16th Australasian Fluid Mechanics Conference
Crown Plaza, Gold Coast, Australia
2-7 December 2007

Cooling by Free Convection at High Rayleigh Number of Cylinders Positioned Above a Plane

B.P. Huynh

Faculty of Engineering
University of Technology, Sydney, NSW 2007 AUSTRALIA

Abstract

Free convection cooling of isothermal circular cylinders positioned above a horizontal plane is investigated numerically, using a commercial Computational Fluid Dynamics (CFD) software package. Computation is performed for high Rayleigh number, in the range $10^9 - 10^{11}$. Chien's turbulence model of low-Reynolds-number K-ε is used, with Prandtl number of 0.707, corresponding to air near standard conditions. Influence of the underlying plane on heat transfer from the cylinders' surface is examined. As the gap between the plane and cylinders is narrowed, a pattern can be seen whereby heat transfer reaches a minimum that moves closer to the cylinder surface with higher Rayleigh number. The plane's thermal condition, adiabatic versus isothermal, produces no significant difference in the heat transfer for the present range of gap ratio, in contrast to laminar case.

Nomenclature

- A surface area per unit length of the whole cylinder
- c_p specific heat at constant pressure
- D cylinder's diameter
- g gravitational acceleration (9.81 m/s^2)
- Gr Grashof number = $g\, \beta\, D^3(T_s - T_\infty) / \nu^2$
- h gap between cylinder and the plane
- $h_{average}$ average heat transfer coefficient = $Q / [A\,(T_s - T_\infty)]$
- k (molecular) thermal conductivity
- K turbulent kinetic energy
- $Nu_{average}$ average Nusselt number = $h_{average}\, D / k$
- p pressure
- Pr (molecular) Prandtl number = $\mu\, c_p / k = \nu / \alpha$
- Pr_t turbulent Prandtl number = $\mu_t\, c_p / k_t = \nu_t / \alpha_t$
- Q heat transfer rate per unit length of whole cylinder surface
- Ra Rayleigh number = $g\, \beta\, D^3\, (T_s - T_\infty) / (\nu\, \alpha)$
- T temperature
- T_s temperature of cylinder's surface (387.52 K)
- T_∞ ambient temperature (300 K)
- u velocity in the x-direction
- u_τ friction velocity = $(\tau_w / \rho)^{1/2}$
- v velocity in the y-direction
- x coordinate in the horizontal direction
- y coordinate in the vertical direction (increasing upward)
- y^+ non-dimensional distance from closest wall = $\delta / (\nu / u_\tau)$
- δ distance from closest wall
- y_{plane} y-coordinate of the horizontal plane
- α (molecular) thermal diffusivity = $k / (\rho\, c_p)$
- β thermal expansion coefficient
- ε turbulent kinetic energy's dissipation rate
- μ (molecular) viscosity
- ν (molecular) kinematic viscosity = μ / ρ
- ρ density
- τ_w wall shear stress

Subscript t: turbulent

Introduction

This paper reports on heat transfer by free-convection at high Rayleigh number from a heated, isothermal horizontal circular cylinder positioned above a horizontal plane. This work is an extension to the turbulent regime of a previous one [7].

Circular cylinder is a very common geometry, and objects of this shape abound. Examples are fluid-carrying pipes, electrical wires, etc. Heat transfer from circular cylinders has been investigated by many authors, especially the situation of an isothermal horizontal cylinder exchanging heat with its surroundings in a totally unobstructed free convection regime. For this configuration, the empirical correlations of Morgan [19] who had reviewed a large body of literature, and of Churchill and Chu [4] have been particularly well accepted [8,17]. This unobstructed free convection around an isothermal, horizontal circular cylinder has also been investigated computationally by other authors [5,14-15,18,22]. The common situation when a cylinder is positioned close to a plane has also been considered. However, focus has been on the case of the heated cylinder being positioned between vertical walls [6,11,16,21], or below a ceiling [1,12,15]. The case of the cylinder having its surface temperature higher than its surroundings' and positioned above a plane (or, equivalently, the cylinder having surface temperature lower than its surroundings' and positioned below a plane) seems to have only been treated experimentally by Jones and Masson [9], and computationally for isothermal underlying plane and low Grashof numbers ($Gr \le 8000$) by Müller and co-workers [10,23] (whose heat transfer results are, however, too low in comparison with correlations of, for example, [4]). In this work, the situation of a heated cylinder positioned above a horizontal plane is investigated using computational method for Rayleigh number in the range of $10^9 - 10^{11}$, using a fluid with Prandtl number 0.707 (corresponding to Gr = 1.41×10^9 − 1.41×10^{11}). Both cases of isothermal and adiabatic plane will be considered.

Modelling and Computation

The physical model is depicted in Figure 1. A horizontal circular cylinder of diameter D is positioned above a solid horizontal plane, with gap h between the two items. The cylinder's surface is assumed to have a uniform temperature T_s, while the surrounding fluid has the constant ambient temperature T_∞, with $T_s > T_\infty$. Here T_∞ is fixed at 300 K, and T_s at 387.52 K. Note that this situation is also equivalent to when the cylinder is positioned below the plane, but with $T_s < T_\infty$.

The underlying plane is assumed to either be of the same temperature as the ambient fluid's (300 K), or be insulated. Free convection would result from temperature difference between the cylinder's surface and the surrounding fluid, and is the subject of this study. Attention will however be given to the total heat transfer rate from the cylinder's surface, which is characterised by an average Nusselt number.

All fluid properties are assumed to be constant and corresponding to those of air at 300 K and standard pressure at sea level (101.3 kPa); but Boussinesq approximation is also assumed for the buoyancy force arising from density variation as a result of temperature change. The following values of molecular properties are used: $\rho = 1.161$ kg/m^3; $\mu = 1.846\times10^{-5}$ N-s/m^2; $\nu = 1.589\times10^{-5}$ m^2/s; $k = 0.0263$ W/m-K; $c_p = 1007$ J/kg-K; $\alpha = 2.25\times10^{-5}$ m^2/s; $Pr = \nu / \alpha = 0.707$; $\beta = 1 / T_\infty = 1/300$ K^{-1}

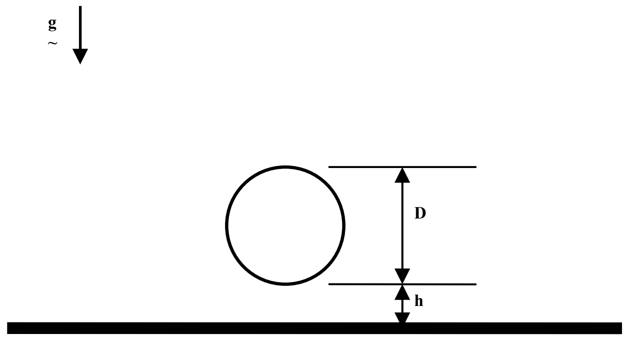

Figure 1. Physical model of the considered problem: free convection about a heated circular cylinder positioned above a horizontal solid plane.

With these values, the Rayleigh number Ra, which is a key parameter in free convection, can be defined and having the following expression

$$Ra = g\, \beta\, D^3 (T_s - T_\infty) / (\nu\, \alpha) = 8.00\times10^{9}\, D^3$$

Thus, Ra can be varied simply by changing the cylinder's diameter D. Turbulent Prandtl number is taken to be constant at $Pr_t = 0.9$, following a suggestion in [2].

Figure 2 shows the computational domain, which is two-dimensional, and in which the following dimensions and coordinates are used

- O: centre of the cylinder; also origin of the used Cartesian coordinate system with x, y axes
- F: highest point of the cylinder's surface; $y = D / 2$
- A: lowest point of the cylinder's surface; $y = -D / 2$
- B: intersection of the solid horizontal plane (represented by BC) with the y-axis; $y = y_{plane}$
- C: outer, lower corner of the computational domain; $x = 5D$
- D: outer, upper corner of the computational domain; $x = 5D$, $y = 5D$
- E: inner, upper corner of the computational domain: $y = 5D$

Since turbulence is expected to have already occurred at the high Rayleigh number considered here, a Reynolds-Averaged Navier-Stokes (RANS) formulation is used, wherein turbulence affects the mean flow through a turbulent viscosity μ_t; turbulent stresses are assumed to be proportional to the mean rates of strain via μ_t. The low-Reynolds-number K-ε turbulence model of Chien [3] is adopted. Thus, governing equations are those of Reynolds-averaged conservation of mass and momentum, and balance of energy, plus the two transport equations for K and ε.

Referring to Figure 2, boundary conditions for the mean variables (velocity components, pressure and temperature) are as follows

- On the cylinder's surface, semi-circle FA: zero velocity and uniform temperature, $u = v = 0$, $T = T_s$
- AB and EF are on the line of symmetry; line-of-symmetry conditions are imposed: $u = 0$, $\partial v/\partial x = 0$, $\partial T/\partial x = 0$
- CD and DE represent the ambient conditions; the fluid has constant ambient pressure and temperature, namely p = 0 (gauge), T = 300 K. However, the thermal condition here applies only on those sections of the boundary where there is inflow; if the computation reveals outflow on any sections, the constant temperature condition there will be ignored; instead, temperature will be computed. Similarly, the constant pressure condition applies only on those sections of the boundary where there is outflow; if the computation reveals inflow on any sections, the constant pressure condition there will be ignored, and pressure will be computed instead
- BC represents the underlying horizontal solid plane: zero velocity $u = v = 0$; as for its thermal condition, two alternatives are used:
  1) isothermal plane with constant temperature $T_{plane} = 300$ K
  2) adiabatic (insulated) plane with $\partial T/\partial y = 0$
- As will be shown below, the situation of totally unobstructed free convection is also considered. In this case, BC will be positioned at a sufficiently large distance from the cylinder, and ambient conditions similar to those on CD and DE are also imposed on BC.

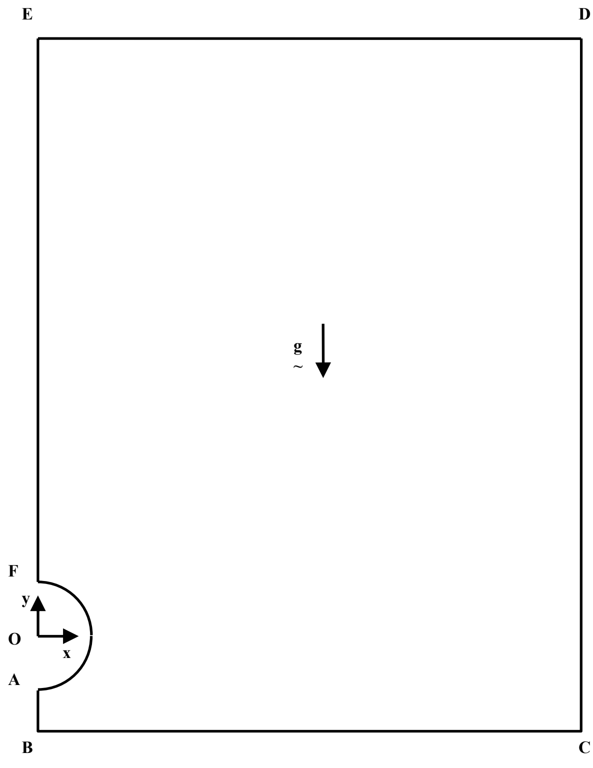

Figure 2. Computational model of the two-dimensional flow field. Cylinder's surface is represented by the semi-circle FA, the underlying solid horizontal plane by BC, lines of symmetry by AB and EF, and ambient conditions by CD and DE.

As regards the turbulence-model variables K and ε, they are prescribed as follows

- On line of symmetry AB and EF: line-of-symmetry conditions are imposed: $\partial K/\partial x = 0$, $\partial \varepsilon/\partial x = 0$
- On solid surfaces FA and BC: default solid-surface condition of the software package (see below) is adopted; this would entail $K = 0$ and $\varepsilon = 0$, following [3]
- On boundaries CD and DE representing ambient conditions, a low level of turbulence is assumed, with constant K =

4.15×10^{-7} m^2/s^2 and ε = 2.20×10^{-10} m^2/s^3. These K and ε values have been chosen so that present computational results of heat transfer agree with correlations from [4] for Ra = 10^{10}. However, if the computation reveals outflow on any sections of these boundaries, the constant K and ε condition there will be ignored; instead, K and ε will be computed

The commercial software package CFD-ACE from the ESI Group is used for the computation. The numerical scheme is the Finite Volume method, and the coupled system of governing equations is solved iteratively for the two mean velocity components, mean temperature and pressure, plus K and ε. Convergence criterion of reduction of residuals in the solved variables by both 3 and 4 orders of magnitude is adopted. This is deemed adequate; a comparison of the solutions with residual reduction of 3 orders of magnitude and those with residual reduction of 4 orders of magnitude shows extremely small difference; thus for example, the relative difference in the total heat transfer rate from the cylinder's surface for the case of totally unobstructed free convection at Ra = 10^{10} is only 1.3×10^{-4}. Computation is done on Pentium 4 machines running Windows 2000 and Unix operating systems. Double precision (64 bits) has been used.

Grid convergence tests have also been performed to ascertain the adequacy of the grid patterns used. Thus, for example, as the number of grid points on the half cylinder's surface of the computational model (semi-circle FA in Figure 2) is increased from 418 to 478, then to 538 and finally to 598, change in heat transfer for the case of totally unobstructed free convection at Ra = 10^{10} is only 1.30%, 0.72%, and 0.32% respectively. Similar variations are also seen with Ra = 10^9 and 10^{11}. From such tests, patterns with 598 grid points on the cylinder's half surface are deemed adequate and used.

Post-solution checks of y^+ values of grid points closest to cylinder's surface show y^+ < about 0.6, thus confirming that the grid pattern there is sufficiently fine for the Chien's model.

Results and Discussion

To provide another solution to what can be seen as a bench mark problem [22], and also to provide further confidence in the software package and the computational scheme used, the case of the totally unobstructed free convection about an isothermal horizontal circular cylinder is also treated. Table 1 thus shows the present computational values of the average Nusselt number $Nu_{average}$ over the whole cylinder surface, along with other results from the literature for Pr ≅ 0.7 (0.707 in the present work).

Average Nusselt number $Nu_{average}$			
Ra	**10^9**	**10^{10}**	**10^{11}**
Present work	108.0	239.8	546.3
Churchill & Chu [4]	115.7	240.5	505.4
Farouk & Güçeri [5]	Very similar to Kuehn & Goldstein [13]		
Kuehn & Goldstein [13]	101	216	465
Matveev & Pustovalov [18]	103	226.2	496
Raithby & Hollands [20]	103.3	222.0	478.1

Table 1. Comparison of the average Nusselt number for totally unobstructed free convection around an isothermal horizontal circular cylinder, with Pr ≅ 0.7 (0.707 for the present work).

It can be seen that in addition to excellent agreement with Churchill & Chu [4] at Ra = 10^{10} , the present values are in fair agreement with other published results.

Figure 3 shows variation of the average Nusselt number $Nu_{average}$ in terms of gap ratio h/D and Rayleigh number Ra, for both when the underlying solid plane has constant temperature equal to that of the ambient fluid (300 K), and when it is insulated and having adiabatic condition.

First, it is seen that for the range of h/D considered, the thermal condition of the plane has virtually no effect on the heat transfer. This is different to the laminar results [7], and indicates that the thermal boundary layer about the cylinder's surface would be very thin. In fact, this is clear in the graphical output of temperature contours (not shown).

As h/D is reduced (from about 4.5), if Ra is also large (here, 10^{10} and 10^{11}), a pattern can be seen whereby $Nu_{average}$ increases to a maximum, and this maximum occurs closer to the cylinder at higher Ra. As h/D is reduced further, $Nu_{average}$ decreases to a minimum, then increases again as the cylinder's surface is approached (except for Ra = 10^{11}; this is discussed below). At smaller Ra value (here 10^9), $Nu_{average}$ decreases to a minimum without passing through a maximum.

That $Nu_{average}$ goes through a maximum at high Ra is believed to be due to increased turbulence in the flow as it has to turn sharply as a result of the plane's presence. This does not happen at lower Ra, in similarity with the laminar situation [7], because the plane also reduces the flow velocity.

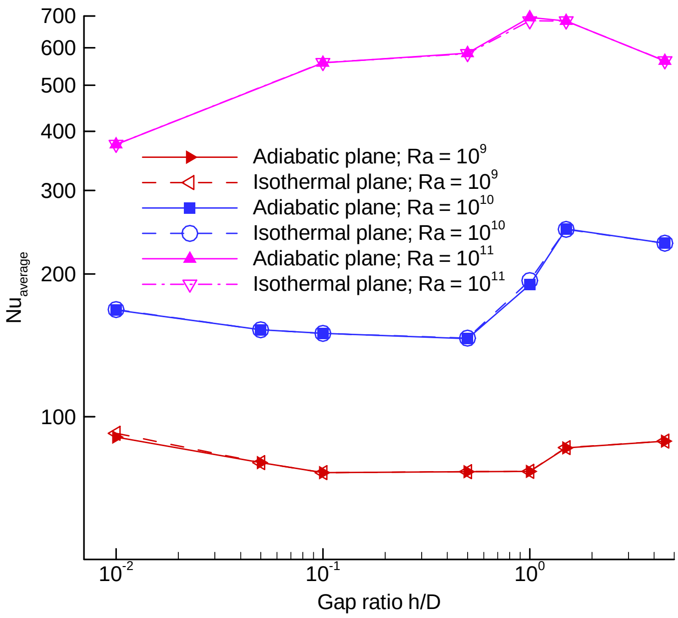

Figure 3. Variation of the average Nusselt number $Nu_{average}$ in terms of Rayleigh number Ra and gap ratio h/D, for both isothermal and adiabatic underlying plane.

With h/D = 0.05, convergence has not been obtained for Ra = 10^{11}, unlike for Ra = 10^9 and 10^{10}. As a result, it is not certain if the minimum in $Nu_{average}$ occurs at around this h/D value, or closer to cylinder, i.e. at h/D < 0.01. This minimum must exist however, because infinite heat transfer by conduction would happen between the cylinder and the plane if it is isothermal and h/D = 0; and as yet no significant difference between results of isothermal and adiabatic planes at h/D = 0.01 is seen. All this shows that the minimum in $Nu_{average}$ moves closer to cylinder's surface as Ra increases. This also agrees with expectation. For, as the gap narrows, convection is reduced, and before conduction can play a significant role, heat transfer is reduced. A more vigorous flow associated with higher Ra would bring the effects of convection closer to the cylinder, resulting in $Nu_{average}$ reaching its minimum closer to the cylinder as well.

Change in the average Nusselt number relative to its values corresponding to the totally unobstructed situation can be seen more clearly in Figure 4, wherein the percentage change in $Nu_{average}$ is defined as

$$\text{Change in } Nu_{average} = 100\times(Nu_{average} - Nu_{ave\_unobstructed}) \;/\; Nu_{ave\_unobstructed}$$

with $Nu_{ave\_unobstructed}$ being the Nusselt number's value corresponding to the totally unobstructed free convection (given as "Present work" in Table 1).

Figure 4 shows that at h/D = 4.5, the effects of the plane on heat transfer is still significant; also, the change in $Nu_{average}$ is more pronounced with larger Rayleigh number. These are in contrast to the laminar case [7], but agree with expectation. For, with the present flow, the conduction-influenced region has been reduced significantly. The plane thus affects the more vigorous flow associated with larger Ra more.

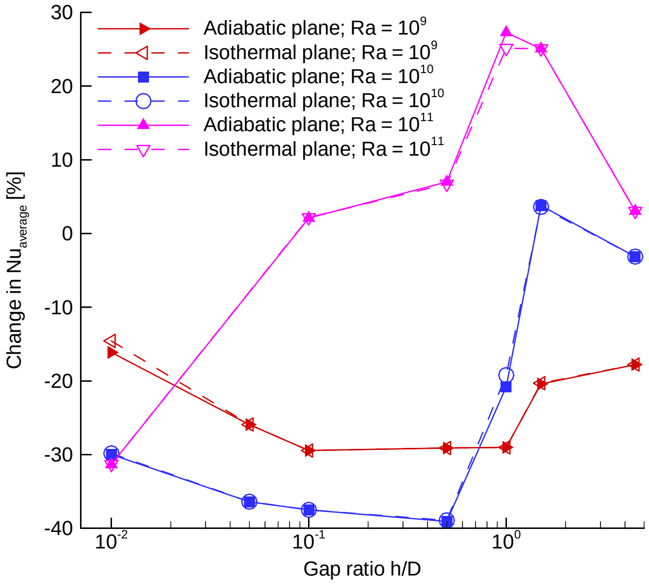

Figure 4. Percentage change of the average Nusselt number $Nu_{average}$ relative to its value of the totally unobstructed free convection, corresponding to both isothermal and adiabatic underlying plane.

Conclusions

Free convection about an isothermal horizontal circular cylinder positioned above a solid plane has been considered computationally, for Ra = $10^9 - 10^{11}$. Thermal condition of the underlying plane has insignificant effect on the heat transfer from cylinder's surface, as gap ratio h/D is reduced to as low as 0.01, thus indicating very thin thermal boundary layer. Change in heat transfer due to the plane's presence is more pronounced at higher Ra for the range of h/D considered, and the plane's influence is still significant at the rather large value of h/D = 4.5. All these are in contrast to the laminar situation.

References

[1] Atmane, M.A., Chan, V.S.S. & Murray, D.B., Natural Convection Around a Horizontal Heated Cylinder: the Effects of Vertical Confinement, *Int. J. Heat Mass Transfer,* **46**, 2003, 3661-3672

[2] Burmeister, L.C., *Convective Heat Transfer, 2nd Ed.*, Wiley, New York, 1993

[3] Chien, K.-Y., Predictions of Channel and Boundary-Layer Flows with a Low-Reynolds-Number Turbulence Model, *AIAA Journal*, **20**, 1982, 33-38

[4] Churchill, S.W. & Chu, H.H.S., Correlating Equations for Laminar and Turbulent Free Convection From a Horizontal Cylinder, *Int. J. Heat Mass Transfer,* **18**, 1975, 1949-1053

[5] Farouk, B. & Güçeri, S.I., Natural From a Horizontal Cylinder – Turbulent Regime, *J. Heat Transfer,* **104**, 1982, 228-235

[6] Farouk, B. & Güçeri, S.I., Natural and Mixed Convection Heat Transfer Around a Horizontal Cylinder Within Confining Walls, *Numerical Heat Transfer,* **5**, 1982, 329-341

[7] Huynh, B.P., Free Convection Cooling of a Horizontal Cylinder Positioned Above a Plane, *Proc. 13th Intern. Heat Transfer Conference*, Sydney, Australia, 13-18/08/2006

[8] Incropera, F.P. and DeWitt, D.P., *Fundamentals of Heat and Mass Transfer, 4th Ed.*, Wiley, New York, 1996

[9] Jones, C.D. & Masson, D.J., An Interferometric Study of Free-Convection Heat Transfer From Enclosed Isothermal Surfaces, *Transactions of the ASME*, **77**, 1955, 1275-1281

[10] Jungbluth, H. & Müller, U., Free Convective Flow Around Cylinders Near Horizontal Walls, *Archives of Mechanics*, **30**, 1978, 859-862

[11] Karim, F., Farouk, B. & Namer, I., Natural Convection Heat Transfer From a Horizontal Cylinder Between Vertical Confining Adiabatic Walls, *J. Heat Transfer*, **108**, 1986, 291-298

[12] Koizumi, H. & Hosokawa, I., Chaotic Behavior and Heat Transfer Performance of the Natural Convection around a Hot Horizontal Cylinder Affected by a Flat Ceiling, *Int. J. Heat Mass Transfer*, **39**, 1996, 1081-1091

[13] Kuehn, T.H. & Goldstein, R.J., Correlating Equations for Natural Convection Heat Transfer Between Horizontal Circular Cylinders, *Int. J. Heat Mass Transfer*, **19**, 1976, 1127-1134

[14] Kuehn, T.H. & Goldstein, R.J., Solution to the Navier-Stokes Equations for Laminar Natural Convection about a Horizontal Isothermal Circular Cylinder, *Int. J. Heat Mass Transfer*, **23**, 1980, 971-979

[15] Lawrence, G.B., Jardin, G.E., Naylor, D. & Machin, A.D., Free Convection From a Horizontal Heated Cylinder Located Below a Ceiling, *Trans. of the CSME*, **23**, 1999, 19-35

[16] Ma, L., van der Zanden,, J., van der Kooi, J. & Nieuwstadt, F.T.M., Natural Convection Around a Horizontal Circular Cylinder in Infinite Space and Within Confining Plates: A Finite Element Solution, *Numerical Heat Transfer - Part A*, **25**, 1994, 441-456

[17] Martynenko, O.G. & Khramtsov, P.P., *Free-Convective Heat Transfer*, Springer, New York, 2005

[18] Matveev, Yu.Ya. & Pustovalov, V.N., Heat Transfer in Turbulent Free Convection Around a Horizontal Nonisothermal Cylinder, *J. Engineering Physics and Thermophysics*, **49**, 1985, 772-777

[19] Morgan, V.T., The Overall Convective Heat Transfer from Smooth Circular Cylinders, *Advances in Heat Transfer,* **11** (Irvine, T.F. & Hartnett, J.P., Eds.), Academic Press, New York, 1975

[20] Raithby, G.D. & Hollands, K.G.T., 1998, Natural Convection, in: *Handbook of Heat Transfer, 3rd Ed.* (Rohsenow, W.M., Hartnett, J.P. & Cho, Y.L., Eds.), McGraw-Hill, New York, 1998

[21] Sadeghipour, M.S. & Razi, Y.P., Natural Convection From a Confined Horizontal Cylinder: the Optimum Distance

Between the Confining Walls, *Int. J. Heat Mass Transfer*, **44**, 2001, 367-374
[22] Saitoh, T., Sajiki, T. & Maruhara, K., Bench Mark Solutions to Natural Convection Heat Transfer Problem Around a Horizontal Circular Cylinder, *Int. J. Heat Mass Transfer*, **36**, 1993, 1251-1259
[23] Sesterhenn, A. & Müller, U., Numerical Calculation of Free Convective Flow Around Cylinders Near Rigid Walls, *Archives of Mechanics*, **28**, 1976, 903-910.