

# Hydraulics of Minimum Energy Culverts and Bridge Waterways

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**SUMMARY** The design of culverts and bridge waterways according to the approach described variously as "minimum energy" and as "constant energy" is summarised. The hydraulics of the flow in such waterways is discussed. Even though the flow occurring can be quite complex, the waterways can be designed with simple calculations within certain limitations. Some of these limitations are discussed. Some results concerning energy losses in the expanding part of the flow are presented.

## 1 INTRODUCTION

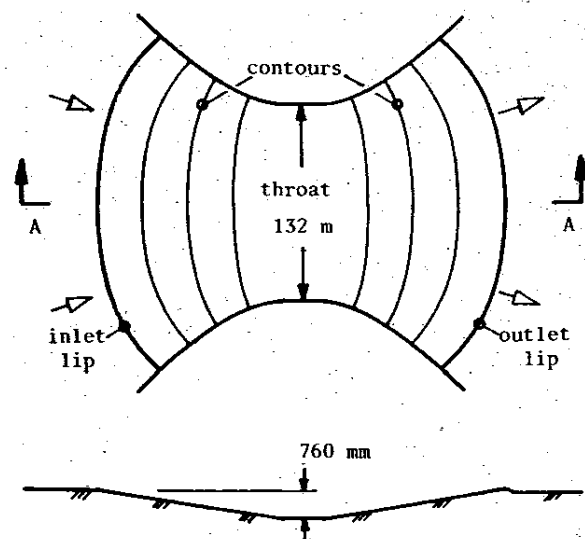
The approach to the design of open channel transitions which is variously described as "minimum energy design" and as "constant energy design" was first expounded by McKay (1971). The title of his report describes the design techniques as being "associated with the concept of constant total energy and compatible specific energy" and, in retrospect, it is a pity that the lengthy title was abbreviated to "Design of Minimum Energy Culverts" (for which the writer must accept responsibility), because this description may have been a source of unnecessary confusion. Indeed, Porter (1978) found it necessary to preface the proceedings of a workshop which discussed these ideas with a note on nomenclature which explained the use of the terms "minimum energy" and "constant energy" in this context. It appears that the confusion which has sometimes arisen from the abbreviated description of the design technique has been a source of some of the controversy which has been associated with it.

In this paper the design technique as applied to culverts and bridge waterways is summarised according to the writer's approach to it. The hydraulics of the flow in such "minimum energy culverts" is then discussed. Although the flow occurring can be quite complex, the waterways can often be designed with simple calculations based on the assumption that the velocity is uniformly distributed across the flow cross-section, within certain limitations. Many have been designed following these procedures and observations of their performance indicate that it has been very satisfactory and generally in accordance with design calculations. Nevertheless, there are limits to the performance of such waterways, particularly with regard to the maximum rates of contraction and of expansion of the flow which can be achieved. If these limits are exceeded the assumption of uniform velocity distribution will not be valid and the simple design procedure will be inadequate.

In essence, the technique under discussion seeks to design culverts and bridge waterways in such a way that the flow in the upstream approach channel is contracted through a streamlined inlet transit-

ion or "fan" into the throat or barrel, where the width of the waterway is smallest, and is then expanded in a streamlined outlet transition or fan before it is finally released into the channel downstream, all this being accomplished with small energy loss. The designed waterway extends from the upstream end of the inlet fan to the downstream end of the outlet fan. A bridge waterway designed and constructed in accordance with these concepts is shown in Figure 1.

An essential aspect of the design is the requirement that the inlet and outlet fans must be shaped in such a way that the flow passes through them without significant form loss. When this requirement is satisfied the energy loss through the structure is due mainly to the boundary shear. It is this aspect which is the basis for the description of such designs as having "constant total energy", the implication being that energy loss due to boundary shear is negligibly small. In fact, either explicitly or implicitly, some energy loss is allowed for in the design. In many cases the assumption has been that the energy gradient through the engineered waterway is the same as that which occurred previously in the natural waterway.



Section AA (Distortion 25:1)

Figure 1 Nudgee Road Bridge Waterway

(Paper C1401 submitted to The Institution of Engineers, Australia).

In many applications it is desirable to achieve a minimum width of waterway at the barrel or throat subject to the over-riding consideration that energy losses are to be kept small. This leads to the strategy of designing for critical conditions to occur in the section of minimum width, i.e. Froude number,  $F = v/(gy)^{1/2} = 1$ , where  $v$  is velocity of flow,  $y$  is depth and  $g$  is the gravitational acceleration. At critical conditions the discharge per unit width is the maximum possible for the available specific energy,  $E = y + v^2/2g$ , and the width of waterway and cross-sectional area of flow will then be a minimum. Typically, the specific energy available at the section of minimum flow area is increased by lowering the level of the invert below the natural ground surface, thereby achieving a further reduction in the minimum cross-sectional area of flow.

The benefits inherent in a waterway designed according to the above principles are:-

- (i) There is no effect on flood levels upstream from the structure if the energy losses are as small as assumed.
- (ii) At the same time the waterway in the barrel or throat is minimised.
- (iii) The flow in the structure is streamlined, with little turbulence; erosion potential is low and surface protection can sometimes be minimal.

Whether the listed benefits lead to such a structure being the "best" for a specific application will depend on many factors which are beyond the scope of this paper. Discussion of them can be found in McKay (1971), Cottman (1976), Porter (1978).

## 2 DESIGN PROCEDURE

It is necessary first to decide on the "design flood"; the discharge and the depth at which it flows at the site of the waterway.

With the assumption of no energy loss through the structure, i.e. constant total head, the waterway is designed to meet the required specifications. Usually, but not always, the design flood is carried through the structure at critical conditions everywhere and the cross-sectional shape is made rectangular. This makes calculation very simple and direct because of the relations at  $F = 1$  viz:-

$$y_c = 2E/3 \quad ; \quad q^2 = gy_c^3 \quad (1)$$

where  $y_c$  is the critical depth and  $q$  is the discharge per unit width. Either the bed profile is specified and the plan widths are calculated from the relations above or the plan shape is specified and the required bed profile is calculated.

In the inlet fan where the flow is converging in plan and in the outlet fan where it is diverging, the widths of flow cross-section calculated by the above process must be measured along the orthogonals to the streamlines, which are usually curved in plan, as illustrated in Figure 1. If the plan shape of a fan is decided in advance the bed profile in the fan can be calculated directly. However, if the longitudinal bed profile in the fan is set in advance, determination of the correct plan shapes of the boundary streamlines and the orthogonals usually involves some graphical adjustment. In either case the orthogonals to the streamlines should coincide with contours of constant bed

elevation. In the design of the fans it has been found that shapes of streamlines and orthogonals determined from flow-nets for two-dimensional irrotational flow give good results. The conditions in the fans are only approximately described by two-dimensional irrotational flow but the results obtained from this approximation are usually significantly better than those obtained by any other simple procedure.

In the final step, the energy loss through the structure is allowed for by tilting the whole geometry downwards in the direction of flow, usually by lowering each part of the ground profile by an amount proportional to its distance from the inlet. More sophisticated adjustments can be made if detailed knowledge of energy losses in the different parts of the waterway is available. An alternative to the procedure described above is to estimate the total energy line at the beginning or as the design proceeds and to use the relations (1) to give the width and final bed profile of the waterway immediately.

Use of the one-dimensional notion of specific energy for design calculations of the waterway implies that the velocity distribution across each cross-section is essentially uniform, that the water surface is horizontal along the orthogonals and that the pressure distribution is hydrostatic. These assumptions will apply with good accuracy provided that,

- (i) the approach conditions upstream from the inlet fan are reasonable and the flow entering the fan is reasonably well distributed;
- (ii) there is no flow separation in any part of the waterway;
- (iii) the radius of curvature of the streamlines in the fans is not too small.

It may be possible to achieve a satisfactory design of waterway in circumstances where one or more of these conditions are not satisfied. However, in such a case the one-dimensional calculations can only be used indicatively and model testing is required to confirm the design.

The calculations of most waterways designed according to these ideas are carried out for the design flow occurring at critical conditions but, in some cases, the flow is required to occur at sub-critical conditions. In such cases the calculations proceed in the same way as described above, the only difficulty being the more complicated calculations required to determine the waterway cross-section at subcritical conditions. Instead of the simple relations (1) which apply at  $F = 1$ , the more general expression for specific energy must be used,

$$E = y + v^2/2g = y + q^2/(2gy^2) \quad (2)$$

For  $F < 1$  there is an unlimited number of pairs of values of  $y$  and  $q$  which satisfy the equation (2) for each value of  $E$ , subject to the restriction,  $2E/3 < y < E$  and  $q < q_c$  and the designer must decide which pair best suit his requirements. Ippen (1950) gave a thorough description of the procedures for designing transitions in subcritical flow using the specific energy diagram. What is novel in the design approach under discussion is its emphasis on designing for flow at critical conditions.

## 3 THE HYDRAULICS OF THE FLOW

In the summary of the design procedure it was noted that certain conditions must be satisfied by the flow in the waterway if the simple one-dimensional calculations are to provide reasonably accurate results. Observations of scale models and of full size structures indicate that these conditions are closely approximated, provided certain limits are observed in the proportions and shape of the waterway. However, the flow occurring in the waterway can become very complex if these limits are exceeded. The simplifying assumptions of one-dimensional flow are invalid if, for example, separation of the flow occurs or very strong curvature of streamlines causes significant departure from hydrostatic pressure distribution. These limitations are discussed below, some of the complications which arise when the design procedures are applied to practical situations are examined and ways of dealing with them are described.

## 3.1 Approach Conditions and the Inlet Fan

In the upstream approach to the inlet fan the flow is in its natural condition and it must find its way into the fan in an "uncontrolled" way. For the flow in the fan to be of satisfactory quality and to be described reasonably accurately by the one-dimensional calculations it is essential that the approaching flow be reasonably well distributed. The flow is converging at this point and the process of convergence will tend to make it more uniform but the amount of convergence is limited and can not be expected to cope with bad approach conditions. If the approach conditions are not satisfactory it is necessary to carry out works in that region to improve them. As pointed out by Cottman (in Porter, 1978), "not trimming and flattening the stream-bed upstream of the entry fan, and over the full width, so that the approach flow can conform to the assumptions made by the designer" will cause unsatisfactory performance of a minimum energy structure. This will manifest itself in the form of increased energy losses or reduced flow capacity and sometimes in the occurrence of large amplitude standing waves on the surface of the flow.

In the approach flow the velocity distribution in the vertical is the fully developed turbulent velocity profile. Calculations in the structure assume the velocity to be essentially uniformly distributed in the vertical and in a waterway which is performing satisfactorily this tailoring of the velocity distribution is produced by the effects of the convergence imposed on the flow as it enters the inlet fan and then as it is further converged into the barrel or throat where the flow cross-section is minimum. The natural approach flow has the characteristics of boundary layer flow in which the boundary layer occupies the full depth. As this flow is converged into and through the inlet fan the velocity distribution is made nearly uniform and a new boundary layer begins to grow from the beginning of the inlet fan. Convergence always improves the uniformity of flow. Nevertheless the amount of convergence available in the free-surface flows occurring in minimum energy structures is definitely limited.

In going from the approach flow to the upstream end of the inlet fan the maximum contraction ratio which can occur in the vertical plane is 1.5:1 for the case of very slow approach flow being accelerated to critical conditions at the beginning of the inlet fan. If the approach flow has significant velocity the vertical contraction ratio is reduced. The contraction ratio in the horizontal plane is,

theoretically, unlimited but it is doubtful whether the horizontal contraction at this point has significant influence on the flow quality where it enters the inlet fan.

In the inlet fan itself, flow is converged in a fully controlled fashion and it is appropriate to consider flow area contraction ratios. If the flow at the beginning of the inlet fan has been accelerated to critical conditions, further acceleration to achieve further reduction in flow area at critical conditions can be achieved only by lowering the bed of the waterway. Thus, as the plan width of the flow cross-section is reduced the depth of flow increases. In a waterway of rectangular section in which  $F = 1$  everywhere and the flow is  $Q$ , the width ( $B$ ), depth ( $y$ ) and cross-sectional area of flow ( $A$ ) are uniquely related to the local specific energy of flow according to:

$$y = y_c = 2E/3 \quad ; \quad B = Qg^{-1/2} (2E/3)^{-3/2} \quad (3)$$

$$A = Q(2gE/3)^{-1/2} \quad ; \quad A \propto E^{-1/2} \propto B^{1/3}$$

The relations (3) show that  $B$  is reduced rapidly as  $E$  is increased by lowering of the bed level but that  $A$  is reduced much more slowly; contraction shapes which are dramatic in appearance in plan are associated with much more modest area contraction ratios. Theoretically, there is no limit to the contraction ratios which can be used but practical considerations set upper limits which are quite modest. Cottman (in Porter, 1978) suggests that the maximum practical value to be used for the ratio  $E_{max}/E_{min}$  is 5, which corresponds to a lowering of the bed of the minimum cross-section by an amount more than four times the depth of the approach flow. For this ratio, the relations (3) show that

$$B_{max}/B_{min} = 11.18 \quad \text{but} \quad A_{max}/A_{min} = 2.24 \quad (4)$$

As stated in Section 2, the orthogonal families of streamlines and equi-potential lines of two-dimensional irrotational flow give shapes which have been found to be suitable for the plan form and bed contour shapes of the inlet fan. However, it is necessary to decide separately the length of the fan since the same contraction ratio can be achieved over a wide range of lengths from the one family of streamlines. The optimum length of inlet fan is the shortest for which the flow approximates the two-dimensional condition without separation or unacceptable transverse water surface slopes. If the ratio of length/width of the fan is made too small the flow is no longer controlled by the plan shape of the fan and enters the fan as an almost parallel flow which chokes before it reaches the section of minimum width. Cottman (in Porter, 1978) suggests that the minimum satisfactory value of  $\text{length}/B_{max}$  for the inlet fan is 0.5 and this is probably a reasonable guide. The inlet fan illustrated in Figure 2 has  $\text{length}/B_{max}$  of 0.44 and performed well in model tests. However, for reasons given in section 3.2.2 this inlet was designed for flow at values of  $F$  less than unity.

## 3.2 The Barrel or Throat

In some applications, the inlet fan leads into a length of waterway in which the width of flow cross-section is constant at its minimum value. This is usually the case for culverts and this length of waterway of constant width is described here as the "barrel". In other applications it is appropriate to converge the flow to a section of minimum width and then to expand it immediately downstream. A waterway of this type is illustrated in Figure 2. The section with minimum width in such a waterway

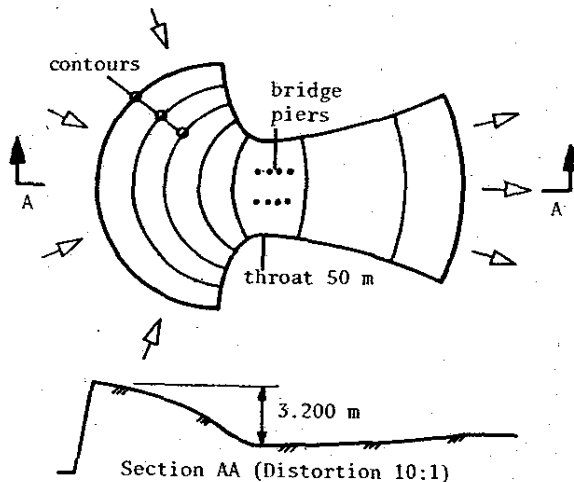


Figure 2 Settlement Shores - Flood Outlet B is described here as the "throat".

### 3.2.1 Minimum slope of barrel

Regardless of assumptions concerning the energy loss through the structure and the allowance made for it, there is a minimum slope which can be used for the invert of the barrel if the flow there is to be at  $F = 1$ . This is referred to as the limit slope (Chow, 1959). For steady uniform flow in a channel it can be shown (Apelt, 1965) that the bed slope  $S_0$  and the Froude number of the flow are related by:

$$S_0 = F^2 g n^2 P^{4/3} (BA)^{-1/3} \quad (5)$$

where  $P$  is the wetted perimeter of flow section and  $n$  is Manning's. For the case  $F = 1$ , the equation (5) gives the value of the slope required if the uniform flow is to be at critical conditions. If the cross-section of the channel is rectangular the expression for  $S_0$  in equation (5) has a minimum at  $y/B = 1/6$  with a value given by,

$$\min S_0 = (8/3) F^2 g n^2 B^{-1/3} \quad (6)$$

The value of  $\min S_0$  given by equation (6) for  $F = 1$  is the limit slope and is the same result as that obtained by Rao & Sridharan (1970). If the slope of the barrel invert does not equal or exceed this minimum value then critical conditions can not be achieved in the barrel and the simple design calculations described in section 2 will be invalid.

The preceding discussion is based on fully developed uniform flow and it is reasonable to enquire to what extent it applies to the flow in the relatively short channel of the culvert barrel where it is probable that the boundary layer is still developing and the flow is not uniform in the strict sense. There is evidence that the theory does apply in this situation in the problems encountered in the model studies of a minimum energy culvert associated with the cooling water circuit of Gladstone Power Station. The barrel of the culvert was rectangular, 3.277 m wide, with bed slope of  $5.78 \times 10^{-3}$ , and was intended to carry a flow of 24.1 m<sup>3</sup>/s at  $F = 1$ , the depth of flow being 1.767 m. However, in model tests at a scale of 1:24 the flow in the barrel was found to be sub-critical at a depth of 2.012 m with  $F$  of 0.82; the flow in the inlet and outlet fans, however, was very close to critical and the water level in the barrel was

noticeably higher than it was immediately upstream and downstream in the fans. The value of  $n$  for the model barrel was estimated at between .012 and .013 and Equation (5) predicts a corresponding range of  $F$  from 0.90 to 0.83, which is in surprisingly good agreement with the observed value. Equation (5) also shows that if the design conditions are to be achieved, the value of  $n$  for the barrel must be not greater than 0.011.

### 3.2.2 Bridge piers in throat

It is clearly desirable that no obstruction be located in the throat of the waterway. However, in the case of large waterways it may be necessary to locate one or more bridge piers in the throat, as illustrated in Figure 2. In such case the bridge piers constitute a further contraction in width. It is not possible for the flow to be at critical conditions in a throat section containing bridge piers and also in the unobstructed sections immediately upstream and downstream unless the bed level in the obstructed section is lower than that immediately upstream and downstream. A similar situation exists in the case of multi-cell box culverts where the effective width of waterway within the culverts is less than that in the fans immediately upstream and downstream because of the presence of the walls of each cell of the set. The change in bed level required to maintain critical conditions throughout the flow in these circumstances is that necessary to provide the minimum specific energy in the obstructed section when critical conditions exist in the unobstructed sections upstream and downstream. If the total effective reduction in the width of the waterway due to the presence of piers or culvert walls is denoted,  $w$ , and the unobstructed width is  $B$ , application of equations (3) shows that the minimum specific energy in the obstructed section,  $E_{ct}$ , is related to that in the unobstructed width,  $E_{ca}$ , as follows,

$$E_{ca}/E_{ct} = (1 - w/B)^{2/3} \quad (7)$$

The required change in bed level can be calculated from equation (7) after allowance has been made for any extra losses in total energy caused by the obstruction. It should be noted that the value of  $w$  should include the effects of any separation caused by lack of stream-lining of the bridge piers or culvert walls.

A practical problem associated with the approach described above is the sudden change in bed level from the unobstructed to the obstructed section. McKay (1971) has shown how to convert this to a progressive change when the obstruction is a continuous diaphragm by raking the upstream and downstream edges of the obstruction over the same distance as the bed level is changed. McKay described this procedure in the context of the dividing walls of multi-cell culvert barrels but exactly the same approach is applicable to the case of a diaphragm bridge pier. However, it is not clear how the technique could be adapted to the case of bridge piers consisting of pile bents, as in the case of Figure 2.

If the bed level in the obstructed section is not made lower than that in the unobstructed section, i.e. if the bed profile is continuous at a grade corresponding to the estimated gradient of the total head line, the Froude number of the flow will be different in the two sections. If the Froude number of the flow in the unobstructed approach channel is  $F_a$  and that in the section containing the obstruction is  $F_t$ , assumption of constant

specific energy and use of the continuity equation leads to the result

$$\frac{F_a^2 (F_t^2 + 2)^3}{F_t^2 (F_a^2 + 2)^3} = \left(1 - \frac{w}{B}\right)^2 \quad (8)$$

If the flow in the section containing the obstruction is critical so that  $F_t = 1$  the Equation (8) simplifies to

$$27 F_a^2 (F_a^2 + 2)^{-3} = \left(1 - w/B\right)^2 \quad (9)$$

Similar results have been presented by Hsieh (1964). The relationship (9) is shown graphically in Figure 3 which shows the limiting Froude number of the approach flow when the conditions are critical in the section containing the obstruction. In this application only the subcritical range would normally be used. It should be noted that the assumption of constant specific energy in derivation of equations (8) and (9) implies that no extra energy loss is caused by the presence of the obstruction. This will be a reasonable assumption only if the obstruction has a well-streamlined shape.

### 3.3 Outlet Fan

The function of the outlet fan is to decelerate the flow and expand it laterally before releasing it into the natural channel downstream. Successful performance of the outlet fan is essential if the minimum energy structure is to operate with small energy loss and if excessive scouring velocities are to be prevented.

If the flow in the outlet fan is critical then the same relations (3) apply as were discussed for the inlet fan in section 3.1, provided, of course, that there is no separation and the velocity distribution is essentially uniform. It is impossible with present knowledge to predict with certainty when these conditions will be satisfied. A research programme is currently underway to define the characteristics of expanding free surface flow at and near critical conditions and some results are available (McMahon, 1979) but the understanding is not yet complete. However, it is clear that it is more difficult to achieve rapid rates of expansion and large expansion ratios in the outlet fan than it is to achieve rapid rates of contraction and large contraction ratios in the inlet fan. The reason for this lies in the difference between the characteristics of the boundary layers in the two fans. In the inlet fan the boundary layer is thin, having started in the vicinity of the upstream end of the fan, and the accelerating flow is associated with a favourable piezometric head gradient which

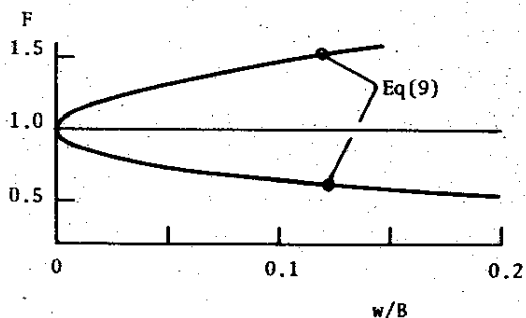


Figure 3 Froude number in approach flow for  $F = 1$  in throat with piers

ensures that the boundary layers will remain thin and energetic. In contrast, in the outlet fan the expanding flow is associated with an adverse gradient of piezometric head which combines with the boundary shear stress to decelerate the flow in the boundary layers. The boundary layers grow rapidly in thickness and, if the outlet fan is long enough, the boundary layer separates and the flow is no longer guided by the geometry of the fan walls. If separation occurs the velocity of flow leaving the fan will be larger than that indicated by the one-dimensional calculations of section 2 and it will also have a non-uniform distribution. Both effects result in increased energy losses through the structure.

As shown by relations (3) the area expansion ratio in the outlet fan,  $A_T$ , i.e.  $A_{max}/A_{min}$ , is much smaller than the width expansion ratio,  $B_T$ , i.e.  $B_{max}/B_{min}$ . For example, in the waterway shown in Figure 1,  $B_T$  is 1.53 but the calculated value of  $A_T$  at design flow is 1.15. The largest value of  $B_T$  used in a minimum energy waterway to date is 8.56 at Newington Bridge (Cottman, 1976) and the corresponding value of  $A_T$  (calculated) is 2.05. The largest value of  $A_T$  which has been measured is 1.78 which is the value at design flood in the model of Settlement Shores Flood Outlet B (Apelt, 1974), illustrated in Figure 2.

#### 3.3.1 Energy losses in outlet fan

The evidence available indicates that energy losses in and at the exit from the outlet fan account for the greater proportion of the losses through the waterway. The energy losses in the outlet fan can be attributed to boundary shear, "form" loss in the expanding flow and exit loss.

The energy loss at the exit from the fan is due to the fact that the velocity at that location is greater than the velocity in the natural channel downstream. The loss of total head at the exit,  $\Delta H_e$ , will be approximately equal to the difference in velocity heads and, if the velocity in the natural channel is very much smaller than that at exit, this loss will be approximately equal to the velocity head at exit. If the flow in the fan is everywhere at  $F = 1$ , use of the relations (3) gives, in this case

$$\Delta H_e / (v_{max}^2 / 2g) = (A_{min} / A_{max})^2 = 1/A_T^2 \quad (10)$$

The energy losses in the fan itself can be determined only by experiment. Some results from McMahon (1979) are shown in Figure 4 together with the relation specified by Equation (10). The data were obtained on expansions with straight side walls and with expansion angles of  $16^\circ$  and  $25^\circ$ . Within the range of the tests there is no significant difference between the data from the two different angles. In all cases except one, the value of  $F$  in the expansion was close to unity, ranging from 0.876 to 0.996. The one exception is the case with the largest value of  $A_T$ , for which  $F = 0.587$ . The experimental data for head loss have been non-dimensionalised with respect to the velocity head at the minimum cross-section (the throat), as in Equation (10). The results for head loss between the throat and the downstream end show relatively small variation in the non-dimensional coefficient over the range of area ratio,  $A_T$ , which includes all likely practical designs. There is a tendency for the coefficient to decrease as  $A_T$  increases. The maximum value is 0.35 near  $A_T = 1$  and the minimum is 0.18 near  $A_T = 2.5$ . The average of all values is 0.28.

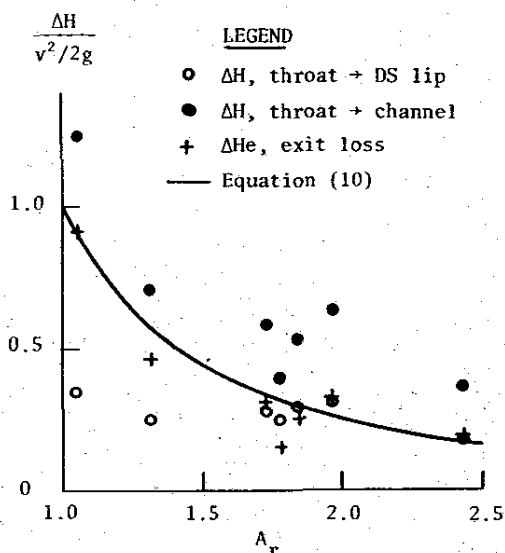


Figure 4 Energy losses in outlet fans

The results for head loss between the throat and the channel downstream from the exit generally follow a pattern similar to but lying above equation (10). The difference between the two sets of values of head loss represents the exit loss,  $\Delta H_e$ , non-dimensionalised as in equation (10). The experimental values for  $\Delta H_e$  follow the predictions of equation (10) quite well. The exit losses could be reduced substantially below those shown in Figure 4 if the outlet fan was extended downstream beyond the outlet rim into the region of sub-critical flow. If minimisation of energy loss is of paramount importance it is often necessary to extend the outlet fan in this way. Theoretically, it is possible to extend and expand the outlet fan downstream until the velocity of flow has been reduced to that of the natural channel and, in this case, there would be no exit loss.

### 3.4 Off Design Flows

All of the discussion to this point relates to the characteristics of the waterway when operating at design flow. It is important also to consider the performance of the structure when carrying flows less than and greater than the design flow. In order to determine the characteristics of the flow when the discharge differs from the design flow it is necessary to consider the stage-discharge relationship which is established in the natural channel downstream and how this affects conditions in the waterway. The following brief discussion is valid for many cases but it does not necessarily apply to all.

At flows significantly less than design the waterway is usually divided into two zones. Most of the waterway is occupied by sub-critical flow which is controlled by the conditions at the exit from the downstream fan. Flow enters the upstream fan through a point of control at the entrance lip and accelerates to supercritical conditions before forming a small hydraulic jump where the deep sub-critical flow is encountered. As the discharge

increases, the zone of supercritical flow reduces in extent and the hydraulic jump is replaced by an undular standing wave. At design flow, the region of supercritical flow and the undular standing wave are eliminated. It is quite likely that the maximum water level in the barrel or throat will be associated with a flow which is less than the design flow. Cottman (1978) has discussed such a case and points out its significance with regard to the design level for the soffit of the bridge which crosses the waterway.

At flows greater than the design flow a choke often occurs in the waterway at, or near, the section of minimum width. If this is the case, upstream from the choke the flow will be sub-critical while, downstream from the choke, the flow will accelerate and a short zone of supercritical flow will exist. Usually, undular standing waves form in this region and provide the means for converting the flow back to the sub-critical conditions which exist in the natural channel downstream. As with the case of flows less than design, calculation of the larger flows is made difficult by lack of data on energy losses and model testing has generally been necessary to check the performance at off-design flows.

"Off-design" conditions also occur at the design flow itself if the estimate of energy loss used in the design of the waterway is significantly in error. If the waterway has been tilted more than is needed to account for energy losses then it is likely that conditions at design flow will be similar to those described above for flows less than design. A short length of supercritical flow is likely to occur in the entrance fan, followed by an hydraulic jump or by an undular standing wave. If the slope given the waterway is too small, choking will occur at the design flow and conditions will be similar to those described above for flows greater than design.

## 4 CONCLUSION

A number of waterways designed with the one-dimensional calculations summarised in section 2 have been built and the evidence available indicates that they perform well, confirming the satisfactory performance observed in model tests. However, it has been shown that in some cases a more sophisticated and detailed approach is necessary to achieve satisfactory design. More data are required concerning energy losses and the characteristics of expanding free surface flow. When these have been obtained the application of the design method can be extended with confidence to even more challenging cases.

## 5 ACKNOWLEDGMENT

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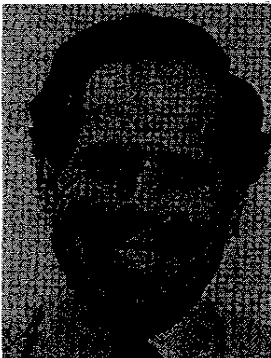
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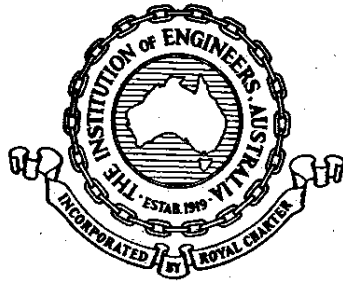
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