 2j\theta}{r^{2j-2}} d\theta, \ j > 1 \\ -\int_0^{2\pi} \ln r . \cos 2\theta d\theta, \ j = 1 \end{cases} \tag{14}$$

And

$$\beta_{jk} = -\frac{i}{4} \oint_{cc'} \left[\frac{2j-1}{\bar{z}^{2k-1} z^{2j}} - \frac{2k-1}{\bar{z}^{2j-1} z^{2k}} \right] dz$$

$$\Rightarrow \beta_{jk} = \frac{(2j-1)(2k-1)}{2(j+k-1)} \int_0^{2\pi} \frac{\cos(2j-2k)\theta}{r^{2j+2k-2}} d\theta \tag{15}$$

Simpson's Rule of integration may be used for the integration of Eq.15. For Ritz condition that S' be a maximum Area, the derivative of S' with b_j should be zero.

$$\frac{\partial S'}{\partial b_j} = 0 \ j = 1,2,.....n \tag{16}$$

Using Eq. 13 we get a set of linear equations with the coefficients b_k.

$$\sum_{k=1}^{n} \beta_{jk} b_k = e_j \tag{17}$$

This system of linear equation is solved to get b's, the values of coefficients of mapping.

The radius of the circle, obtained by the mapping is the mean radius of the curve cc'

$$r_o = \frac{1}{2\pi} \int_0^{2\pi} l d\theta \tag{15}$$

Substituting mean radius, Area, b_1 and half beam values in Eq. 6 gives the vertical added mass coefficient of the two-dimensional section.

The procedure included obtaining the series of added-mass coefficients at each station. By incorporating strip theory we can calculate the added mass coefficients of three-dimensional tanker-shaped offshore production platform from the added masses of the two dimensional cross sections of the tanker at each section as shown in Figure 2. Heave-induced heave added mass (A_{33}), pitch-induced pitch added mass (A_{55}), and heave-induced pitch added mass (A_{53}) or pitch-induced heave added mass (A_{35}) are obtained from the two-dimensional heave added mass (a_{33}) by the strip theory formulations as below

$$A_{33} = \int a_{33} dx \tag{16}$$

$$A_{53} = A_{35} = -\int a_{33} x dx \tag{17}$$

$$A_{55} = \int a_{33} x^2 dx \tag{18}$$

Implementation & validation

The above formulation has been implemented for the calculations of added mass coefficients of a Floating Production Storage and Offloading (FPSO) tanker with dimensions shown in Table 1. the draft was varied to obtain a range of B/2T ratios. The sectional heave added mass thus evaluated was compared against published data for ship-shaped sections by Vugts [3] in Figure 4.

The data from conformal mapping shows approximately constant value for various B/2T with a slight decrease in the limit of B/2T $\rightarrow$ 0 i.e. a very deep structure. In contrary Vugts data shows a very small value of a_{33} as B/2T$\rightarrow$0. The agreement is seen to be

Table 1. FPSO tanker particulars

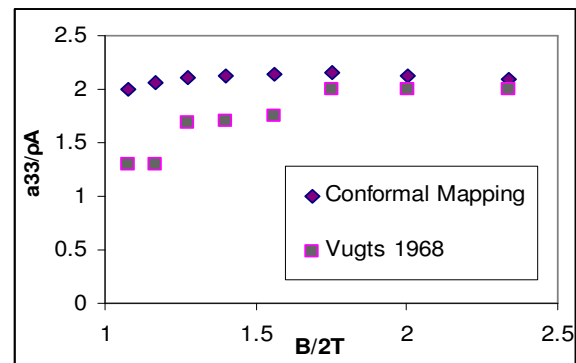| Parameter	Value	
Length Over All (LOA)	329	meters
Length between perpendiculars (Lpp)	318	meters
Beam (b)	57.24	meters
Depth (d)	28.2	meters
Draft (D)	variable	meters
Displacement (W)	145800	Tonnes

Figure: 4 Non-dimensional heave added mass ($a_{33}/\rho A$) Vs B/2T

1390

reasonable at realistic values of *B*/2*T* of around 2 (as shown in Figure 4). At very high values of *B*/2*T* (shallow draft) the conformal mapping shows an increase in the added mass coefficient, which is contrary to Vugt's data. The shallow draft limitation of strip theory is well known (for example, Lewis [9]) and is being studied further. Landweber and Macagno [4] mentioned that the mapping twice gives more accuracy in the added mass coefficient values. This aspect will be considered in the future work.

Figures 5 and 6 show curves of heave-induced pitch (A_{53}) pitch-induced pitch added mass (A_{55}) and pitch induced heave (A_{35}) vs. *B*/2*T*. Since these are obtained from Eqs. 17 and 18, they follow an amplified monotonic trend derived from Figure 4.

The added mass values and corresponding coefficients of the entire hull were independently computed using both AQWA and SESAM for two different drafts. As is well known, computations with boundary element software becomes quite unreliable at very low wave lengths, corresponding to higher frequencies. For realistic offshore structures this limits application of these methods to periods less than 5 seconds (frequencies of 0.2 Hz). The computational results at the highest frequency are compared with the present strip theory results in Tables 2 and 3. The agreement is shown to be very reasonable with maximum errors of less than 10% in all cases.

Conclusions

We have presented the theory and implementation of a conformal mapping technique and compared with published data as well as with industry standard software results. Following conclusions have been drawn.

- Conformal mapping method of calculating added mass coefficients is quicker for calculations at varying drafts as compared to the other two industry accepted methods.
- It achieves the level of accuracy in calculating added mass of the three dimensional floating tanker.

Further studies in progress:

- Analysis of the increase in the 2-D sectional added mass coefficient at higher *B*/2*T*, which is contrary to the Vugts data.
- Repetitive application of conformal mapping to achieve higher level of accuracy
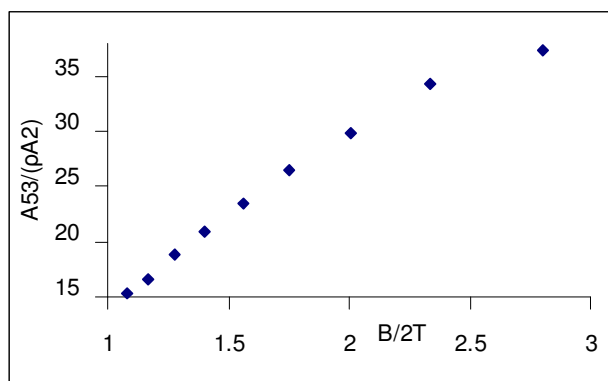
Acknowledgements

Figure 5. Heave induced pitch added mass ($A_{53}/\rho A^2$) vs. *B/2T*.

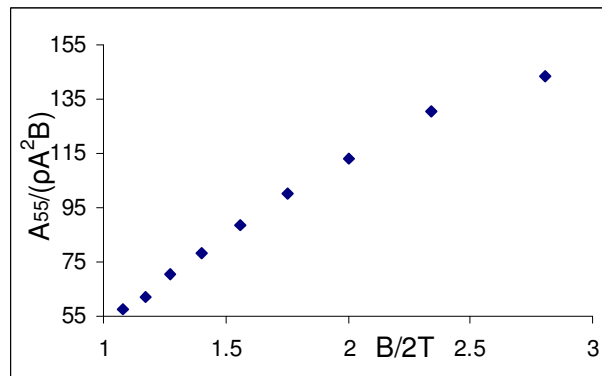**Figure 6.** Two-dimensional pitch induced pitch added mass ($a_{55}/\rho A^2 B$) vs. *B/2T*.

Table 2. Comparison of added mass

At draft 10 m

Method	A_{33} (kg)	A_{53} (kg – m)	A_{55} (kg – m^2)
Draft = 10 m			
Conformal Mapping	2.57E+08	-4.70E+10	1.01E+13
AQWA	2.66E+08	-4.71E+10	9.36E+12
Draft = 10.56 m			
Conformal Mapping	2.82E+08	-5.16E+010	1.11E+013
AQWA	2.86E+08	-6.07E+10	1.05E+13
SESAM	2.79E+08	-4.86E+10	8.29E+12

Table 3. Comparison of Added Mass Coefficients

Method	C_{33}	C_{53}	C_{55}
Draft = 10 m			
Conformal Mapping	1.85	-3.38E+02	7.27E+04
AQWA	1.91	-3.39E+02	6.73E+04
Draft = 10.56 m			
Conformal Mapping	1.96	-3.59E+02	7.73E+04
AQWA	1.99	-4.23E+02	7.31E+04
SESAM	1.94	-3.38E+02	5.77E+04

References

[1] AQWA User manual, Horsham (UK), Century Dynamics Ltd., RH12 2DT, 2004
[2] *www.dnv.com/software*
[3] Vugts J H, The hydrodynamics coefficients for swqying heaving and rolling cylinders in a free surface, *Int. Shipbldg, Prog*. 15, 1968.
[4] Landweber, L. and Macagno, M., Added Mass of Two-Dimensional Forms Oscillating in a Free Surface, *J. Ship Research,* 1, (3), 1957.
[5] Landweber, L. and Macagno, M., Added Mass of a Three Parameter Family of Two-Dimensional Forms Oscillating in a Free Surface, *J. Ship Research,* 2, (4), 1959.
[6] Landweber, L. and Macagno, M., Added Mass of Two Dimensional Forms by Conformal Mapping, *J. Ship Research,* April 1967.
[7] Macagno, M., A Comparison of Three Methods For Computing Added Mass of Ship Sections, .*J Ship Research,* April 1967.
[8] Lewandowski, E M, *The Dynamics of Marine Craft,* Advanced Series on Ocean engineering, 22, 2004.
[9] Lewis, E V, *Principles of Naval Architecture*, 3, SNAME Publ,1988.