Welfare Enhancing Mergers under Product Differentiation

Tina Kao & Flavio Menezes. Discussion Paper No. 350, September 2007, School of Economics, The University of Queensland. Australia.

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EPrint Type:	Departmental Technical Report
Keywords:	merger, differentiaated products, cost asymmetry
Subjects:	340000 Economics;
ID Code:	JEL Classification L11, L12
Deposited By:	

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Welfare Enhancing Mergers under Product Differentiation*

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April 2 2007

Abstract

We follow the duopoly framework with differentiated products as in Singh and Vives (1984) and Zanchettin (2006) and examine the welfare effects of a merger between two asymmetric firms. We find that for quantity competition, the merger increases total welfare if the cost asymmetry falls into a specific range. Furthermore, this parameter range widens if the products are closer substitutes. On the other hand, mergers are never welfare enhancing in this setting when firms compete in prices.

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1 Introduction

In this paper, we focus on differentiated goods and derive the analytical condition for a merger to enhance welfare. The possibility of welfare enhancing merger is not new. Textbook examples show that for homogenous goods, when the cost difference is substantial, a merger can increase social welfare by improving allocative efficiency in the Cournot equilibrium. In a homogenous product set-up, Farrell and Shapiro (1990) provide sufficient

*Flavio Menezes acknowledges the financial support of the Australian Research Council (Grant DP 0663768). We thank Luke Boosey for useful comments.

conditions for profitable mergers to raise welfare with general demand and cost conditions.

We follow the set up in Singh and Vives (1984) and Zanchettin (2006) and derive the analytical condition for a merger to enhance welfare for markets with differentiated products and asymmetric firms. Moreover, we obtain the welfare enhancing condition for Cournot competition with homogenous goods in the limit, i.e., as goods become perfect substitutes. In the same framework, we show that if firms compete in prices, mergers always reduce total welfare. The positive welfare effect of merger comes from improved efficiency by allocating more output to the more efficient firm.

As Zanchettin (2006) points out, the efficient firm always produces more under price competition than under quantity competition. It follows then that the efficiency gains from a merger are lower under price competition than under quantity competition. Moreover, unlike quantity competition, the increase in profits due to efficiency gains under price competition is not enough to outweigh the decline in consumer surplus from higher prices.

We present the model set up in the next section and solve first for the optimisation problem the merged entity faces. Sections 3 and 4 analyse the quantity competition and price competition games in turn and derive the welfare results of mergers. While the main body of this paper deals with substitute goods, Section 5 briefly discusses the case of complementary goods. The final section provides some concluding remarks.

2 The Model

To facilitate comparison, we use the same notation as Zanchettin (2006). Consider a two-sector model with one monopoly firm in each sector. The inverse demand curves for the two goods are

$$p_1 = \alpha_1 - (q_1 + \gamma q_2) \tag{1}$$

and

$$p_2 = \alpha_2 - (\gamma q_1 + q_2)\,. \tag{2}$$

The parameter γ measures the degree of product differentiation. If $\gamma = 0$, the demand for the two goods are independent. For the main body of the paper, we assume that the two goods are substitutes, $0 \leq \gamma \leq 1$.

We assume that the marginal costs of production in markets 1 and 2 are equal to c_1 and c_2, respectively, and that there are no fixed costs. Following Zanchettin (2006), we define an index a to measure the asymmetry between the two firms.

Definition 1 *Let $a \equiv (\alpha_1 - c_1) - (\alpha_2 - c_2)$ and $\alpha_1 - c_1 = 1$. Without loss of generality, assume $a \geq 0$.*

The index a summarises the asymmetry between the two markets and firms. For $a = 0$, two firms are symmetric. For $a \geq 1 - \frac{\gamma}{2}$, the asymmetry between the firms is so large that firm 2 does not exert any competitive pressure on firm 1, and in equilibrium firm 1 sets its quantity at the monopoly level, $q_1 = q_1^M$, and $q_2 = 0$. That is, the limit price required for firm 1 to drive firm 2 out of the market is greater than the monopoly price firm 1 would like to charge at $q_2 = 0$. This is explained in more detail in Sections 3 and 4 for quantity game and price game respectively. We focus on the case $a \leq 1 - \frac{\gamma}{2}$ in this paper.

Note that for the merged entity, without strategic interaction, setting quantity is equivalent to setting price. Before analysing the duopoly game, we solve the merged entity's optimisation problem:

$$\max_{\{Q_1, Q_2\}} (p_1 - c_1) Q_1 + (p_2 - c_2) Q_2. \tag{3}$$

The first order conditions yield

$$Q_1 = \frac{1 - 2\gamma Q_2}{2} \text{ and } Q_2 = \frac{(1-a) - 2\gamma Q_1}{2}. \tag{4}$$

We will denote by Q the quantity choices of the merged entity and by q the quantity choices of each of the firms when they are in competition.

For $a < 1 - \gamma$, the solution is interior:

$$Q_1^* = \frac{1 - \gamma(1-a)}{2(1-\gamma^2)} \quad \text{and} \quad Q_2^* = \frac{(1-a) - \gamma}{2(1-\gamma^2)}. \tag{5}$$

This gives the merged entity's profit equal to

$$\Pi = \frac{1 - 2\gamma(1-a) + (1-a)^2}{4(1-\gamma)(1+\gamma)}, \tag{6}$$

and the resulting consumer surplus is

$$CS = \frac{(1-a)^2 - 2\gamma(1-a) + 1}{8(1+\gamma)(1-\gamma)}. \tag{7}$$

For $a \geq 1-\gamma$, the first order conditions yield $Q_2^* = 0$ and $Q_1^* = \frac{1}{2}$. The merged firm profit is equal to $\frac{1}{4}$, and the consumer surplus is equal to $\frac{1}{8}$.

3 Quantity competition

If the two firms are in competition, each firm i solves $\max_{q_i}(p_i - c_i)q_i$. This yields the following best response function

$$q_i = \frac{\alpha_i - \gamma q_j - c_i}{2}, i, j = 1, 2. \tag{8}$$

For $a \leq 1 - \frac{\gamma}{2}$, both firms produce positive output. This gives

$$q_1^C = \frac{2-\gamma(1-a)}{4-\gamma^2} \text{ and } q_2^C = \frac{2(1-a)-\gamma}{4-\gamma^2}. \tag{9}$$

The resulting profits are

$$\pi_1 = \frac{(2-\gamma(1-a))^2}{(4-\gamma^2)^2} \text{ and } \pi_2 = \frac{(2(1-a)-\gamma)^2}{(4-\gamma^2)^2}. \tag{10}$$

This gives consumer surplus

$$CS = \frac{(4-3\gamma^2)(1-a)^2 + 2(1-a)\gamma^3 + 4 - 3\gamma^2}{2(\gamma+2)^2(2-\gamma)^2}. \tag{11}$$

Notice that, as analysed above, for $a \geq 1-\gamma$, the merged entity would choose to produce only good 1. We plot the different cases, depending on whether or not there is a corner solution, in the following diagram. Our analysis focuses on cases 1 and 2. In particular, we will show that it is possible for a merger to increase welfare in case 1.

3.1 Welfare Results

For the entire parameter range, as expected, industry profit always increases and consumer surplus decreases after the merger. In any two-to-one merger, the merged entity can always mimic the pre-merger behaviour of the firms and, therefore, profits have to be (weakly) higher. For substitute goods, this

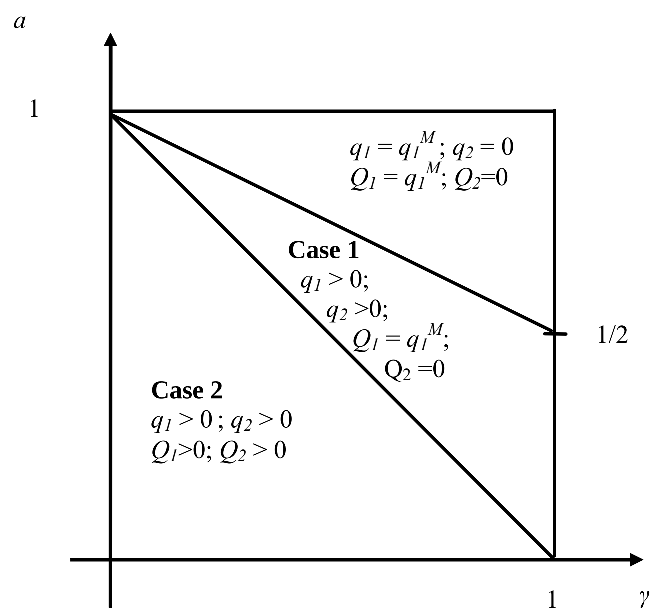

Figure 1: Output equilibirum in (γ, a) space under quantity competition.

means (weakly) higher price and, therefore, lower consumer surplus. However, we show below that under quantity competition, for a given parameter range, two-to-one mergers can be welfare improving; the increase in profits dominate the fall in consumer surplus.

Proposition 1 *For the parameter range,* $\frac{(2-\gamma)(12-4\gamma-3\gamma^2)}{(24-2\gamma^2)} \leq a \leq 1-\frac{\gamma}{2}$, *the total surplus increases post merger under quantity competition.*

Proof. For $1-\gamma \leq a \leq 1-\frac{\gamma}{2}$, if firms are in competition, $q_1 = q_1^C$ and $q_2 = q_2^C$. This gives

$$\pi_1 = \frac{(2-\gamma(1-a))^2}{(4-\gamma^2)^2} \text{ and } \pi_2 = \frac{(2(1-a)-\gamma)^2}{(4-\gamma^2)^2}. \tag{12}$$

The resulting consumer surplus is

$$CS = \frac{\left(4-3\gamma^2\right)(1-a)^2 + 2(1-a)\gamma^3 + 4 - 3\gamma^2}{2(\gamma+2)^2(2-\gamma)^2}. \tag{13}$$

On the other hand, for the merged entity, $Q_1 = \frac{1}{2}$ and $Q_2 = 0$. The merged firm profit is equal to $\frac{1}{4}$ where the consumer surplus is equal to $\frac{1}{8}$.

The total surplus post merger increases if

$$\frac{3}{8} \geq \frac{(2-\gamma(1-a))^2}{(4-\gamma^2)^2} + \frac{(2(1-a)-\gamma)^2}{(4-\gamma^2)^2} + \frac{\left(4-3\gamma^2\right)(1-a)^2 + 2(1-a)\gamma^3 + 4 - 3\gamma^2}{2(\gamma+2)^2(2-\gamma)^2}. \tag{14}$$

This holds for

$$a \geq \frac{(2-\gamma)\left(12-4\gamma-3\gamma^2\right)}{(24-2\gamma^2)}. \tag{15}$$

Finally, note that

$$1-\gamma \leq \frac{(2-\gamma)\left(12-4\gamma-3\gamma^2\right)}{(24-2\gamma^2)} \leq 1-\frac{\gamma}{2}. \tag{16}$$

■

For $1-\gamma \leq a \leq 1-\frac{\gamma}{2}$, the total market output decreases post merger, $q_1^C + q_2^C < \frac{1}{2}$, and the average market up increases as

$$\left(p_1\left(q_1^C, q_2^C\right) - c_1\right)\frac{q_1^C}{q_1^C + q_2^C} + \left(p_2\left(q_1^C, q_2^C\right) - c_1\right)\frac{q_2^C}{q_1^C + q_2^C} > \frac{\alpha_1 + c_1}{2} - c_1,$$

where $p_1\left(q_1^C, q_2^C\right)$ and $p_2\left(q_1^C, q_2^C\right)$ are the prevailing market prices when firms set quantities equal to q_1^C and q_2^C respectively.

Therefore, consumer surplus decreases. The possibility of welfare gain comes from the efficiency gain of shutting down production of the high cost product. It follows that there is welfare gain only if the asymmetry between firms is sufficiently large.

As emphasised previously, for $a > 1-\frac{\gamma}{2}$, firm 2 is so relatively inefficient that in either duopoly competition or post-merger, $q_2 = 0$. A merger of the two firms in this case has no effect on market outcomes.

The possibility of a welfare enhancing merger only occurs in Case 1 where both firms produce positive amounts if they are in competition while the merged entity only produces good 1. The efficiency gain from shutting down the inefficient production outweighs the loss resulting from a higher market price.

Note that the band for a welfare increasing merger is equal to

$$\Delta = 1 - \frac{\gamma}{2} - \frac{(2-\gamma)\left(12-4\gamma-3\gamma^2\right)}{(24-2\gamma^2)}. \tag{17}$$

As the products become closer substitutes, the parameter range for welfare enhancing merger widens:

$$\frac{\partial\Delta}{\partial\gamma} = \frac{\left(\gamma^4 - 32\gamma^2 + 48\right)}{\left(12-\gamma^2\right)^2} \geq 0. \tag{18}$$

Since homogenous good Cournot competition is a special case of our differentiated products framework with $\gamma = 1$, we have the following corollary.

Corollary 1 *For homogenous good Cournot competition, a merger between duopolists yields higher social welfare if* $\frac{5}{22} \leq a \leq \frac{1}{2}$.

While two-to-one mergers can be welfare enhancing under quantity competition, the next section shows that this is not the case for price competition.

4 Price competition

From the inverse demand curves given in Equations 1 and 2, we obtain the demand curves $q_1 = \frac{(\alpha_1-p_1)-\gamma(\alpha_2-p_2)}{(1-\gamma^2)}$ and $q_2 = \frac{(\alpha_2-p_2)-\gamma(\alpha_1-p_1)}{(1-\gamma^2)}$. The firm's optimisation problem can be written as $\max_{p_i} (p_i - c_i)\, q_i$. For $p_1 > c_1$ and $p_2 > c_2$, this yields the following best response functions:

$$p_1 = \frac{\alpha_1 + c_1 - \gamma\,(\alpha_2 - p_2)}{2} \text{ and } p_2 = \frac{\alpha_2 + c_2 - \gamma\,(\alpha_1 - p_1)}{2}. \tag{19}$$

This yields the interior solutions

$$p_1^B = \frac{2\alpha_1 + 2c_1 - \gamma^2\alpha_1 - \gamma\,(1-a)}{(2+\gamma)\,(2-\gamma)} \text{ and } p_2^B = \frac{2\alpha_2 + 2c_2 - \gamma^2\alpha_2 - \gamma}{(2+\gamma)\,(2-\gamma)}. \tag{20}$$

Note that the assumption $a \geq 0$ implies that $p_1 - c_1 \geq p_2 - c_2$.

In contrast to quantity setting game, for differentiated Bertrand, the efficient firm may be able to charge a low enough price to drive the other firm out of the market even if $a < 1 - \frac{\gamma}{2}$. This will occur when $q_2 \leq 0$ or

$$\frac{(\alpha_2 - p_2) - \gamma\,(\alpha_1 - p_1)}{(1-\gamma^2)} \leq 0. \tag{21}$$

This holds for

$$p_2 \geq \alpha_2 - \gamma\,(\alpha_1 - p_1)\,. \tag{22}$$

To enforce this price below c_2, firm 1 needs to choose a price such that:

$$p_1 \leq \alpha_1 - \frac{1-a}{\gamma}. \tag{23}$$

When $p_1 \leq \alpha_1 - \frac{1-a}{\gamma}$, $q_2 = 0$. For $q_2 = 0$, the unconstrained profit maximising price for firm 1 is $p_1 = \frac{\alpha_1 + c_1}{2}$. Therefore, in this case, firm 1's best reply is to charge $p_1 = \min\left\{\frac{\alpha_1+c_1}{2}, \alpha_1 - \frac{1-a}{\gamma}\right\}$.

For firm 2, being the inefficient firm, it charges according to the best response function specified above or if the best response prescribes below marginal cost pricing, it sets $p_2 = c_2$. That is, it charges $p_2 = \max\left\{c_2, \frac{\alpha_2+c_2-\gamma(\alpha_1-p_1)}{2}\right\}$. The kink in the best response occurs when

$$c_2 = \frac{\alpha_2 + c_2 - \gamma(\alpha_1 - p_1)}{2} \text{ or } p_1 = \frac{\gamma\alpha_1 - (1-a)}{\gamma}. \tag{24}$$

Note that

$$\frac{\alpha_1 + c_1}{2} \leq \frac{\gamma\alpha_1 - (1-a)}{\gamma} \text{ if } a \geq 1 - \frac{\gamma}{2}. \tag{25}$$

Therefore, as under quantity competition, for $a \geq 1 - \frac{\gamma}{2}$, firm 1 acts as an unconstrained monopoly and charges $p_1 = \frac{\alpha_1+c_1}{2}$ and $p_2 - c_2 = q_2 = 0$.

For $1 - \frac{\gamma}{2-\gamma^2} \leq a \leq 1 - \frac{\gamma}{2}$, the equilibrium is that

$$p_1 = \alpha_1 - \frac{1-a}{\gamma} \text{ and } p_2 - c_2 = q_2 = 0. \tag{26}$$

Firm 1 charges a price just low enough to drive firm 2 out of the market. Zanchettin (2006) terms the pricing behaviour in this parameter range the limit-pricing equilibrium. Note that in this parameter range, for quantity competition, both firms produce positive output. The ability of firm 1 to exercise limit pricing is the key for Zanchettin's result that the efficient firm prefers price competition.

For $a < 1 - \frac{\gamma}{2-\gamma^2}$, we have the usual interior solution for differentiated Bertrand with the equilibrium $p_1 = p_1^B$ and $p_2 = p_2^B$. We distinguish different cases by if the equilibrium outcome is given by interior solution, corner solution, or limit pricing behaviour. The different cases are plotted in the following diagram.

4.1 Welfare Results

While two-to-one mergers can be welfare enhancing under quantity competition, the next result shows that this is not the case for price competition.

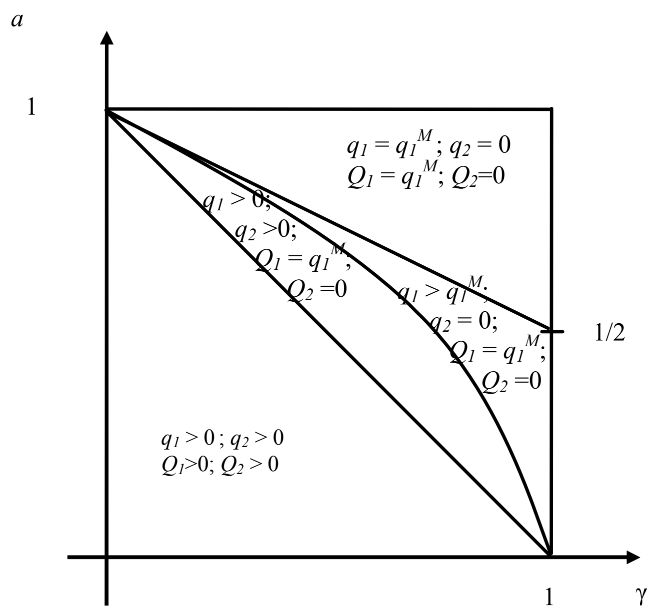

Figure 2: Output equilibirum in (γ, a) space under price competition.

Proposition 2 *When goods are substitutes, a merger from duopoly to monopoly always reduces total welfare if firms compete in prices.*

Proof. Case 1:$1 - \frac{\gamma}{2-\gamma^2} \leq a \leq 1 - \frac{\gamma}{2}$:

If firms 1 and 2 are in competition, $p_1 = \alpha_1 - \frac{1-a}{\gamma}$, $\pi_1 = \frac{(\gamma+a-1)(1-a)}{\gamma^2}$, and $p_2 - c_2 = q_2 = 0$. Consumer surplus is $CS = \frac{(1-a)^2}{2\gamma^2}$. For the the merged entity, since setting price is the same as setting quantity, the equilibrium is the same as the corresponding case for quantity competition, $Q_1 = \frac{1}{2}$ and $Q_2 = 0$.

Total surplus goes down post merger if

$$\frac{3}{8} \leq \frac{(\gamma + a - 1)(1 - a)}{\gamma^2} + \frac{(1 - a)^2}{2\gamma^2}. \tag{27}$$

This holds since in this case

$$(2a + 3\gamma - 2)(2a + \gamma - 2) \leq 0. \tag{28}$$

Case 2: $1 - \gamma \leq a \leq 1 - \frac{\gamma}{2-\gamma^2}$:

If firms 1 and 2 are in competition, $p_1 = p_1^B$ and $p_2 = p_2^B$. This gives

$$\pi_1 = \frac{\left(a\gamma - \gamma - \gamma^2 + 2\right)^2}{(\gamma+2)^2 (2-\gamma)^2 (1-\gamma^2)} \text{ and } \pi_2 = \frac{\left(a\gamma^2 - \gamma - \gamma^2 - 2a + 2\right)^2}{(\gamma+2)^2 (2-\gamma)^2 (1-\gamma^2)}. \quad (29)$$

Consumer surplus is

$$CS = \frac{a^2\left(4 - 3\gamma^2\right) - 2(1-\gamma)(2+\gamma)^2 a + 2(1-\gamma)(2+\gamma)^2}{2(\gamma+2)^2(2-\gamma)^2(1+\gamma)(1-\gamma)}. \quad (30)$$

The merged entity produces $Q_1 = \frac{1}{2}$ and $Q_2 = 0$.

The total surplus goes down post merger if

$$\begin{aligned}\frac{3}{8} \leq\ & \frac{\left(a\gamma - \gamma - \gamma^2 + 2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} + \frac{\left(a\gamma^2 - \gamma - \gamma^2 - 2a + 2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} \\ & + \frac{a^2\left(4-3\gamma^2\right) - 2(1-\gamma)(2+\gamma)^2 a + 2(1-\gamma)(2+\gamma)^2}{2(\gamma+2)^2(2-\gamma)^2(1+\gamma)(1-\gamma)}. \quad (31)\end{aligned}$$

This holds since

$$\begin{aligned} & 4\left(2\gamma^4 - 9\gamma^2 + 12\right)a^2 + 8(1-\gamma)(2\gamma-3)(\gamma+2)^2 a \\ & -(1-\gamma)\left(16\gamma - 9\gamma^2 + 3\gamma^3 - 12\right)(\gamma+2)^2 \\ \geq\ & 0. \quad (32)\end{aligned}$$

Case 3: $a \leq 1-\gamma$:

If firms 1 and 2 are in competition, $p_1 = p_1^B$ and $p_2 = p_2^B$. The merged entity produces $Q_1 = Q_1^*$ and $Q_2 = Q_2^*$ as given in Equation 5.

The total surplus goes down post merger if

$$\begin{aligned} & \frac{1 - 2\gamma(1-a) + (1-a)^2}{4(1-\gamma)(1+\gamma)} + \frac{(1-a)^2 - 2\gamma(1-a) + 1}{8(1+\gamma)(1-\gamma)} \\ \leq\ & \frac{\left(a\gamma - \gamma - \gamma^2 + 2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} + \frac{\left(a\gamma^2 - \gamma - \gamma^2 - 2a + 2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} \\ & + \frac{a^2\left(4-3\gamma^2\right) - 2(1-\gamma)(2+\gamma)^2 a + 2(1-\gamma)(2+\gamma)^2}{2(\gamma+2)^2(2-\gamma)^2(1+\gamma)(1-\gamma)}. \quad (33)\end{aligned}$$

Or if

$$\gamma\left(12 - 5\gamma^2\right)a^2 + 2(1-\gamma)(4-3\gamma)(\gamma+2)^2 a - 2(1-\gamma)(4-3\gamma)(\gamma+2)^2 \leq 0. \quad (34)$$

This holds for

$$a \leq \frac{-(1-\gamma)(4-3\gamma)(\gamma+2)^2 + \sqrt{(1-\gamma)(3\gamma+4)(4-3\gamma)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}}{\gamma(12-5\gamma^2)}. \tag{35}$$

Since $\frac{-(1-\gamma)(4-3\gamma)(\gamma+2)^2+\sqrt{(1-\gamma)(3\gamma+4)(4-3\gamma)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}}{\gamma(12-5\gamma^2)} \geq 1-\gamma$, total welfare always goes down post merger in this case. ■

As pointed out by Zanchettin (2006), the efficient firm produces more under price competition than under quantity competition over the entire parameter space. This is the most apparent in the limit pricing range where the inefficient firm is driven out of the market under price competition while it remains active under quantity competition. Therefore, the efficiency gain from a merger is lower under price competition and not sufficient to outweigh the decline in consumer surplus post merger.

5 Complementary Goods

When the two goods are complements, for the merged firm, the first order conditions with respect to good i increases in the quantity of good j. If the firms compete in quantities, the best responses are upward sloping. There is no corner solution and in equilibrium $Q_1 = Q_1^*$, $Q_2 = Q_2^*$, $q_1 = q_1^C$, and $q_2 = q_2^C$ as given in Equations 5 and 9.

When firms compete in prices, similarly, there is no corner solution and both firms produce positive amounts in equilibrium. Note that $p_2^B > c_2$ for $\gamma < 0$.

Remark 1 *When goods are complements, the inefficient firm always produces positive output in equilibrium. Both industry profit and consumer surplus go up after the merger.*

When goods are complements, firms produce too little since they do not take into consideration the positive externality their increased production has on the other firm. Therefore, after the merger, once the externality is internalised, production of each good increases and industry profit increases. Consumer surplus also increases due to expanded output. Note that for this to hold, we do not require that goods are perfect complements. This is true even for an arbitrarily small degree of complementarity.

Finally, the welfare comparison between price and quantity competition for the two firms has been analysed by Zanchettin (2006) for the case of substitute goods. Zanchettin found that price competition could give the efficient firm higher profit if the cost difference is high and the products are close substitutes. Next we extend his analysis to the case of complements.

Remark 2 *Both firms as well as consumers prefer price competition to quantity competition when goods are complements.*

Proof. See Appendix. ■

When goods are complements, the prices are higher, and quantity produced lower, under quantity competition than under price competition. The ranking of the output under different competition modes is: $q_i^C < q_i\left(p_i^B\right) < Q_i$, where $i = 1, 2$, q_i^C represents the interior solution when firms compete in quantity, $q_i\left(p_i^B\right)$ is the output level under price competition given that the equilibrium price is q_i^B, and Q_i is the output for the merged entity.

6 Conclusions

In this paper, we develop the analytical condition for a merger of duopolists to be welfare enhancing if firms compete in quantities. A merger between goods which are substitutes increases social welfare if the cost difference is substantial. Furthermore, the parameter range for the merger to be welfare enhancing widens if the products are closer substitutes. If firms compete in prices, on the other hand, a merger between duopolists is never welfare improving. Finally, we show that mergers are always welfare improving for any (even very small) degree of complementarity.

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7 Appendix

Proof. of Remark 2: Firm 1 gets higher profit in the price game if

$$\frac{\left(a\gamma-\gamma-\gamma^2+2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} \geq \frac{(2-\gamma(1-a))^2}{(4-\gamma^2)^2}.$$

This holds since $a^2\gamma+2(1-\gamma)a-2(1-\gamma)\leq 0$. Similarly, firm 2 gets higher profit in the price setting game.

Consumer surplus is higher in the price game if

$$\begin{aligned} & \frac{\left(3\gamma^2-4\right)a^2+2(1-\gamma)(\gamma+2)^2a-2(1-\gamma)(\gamma+2)^2}{2(\gamma+2)^2(2-\gamma)^2(\gamma+1)(\gamma-1)} \\ \geq\ & \frac{\left(4-3\gamma^2\right)a^2-2(1+\gamma)(2-\gamma)^2a+2(1+\gamma)(2-\gamma)^2}{2(\gamma+2)^2(2-\gamma)^2}. \end{aligned}$$

This holds since

$$\begin{aligned} & \left(3\gamma^2-4\right)a^2+2(1-\gamma)\left(4-\gamma^2+2\gamma\right)a \\ & -2(1-\gamma)\left(4-\gamma^2+2\gamma\right) \\ \leq\ & 0. \end{aligned}$$

■