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How expensive space-zero-gravity convection experiments can be carried out in terrestrial conditions – magnetic convection of a paramagnetic fluid

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Abstract

Over the last decade or so it has became possible to build high-temperature super-conducting magnets that operate in a laboratory environment. Many new phenomena connected with strong magnetic fields have been reported (e.g. promotion of combustion, magnetic levitation, separation methods for weakly magnetic materials etc.). There are many applications of the use of magnetic force on the Earth. For instance, knowing how to control such a force makes it possible to negate the influence of the gravitational force and study a particular phenomenon as it would occur in the Cosmos, but under terrestrial conditions, avoiding the need for expensive space travel. The use of a magnetic field may also help in many processes such as crystal growth, mixing and material processing.

The present work is concerned with magnetic convection of a paramagnetic fluid in a cubical enclosure heated and cooled from the sidewalls. The influence of a 10-T magnetic field on the convection mode of the paramagnetic fluid and the heat transfer rate were investigated numerically and experimentally, and compared with gravitational natural convection. The present study clearly shows that natural convection can be enhanced, and the direction of the convection flow can be changed using a strong magnetic field in terrestrial conditions.

Introduction

The control of convection phenomena in industrial and scientific applications is an interesting concept. However, before it can be applied, fundamental studies and experiments have to be carried out. Enhancement or suppression of convection phenomena and improvement of the heat and mass transfer has been a long-term subject investigated by many researchers. In terrestrial condition, the gravitational acceleration is constant and uniform everywhere. Therefore, controlling natural convection is difficult. One of the simplest ways to provide an additional acceleration to a fluid is to rotate it [1]. This method has been extensively studied; however, it is not always simple to apply this to real situations. An alternative way of controlling the gravitational effect came across only recently when it became possible to build high-temperature, super-conducting magnets that operate in laboratory environments. The strength of the magnetic force has become sufficiently large to reactivate sustained investigations of magnetic force which were started by Faraday in 1846. The present super-conducting magnets can generate high magnetic fields, up to 30 T at room temperature. Many new interesting phenomena have been reported concerning materials of low magnetic susceptibilities. For instance, it becomes possible to levitate diamagnetic materials under terrestrial conditions. One of the most spectacular examples is the levitation of a famous frog called Larry and the kissing water droplets in the bore of a super-conducting magnet [2].

A quantitative treatment of fluid convection under a magnetic field appears to have been initiated by Bai et al. [3]. Braithwaite et al. [4] used a magnetic field to enhance and suppress the Rayleigh-Benard convection in a solution of gadolinium-nitrate, and showed that the effect depends on the relative orientation of the magnetic force and the temperature gradient. Huang et al. [5] studied thermo-convection in a horizontal fluid layer by classical linear stability analysis. They created a mathematical model to describe the convection of para- and diamagnetic fluids. Tagawa et al. [6] employed a procedure similar to the Boussinesq approximation, and developed a simple model equation for convection resulting from a magnetic field. Most of these works are summarized in a recent book by Ozoe [7].

Therefore, knowing how to control magnetic force makes it possible to suppress the influence of gravity, and study particular phenomena under terrestrial conditions as though they were occurring in the Cosmos. Conducting experiments on Earth is considerably more economic than attempting to do so in space. Moreover, the use of a magnetic field to suppress convection may also be useful in many industrial applications such as crystal growth, mixing and material processing. Therefore, fundamental studies of magnetic convection are very necessary at the present time.

Experimental Apparatus

Figure 1 shows a schematic view of the experimental enclosure. Four walls of the cube are made of transparent Plexiglas. The left-hand-side copper plate was cooled by running water from a constant temperature circulator and the right-hand-side copper plate was heated with rubber-coated nichrome wire. The electric voltage and current of the nichrome wire were measured with two multi-meters. The temperatures of the copper plates were measured with six T-type thermocouples inserted into small and deep holes in each copper plate.

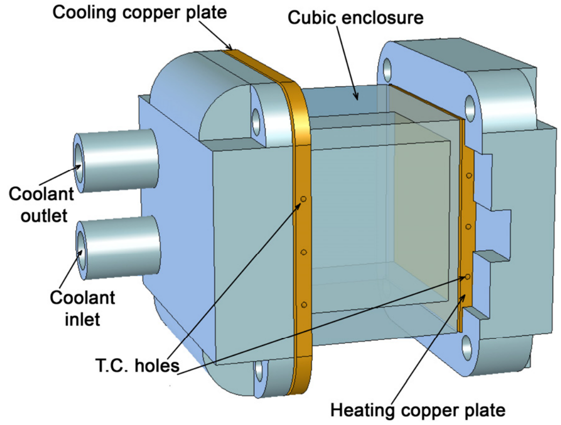

Figure 1. Experimental apparatus. The internal length of the cubic enclosure is 0.032 m.

The enclosure was filled with an 80% mass aqueous solution of glycerol in which gadolinium nitrate hexahydrate Gd(NO3)3·6H2O was dissolved to 0.8 mol/kg in order to make the working fluid paramagnetic. The measurement of the fluid properties such as the magnetic susceptibility, density, viscosity and thermal expansion coefficient were described in detail in [8, 9], and are not repeated here. Other properties (those shown with an asterisk) are estimated for the 80% mass of aqueous solution of glycerol. The major properties of the working fluid are listed in Table 1.

Property	Value	Unit
α^*	1.01×10^{-7}	m^2/s
β	0.52×10^{-3}	1/K
λ^*	0.397	W/(m·K)
μ	86.89×10^{-3}	Pa·s
ν	5.9×10^{-5}	m^2/s
ρ	1463	kg/m^3
χ	23.09×10^{-8}	m^3/kg
Pr	584	-

Table 1. Physical properties of the working fluid.

Flow Visualization

In order to visualize and measure the temperature field, a very small amount of thermo-chromic liquid crystal slurry (KWN-20/25, Japan Capsular Products) was added to the working fluid. The concentration of the slurry was adjusted for each experiment, and was in general less than 1 ppt. Quantitative temperature information was extracted from the experimental photographs using the Particle Image Thermometry (PIT) technique [8, 9]. Figure 2 shows a photograph of the enclosure position in the experiment. The 10-Tesla super-conducting magnet was placed horizontally with the experimental model located just outside the bore of the magnet. Experimental photographs were taken for two middle cross-sections: vertical and horizontal respectively, in two separate runs. In order to generate a vertical light-sheet, a mirror was placed above the enclosure to redirect the white light sheet coming from a projector lamp located at a distance of 3 m from the system to the downward direction. In this way, a vertical cross-section of the cube was illuminated, giving a colour map of the temperature field. For photographing a horizontal cross-section, a horizontal light-sheet was directed towards the mid height of the enclosure, and the image of a colour map was reflected by the mirror and then captured by a camera. All experimental images were taken by a Canon EOS 10D digital camera with a lens Canon EF 70-200mm f2.8 IS after a steady state was attained for each case.

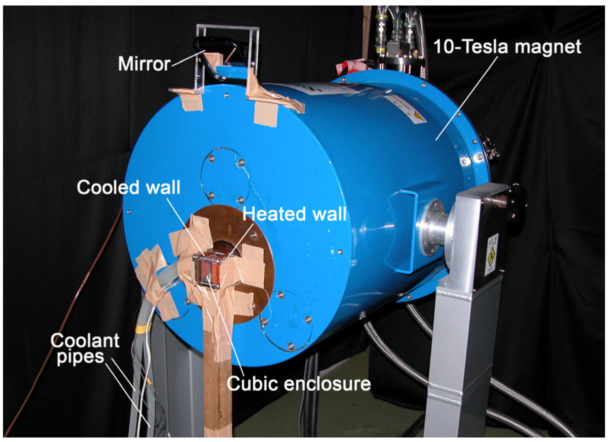

Figure 2. Photograph of the enclosure position in the experiment.

Numerical Model

Corresponding numerical simulations were carried out in order to compare the numerical predictions with the experimental results. The numerical model for the magnetic convection of a paramagnetic fluid is based on the work of Tagawa et al. [6]. It employs Curie's law under which the magnetic susceptibility of a paramagnetic substance is inversely proportional to its absolute temperature. For not very large temperature differences, the magnetic susceptibility may be represented by a Taylor expansion and the magnetic force term in the momentum equation becomes proportional to the temperature T. When a temperature difference occurs in a paramagnetic fluid, a magnetic buoyancy force is generated in the presence of a magnetic field. Examples of our previous computations using the same numerical model can be found in [8, 9, 10, 11].

The non-dimensional forms of the governing equations are written as follows:

- Continuity equation

$$\nabla \cdot \boldsymbol{U} = 0 \qquad (1)$$

- Momentum equation

$$\frac{D\boldsymbol{U}}{D\tau} = -\nabla P + \Pr \nabla^2 \boldsymbol{U} + \mathrm{Ra} \Pr T\left[\boldsymbol{e}_z - \gamma \frac{C}{2} \nabla B^2\right] \qquad (2)$$

- Energy equation

$$\frac{DT}{D\tau} = \nabla^2 T \qquad (3)$$

- Momentum parameter for paramagnetic fluids

$$C = 1 + \frac{1}{\beta \theta_0} = 7.56 \qquad (4)$$

- Prandtl number

$$\Pr = \frac{\nu}{\alpha} = 100 \qquad (5)$$

- Rayleigh Number

$$Ra = \frac{g\beta l^3 (\theta_h - \theta_c)}{\alpha \nu} = 28040 \cdot \Delta\theta \qquad (6)$$

- Gamma parameter

$$\gamma = \frac{\chi_0 b_0{}^2}{\mu_m g l} = 0.586 \cdot b_0{}^2 \qquad (7)$$

The equations were normalized using the following formulae:
$X = x/x_0$, $Y = y/y_0$, $Z = z/z_0$, $U = u/u_0$, $V = v/v_0$, $W = w/w_0$, $\tau = t/t_0$, $P = p/p_0$, $\boldsymbol{B} = \boldsymbol{b}/b_0$, $T = (\theta - \theta_0)/(\theta_h - \theta_c)$, $t_0 = l^2/\alpha$, $x_0 = y_0 = z_0 = l$, $u_0 = v_0 = w_0 = \alpha/l$, $b_0 = \mu_m i/l$, $p_0 = \rho_0 \alpha^2/l^2$.

The boundary conditions for this system are given as follows:

$U = V = W = 0$	at all walls of the cube,
$T = 0.5$	at $X = -0.5$,
$T = -0.5$	at $X = 0.5$,
$\partial T/\partial Y = 0$	at $Y = -0.5, 0.5$,
$\partial T/\partial Z = 0$	at $Z = -0.5, 0.5$.

The initial condition is the conduction state:
$U = V = W = 0$ and $T = -X$ for $-0.5 \le X \le 0.5$.

All the above partial differential equations were approximated by a finite difference method. The HSMAC (Highly Simplified Marker and Cell) method was used to concurrently solve for the pressure and the velocity fields [12]. The number of meshes was chosen to be 40×40×40. Distribution of the magnetic field was computed using the Biot-Savart law for a multi-coil system with two solenoids (as for our real 10-Tesla super-conducting magnet).

The average Nusselt number on the hot wall was computed as follows:

$$Nu = \frac{\int_{-0.5}^{0.5}\int_{-0.5}^{0.5} \left(\partial T/\partial X\right)_{X=-0.5}^{convection} dYdZ}{\int_{-0.5}^{0.5}\int_{-0.5}^{0.5} \left(\partial T/\partial X\right)_{X=-0.5}^{conduction} dYdZ} \qquad (8)$$

Results and discussion

As indicated above, flow visualization was made with a 10-Tesla super-conducting magnet with the experimental model placed just outside the bore of the horizontally located magnet. In such a configuration, the magnetic force acts in the perpendicular direction to the gravitational force and always attracts paramagnetic fluids. However, due to the difference in the magnetic susceptibility caused by the temperature change (according to the Curie's law), cold fluid has a larger value of magnetic susceptibility and is attracted more. On the other hand, hot fluid has a smaller value of the magnetic susceptibility and is attracted less. Because the fluid is enclosed in a cavity, and the continuity equation has to be satisfied, it always appears that the relatively cold fluid is attracted to the magnet, and the relatively hot fluid is repelled from the magnet, as will be seen later.

Table 2 lists the experimental cases presented in this report. Two experimental runs were carried out for 0 and 10 Tesla respectively, and the photographs were taken for vertical middle cross sections (Cases F0 and F10) and horizontal middle cross sections (Cases T0 and T10) respectively.

The experimental and numerical results are shown in Figures 3, 4, 5 and 6 respectively. Figure 3 shows the experimental photographs (on the left) of the vertical middle cross-section, and the corresponding numerical simulations (on the right) for Cases F0 and F10. Figure 4 shows experimental photographs (on the left) of the horizontal middle cross-section and the corresponding numerical simulations (on the right) for Cases T0 and T10. It is worth noting that the experimental photographs in Figure 4 were taken through the mirror, and the magnet position is the closest to the bottom of the photographs. Figure 5 plots isothermal surfaces and Figure 6 long-time streak-lines for Cases T0 and T10 (the figures correspond to the mirror projection of Figure 4).

Case	b_0 [T]	γ [-]	T_h [ºC]	T_c [ºC]	ΔT [ºC]	Ra [-]
F0	0	0	21.54	17.91	3.68	1.02×10^5
F10	10	58.60	21.18	17.92	3.26	9.14×10^4
T0	0	0	21.59	17.91	3.68	1.03×10^5
T10	10	58.60	21.07	17.94	3.13	8.78×10^4

Table 2. Experimental parameters (*F* denotes that the experimental photo was taken from the front of the cube, *T* denotes that the photo was done from the top through the mirror).

At a magnetic induction of 0 T, the magnetic buoyancy force is not acting, and thus pure gravitational natural convection can be observed. In the upper part of the enclosure, relatively hot fluid prevails, in contrast to the lower part of the cube where the fluid is relatively cold, as seen in Figures 3a and 5a. A large roll is generated as is seen from the long-time streak lines in Figure 6. The hot fluid flows upward along the right heated wall and along the top ceiling toward the left cold wall. Colder fluid flows downward along the cold wall and along the bottom adiabatic wall toward the hot wall. Figure 4a shows the temperature field in the horizontal middle cross-section. It is clear from this figure that the flow is symmetrical. Close to the adiabatic walls, wall effect can be observed. This observation is also confirmed by the isothermal surfaces computed for the corresponding case in Figure 5a. The average Nusselt number computed for Case T0 was 4.712.

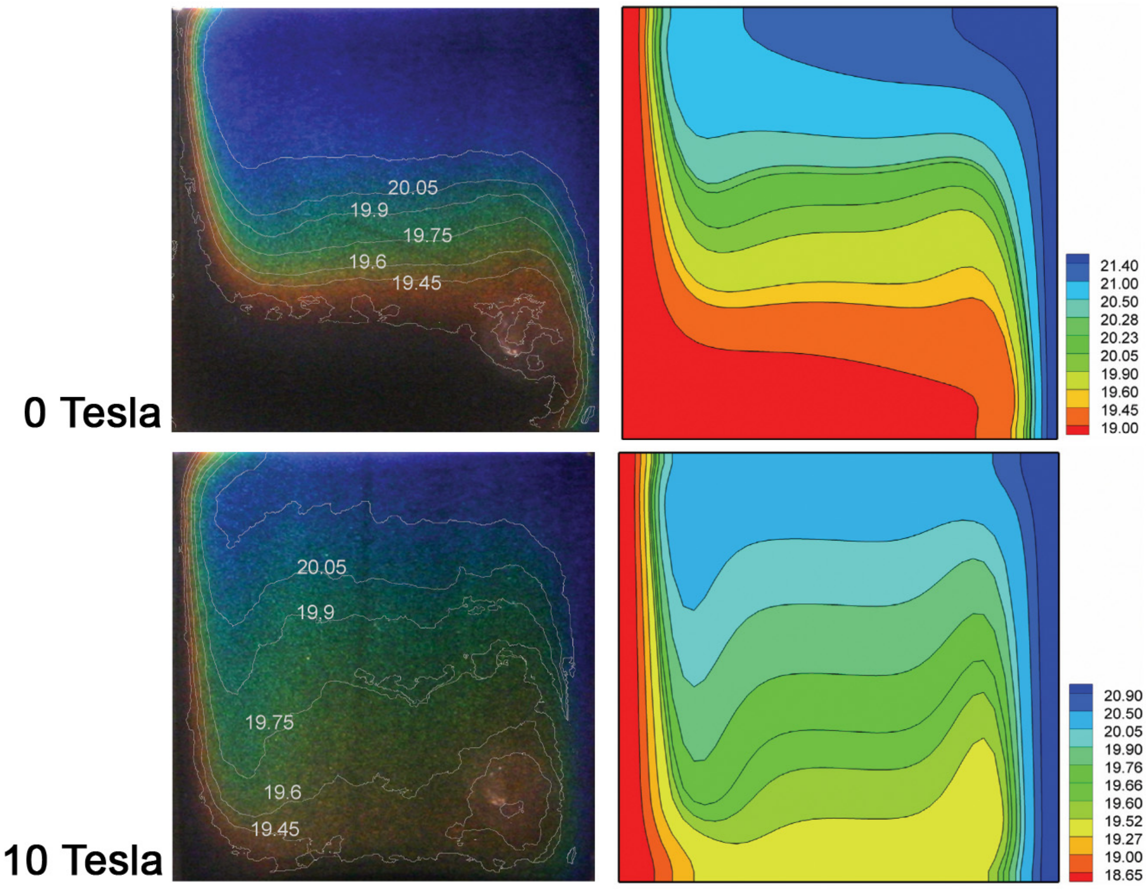

Figure 3. Isotherms on the vertical mid cross-sectional plane for Cases *F0* and *F10*. Left: experiment + PIT; Right: corresponding numerical simulations.

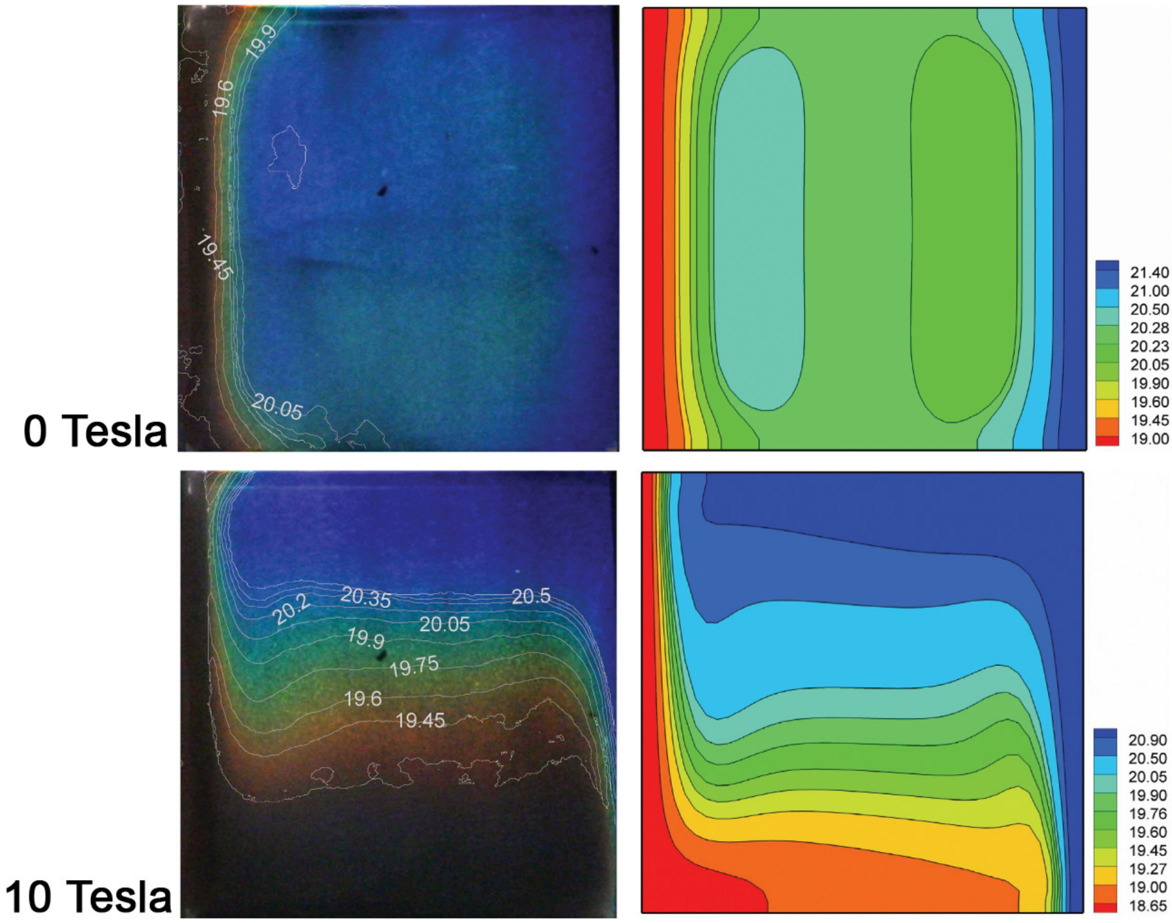

Figure 4. Isotherms on the horizontal mid cross-sectional plane for Cases *T0* and *T10*. Left: experiment + PIT; Right: corresponding numerical simulations.

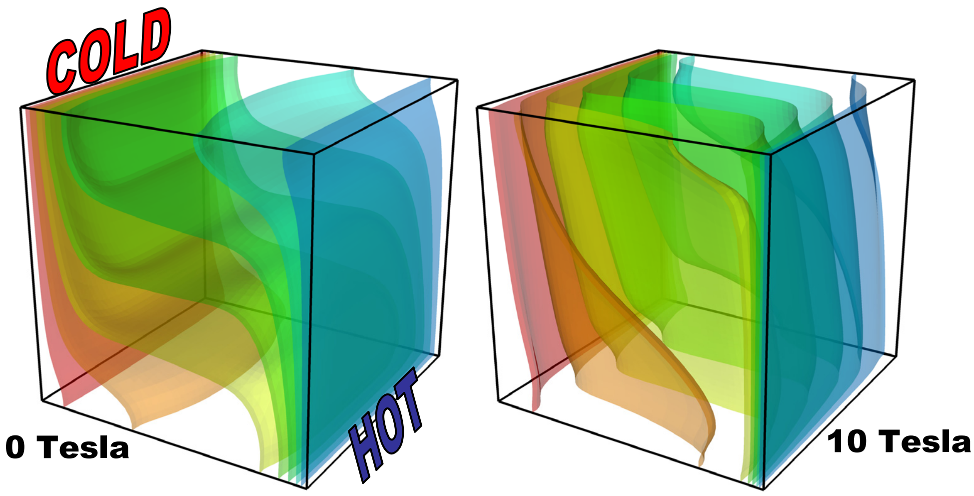

Figure 5. Isothermal surfaces computed for Cases *T0* (left) and *T10* (right).

At a magnetic induction of 10 T, the situation changed. The gravitational buoyancy force acted together with the magnetic buoyancy force. As a consequence, a nearly horizontal circulation of the fluid was observed. The symmetrical profile of the temperature in the horizontal cross-sectional plane was destroyed. This can be clearly seen in Figure 4b. The relatively colder fluid (in brown/black colours) was "attracted" toward the magnet location and relatively warmer fluid was "repelled" from the magnet location. Figure 6b shows this flow behaviour more clearly as a skewed and almost horizontal circulation. The gravitational force was overcome by the magnetic force in this case; however the effect of the gravitational buoyancy still can be seen in Figure 5b in which the relatively colder fluid falls downwards along the lower part of the front adiabatic wall. The calculated average Nusselt number for case *T10* was 6.549, indicating enhancement of the convection and thus heat transfer.

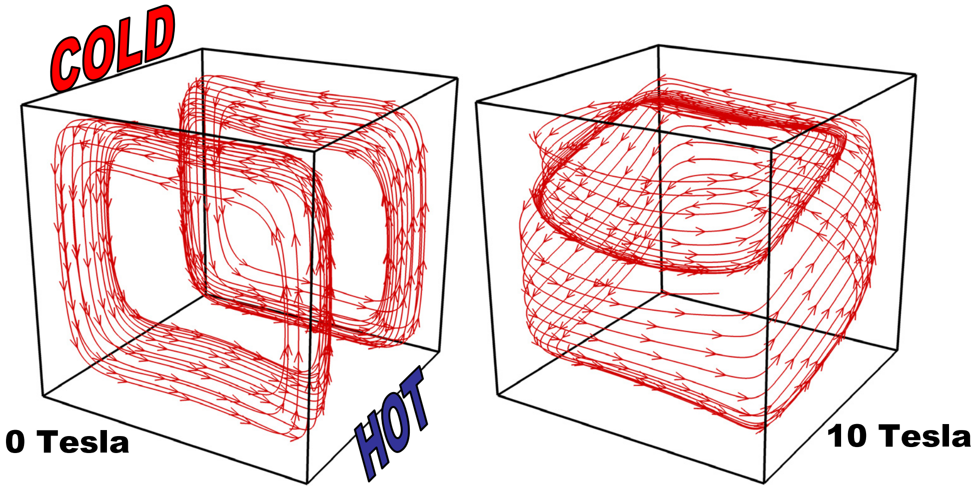

Figure 6. Long-time streak-lines computed for Cases *T0* (left) and *T10* (right).

Conclusions

In this paper, the convection of a paramagnetic fluid under a strong magnetic field in a cubical enclosure was investigated experimentally and numerically. The enclosure was placed horizontally with the rear adiabatic wall close to the magnetic coil. The experiment was carried out for the case with and without a magnetic field. The objective of this study was to show that gravitational convection can be overcome in terrestrial conditions with the application of a magnetic field. It was shown that at 10 T, with the magnetic force acting perpendicularly to gravitational force, almost horizontal convective circulation was established. If an even stronger magnetic field is imposed, the gravity effect will become negligible. Other examples of suppression or enhancement of the gravity effect on natural convection flows can be found in [6-11]. The present three-dimensional numerical computation carried out for comparison purposes verified the numerical code for the study of convection phenomena in a strong magnetic field.

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