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Numerical Simulation of a Ludwieg-tube Fuel Delivery System for Scramjet Experiments in Shock Tunnels

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Abstract

The T4 shock tunnel at The University of Queensland is regularly used to perform supersonic combustion experiments. The fuel for the test model is supplied using a Ludwieg-tube delivery system. A combination of theoretical modelling and calibration tests is used to determine the mass flow-rate of fuel for given Ludwieg-tube initials conditions and the measured pressure in the plenum chamber that supplies the fuel to the model. The theoretical model used in the calibration procedure is presented. The goal of this project is to check the suitability of the modelling assumptions by simulating the complete Ludwieg-tube system using the one-dimensional Lagrangian computer code, L1d. Simulation of the fuel delivery system is done separately for viscous and inviscid flow with conditions replicating those used in supersonic combustion tests in T4. The steady plenum pressures from the simulations of inviscid and viscous flow are, on average, within ±7% and ±4%, respectively, of the values measured experimentally. Further, the fuel mass flow-rates obtained from viscous simulations are, on average, within ±13% of the experimental values.

Nomenclature

Roman symbols

a	local speed of sound;
A	valve orifice area;
A^*	effective throat area of holes in the plenum;
d	valve orifice diameter;
G	constant that relates pressure drop across the valve;
K	valve loss coefficient;
L	length;
$\dot{m}_f$	fuel mass flow-rate;
p	pressure;
R	gas constant;
T	temperature;
u	flow speed of gas;
V	total volume of Ludwieg-tube and extended tube upstream the valve;

Greek symbols

α	constant that relates initial pressure in the Ludwieg-tube and the plenum steady pressure to the fuel mass flow-rate;
γ	specific heat ratio;
ρ	density;

Subscripts

p	plenum chamber conditions;
0	Ludwieg-tube stagnation conditions;
0_i	Ludwieg-tube initial conditions;
0_f	Ludwieg-tube final conditions;
tu	extended tube upstream the valve;
td	extended tube downstream the valve;

Abbreviations

L1d	Lagrangian one-dimensional;
T4	T4 shock tunnel;

1. Introduction

Ludwieg-tubes can provide clean uniform flow at economical cost. The concept of such a tube was first proposed in 1955 by the German scientist Hubert Ludwieg. A Ludwieg-tube basically consists of a long high pressure tube closed at one end and a converging-diverging nozzle at the other end. Downstream of the nozzle, a diaphragm is used to separate the high pressure gas in the Ludwieg-tube from the low pressure gas in the dump tank. When the diaphragm ruptures (generally done by a piercing device), a shock propagates into the low pressure region of the dump tank while an unsteady expansion wave moves upstream into the long high pressure tube, choking the nozzle throat [1]. Subsequently, a sonic flow is established, and the flow remains steady until the reflected expansion waves from the closed end of the high-pressure tube arrive back at the nozzle. The duration of steady flow is relatively short because the expansion waves travel at the local speed of sound. However, this period can be increased by increasing the length of the high pressure tube. More information on the operation of Ludwieg-tubes can be found in reference [2].

The fuel delivery system in T4 works on the same principle as a Ludwieg-tube. The fuel delivery system consists of a long, high-pressure tube filled with the fuel (usually hydrogen), a fast acting solenoid valve and a plenum chamber with holes. The holes serve as sonic exhaust nozzles for releasing gas into the combustion chamber of the scramjet model. The solenoid valve has extended tubes on either side that connect to the Ludwieg-tube and the plenum. The valve, when closed, separates the high pressure side from the low pressure side. The increase in cross-sectional area in the plenum chamber reduces the speed of gas flow there to achieve near uniform conditions to feed the injection holes.

With the holes acting as sonic nozzles to discharge gas from the plenum, the measurements of the plenum total pressure and total temperature can be used to determine the fuel mass flow-rate through the holes if the discharge coefficient for the holes is known. But the measurement of plenum total temperature is not simple. The response time of typical thermocouples that might be used for temperature measurement is usually insufficient for the short duration flows in the fuel-delivery Ludwieg-tube (of order 20 ms). Therefore, the plenum total temperature is not measured.

To address this problem in determining the fuel mass flow-rate, a theoretical model is proposed. The theoretical model can be used to calculate the mass flow-rate without the requirement to measure the plenum total temperature. The theoretical model is developed on two critical assumptions. First, it is assumed that the flow from the Ludwieg-tube to the plenum is adiabatic. This condition is checked in the numerical simulations by comparing the stagnation temperature in the plenum (T_p) and Ludwieg-tube (T_0). To include the pressure loss across the valve, it is assumed that the pressure in the plenum is a constant fraction of the pressure in the Ludwieg-tube, i.e., $p_p = Gp_0$. The simulation quantifies the influence of this assumption by checking the effect of different valve loss coefficients on plenum pressure for particular initial conditions.

This paper includes the theoretical model used to calculate the mass flow-rate, a brief description of the numerical simulation, the results obtained from the simulation and a comparison of the results from the simulation with those from the experiments.

2. Theoretical Model

The following theoretical model is used to calculate the fuel mass flow-rate without a direct measurement of the plenum total temperature. This model starts with the assumption that the gas in the Ludwieg-tube expands isentropically from the initial fill conditions. Therefore

$$\frac{p_0}{p_{0_i}} = \left(\frac{T_0}{T_{0_i}}\right)^{\frac{\gamma}{\gamma-1}}, \text{ and}$$

$$T_0 = T_{0_i}\left(\frac{p_0}{p_{0_i}}\right)^{\frac{\gamma-1}{\gamma}}, \qquad (1)$$

where, γ = 1.41 for Hydrogen. The flow is assumed adiabatic from the Ludwieg-tube to the plenum, which is reasonable considering that the initial gas temperature in the Ludwieg-tube and the plenum is around 300 K and 296 K respectively. Further, as the Ludwieg-tube fuel delivery system is exposed to standard atmospheric conditions, the amount of heat transfer from the fuel system to the surrounding is not significant.

$$T_p = T_0 . \qquad (2)$$

It is assumed that the flow is choked at the plenum holes. Therefore the fuel mass flow-rate is

$$\dot{m}_f = \sqrt{\left(\frac{\gamma}{R}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\,\frac{p_p A^*}{\sqrt{T_p}} = \frac{p_p A^* C}{\sqrt{T_p}} ,$$

where, C = 0.0107 $K^{1/2}sm^{-1}$, for H_2 at 1 atm and 20°C. From equation (2), $T_p = T_0$. Therefore

$$\dot{m}_f = \frac{p_p A^* C}{\sqrt{T_0}} . \qquad (3)$$

But from equation (1), T_0 is related to T_{0_i} so that

$$\dot{m}_f = \frac{p_p A^* C}{\sqrt{T_{0_i}}}\left(\frac{p_{0_i}}{p_0}\right)^{\frac{\gamma-1}{2\gamma}} .$$

There will be a loss of total pressure as the fuel flows from the Ludwieg-tube to the plenum through the solenoid valve. It is assumed that the pressure in the plenum is a constant fraction of the pressure in the Ludwieg-tube. Therefore,

$$p_p = Gp_0 ,$$

or

$$p_0 = (1/G)\,p_p , \qquad (4)$$

where, G is a constant, which depends upon the valve loss coefficient. Using equation (4) in equation (3),

$$\begin{aligned}\dot{m}_f &= \frac{p_p A^* C}{\sqrt{T_{0_i}}}\left(G\,\frac{p_{0_i}}{p_p}\right)^{\frac{\gamma-1}{2\gamma}} \\ &= \frac{p_p A^* B}{\sqrt{T_{0_i}}}\left(\frac{p_{0_i}}{p_p}\right)^{\frac{\gamma-1}{2\gamma}}\end{aligned}$$

where, B is a constant $= C\left(G^{\frac{\gamma-1}{2\gamma}}\right)$. Therefore

$$\begin{aligned}\dot{m}_f &= \frac{BA^*}{\sqrt{T_{0_i}}}\left(p_{0_i}\right)^{\frac{\gamma-1}{2\gamma}}\left(p_p\right)^{\frac{\gamma+1}{2\gamma}} \\ &= \alpha\left(p_{0_i}\right)^{\frac{\gamma-1}{2\gamma}}\left(p_p\right)^{\frac{\gamma+1}{2\gamma}}\end{aligned} \qquad (5)$$

where, $\alpha = \frac{BA^*}{\sqrt{T_{0_i}}}$. The mass that leaves the system during the period of time for which the valve is open can be written as

$$\Delta m = \int_0^t \dot{m}_f\,dt = \alpha\left(p_{0_i}\right)^{\frac{\gamma-1}{2\gamma}}\int_0^t \left(p_p\right)^{\frac{\gamma+1}{2\gamma}}\,dt . \qquad (6)$$

Note that the initial mass of gas in the tube is $m_i = \rho_{0_i} V = \frac{p_{0_i} V}{RT_{0_i}}$ and the final mass of gas in the tube is $m_f = \rho_{0_f} V = \frac{p_{0_f} V}{RT_{0_f}}$.

Therefore, the total amount of gas that leaves the Ludwieg-tube when the valve is open is

$$\Delta m = m_i - m_f = \left(\frac{p_{0_i}}{T_{0_i}} - \frac{p_{0_f}}{T_{0_f}}\right)\left(\frac{V}{R}\right).$$

From equation (1),

$$T_{0_f} = T_{0_i}\left(\frac{p_{0_f}}{p_{0_i}}\right)^{\frac{\gamma-1}{\gamma}} .$$

Therefore,

$$\begin{aligned}\Delta m &= \left(\frac{V}{R}\right)\left(\frac{p_{0_i}}{T_{0_i}} - \frac{p_{0_f}}{T_{0_i}}\left(\frac{p_{0_i}}{p_{0_f}}\right)^{\frac{\gamma-1}{\gamma}}\right) \\ &= \left(\frac{V}{RT_{0_i}}\right)\left(p_{0_i} - p_{0_f}\left(\frac{p_{0_i}}{p_{0_f}}\right)^{\frac{\gamma-1}{\gamma}}\right)\end{aligned} \qquad (7)$$

Equating equations (6) and (7),

$$\alpha\left(p_{0_i}\right)^{\frac{\gamma-1}{2\gamma}}\int_0^t\left(p_p\right)^{\frac{\gamma+1}{2\gamma}}dt=\left(\frac{V}{RT_{0_i}}\right)\left(p_{0_i}-p_{0_f}\left(\frac{p_{0_i}}{p_{0_f}}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

Therefore,

$$\alpha=\left(\frac{V}{RT_{0_i}}\right)\left(\frac{\left(p_{0_i}\right)^{\frac{1-\gamma}{2\gamma}}}{\int_0^t\left(p_p\right)^{\frac{\gamma+1}{2\gamma}}dt}\right)\left(p_{0_i}-p_{0_f}\left(\frac{p_{0_i}}{p_{0_f}}\right)^{\frac{\gamma-1}{\gamma}}\right) \qquad (8)$$

The value of α obtained from equation (8) can be substituted into equation (5) to determine the fuel mass flow-rate at any point in time during a test.

3. Numerical Simulation

The numerical modelling used for the simulation of the fuel delivery system is a Lagrangian one-dimensional code called L1d. L1d has been used successfully for simulation of transient-flow facilities such as the light-gas launchers and free-piston driven shock tunnels. The numerical modelling embodied within L1d is based on a quasi-one-dimensional Lagrangian description of the gas dynamics coupled with engineering correlations, to estimate gas behaviour. More information on L1d can be found in reference [3].

A representation of the geometry of the fuel delivery system is required for the simulation. Noting that L1d is a Lagrangian one-dimensional code, a hydraulic diameter for the plenum is used in the simulation to represent its actual rectangular cross-section. Also, a single hole is used to simulate all the fuel injection holes. The area of the single simulated injection hole has the same area as that of the multiple injection holes in the actual model. L1d performs best when changes in cross-sectional area occur gradually rather than rapidly. Therefore in the simulation, the transition between tubes of different area is taken as a cubic with the transition length being one diameter of the larger tube. Also, while the fuel injection holes from the plenum chamber are choked sonic orifices, the single hole is modified as a convergent-divergent hole with the throat area equal to the combined area of the sonic orifices. The dimensions of the entire fuel delivery system are shown in figure 1.

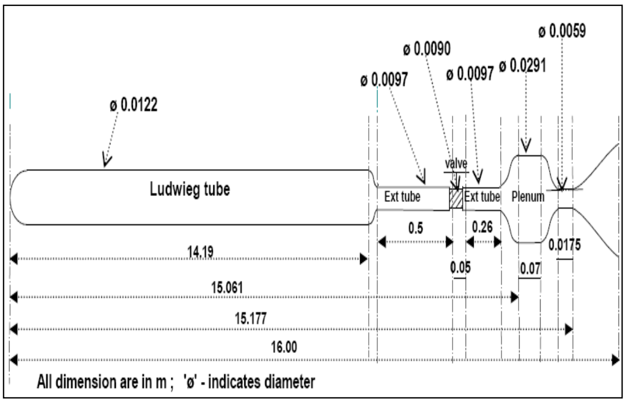

Figure 1. Geometry of the fuel delivery system used for the simulation

Experimental data for the fuel delivery system were obtained from four shots in the T4 Shock Tunnel that involved combustion tests of a scramjet model. For all experiments $T_{0_i} \approx 300$ K. Experimental data were also available from three calibration shots. Data were obtained from the experimenter for the fuel delivery system, Dr. Milinda Suraweera. The experimental data in Table 1 show the initial and final Ludwieg-tube pressures and the mean plenum pressures during the tests.

In the experiments and simulations, the Ludwieg-tube was initially filled with hydrogen and the gas downstream of the valve was air. The initial fill conditions for the Ludwieg-tube in the simulations (p_{0_i} , T_{0_i}) were set to the conditions for each T4 shot or calibration test. The air downstream of the valve was set to 130 Pa at 295 K. Since L1d does not have features to replicate the operation of the solenoid valve, a gas-interface was incorporated in the code to separate the two gas slug. Therefore valve delays in opening and closing were not replicated in the simulation. However, L1d includes features to incorporate loss regions to account for pressure losses through the valve and the losses caused by sudden expansion or contraction at changes in the cross-sectional area of the pipe. The extent of pressure loss is specified by an appropriate minor loss coefficient for a particular region.

T4 shot no.	p_{0_i} (kPa)	p_{0_f} (kPa)	p_P (kPa)
9291	1680	1300	1250
9292	1315	1080	950
9293	1005	790	740
9294	1005	810	765
Calibration 3	970	760	800
Calibration 4	1120	940	890
Calibration 5	2330	1775	1775

Table 1. Experimental data for the Ludwieg-tube fuel delivery system

In L1d, increasing the number of cells provides better resolution of the results from the simulation, but increasing the number of cells also increases the computational time required to finish a simulation. The appropriate number of cells was determined using a grid refinement study. For the simulations done in this project 1500 cells were used for the slug upstream of the gas-interface and 200 cells were used for the slug downstream of the gas-interface. The simulations were run for a minimum of 30 ms, which is approximately the time for which the valve was left opened in the T4 shots and fuel system calibration tests.

4. L1d Simulation Results

In a Ludwieg-tube, if the growth of boundary layers on the tube walls is neglected, several steady periods of constant stagnation conditions are expected [4]. An inviscid L1d simulation of the T4 fuel Ludwieg-tube for Shot 9293 was run for a total of 60 ms to show two such regions of approximately steady plenum pressure. Figure 2 shows results from this simulation in the form of a wave diagram for the Ludwieg-tube and the pressure trace in the plenum chamber. The results show that the second period of steady flow is shorter in duration than the first.

The arrival of the unsteady expansion waves at the valve causes the pressure in the plenum to drop as the injector unchokes. However, as the expansion waves reflect back, the injector chokes again giving rise to the next steady stage.

Inviscid simulations were done for the conditions of the experiments listed in Table 1. Plenum conditions obtained from the simulations were then compared with the corresponding experimental data. The uncertainty in the plenum pressure for the experiment is calculated to be ±4%. Figure 3 compares the pressure in the plenum chamber for the inviscid simulation and experiment for shot 9291.

The simulation shows an overshoot in pressure in the plenum chamber that is not evident in the experiment. There is also a larger drop in pressure associated with the arrival of the unsteady expansion in the simulation than in the experiment. Similar results were obtained for other conditions. For all inviscid simulations the plenum pressures from the simulations were higher than the experimental values. This is attributed to the absence of complete simulation of losses in the inviscid simulations.

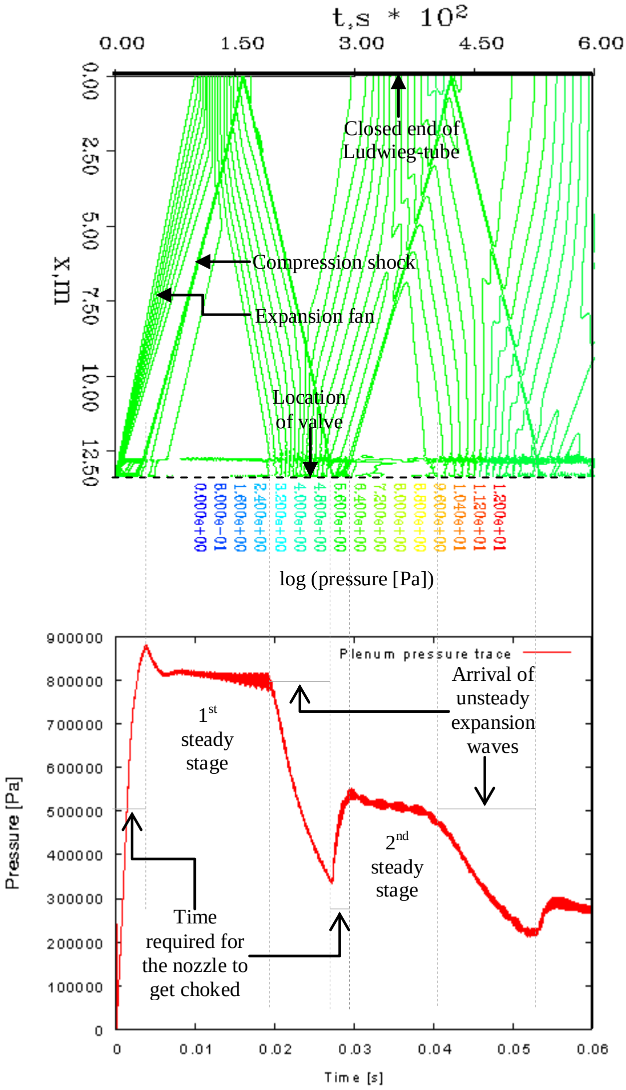

Figure 2. Plenum pressure trace relative to the expansion fan

Viscous simulations were also done for the conditions listed in Table 1. Loss regions can be included in viscous simulations. The loss for the valve was simulated by including a loss region, with loss factor, K = 0.5, over a length of 50 mm around the valve location. The value of K was chosen based on table 6.5 of White [5].

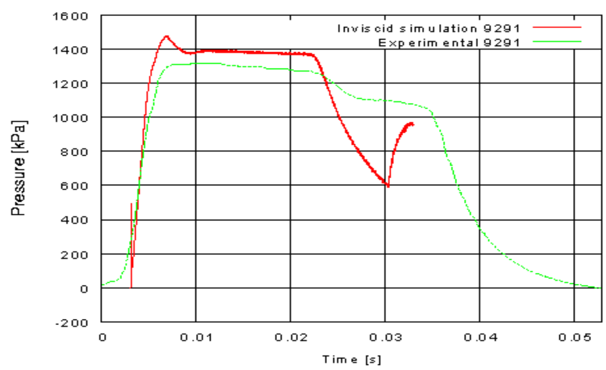

Figure 3. Experimental (green) and inviscid simulation (red) plenum pressure traces for conditions of shot 9291

The plenum pressure traces from the viscous simulations for all conditions showed good agreement with the corresponding experimental traces. Figure 4 shows the comparison for shot 9291.

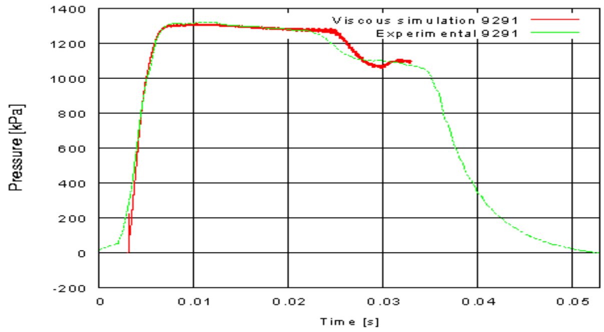

Figure 4. Experimental (green) and viscous simulation (red) plenum pressure trace for shot 9291 conditions

The viscous simulations can be used to quantify the influence of the assumption $p_p = Gp_0$ made in the theoretical model. Viscous simulations were done with various loss coefficient factors K to check the pressure loss across the valve (gas-interface). The results are as shown in the table 2.

K	**Stagnation pressure (kPa)**		**Pressure drop (kPa)**
	Start of valve	End of valve	
0.25	714	690	24
0.50	717	684	33
0.75	720	680	40
1.00	724	675	49

Table 2. Pressure drop for different valve loss coefficients from viscous simulation of shot 9293 conditions

Note that G will vary with the applied loss coefficients. From the results shown in Table 2, the average value of G is 0.95. Table 3 shows the comparison of the plenum pressure from both inviscid and viscous simulations and the experiment.

The simulation pressures indicated in Table 3 are average values taken over the period of steady flow. The pressures from the inviscid and viscous simulations are, on average, within ±7% and ±4% respectively of the corresponding experimental values. Since the viscous simulations show better agreement with experiments than do the inviscid simulations, viscous flow simulations have been used in further analysis.

T4 shot no.	**Plenum pressure (kPa)**		
	Inviscid simulation	**Viscous simulation**	**Experimental**
9291	1352	1290	1250
9292	1059	1008	950
9293	809	769	740
Calibration 3	783	743	800
Calibration 4	904	859	890
Calibration 5	1881	1795	1775

Table 3. Overview of steady plenum pressures from the simulations and experiments

The fuel mass flow-rates from the viscous simulations are calculated using the basic mass flow-rate equation, $\dot{m}_f = \rho A u$. These results are compared with the actual mass flow-rate calculated by using α obtained from equation (8) for each shot and then substituting it in equation (5). Comparisons between the mass flow-rates are shown in Table 4.

T4 shot no.	p_{0_i} (kPa)	Simulation		Experimental	
		p_p (kPa)	$\dot{m}_f$ (kg/s)	$\alpha \times 10^8$ (m^2/s)	$\dot{m}_f$ (kg/s)
9291	1680	1290	0.0180	1.360	0.0182
9292	1315	1008	0.0130	1.179	0.0123
9293	1005	769	0.0108	1.186	0.0094
Calibration3	970	743	0.0108	1.136	0.0087
Calibration4	1120	859	0.0116	1.010	0.0090
Calibration5	2330	1795	0.0252	1.235	0.0230

Table 4. Fuel mass flow-rate obtained from simulations and experiments

Table 4 shows that the fuel mass flow-rates from the viscous simulations are, on average, within ±13% of the corresponding experimental values. The uncertainties in the mass flow rates from the viscous simulations and for the experiments are calculated be ±9% and ±7% respectively.

5. Conclusions

The results indicate that the assumptions made in the theoretical model are reasonable and that the L1d simulations are able to give approximate values of the fuel mass flow-rates for the experiments. Viscous L1d simulations show better agreement with measurements for the present Ludwieg-tube fuel delivery system than do inviscid simulations.

6. References

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