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Laminar Spirals in the Outer Stationary Cylinder Couette-Taylor System

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Abstract

We present numerical simulations to demonstrate the existence of laminar spiral flows between both finite and infinite length concentric cylinders and finite truncated cones where only the inner wall rotates. The velocities and pressure are calculated by a spectral element/Fourier method. Different gap ratios are investigated. Convergence of the numerical results is shown with reference to flows between infinite cylinders. The presence of top and bottom endplates results in vortex dislocations that are observed at the frontiers between the Ekman vortices present at each end and the spiral vortices.

Introduction

The last century has seen hundreds of works devoted to the flow confined between circular cylinders since the pioneering investigation of Taylor [1], but even now, new flow behaviours are reported both experimentally and theoretically [2-4]. Laminar spirals have been studied extensively and were reported when both cylinders were oppositely rotated [5-10]. With such an experimental system, Andereck *et al*. [5] obtained a stability diagram where laminar spirals occupied a very narrow region. In the Couette system the basic flow is naturally unstable to non-axisymmetric spiral vortices near the inner cylinder when the outer cylinder rotates in the opposite direction as reported by Coles [11], Krueger *et al*. [12] and Snyder [13-14]. Spiral vortices are strongly dependent on initial conditions, as they exist in different flow modes, and may even co-exist with steady Taylor vortices [5]. Laminar spirals have been also observed in the system of conical cylinders. Wimmer [15] found that helical flow structures could be generated between the conical cylinders when only the inner body was rotated. These structures were very sensitive to initial conditions. Noui-Mehidi *et al*. [16] have reported that the helical structure between conical cylinders could be generated only for very slow acceleration rates of the inner body.

Andereck *et al*. [5], working in a system with counter-rotating finite cylinders, related the presence of laminar spirals to the presence of top and bottom boundaries. However, Antonijoan *et al*. [4] predicted via numerical simulation that spiral regimes may exist in infinite Taylor-Couette flow. They reported that spiral regimes (which in general are temporally periodic in a fixed frame of reference) arose through a Hopf bifurcation from the circular Couette flow in a system with counter-rotating cylinders. Their results also included solutions with laminar spirals in a system where only the inner cylinder could rotate, but this possibility was not dealt with in detail.

The present paper reports on numerical simulations of laminar spirals in circular Couette flow. Motivation for this work initially emanated from an investigation of the helical flow between finite conical cylinders of same apex angle, and where only the inner cylinder rotates. Study of the effect of the conical apex angle on the stability of the helical structure showed that spiral vortices were maintained even at very small apex angle. Then, the numerical projection of the solution obtained between conical cylinders into a cylindrical annular gap showed that spiral vortices were also maintained in a configuration of circular Couette flow where the outer cylinder is stationary, and where endwalls are present. In agreement with the results of Antonijoan *et al*. [4], spiral vortices are also found to be stable when the cylinders are infinite in length for both counter-rotating cylinders, and when the outer cylinder is fixed. Thus the results show that laminar spirals can exist between both infinite and finite circular cylinders when the outer cylinder is stationary and that these spirals are stable over a range of Reynolds numbers.

Numerical method

The direct numerical simulation formulation used here has been described in detail by Blackburn & Sherwin [17] and is reported briefly in the following. In the cylindrical geometry the velocity u (z, r, θ) is projected by Fourier transformation in the azimuthal direction onto a set of two-dimensional complex modes $\breve{u}_k$ (z, r). This Fourier basis is coupled with a spectral element discretization in the meridional plane. Time integration is carried out with a second-order semi-implicit scheme. The meridional computational domain was discretized into 231 spectral elements with Gauss-Lobatto-Legendre (GLL) nodal basis functions. Through numerical experiments, it was found that 16 planes of data in the azimuthal direction were sufficient to generate a stable and smooth solution. The accuracy of the results is also checked with regards of the number of element np as shown in Table 1. A tensor-product polynomial order of 7 within each spectral element was found sufficient to achieve a good accuracy.

$\mathbf{n_p}$	**3**	**5**	**7**	**9**
U_{max}	0.56513	0.56574	0.56566	0.56563

Table 1. Comparative analysis of the accuracy of the numerical results: n_p is the GLL polynomial order in each direction within each spectral element, U_{max} is the averaged value of the maximum axial velocity component, normalized by Ωd.

Results

In our investigation, in order to validate the numerical results against those of Antonijoan *et al*. [4], calculations with a gap ratio of η=0.8 were conducted in the case of infinite counter-rotating cylinders. The outer Reynolds number Ro= b Ω d / ν was fixed to –50 and the inner Reynolds number Ri= a Ωd / ν was varied between 106 and 120 (a is the inner cylinder radius and b is the outer cylinder radius, Ω the rotational speed, d the gap and ν the kinematic viscosity).

A good agreement was found by determining the bifurcating critical value of Ri=106.1 with an accuracy of 0.6% compared to the results of Antonijoan *et al.* [4]. The study of the energy distribution at the different modes showed that the predominant mode in the spiral regime is the mode energy with k=1 as can be seen in Fig. 1.

In the study of laminar spirals between conical cylinders, the first computations started with initial conditions that were a rest state to which was added a random perturbation of order 10^{-6} in the mode k=1. In the geometry of conical cylinders, both conical cylinders had the same apex angle α of 24, the inner conical cylinder rotating and the outer one stationary. Fixed endplates were imposed at the top and bottom of the flow system and the Reynolds number defined by: Ri = a Ωd / ν was fixed to 200. When the solution reached steady state as ascertained by constant time series of the energies in all the azimuthal Fourier modes, the apex angle was decreased to 16 degrees and the simulations started again from the previous obtained solution. The procedure was repeated for an apex angle of 4 degrees until steady state was also reached. Figure 2 shows the flow structure obtained for an apex angle of 16 degrees and a Reynolds number of 200. At the final stage, for the same latter Reynolds number the apex angle was set to zero, which yields a cylindrical annulus configuration and the same procedure was adopted. The spiral structure obtained in the circular Couette case is presented in Fig. 3 for a Reynolds number Ri=300.

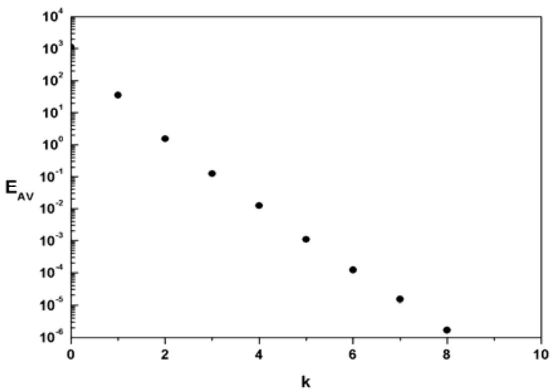

Fig.1. Distribution of the average azimuthal Fourier mode energy in the case of infinite counter-rotating cylinders with η=0.8, Ro=-50 and Ri=110.

Calculations conducted with the outer cylinder fixed have revealed the persistence of laminar spirals in the infinite cylinder geometry. Figure 4 presents the spiral regime calculated at Ri=110 and Ro=0 for a gap ratio η=0.8.

Time series of the velocity component revealed that the axial periodic drift of the spiral structure proceeds with a frequency f_s that has a ratio to the rotational frequency f_r of the inner cylinder, which decreases as Reynolds number increases. The ratio f_s/f_r has a value of 0.35 at Ri=220 and decreases to 0.32 at Ri=300 in the infinite cylinder geometry.

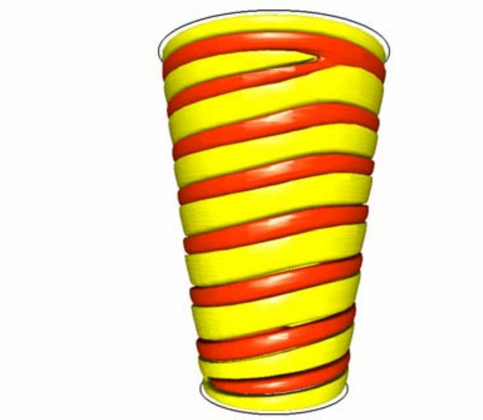

Fig.2. Laminar spirals in the system of conical cylinders. Ri=200, α=16 degrees, η=0.84 at the top of the system, Γ=H/d=12.50. (H is the height of the fluid column, d the gap and α the total (apex) opening angle of the conical cylinders). Isosurfaces show +/- radial velocity

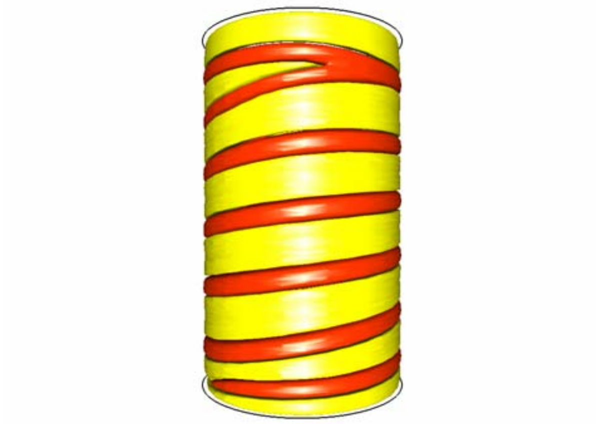

Fig.3. Laminar spirals in the system of concentric cylinders with endplates. Re=300, η=0.75, Γ=12.98, fs / fr = 0.35.
Isosurfaces show +/- radial velocity

With the presence of the top and bottom endplates, Ekman vortices are present near the fixed plates due to the pumping fluid layers. The spiral vortices become confined between the Ekman vortices and are separated at each end from the Ekman layer by a vortex dislocation. These dislocations observed also between conical cylinders, can be seen in Figs. 2 and 3 presenting isosurfaces of the axial velocity component for conical cylinders and concentric cylinders respectively.

The analysis of the flow properties at the dislocation showed some interesting results. Figure 5 presents a contour plot of the radial component of the vorticity taken at an axial cross section of the cylinders. The section is located at the middle of the spiral vortex where the dislocation occurs.

It can be noticed from Fig.5 that at the dislocation the vorticity of the spiral vortex is higher than the vorticity of the remaining vortex, which belongs to the Ekman layer. The spiral vortex occupies 3/8 of the cross section and is located near the inner cylinder.

Fig.4. Laminar spirals in the system of infinite counter-rotating cylinders. Ri=110, Ro=0, η=0.8.

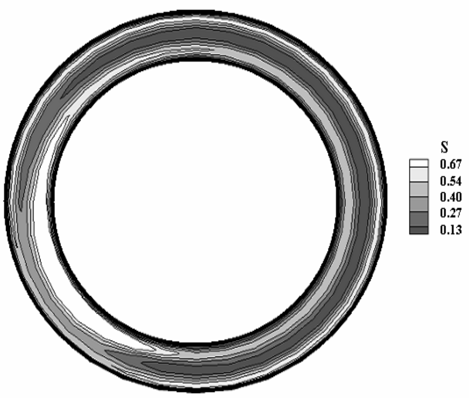

Fig.5. Representation of vorticity contour lines at an axial cross section in finite cylinders geometry. The section is taken at the location of the vortex dislocation separating the top Ekman vortex from the spiral vortices taking birth below. Ri=300, Ro=0, η=0.75, Γ=12.98.

Conclusions

The present results have shown that spiral vortices between concentric cylinders in a finite geometry have properties different from spiral vortices between infinite cylinders when the outer cylinder is stationary. There is need to pursue furthermore both numerical and experimental investigations to better understand the mechanisms behind the formation of spiral vortices and highlight the characteristic properties associated with this kind of flow regimes.

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