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The development of turbulent pipe flow

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Abstract

Whilst turbulence still remains one of the great mysteries of classical physics, its reputation for a chaotic lack of structure is under intense scrutiny. Research now points to the existence of highly organized large-scale structures within turbulent flows. Much of this research have almost exclusively been carried out within a so called fully-developed region of flow. This region is downstream of a point beyond which the flow field's behavior is invariant. However, it has not been conclusively proven that a fully developed region actually exists. A literature survey found that most facilities are cited as being 'sufficiently long', yet no formal definition or documentary evidence of the fully developed condition is widely accepted. The aim of the study is to produce a detailed analysis of the flow from the uniform inlet conditions through to the fully-developed turbulent state, along with an accurate definition of what constitutes fully developed flow. This investigation concerns the affect of both the growth of large-scale structures and their role in the evolution of the flow to the fully-developed condition. With the increasing acceptance of the turbulent large-scale structures model, previous research has not yet shown how large-scale structures affect this development length. These aims were achieved by the design, development and deployment of a carriage to transport hot-wires within the pipe allowing measurements at any stream wise point within the working section

Introduction

The study of development length of duct flows has been approached many ways over the years, however recent developments in the understanding of the nature of turbulent flows including the existence of highly ordered eddy trains or large-scale structures [4] within the flow have led to the authors' renewed interest in development length work. Previous investigations of internal flows such as pipes, have almost exclusively been carried out within a so-called fully-developed region of flow. This region of flow is believed to be streamwise invariant. However, it has not been conclusively proven that a fully developed region actually exists, or how far downstream if it does. A literature survey found that most facilities are cited as being 'sufficiently long', yet no formal definition or documentary evidence of the fully developed condition is widely accepted, this is however not a product of inactivity on the part of researchers. As data acquisition and analysis methods have advanced, more and more stringent conditions have been placed on the definition of fully developed flow. Each subsequent work on the matter has indeed shown the existence of a fully developed flow satisfactory to their own conditions often adding a subsequent measure to the work preceding theirs. Despite this extensive time and effort spent on pipe flow development research, the lack thereof is mainly attributed to the difficulty in gaining access to sufficient stream-wise positions within the working section for data collection. Taking, for example, the pipe flow apparatus used in the current study measuring stations were originally only available at $60D$ intervals, where D is the pipe diamter.

Early judgements of development length took only mean velocity into consideration. For example, [7] suggested fully developed mean flow was achieved in his experiment at $40D$. More detailed work by [2] showed a mean velocity overshoot within the inlet region of the pipe before the centerline velocity settled to its final fully developed value. This 'overshoot' phenomenon was shown to be subtle and of short streamwise duration. The latter observation would imply that any detailed study of developing pipe flow will likely miss important flow changes due to the aforementioned difficulty of having good streamwise resolution of measurement stations.

In later work by Perry & Abell [8] at higher Reynolds numbers up to 175×10^3, a development length of $71.9D$ was suggested for the flow to become fully developed (Reynolds number is defined as $Re = U_bD/\nu$, where U_b is the bulk velocity and ν is kinematic viscosity). Their definition of fully developed flow required that the mean velocity *and* the turbulence intensity, $\overline{u'^2}$ profiles were the same for two consecutive measurement stations. Even more stringent conditions were imposed by Zagarola & Smits [9] who suggested an empirical model based on the characteristic time scale for the development of large scale structures. This empirical form was calibrated using measurements by Dean & Bradshaw [3], again acquired at only two points.

Recent investigations into the large-scale features of turbulence (e.g. Adrian [1], Hutchins & Marusic [4] and Monty *et al.* [6]) have revealed coherent structures up to $20R$ [6], where R is the pipe radius. With such large stream-wise lengths, it seems reasonable to assume that their development to an invariant state may be critical in the development length problem [9].

Clearly then, the development length required to ensure fully developed turbulent flow in a pipe is neither certain nor is the development process well documented. Therefore, the aim of the study is to produce a highly detailed analysis of the flow from the uniform inlet conditions through to the fully-developed turbulent state, along with an accurate definition of what constitutes fully developed flow.

Experimental apparatus

Pipe flow facility

The pipe flow facility is located in the Walter Bassett Aerodynamics Laboratory at the University of Melbourne. A radial fan located at the exit sucks air through a honeycomb screen into a settling chamber interspersed with 5 mesh screens at the entrance to the pipe. Immediately following the settling chamber is a 5.2:1 circular contraction. These provisions are intended to introduce a uniform, low turbulence intensity flow into the pipe. The entrance to the 0.0988 m diameter, drawn brass working section is lined with a 150 mm long, 60*grit* sand paper trip. Each section is nominally 6 metres in length and the overall pipe length is approximately $400D$. Five pressure tappings located at approximately six meter intervals enable the pressure gradient to be measured accurately. Results were obtained at $Re \approx 100\times10^3$ and $Re \approx 200\times10^3$.

Carriage

In order to obtain the measurements required to improve our understanding of turbulent pipe flow development, it was required that a probe be built capable of *any* streamwise movement within the pipe working section whilst minimizing the flow disruption. Furthermore it was required that the positioning of the probes was accurate such that a figure for the fully developed length could be determined with minimal error. Following much discussion, the authors realised that the desired measurements could only be obtained with an internal carriage. The final result is illustrated in Figure 1. The slender design features a narrow body and two sets of three legs spaced equally about the centerline (i.e., 120°). The upper legs are equipped with two magnets which are attracted to magnets housed in an external guide apparatus, through the brass wall of the pipe. Magnetic coupling between the two components allows the probe assembly to be moved within the working section without the need to stop the flow or disassemble the working section. Five hot-wire anemometers are arranged vertically, each attached to 15*cm* tubes protruding from a vertical bar mounted on the front section of the carriage. Measurements were taken simultaneously at $0.3R, 0.5R, 0.7R$ and $0.8R$ from the pipe wall with the fifth hot-wire mounted at the centerline R. To enable in situ calibration of the hot-wire anemometers, a custom built pitot-static tube was mounted at $0.2R$. A seven-meter-long, custom-made umbilical cable attached the carriage to an AA Labs AN1003 anemometry system and MKS 680 pressure transducer. A *Microstar Laboratories 4000a* data acquisition board allowed simultaneous sampling on all six channels (five hotwires and one pressure transducer) at 20 kHz for two minutes. This total measurement time corresponds approximately to the time taken for 36000 and 72000 pipe radii to pass the hot-wire probes for the low and high *Re* respectively.

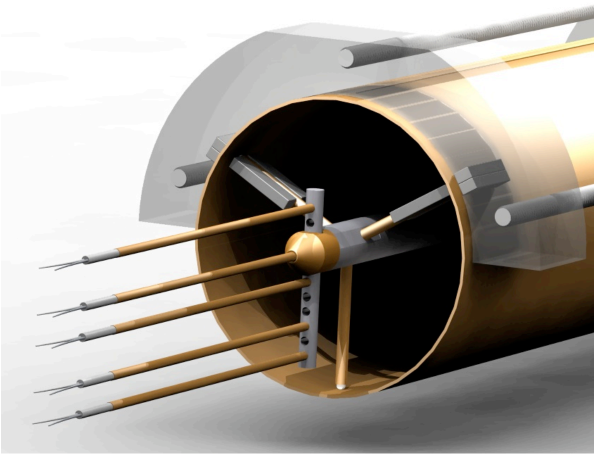

Figure 1: Hot-wire probe carriage illustrated within the working section of the pipe. The external magnetic guide used to position the probes during measurement is also shown.

Experimental Procedure and Data Analysis Methods

Measurement procedure

Utilizing the aforementioned pitot static setup, calibration was performed before each length of pipe was measured. Each of the hot-wires was calibrated in situ at the end of a fully-developed channel flow. The resulting calibration data were subsequently fitted with a fourth order polynomial, allowing the velocity of the data sampled from the pipe to be calculated. The inherent delicacy of the hot-wire array and the time required to traverse each six metre section of pipe required that only one 6 metre section could be sampled before re-calibration. For each section of the pipe, velocity measurements were taken at intervals of $2.5D$, spanning from the pipe entrance to a distance $228D$ downstream. The pressure drop in the streamwise direction, the air temperature and pressure were measured before and after the data collection for each 6-metre section.

Non-dimensionalised quantities

The inability to traverse the entire $\sim 200D$ length of pipe in one sitting led to slight mismatching of Reynolds number between each $\sim 60D$ experiment. This led to the need to scale data from separate runs to produce consistency in the statistical and spectral analysis contained in later sections. For this reason, the friction velocity, derived from the the wall shear stress, τ_w was required:

$$\rho {U_\tau}^2 = \tau_w = \frac{D}{4}\left|\frac{dp}{dx}\right|. \qquad (1)$$

The pressure gradient is reported to become constant very early in the development of the flow [2], meaning that pressure tappings located more than $100D$ from the inlet should provide an accurate value of U_τ.

Statistical analysis

As discussed in the introduction, previous research has focussed on the convergence of flow statistical quantities as an indication of the development length. Thus, for each measurement location these same statistical quantities were calculated for each hot-wire from the data collected over two minutes of sampling time. Included in the analysis are the local mean velocity, variance, $\overline{u'^2}$, skewness, $\overline{u'^3}$ and flatness, $\overline{u'^4}$.

With a renewed importance being placed upon the role of large-scale structures, the main[1] analysis performed was that of spectral analysis via Fourier transforms. The benefit of this technique is that it identifies the wavenumbers of structures contributing most to the kinetic energy of the flow. As such, the development of the spectra was analysed, paying particular attention to the energy associated with the low wavenumber (large-scale) structures. Fourier spectral analysis is a common technique employed to detect these structures, however it should be noted that it has the limitation of assuming the structures have sinusoidal velocity signatures. For this comparative study, the limitations of the technique are not expected to impact the conclusions drawn.

Results and Discussions

Azimuthal invariance

The rotational mobility of the probe enabled flow measurements to be taken at radial positions 90 degrees apart. This enabled in situ measurement of the flow at different azimuthal orientation with the same experimental rig which permitted a measure of the symmetry of the flow. The data recorded at various azimuthal positions indicate axis-symmetry to within ±2%. These measurements were taken to help establish the quality of the flow and ensure all conclusions were due to axis-symmetric flow development only.

Velocity profiles

Figure 2 shows the mean velocities at the centreline, U_{CL} with dashed lines marking ±1% from the final value. All velocities were scaled with the average U_{CL}, except over the first $45D$, where U_{CL} was taken to be the mean of the last three data points

[1] in terms of CPU hours required

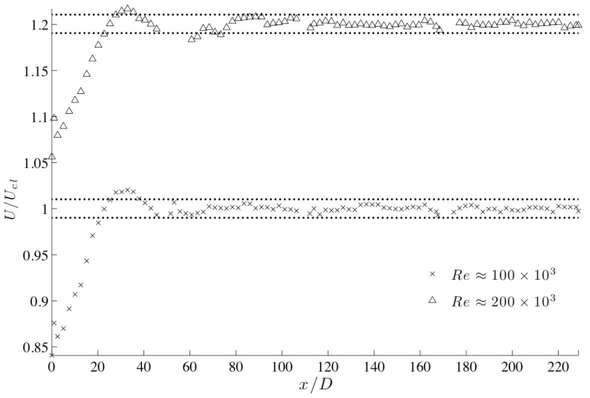

Figure 2: Centreline mean velocity along the streamwise direction for $x/D = 0$ to 230 for $Re \approx 100 \times 10^3$ and 200×10^3. Data points for $Re \approx 200 \times 10^3$ vertically shifted 0.2 units for clarity.

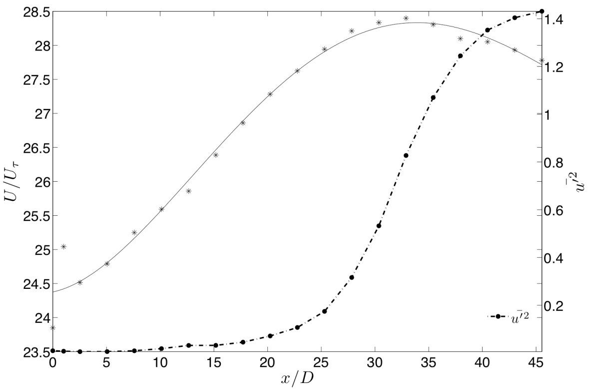

Figure 3: Centreline mean velocity scaled with U_τ and the corresponding turbulence intensity $\overline{u'^2}$ plotted against streamwise distance, x/D.

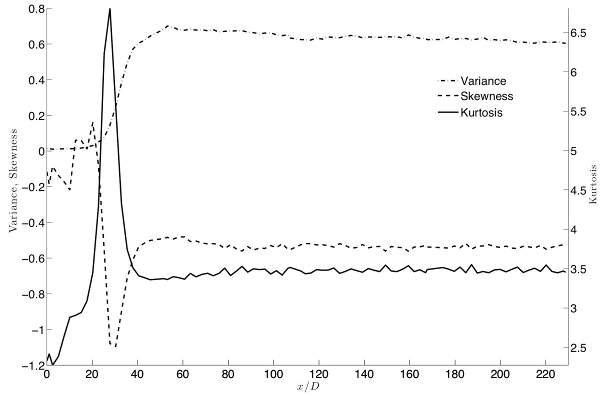

Figure 4: Variance scaled with $U_\tau{}^2$, skewness, and kurtosis for $Re \approx 100 \times 10^3$, in the streamwise direction.

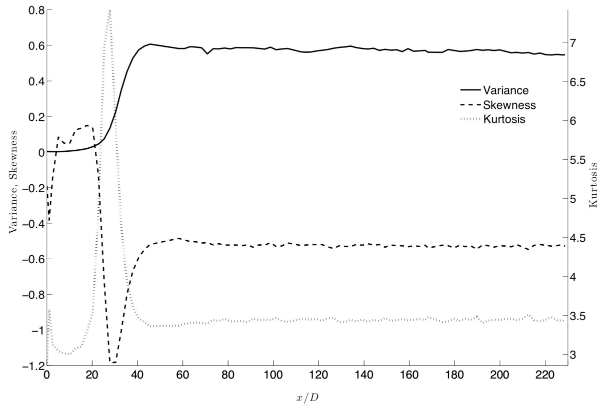

Figure 5: Variance scaled with the fully develop $U_\tau{}^2$, skewness, and kurtosis for $Re \approx 100 \times 10^3$, along the streamwise direction for $x/D = 0 \rightarrow 230$.

of the section (due to the lack of development). These results agree reasonably well with the findings of Abell and Perry [8] who suggested that the mean velocity was invariant after $71.9D$. In fact, as this study provided measurements taken at very small streamwise intervals, the mean velocity appears to be invariant at a streamwise distance of $x/D \approx 60$ for $Re \approx 100 \times 10^3$. The required length for invariant mean velocity was extremely similar for $Re \approx 200 \times 10^3$, although no conclusions can be drawn about Reynolds number effects since the two *Re* are not significantly different in magnitude.

The two velocity variations shown in figure 2 support the velocity peak observations of [2]. Centerline velocity distributions for both Reynolds numbers indicate a distinct overshoot before the mean velocity becomes invariant. It should be remarked that this overshoot occurs over a relatively short streamwise distance of approximately $10D$. As previously discussed, measurements performed at larger intervals by past workers may well have missed the peak all together. A plausible physical argument for the existence of the velocity peak may be: as the flow enters the pipe and slows due to shear at the wall, the central core of the flow speeds up to maintain a constant mass efflux. The centreline velocity must then reach a peak value as the core diminishes, before settling to a lower mean velocity. Comparison of the variance with the position of the mean velocity peak is shown in figure 3 and suggests that the peak velocity is due to the reduction in cross-sectional area of the laminar core as the axially symmetric boundary layer encroaches on the center of the pipe; the peak occurring just before the boundary layer reaches the pipe centerline and the transition to a fully turbulent mean velocity value is complete. The lagging peak of the intensity statistic is indicative of the peak centreline velocity occurring within the intermittently turbulent core region with the centreline velocity decreasing as the transition to fully turbulent flow is made.

Higher-order statistics

Figures 4 and 5 display development of the higher-order turbulence quantities (variance, skewness, kurtosis). Referring to kurtosis in particular, peaks are associated with sudden large fluctuations about the mean, which is indicative of the transition from flow with intermittent turbulence to fully turbulent. Shortly after the peak, $\overline{u'^4}$ becomes invariant for the remaining length of the pipe. The skewness displays much the same behaviour. For the variance, the higher Reynolds number data show that this quantity reaches its fully developed value before the higher-order statistics which might be expected. However, for the lower *Re* the variance slowly drops to its final value at $\sim 110D$. However, since this appears to be an anomalous result, further investigation will be required to confirm it is accurate. Setting this result aside, it would appear that the highest-order statistics are fully developed by 80D as suggested by Perry & Abell [8].

Spectral development

Plots of pre-multiplied spectra $k_x\Phi_{uu}$ versus k_xR for increasing values of x/D yields a more complete picture of the structure lengths present within the early stages of the transition to fully developed flow. Due to the unprecedented amount of data being generated and the investigation focussing on *how* the flow physically develops, a more intuitive method was required for the presentation of data. Conventional methods of plotting $k_x\Phi_{uu}$ versus k_x on two-dimensional log-normal axes did not allow for the streamwise development to be clearly visualised. As such, the maps presented in figures 6 and 7 below attempt to convey the desired information. A top down view of the contour map of the pre-multiplied energy spectra enables the rapid identification of important flow features. The horizontal axis is the streamwise position within the pipe working section with the vertical axis denoting wave number, k_x; red colouring indicates high energy regions and blue the low energy regions. Both figures illustrate the growth of the turbulence from the laminar inlet conditions. With laminar conditions existing at the inlet the plots show extremely low energy for all wavenumbers, as expected. The region of the flow where the core becomes intermittently turbulent is indicated by the growth of the energy peak between $k_xR = 80$ and 100. It is thought that this method for the presentation of large amounts of spectral data enables better isolation of trends within the development of turbulent flow than would have otherwise been possible.

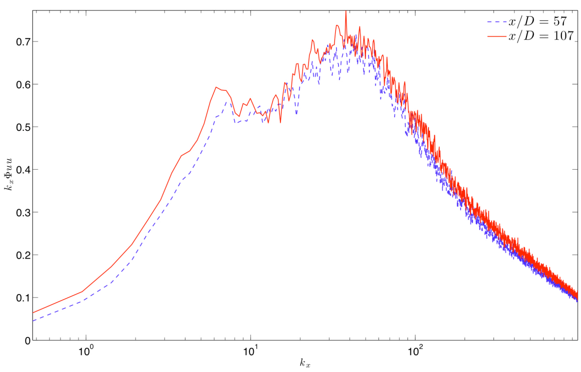

Figure 8: Pre-multiplied spectra $k_x\Phi_{uu}$ at $y/R = 0.5$ for $x/D = 57$ and 107. Note the shift in the position of the secondary, low wavenumber peak.

As x/D increases past 50, the bulk shape of the pre-multiplied spectra begins to converge to a final shape. This initial transition is well defined and easily identified, implying that the turbulent flow quickly reaches a fully developed condition with respect to the energy spectrum. However, a closer look in figure 8 illustrates the divergence towards lower wave numbers of the low wave number energetic peak centred roughly around $k_xR = 5$. As the flow develops, the eddy packet hypothesis [5] suggests that larger numbers of eddies, termed hairpin vortices are falling into line. These large-scale structures continue to entrain eddies as the flow develops, thus shifting energy further into the smaller wavenumbers. As these eddies are merely becoming more aligned, the contribution made to the overall flow energy is not substantially increased (as evidenced by constant variance shown in figure 5). This therefore represents a redistribution of the energy of the turbulent motions of the flow, with the prevailing tendency towards development of the longer structures.

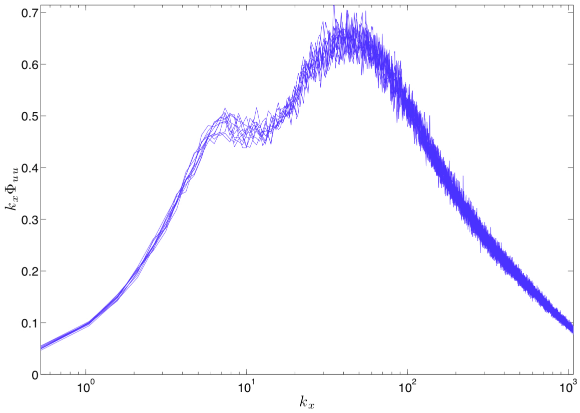

Figure 9: Illustration of the collapse of ten individual Energy Spectra lines $k_x\Phi_{uu}$ versus k_xR from between $x/D = 184$ and 210.

If the fully developed condition is to actually exist it is important that the movement of this peak be traced to an invariant position in line with the definition of fully developed flow. Figure 9 illustrates the collapse of ten separate energy spectra curves taken at a wall normal distance of 0.5R. Note that the plots come from between $x/D = 184$ and 210 to ensure that fully developed conditions were prevailing. The invariance of the position of the low wavenumber energetic peak, indicative of the large-scale structures, would suggest that the long eddy trains have reached a critical length beyond which they do not grow. This entrainment process would then present the final hurdle in the process of the flow becoming fully developed.

Conclusion and further research

To bridge the 60D gap between measurement stations, the successful design and construction of a novel internal carriage housing five hot-wire anemometry probes has enabled the first pipe flow measurements to be made with high streamwise resolution within the working section. The results have helped provide a much better understanding of how the turbulent boundary layer develops from the trip; in particular, the growth of large-scale structures and their role in the evolution of the flow to the fully-developed condition.

This preliminary study shows the gradual development of the centreline velocity, with indication of the location where the boundary layers may have merged, and the interaction between them. Noting that the mean velocity profile requires development over approximately 50D to become invariant, statistical analyses reveal that when consideration is made of the higher-order statistics, a longer length of approximately 80D is required. However, this length was further increased by the addition of the development of the large-scale flow structures as a criteria for fully developed flow. It was shown these structures grow relatively slow, even though their presence was noted very early in the working section. It was found that large scale structures were continuing to grow in size past the points earlier thought to be fully developed. As the large-scale structures require a longer development length, it is proposed that the main criteria for a fully-developed pipe flow should focus on the development of these structures, rather than on the mean velocity profile or higher order statistics.

References

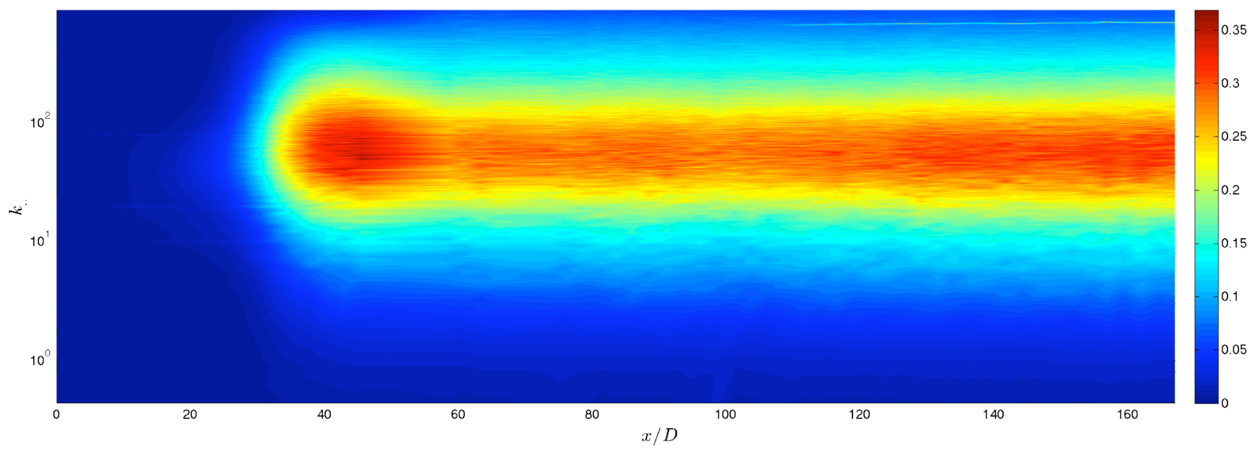

Figure 6: Pre-multiplied pipe spectra $k_x\Phi_{uu}$ for $y/R = 1$, plotted against distance from inlet x/D and k_xR for $Re = 200 \times 10^3$

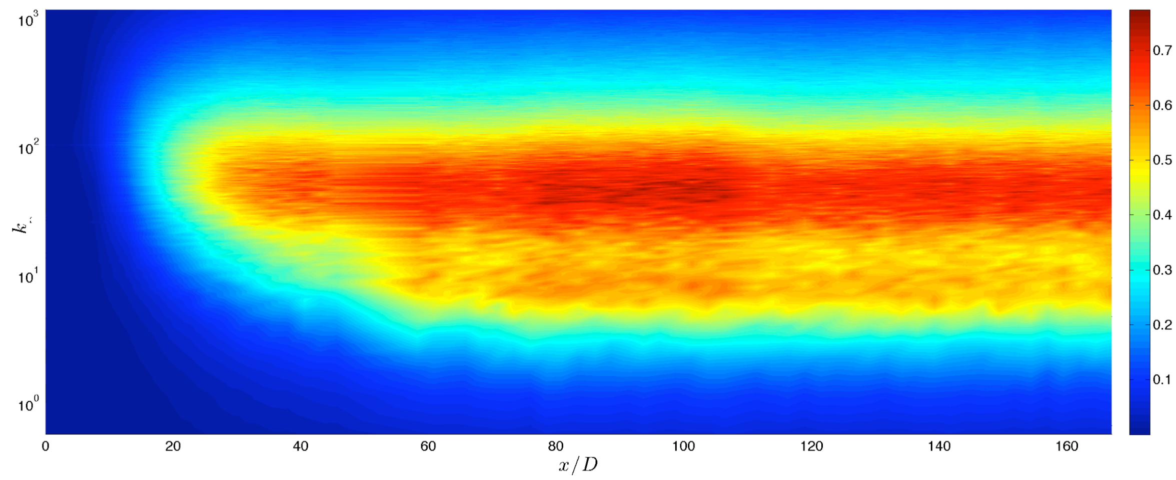

Figure 7: Pre-multiplied pipe spectra $k_x\Phi_{uu}$ for $y/R = 0.5$, plotted against distance from inlet x/D and k_xR for $Re = 200 \times 10^3$

[1] Adrian, R. J., Hairpin vortex organization in wall turbulence, Physics of Fluids, **19**, 2007, 041301.

[2] Barbin, A. R. and Jones, J. B., Turbulent flow in the inlet region of a smooth pipe, Transactions of the ASME, **85**, 1963, 29 to 34.

[3] Dean, R. B. and Bradshaw, P., Measurements of interacting turbulent shear layers in a duct, Journal of Fluid Mechanics Digital Archive, **78**, 2006, 641–676.

[4] Hutchins, N. and Marusic, I., Evidence of very long meandering features in the logarithmic region of turbulent boundary layers, Journal of Fluid Mechanics, **579**, 2007, 1–28.

[5] Kim, K. C. and Adrian, R. J., Very large-scale motion in the outer layer, Physics of Fluids, **11**, 1999, 417–422.

[6] Monty, J. P., Stewart, J. A., Williams, R. C. and Chong, M. S., Large-scale features in turbulent pipe and channel flows, Journal of Fluid Mechanics Digital Archive, **589**, 2007, 147–156.

[7] Nikuradse, J., Gesetzmassigkeiten der turbulenten stromung in glatten rohren, ,, Forschung auf dem Gebiet des Ingenieurwesens, Translated in NASA TT F-10, 359, 1966, **3**, 1932, 1–36.

[8] Perry, A. E. and Abell, C. J., Scaling laws for pipe-flow turbulence, Journal of Fluid Mechanics, **67**, 1978, 257–271.

[9] Zagarola, M. V. and Smits, A. J., Mean-flow scaling of turbulent pipe flow., Journal of Fluid Mechanics, **373**, 1998, 33–79.