Fringe spacing and phase of interfering matter waves

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We experimentally investigate the outcoupling of atoms from Bose-Einstein condensates using two radio-frequency (rf) fields in the presence of gravity. We show that the fringe separation in the resulting interference pattern derives entirely from the energy difference between the two rf fields and not the gravitational potential difference between the two resonances. We subsequently demonstrate how the phase and polarization of the rf radiation directly control the phase of the matter wave interference and provide a semiclassical interpretation of the results.

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I. INTRODUCTION

One of the defining properties of a Bose-Einstein condensate (BEC) is its long-range phase coherence. The first clear demonstration of the coherence of Bose gases at low temperature was the observation of interference fringes in the expansion of two initially spatially separated condensates [1]. Since then, other experiments have studied the coherence properties both near $T=0$ and at finite temperatures using a variety of techniques. Some of these include using output-coupled atoms from an array of tunnel coupled condensates in an optical standing wave [2], Bragg spectroscopy [3], Bragg interferometry [4–6], and density fluctuations following ballistic expansion [7]. More recently, it has been shown that atoms extracted from a condensate by radio frequency (rf) fields carry the phase coherence properties of the parent condensate, as demonstrated by measurement of the second-order correlation function [8].

An elegant scheme to probe the spatial coherence of a trapped BEC based on the interference of two outcoupled matter waves was reported by Bloch *et al.* [9,10]. This used two rf fields to outcouple atoms from different locations within a Bose gas system. These matter waves then interfere while falling under gravity in a situation that is somewhat analogous to a textbook Young's double-slit experiment with light. A high contrast matter-wave interference pattern was observed at temperatures below the BEC transition temperature for arbitrary slit separation (within the condensate), confirming the phase coherence of the BEC. A numerical model of two outcoupled modes based on the Gross-Pitaevskii equation at zero temperature agreed with the experimental observations [11]. More recently, the atom-by-atom build up of a matter wave interference pattern has been observed using single-atom detection [12].

The experiments reported in Refs. [9,10,12] focused mainly on the visibility of the interference patterns and their relation to the coherence length of the Bose gas system. However, they did not fully address the issues of the fringe spacing and phase of the interference pattern for general two-mode outcoupling from a BEC. In this paper we explicitly demonstrate the origins of the phase and spacing of the interference fringes, and provide a useful semiclassical interpretation of the interference. However, we would like to emphasize that none of our results are in contradiction to those of Refs. [9,10,12].

This paper is organized as follows. In Sec. II we give an overview of the theory of outcoupling atoms from a BEC using two rf fields. Section III details experimental measurements of the interference fringe spacing. Section IV describes measurements of the phase of the interference patterns and how these are determined by the phase and polarization of the rf fields from both a semiclassical and quantum viewpoint. We conclude with a discussion of the regime of validity for the semiclassical picture and its limitations.

II. DUAL RF OUTPUT COUPLING

Outcoupling atoms from Bose-Einstein condensates with rf fields has been used extensively to produce beams of atoms, generally referred to as "atom lasers" [13–18]. The rf radiation of frequency ω_{rf} drives resonant (stimulated) transitions from a trapped Zeeman sublevel to an untrapped state in which the atom falls under gravity. Outcoupling occurs at locations where the total energy difference between the trapped and untrapped states is equal to $\hbar\omega_{\mathrm{rf}}$. This is usually determined by the Zeeman potential so that the resonance condition may be written

$$\hbar\omega_{\mathrm{rf}} = \mu_B g_F |B(\mathbf{r})|, \tag{1}$$

where μ_B is the Bohr magneton, g_F is the Landé g-factor, and $B(\mathbf{r})$ is the magnetic field.

A typical starting point for atom laser experiments is a condensate trapped in a cigar-shaped magnetic potential of the form $U(\mathbf{r})=m\omega_z^2(\kappa^2x^2+y^2+z^2)/2$ where m is the mass of the atom, $\omega_z=\omega_y$ is the trapping frequency in the tight directions of the trap, and $\kappa=\omega_x/\omega_z$. Atoms can be outcoupled from the surface of an ellipsoid of the magnetic equipotential which satisfies the resonance condition (1). However, gravity will cause a displacement of the minimum of the total potential from the magnetic field minimum. This gravitational sag

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means a harmonically trapped condensate will be displaced from the magnetic field minimum by a distance, $z_0 = -g/\omega_z^2$, where ω_z is the trapping frequency in the direction of gravity. This displacement is typically greater than the size of the condensate, so that the ellipsoidal equipotential surfaces can be approximated by planes which intersect the condensate at different heights, z. In some situations, such as the one we are discussing here, the dependence on the x and y coordinates can be neglected for quantitative purposes and only the z dimension need be considered. However, this is not always true and the transverse profile of the condensate and trapping potential do impact on the transverse mode of an atom laser [19,20].

In previous work [9,10] two rf fields of frequencies ω_1 and ω_2 were used to outcouple atoms from a condensate. The two spatially separated resonances were interpreted as creating two slits from which atoms were extracted from the condensate. The outcoupling points, z_1 and z_2, were chosen to be centered around the middle of the condensate located at z_0, the minimum of the combined magnetic (harmonic) and gravitational (linear) potential.

Under this condition the gravitational energy difference between the two outcoupling points, determined by the slit separation $\Delta z = z_1 - z_2$, is exactly equal to the difference in energy between the two applied rf fields. This can easily be seen from the derivative of the magnetic potential, where $\Delta E \approx m\omega_z^2 z \Delta z$. At the central position, z_0, we find $\Delta E = \hbar(\omega_1 - \omega_2) = mg\Delta z$.

However, this result is only true when the midpoint between the two resonances $\bar{z} = (z_1 + z_2)/2$ coincides with the center of the BEC z_0. If the two resonant points are not symmetrically located about the center of the trap, the z^2 dependence of the magnetic potential means that the slit separation for a fixed $\Delta\omega = \omega_1 - \omega_2$ varies inversely with $\bar{z}$. Thus the gravitational energy difference between the two resonance positions is not necessarily equal to $\hbar\Delta\omega$ and Δz may change significantly across the width of a condensate. For a harmonic potential the slit separation is approximately

$$\Delta z \approx \frac{\hbar\Delta\omega}{m\omega_z^2 \bar{z}}. \tag{2}$$

In a Young's double slit experiment the fringe spacing, λ, of the interference pattern is proportional to Δz^{-1}. However, in dual rf outcoupling experiments with a fixed $\Delta\omega$ the fringe spacing λ is independent of Δz. The gravitational energy difference $mg\Delta z$ between the two resonant locations does vary with $\bar{z}$. However, the fringe spacing of the interference pattern depends on the total energy difference between the two indistinguishable outcoupling paths and must always equal $\hbar\Delta\omega$ to satisfy energy conservation. This is a general result that we feel is important to clarify as it was not clearly stated in previous work [9,10] but can be seen in the theoretical analysis of Ref. [11].

Here we discuss the specific case of an $F=1$ ^{87}Rb condensate in the Thomas-Fermi (TF) regime. The total energy of the trapped state $|F=1, m_F=-1\rangle$ consists of the sum of its magnetic, gravitational, and mean field energies. The untrapped (outcoupled) state $|1,0\rangle$ experiences negligible magnetic potential, but, while still within the condensate, experiences both the mean field and gravitational potentials. In one-dimension the total energy of a particular substate can be written as

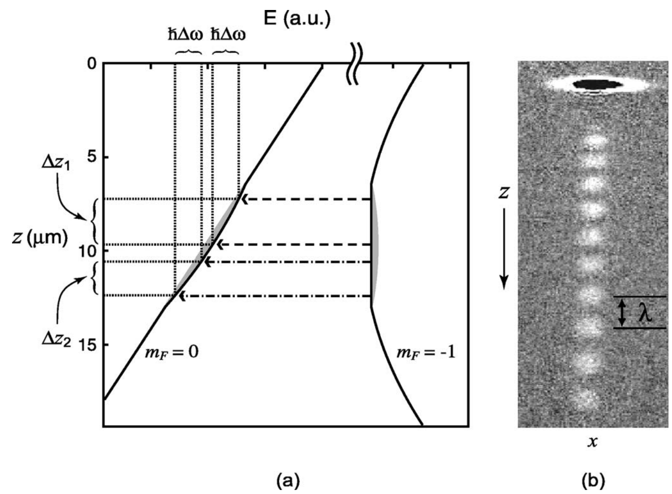

FIG. 1. (a) Total energy (solid lines) of atoms in the trapped $m_F = -1$ and untrapped $m_F = 0$ states for the parameters used in our experiments. The dashed and dash-dotted lines represent two pairs of rf fields with equal $\Delta\omega$ used to outcouple atoms from the BEC at different locations. The gravitational energy difference $mg\Delta z$ may vary for a fixed $\Delta\omega$, however, when the interaction energy (shaded) of atoms in the untrapped state is included the total energy difference between the two outcoupled matter waves is always equal to $\hbar\Delta\omega$. (b) The measured outcoupled matter wave beams display an interference pattern with a constant fringe spacing $\lambda(z)$ given by Eq. (4) for a fixed $\Delta\omega$ independent of Δz. For this image $\Delta\omega = 2\pi \times 1000\ \mathrm{s}^{-1}$.

$$E_{m_F}(z) = -m_F\left(\tfrac{1}{2}m\omega_z^2 z^2 + \mu_B g_F B_0\right) - mgz + g_{1D}|\psi(z)|^2, \tag{3}$$

where B_0 is the magnetic field at the minimum of the trap, g_{1D} is the 1D effective interaction strength (assumed to be the same for all m_F, which is approximately true but not an essential point in this discussion) and $|\psi(z)|^2 = \Sigma_{m_F}|\psi_{m_F}(z)|^2$ is the total atomic density. The energies of the trapped $|1,-1\rangle$ state and the untrapped $|1,0\rangle$ state are plotted (solid lines) in Fig. 1(a) for the parameters used in our experiments. The shaded regions indicate the mean field contribution to the total energy. Also shown are two pairs of rf fields with the same $\Delta\omega$ chosen to lie within the width of the condensate (dashed and dash-dotted lines). As the two pairs are centered around different $\bar{z}$, the Δz for each pair is distinct.

In the TF limit the interaction energy between the condensate and the outcoupled state exactly compensates for the difference in gravitational potential at different slit locations. The density profile, $|\psi(z)|^2$, mirrors the shape of the magnetic trapping potential so that the energy splitting between the two states is always equal to the difference in their magnetic potentials. Additionally, the energy of trapped atoms within the condensate is independent of z so that only the final energies on the $m_F=0$ curve determine the energy difference between the two outcoupled beams. For the case of a noninteracting BEC, the energy difference between the two outcoupled beams is equal to the difference of the magnetic and gravitational potentials at the two resonant locations.

Having established that the fringe spacing, λ, depends only on $\Delta\omega$, it can easily be shown that

$$\lambda(z) = \frac{\sqrt{2g(z-z_0)}}{\Delta f}, \quad (4)$$

where $\Delta f = \Delta\omega/2\pi$. The fringe spacing λ is a function of z as the outcoupled atoms accelerate in the z direction under gravity, as can be seen in Fig. 1(b).

The outcoupled matter waves are well described by a superposition of Airy functions [21] whose energy difference, $E_1 - E_2$, is equal to $\hbar\Delta\omega$. It should be noted that the phase of the rf fields is imprinted on to each of the atomic outcoupled matter waves, a fact that is immaterial for single mode output coupling, but determines the phase of the fringes for the two mode case. With equal rf powers the measured output density at time t is (in the asymptotic limit) [9–11]

$$\begin{aligned}|\psi_0(z,t)|^2 &= |\psi_{E_1}(z,t) + \psi_{E_2}(z,t)|^2 \\ &\propto \frac{1}{\sqrt{|z-z_0|}}\{2 + 2V\cos(\Delta\omega(\sqrt{2(z-z_0)/g} + t) \\ &\quad + \Delta\phi_{\mathrm{rf}})\}, \end{aligned} \quad (5)$$

where the visibility, V, is given approximately by $\sqrt{n(z_1)n(z_2)}/(n(z_1)+n(z_2))$ at zero temperature [11]. This equation is the generalized version of Eq. (1) in Ref. [9] and equivalent to Eq. (14) in Ref. [11] where we have explicitly included the phase term $\Delta\phi_{\mathrm{rf}} = \phi_{\mathrm{rf1}} - \phi_{\mathrm{rf2}}$ to show that the rf phase difference appears in the observed interference pattern. The $\sqrt{2(z-z_0)/g}$ term simply represents the time taken t_z, for an atom falling from the BEC under gravity to reach position z. The outcoupled matter waves appear as pulses emitted from the condensate at a frequency of $\Delta f = \Delta\omega/2\pi$ as noted by Ref. [11], with a phase determined by the relative phase of the two rf fields. This suggests a semiclassical interpretation of the outcoupling that we discuss further in Sec. IV.

III. FRINGE SPACING MEASUREMENTS

To explicitly demonstrate that the fringe spacing does not depend on the gravitational energy difference $mg\Delta z$, we have performed a number of experiments with several values of Δz. Our experimental procedure for producing condensates has been described elsewhere [22] and was used here with only slight modifications. An atom chip is used to produce near pure condensates containing 2×10^5 ^{87}Rb atoms in the $|1,-1\rangle$ ground state. Our chip design facilitates the production of relatively large condensates in highly stable trapping fields. The final trapping frequencies are 160 Hz in the tight direction and 6.7 Hz in the weak direction. The elongated geometry of the trap means we must cool well below the three-dimensional critical temperature T_c to produce fully phase coherent condensates (typically T_ϕ for our parameters is less than $T_c/2$ [23]). Weak outcoupling is induced by applying two rf fields of the same amplitude, with frequencies ω_1 and ω_2 tuned to be resonant with atoms in the condensate, and Rabi frequencies, $\Omega = \mu_B g_F |B|/2\hbar$, of approximately 50 Hz for each rf source. After outcoupling for 10 ms, the trap is left on for a further 3 ms before being turned off abruptly. An absorption image is taken after 5.3 ms of free expansion.

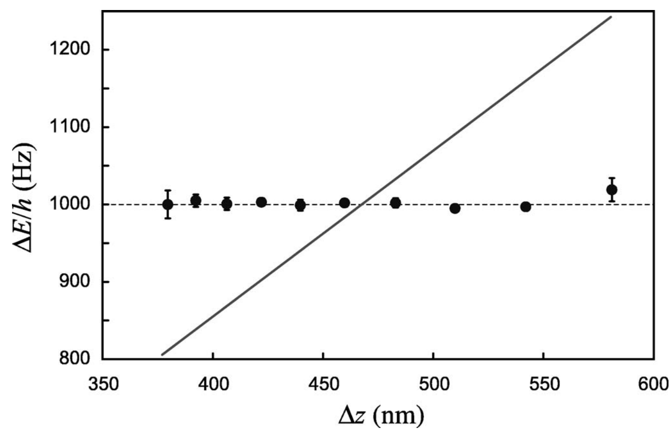

FIG. 2. Measured beat frequency of interfering atom lasers as a function of the slit distance $\Delta z = z_1 - z_2$ for $\omega = 2\pi\times 1000$ s^{-1}). The closed circles are experimental data, the dashed line is a plot of $\Delta f = 1000$ Hz, and the solid line is a plot of the gravitational potential energy difference $mg\Delta z/h$. The fringe spacing of interfering atom laser beams is fixed by $\Delta\omega = 2\pi\times\Delta f$.

We have measured the fringe spacing of the interference pattern with a fixed $\Delta\omega$ of $2\pi\times 1000$ s^{-1} as Δz was varied over the range 380–580 nm by varying $\bar{z}$. A series of experimental images similar to that in Fig. 1(b) were taken for ten different values of $\bar{z}$. These images were then integrated over x and a function of the form of Eq. (5) was fitted to the data, with the visibility V, beat frequency $\Delta\omega$, and phase $\Delta\phi$ as fitted parameters. The fitted beat frequency relates directly to the energy difference of the two outcoupled matter waves. We choose to compare beat frequencies rather than the spatial fringe period as the beat frequency is constant in time while the fringe spacing varies with $\sqrt{z}$.

Results of these measurements are plotted in Fig. 2. The experimentally measured beat frequencies (closed circles) fit very well on the dashed line which is a plot of $\Delta f = 1000$ Hz and do not depend on Δz. For comparison, the solid line is the Young's double slit prediction ($\Delta f = mg\Delta z/h$). The point where the solid line crosses 1000 Hz corresponds to the special case used in Refs. [9,10] where the energy difference between the two rf fields is equal to the difference in gravitational potentials.

IV. PHASE MEASUREMENTS

In this section we explicitly demonstrate that the location of the interference fringes is determined by the phase difference of the two rf fields. A sequence of dual rf outcoupling experiments was performed using fixed values ω_1 and ω_2 ($\Delta\omega = 2\pi\times 500$ s^{-1}) but varying the relative phase of the two rf fields. This has the effect of shifting the phase of the rf beat note. All other experimental parameters were kept fixed.

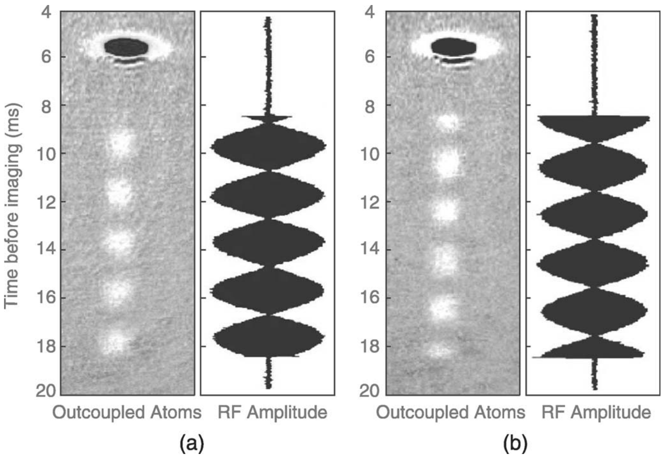

FIG. 3. The phase of the interference pattern is determined by the relative phase of the rf fields used to outcouple the matter waves. (a) and (b) Different runs of the experiment under identical conditions apart from different phases of the applied rf fields. On the left are absorption images of the condensate (top) and outcoupled atoms and on the right is the beat note of the corresponding rf used to drive the outcoupling measured on an oscilloscope. The vertical axis indicates the time before the image was taken and was obtained for the atom images through the relationship, $t_z = \sqrt{2(z-z_0)/g}$.

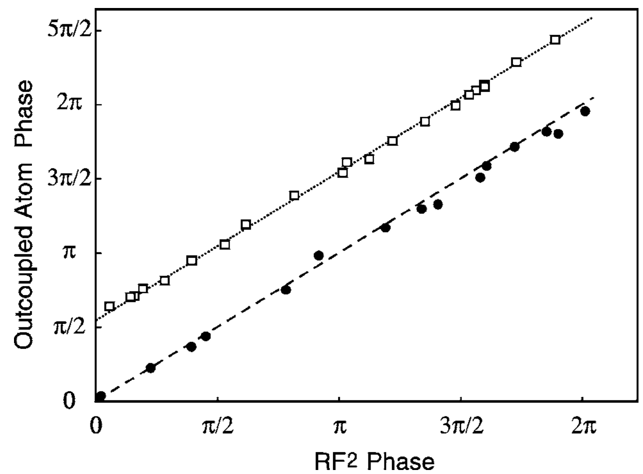

FIG. 4. Plot of the phase of the modulated atom beam against the phase of the beating rf field. Closed circles represent experimentally measured data using a single coil and the dashed line is a plot of $y=x$. Open squares are experimentally measured phases when the rf is produced by two separate (near) orthogonally mounted coils and the dotted line is a plot of $y=x+0.55\pi$.

Two examples of the data obtained are shown in Fig. 3, where absorption images of the outcoupled atoms appear on the left and the beat note of the corresponding rf fields used for outcoupling (measured on an oscilloscope) are shown on the right. The z axis of the absorption images has been rescaled by $t_z = \sqrt{2(z-z_0)/g}$ to linearize the axis in time for ease of comparison with the rf. It is clear from these images that the pulses of outcoupled atoms correspond to the largest amplitude of the rf beat note.

In order to quantify this we have analyzed a series of similar data in which the relative phase of the rf fields was allowed to vary randomly over the range 0 to 2π (the two rf generators were not phase locked in these experiments). A similar fitting procedure to that described earlier (except with the beat frequency fixed at 500 Hz) was applied to all of the absorption images to determine the phases of the interference patterns. A function of the form $A[1+\cos(\Delta\omega t+\Delta\phi_{\mathrm{rf}})]$ with fixed $\Delta\omega$ was also fitted to the envelope of the square of the measured rf beat note to establish its phase. The two phases are plotted against each other (closed circles) in Fig. 4. The dashed line through this data is a plot of $y=x$. The phase of the modulated atom beam matches very well the phase of the applied rf field, demonstrating that the rf phase is imprinted onto the outcoupled matter waves [$\Delta\phi_{\mathrm{rf}}$ in Eq. (5)].

A. Semiclassical interpretation

When both rf fields are resonant at locations with a similar density of condensed atoms (as in this paper and previous experiments [9,10,12]) it is possible to apply a semiclassical interpretation which reproduces the experimental observations and provides helpful insights. Defining the mean frequency, $\bar{\omega}=(\omega_1+\omega_2)/2$, and the beat frequency, $\delta=(\omega_1-\omega_2)/2$, and recalling the standard trigonometric identity

$$\sin \omega_1 t + \sin \omega_2 t = 2 \sin \bar{\omega} t \cos \delta t, \tag{6}$$

we see that the sum of two oscillating fields is equivalent to an amplitude modulated carrier wave. The carrier frequency, $\bar{\omega}$, is typically three orders of magnitude higher than the beat frequency, δ. Adding a phase, ϕ, to one of the rf fields shifts the phase of both the carrier and beating terms by half this amount.

This form of amplitude modulation results in all of the rf power being carried in the sidebands at $\bar{\omega}\pm\delta$. We may now consider the condensate interacting with this amplitude modulated rf field. The number of atoms outcoupled is proportional to the Rabi frequency squared (i.e., proportional to the amplitude squared of the rf field at time t) and is modulated in time at a rate $2\delta=\Delta\omega$. Once outcoupled the atoms fall under gravity and, as they have a low spread of initial momenta, the outcoupled density will be modulated in time (Fig. 3).

B. Effect of rf polarization

In the above described experiments, both rf fields were generated by passing the two rf currents through the same coil. This means the rf field in the vicinity of the BEC was linearly polarized perpendicular to the quantization axis, defined by the trapping fields to be oriented along x. We have also performed experiments using separate orthogonally mounted coils where one rf current was sent through each coil. The rf field at the location of the BEC is no longer a beating linearly polarized field but rather one which has polarization that varies from vertical linear, to left-hand circular, to horizontal linear, to right-hand circular and back to vertical linear within a single beat period ($\tau=1/\Delta\omega$). This type of beating polarization is analogous to the optical field used in lin $\perp$ lin sub-Doppler (Sisyphus) laser cooling [24] but the field is periodic in time rather than space.

The outcoupling transition from $|1,-1\rangle$ to $|1,0\rangle$ requires the transfer of $+\hbar$ of angular momentum to the atom. As the outcoupling process relies on stimulated transitions, this means an atom emits a σ^- photon. When the rf field is derived from a single coil, it is easy to see that the maximum amplitude of σ^- radiation (and hence the maximum amplitude for atoms to undergo stimulated transitions) occurs when the two rf fields are in phase. The linearly polarized field may be decomposed into two counter-rotating circular fields and the σ^- component of this drives the outcoupling. With perpendicularly oriented coils however, the maximum amplitude of σ^- radiation occurs during one of the circularly polarized phases of the beat note. This corresponds to the times when the rf fields are $\pi/2$ out of phase.

Also shown in Fig. 4 (open squares) is a plot of the experimentally measured phase of the matter wave interference pattern versus the phase of the beating rf fields performed with (near) perpendicularly oriented coils. The atomic output is phase shifted by approximately $\pi/2$ from the rf as expected. The slight mismatch between the measured shift of 0.55π and the expected shift of 0.5π was due to imperfect alignment of the rf coils (precise perpendicular alignment would have impeded optical access in our setup). From the quantum viewpoint this phase shift is equivalent to a $\pi/2$ phase shift of the σ^- component of one of the linearly polarized fields with respect to the other, which is then imprinted onto the phase of one of the outcoupled beams. It follows that for coils mounted antiparallel maximum outcoupling would occur when the two rf fields are π out of phase.

V. CONCLUSIONS

In conclusion, we have studied the fringe spacing and phase of matter wave interference patterns produced by outcoupling atoms from a BEC with two rf fields. We have shown that the energy difference between the two rf fields determines the spacing of the interference pattern, not the gravitational potential difference determined from the classical slit separation. When both rf fields are resonant within the condensate, semiclassical arguments based on interfering rf fields reproduce the experimental observations. These also provide a simple physical picture of how the phase and polarization of the rf field determine the phase of the observed matter wave interference pattern. It is important to note, however, that the semiclassical picture fails when one of the fields is not resonant with the condensate, even if the carrier lies within the condensate. In such situations, the atom laser output will not be modulated in time as there is no power at the carrier frequency.

Our experiments extend previous work which examined the fringe visibility for the specific case where $\hbar\Delta\omega = mg\Delta z$ [9,10]. We wish to emphasize that our findings in no way contradict the phase coherence studies reported in Refs. [9,10]. We have also performed experiments with cold thermal atoms and see (as in Refs. [9,10]) that the visibility of interference pattern diminishes, due to the thermal spread of velocities in the trapped gas.

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