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Hydraulics of skimming flows on stepped chutes: The effects of inflow conditions?

L'hydraulique des écoulements écumants sur des chutes en marches d'escalier: les effets des conditions d'alimentation?

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ABSTRACT
Modern stepped spillways are typically designed for large discharge capacities corresponding to a skimming flow regime for which flow resistance is predominantly form drag. The writer demonstrates that the inflow conditions have some effect on the skimming flow properties. Boundary layer calculations show that the flow properties at inception of free-surface aeration are substantially different with pressurized intake. The re-analysis of experimental results highlights that the equivalent Darcy friction factor is $f \sim 0.2$ in average on uncontrolled stepped chute and $f \sim 0.1$ on stepped chute with pressurized intake. A simple design chart is presented to estimate the residual flow velocity, and the agreement of the calculations with experimental results is deemed satisfactory for preliminary design.

RÉSUMÉ
Les déversoirs en marches d'escalier modernes sont typiquement conçus pour de grandes capacités de débits correspondant à un régime d'écoulement écumant dont la résistance à écoulement est principalement de traînée. L'auteur montre que les conditions d'alimentation ont un certain effet sur les propriétés de l'écoulement. Les calculs de couche de limite montrent que les propriétés d'écoulement au commencement de l'aération de la surface libre sont considérablement différentes avec une alimentation en charge. La re-analyse des résultats expérimentaux fait ressortir que le facteur de frottement équivalent de Darcy est $f \sim 0.2$ en moyenne sur les chutes sans contrôle et $f \sim 0.1$ avec une alimentation en charge. Un diagramme simple de conception est présenté pour estimer la vitesse résiduelle d'écoulement, et l'accord des calculs avec des résultats expérimentaux est considéré comme satisfaisant pour une conception préliminaire.

Keywords: Stepped spillway, skimming flow, inflow conditions, inception of air entrainment, flow resistance.

1 Introduction

Research and development into stepped spillway hydraulics has been very active for the past two decades with 73 journal publications (Web of Science^{TM} 1985–2003), some specialized workshops (Ohtsu and Yasuda, 1998; Minor and Hager, 2000; Mossa *et al*., 2004) and two books (Chanson, 1995, 2001). Modern stepped spillways are typically designed for large discharge capacities corresponding to a skimming flow regime as illustrated in Fig. 1. Flow resistance is predominantly form drag. The flow is non-aerated at the upstream end. Free-surface instabilities are however observed and strong air–water mixing occurs downstream of the inception point of free-surface aeration. Detailed air–water flow measurements highlighted large amounts of entrained air further downstream and very-strong interactions between main stream turbulence, step cavity recirculation zones and free-surface (e.g. Chanson and Toombes, 2003; Yasuda and Chanson, 2003).

Previous studies were conducted with a wide range of inflow conditions (Table 1, Fig. 1), although there are basic differences. With an uncontrolled ogee profile, the pressure distribution is quasi atmospheric in the entire flow at design flow conditions by definition of the ogee development (Henderson, 1966; Chanson, 2004), A further subdivision may be made between an entire smooth ogee profile and an ogee development with small first steps in the profile. With an uncontrolled broad-crest, the pressure distribution is hydrostatic at the crest. For a pressurized intake, the inflow pressure distribution is greater than hydrostatic. The inflow conditions may affect the entire flow field as this is known in flows behind bluff body (e.g. Silberman and Song, 1961; Laali and Michel, 1984; Michel, 1984; Verron and Michel, 1984).

In this note, the writer argues that differences in inflow conditions have some effect on the skimming flow properties. A careful analysis of boundary layer equations demonstrates that the inflow conditions affect the flow properties at inception of free-surface aeration. A re-analysis of large-size experimental results suggests lower flow resistance in experimental facilities with pressurized intake.

2 Developing flow region

In skimming flows down stepped chutes, the flow is non-aerated at the upstream end and the free-surface is relatively smooth and

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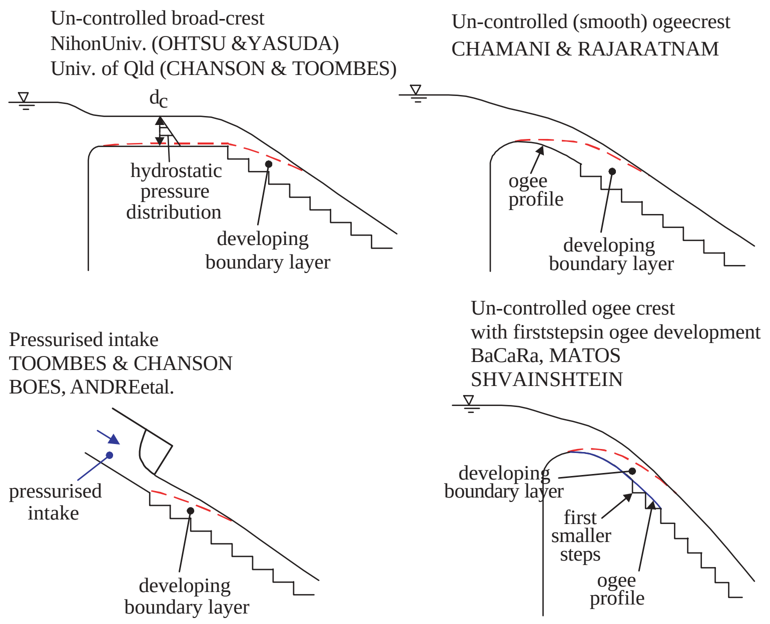


Figure 1 Sketch of skimming flows on stepped chutes for different inflow conditions.

Table 1 Re-analysed experimental data of flow resistance

Reference	Flow conditions	Remarks
André *et al*. (2003)	$\theta = 30°$, $h = 0.06$ m, $W = 0.5$ m.	Air–water flow measurements Pressurized intake inflow
BaCaRa (1991)	$\theta = 53.1°$, $h = 0.06$ m, $W = 1.5$ m $\theta = 53.1°$, $h = 0.024$ m $\theta = 59°$, $h = 0.024$ m $\theta = 63.4°$, $h = 0.024$ m	Clear-water (non-aerated) flow Uncontrolled ogee inflow with small steps in ogee development
Boes (2000)	$\theta = 30°$, $h = 0.046, 0.092$ m, $W = 0.5$ m $\theta = 50°$, $h = 0.031, 0.093$ m, $W = 0.5$ m	Air–water flow measurements Pressurized intake inflow ($3 \leq Fr_1 \leq 10$)
Chamani and Rajarathnam (1999)	$\theta = 51.3°$, $h = 0.313, 0.125$ m, $W = 0.3$ m $\theta = 59°$, $h = 0.313$ to 0.125 m, $W = 0.3$ m	Air–water flow measurements Uncontrolled smooth ogee crest inflow
Chanson and Toombes (2002a)	$\theta = 21.8°$, $h = 0.10$ m, $W = 1$ m	Air–water flow measurements Uncontrolled broad-crested weir inflow
Gonzalez and Chanson (2004)	$\theta = 15.9°$, $h = 0.05, 0.10$ m, $W = 1$ m	Air–water flow measurements Uncontrolled broad-crested weir inflow
Matos (2000)	$\theta = 53.1°$, $h = 0.08$ m, $W = 1.0$ m	Air–water flow measurements Uncontrolled ogee inflow with small steps in ogee development
Shvajnshtejn (1999)	$\theta = 38.7°$, $h = 0.05$ m, $W = 0.48$ m $\theta = 51.3°$, $h = 0.0625$ m, $W = 0.48$ m	Clear-water (non-aerated) flow Uncontrolled ogee inflow with small steps in ogee development
Toombes and Chanson (2000)	$\theta = 3.4°$, $h = 0.143$ m, $W = 0.25, 0.5$ m	Air–water flow measurements Pressurized intake inflow ($2.5 \leq Fr_1 \leq 10$)
Chanson and Toombes (2002b)	$\theta = 3.4°$, $h = 0.0715$ m, $W = 0.5$ m	Air–water flow measurements Pressurized intake inflow ($2.5 \leq Fr_1 \leq 10$)
Ohtsu *et al*. (2000)	$\theta = 55°$, $h = 0.025$ m, $W = 0.4$ m.	Air–water flow measurements Uncontrolled smooth WES ogee crest inflow
Yasuda and Ohtsu (1999)	$\theta = 5.7°$, $h = 0.006$–0.10 m, $W = 0.4$ m $\theta = 11.3°$, $h = 0.006$–0.10 m, $W = 0.4$ m $\theta = 19°$, $h = 0.002$–0.08 m, $W = 0.4$ m	Measurements in downstream stilling basin Uncontrolled broad-crested weir inflow
	$\theta = 30°$, $h = 0.004$–0.07 m, $W = 0.4$ m $\theta = 55°$, $h = 0.003$–0.064 m, $W = 0.4$ m	Measurements in downstream stilling basin Uncontrolled smooth Waterways Experimental Station ogee crest inflow

Notes: θ, bed slope; Fr_1, inflow Froude number; h, step height; W, channel width.

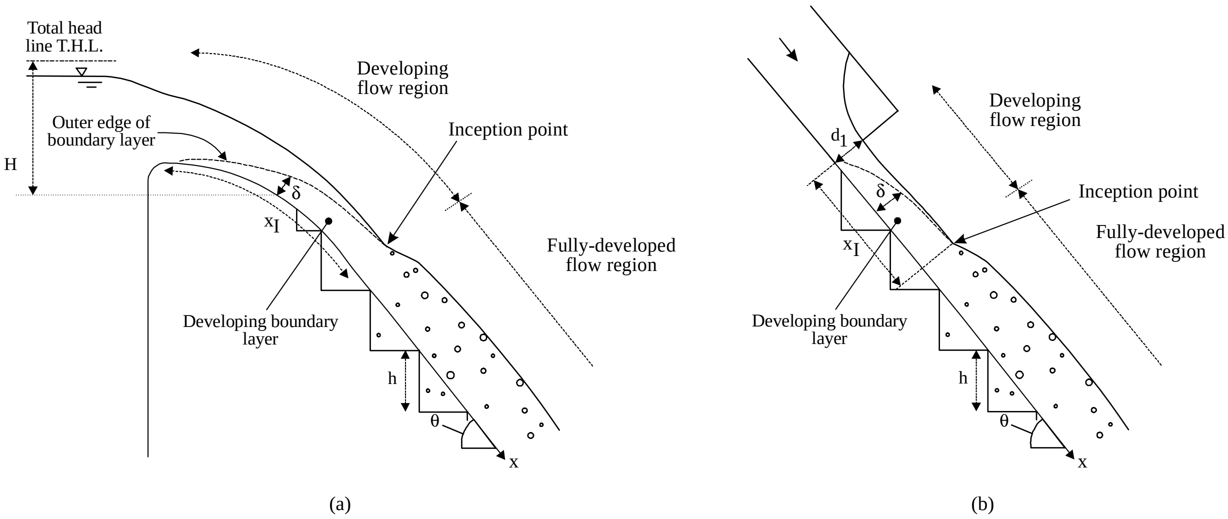


Figure 2 Sketch of developing flow region above stepped chute. (a) Uncontrolled inflow conditions; (b) Pressurized intake inflow conditions.

glassy. Turbulence is generated however at the invert and a bottom boundary layer develops (Fig. 2). When the outer edge of the boundary layer is close to the free-surface, interactions between the boundary layer turbulence and the free-surface induce significant free-surface aeration. That location is called the inception point of air entrainment. Its position is defined as the distance x_I from the start of the growth of the boundary layer, and the water depth at inception is d_I.

Simple calculations of boundary layer growth may be developed (e.g. Chanson, 1995). A summary is given in Appendix 1 for steady flows in rectangular prismatic channels with flat horizontal steps. For uncontrolled stepped chutes, Chanson (1995) compared successfully these with a large number of experimental data obtained in laboratory models and prototype. (In that study, the location of inception point was defined as the first apparition of "white waters" at the free-surface.) A statistical analysis of the data indicated that the inception point location and the flow depth at inception were best correlated by:

$$\left(\frac{(x_I)_{uc}}{h\cos\theta}\right)^{1.4} = 24.14(\sin\theta)^{0.111} F_* \qquad (1)$$

$$\frac{(d_I)_{uc}}{h\cos\theta} = \frac{0.4034}{(\sin\theta)^{0.04}}(F_*)^{0.592} \qquad (2)$$

where the subscript "uc" refers to uncontrolled inflow conditions (Fig. 2a), h is the vertical step height, θ is the angle between the pseudo-bottom formed by the step edges and the horizontal, $F_* = q_w/\sqrt{g\sin\theta(h\cos\theta)^3}$, q_w is the discharge per unit width and g is the gravity acceleration. A comparison between Eqs (1) and (2), and experimental data, is presented in Fig. (3a, b), respectively.

With a pressurized intake, the outflow is thinner and faster than with an uncontrolled crest. In turn the outer edge of the boundary layer may be expected to reach the free-surface more rapidly than on an uncontrolled chute for an identical flow rate and stepped geometry. Analytical calculations (Appendix 1) demonstrate that:

$$\left(\frac{(x_I)_{pi}}{h\cos\theta}\right)^{1.4} = \left(\frac{(x_I)_{uc}}{h\cos\theta}\right)^{1.4} \times \frac{1}{\sqrt{1+\frac{F_*^{2/3}}{\frac{(x_I)_{pi}}{h\cos\theta}\sin\theta^{1/3}}\left(\mathrm{Fr}_1^{-2/3}+\frac{1}{2}\mathrm{Fr}_1^{4/3}\right)}} \qquad (3)$$

$$\left(\frac{(d_I)_{pi}}{h\cos\theta}\right)^{1.57} = \left(\frac{(d_I)_{uc}}{h\cos\theta}\right)^{1.57} \times \frac{1}{\sqrt{1+\frac{F_*^{2/3}}{\frac{(x_I)_{pi}}{h\cos\theta}\sin\theta^{1/3}}\left(\mathrm{Fr}_1^{-2/3}+\frac{1}{2}\mathrm{Fr}_1^{4/3}\right)}} \qquad (4)$$

where the subscript "pi" refers to pressurized intake conditions and Fr_1 is the intake flow Froude number. In Eqs (3) and (4), right-hand side, the last term is a correction factor taking into account the intake flow conditions. Note that the correction factor is a function of the flow rate, invert slope, step height and inflow Froude number

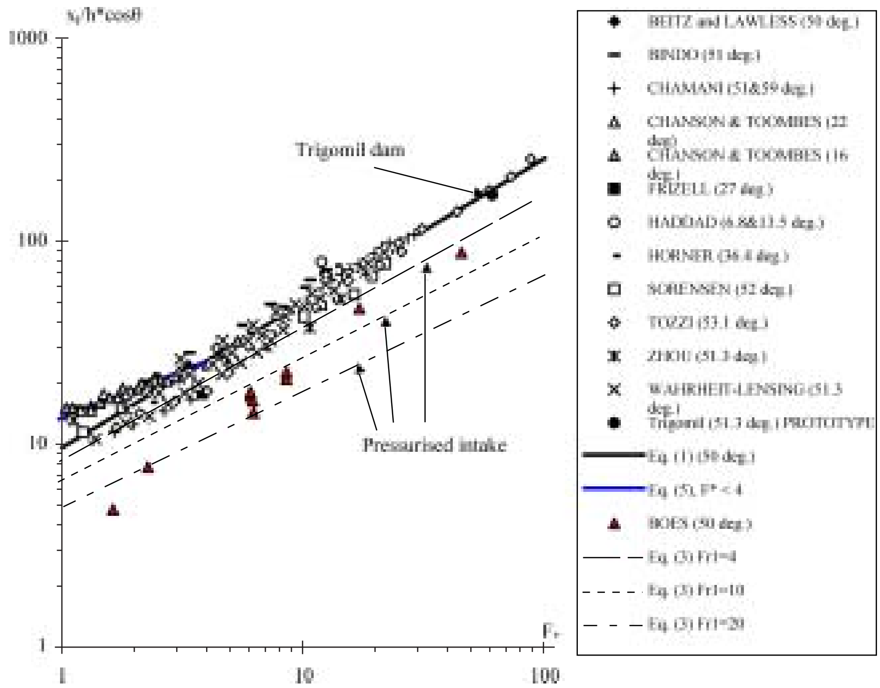
Equations (3) and (4) are presented in Fig. 3. They are compared with experimental data obtained with pressurized intake. Figure 3 shows a good agreement between Eqs (3) and (4), and Boes' (2000) data. The latter were obtained with inflow Froude numbers ranging from 3 to 10. The results (Fig. 3) demonstrate that the inception point is located significantly more upstream with pressurized intake inflows than with uncontrolled chutes, while the flow depth at inception is smaller with pressured intake. According to Eqs (3) and (4), the differences must increase with increasing inflow Froude number Fr_1; the data of Boes tend to agree with the trend, but the number of data is too limited to be statistically meaningful.

2.1 Discussion

Chanson and Toombes (2002a) presented in addition experimental observations of inception obtained in transition and skimming

(a)

Inflow conditions	References
Uncontrolled intake (model data)	Beitz and Lawless (1992), Bindo *et al.* (1993), Chanson and Toombes (2002), Frizell (1992), Haddad (1998), Horner (1969), Sorensen (1985), Tozzi (1992), Zhou (1996), Warheit-Lensing (1996)
Uncontrolled intake (prototype data)	Sanchez-Bribiesca and Gonzalez-Villareal (1996)
Pressurized intake (model data)	Boes (2000)



(b)

Inflow conditions	References
Uncontrolled intake (model data)	BaCara (1991), Bindo *et al.* (1993), Frizell (1992), Haddad (1998), Horner (1969), Sorensen (1985), Tozzi (1992), Zhou (1996), Warheit-Lensing (1996)
Pressurized intake (model data)	Boes (2000)

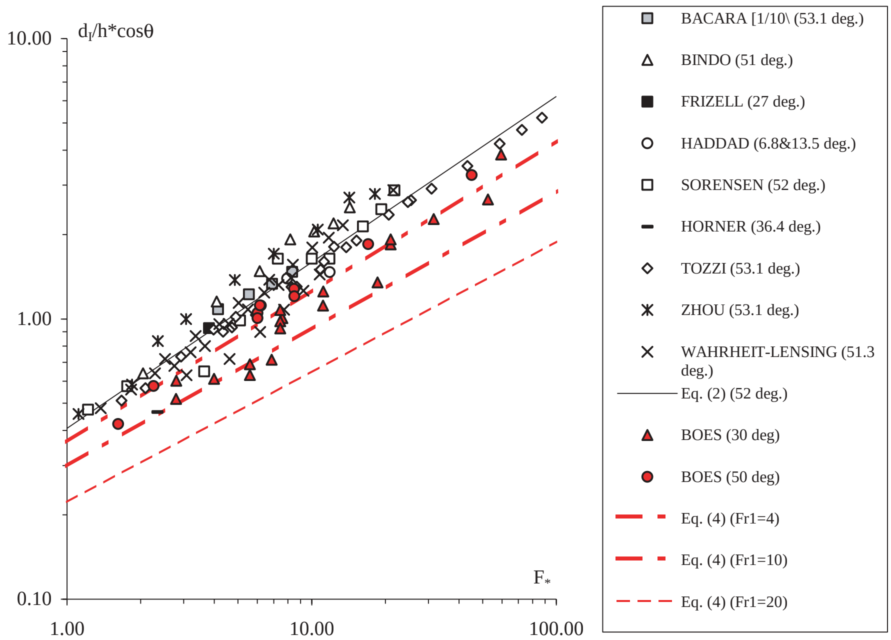


Figure 3 Flow conditions at the inception point of free-surface aeration. (a) Dimensionless location of the inception point of free-surface aeration—comparison between experimental data and Eqs (1) and (3) for $\theta = 50°$. (b) Dimensionless flow depth at the inception point of free-surface aeration—comparison between experimental data and Eqs (2) and (4) for $\theta = 50°$.

flow regimes for 16° and 22° slopes with uncontrolled broad-crest inflow. For $F_* < 4$, their data did not follow Eq. (1) and were best correlated by:

$$\frac{x_I}{h\cos\theta} = \frac{12.34}{(\sin\theta)^{0.0796}} F_*^{0.465}, \quad F_* < 4 \tag{5}$$

Equation (5) is shown in Fig. 3(a) for completeness.

3 Flow resistance in skimming flows

Skimming flows are characterized by significant form losses with formation of recirculating vortices between the main flow and the step corners. A comprehensive re-analysis of flow resistance included more than 38 model and four prototype studies with channel slopes ranging from 5.7° to 55° (Chanson *et al*., 2002). The equivalent Darcy friction coefficient f was typically between 0.1 and 0.35. Different research facilities yielded different results however and researchers continue to disagree on the reasons for these differences (Chanson, 2000).

Flow resistance data for large-size model data ($h > 0.020$ m, $\mathrm{Re} > 1\mathrm{E}+5$) are presented in Fig. 4(a) in terms of the equivalent Darcy friction factor f as function of the dimensionless step roughness height, where Re is the flow Reynolds number defined in terms of the hydraulic diameter D_H. Details of experiments are summarized in Table 1. For steep chutes ($\theta > 15°$), the friction factor data presented no obvious correlation with the relative step roughness height $h\cos\theta/D_H$, Reynolds, Froude nor Weber numbers (Chanson *et al*., 2002). They compared however favourably with a simplified analytical model of the pseudo-boundary shear stress which may be expressed, in dimensionless form, as:

$$f_d = \frac{2}{\sqrt{\pi}} \cdot \frac{1}{K} \tag{6}$$

where f_d is an equivalent Darcy friction factor estimate of the form drag, $1/K$ is the dimensionless expansion rate of the shear layer. The coefficient $1/K$ is assumed to be constant in a Prandtl mixing length model (Rajaratnam, 1976; Schlichting, 1979). Equation (6) predicts $f_d \approx 0.2$ for $K = 6$ that is close to observed friction factors (Fig. 4a). Skimming flow resistance data appeared to be distributed around three dominant values: $f \approx 0.105$, 0.17 and 0.30 as shown in Fig. 4(b). Figure 4(b) presents the probability distribution function of Darcy friction factor where the histogram columns represent the number of data with friction factors within the interval: e.g., the probability of friction factors from 0.18 to 0.20 is represented by the column labelled 0.18. The intervals were selected with a constant logarithmic increment. The first and last columns indicate the number of data with friction factors less than 0.08 and greater than 1.0, respectively.

The writer proposes that flow resistance in skimming flows is not an unique function of flow rate and stepped chute geometry. It is hypothesized that the form drag process may present several modes of excitation that are functions of the inflow conditions. At each step edge, shear instabilities may generate different cavity wake regimes, associated with different drag coefficients. In Fig. 4(b), the dominant values $f \approx 0.105$, 0.17 and 0.30 would correspond to three dominant modes (or regimes) of excitation induced by different inflow conditions sketched in Fig. 1.

Figure 4(b) shows that experiments with pressurized intake yielded lower flow resistance than for uncontrolled inflow conditions. For example, the re-analysis of data from Boes (2000) and André *et al.* (2003) gives $f \sim 0.10$ that is about three times smaller than the third dominant value ($f = 0.30$, Fig. 4b). Similarly, skimming flow experiments by Chanson and Toombes (2002b) down a flat slope ($\theta = 3.4°$, $h = 0.07$ m) with pressurized intake yielded friction factors three times smaller than data of Yasuda and Ohtsu (1999) on a 5.7° stepped slope with uncontrolled broad-crest. Larger flow resistance was observed on stepped chutes with uncontrolled inflow conditions with $f \sim 0.21$ in average for uncontrolled ogee crest and $f \sim 0.15$ in average for uncontrolled broad-crest inflow conditions (for all data in Fig. 4).

4 Application

On steep chutes ($\theta > 15°$), the flow acceleration and boundary layer development affect the flow properties. Complete flow calculations are tedious. Calculations of developing flow and uniform equilibrium flow may be combined to provide a general trend which may be used for a preliminary design (Chanson, 2004). The ideal fluid flow velocity at the downstream end of the chute is:

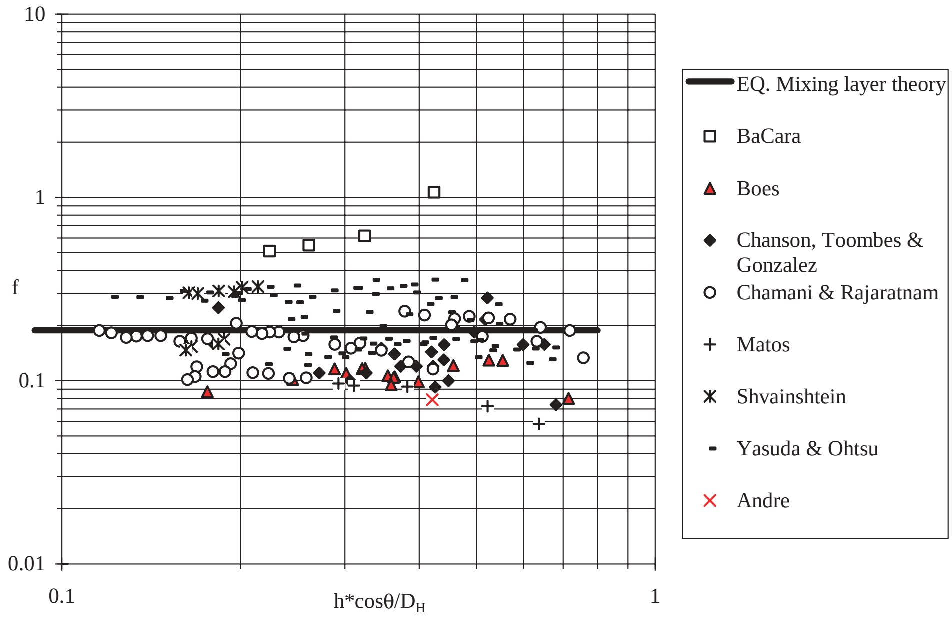
$$V_{max} = \sqrt{2g(H_{max} - d\cos\theta)} \tag{7}$$

where H_{max} is the upstream total head and d is the downstream depth of the ideal flow (Fig. 5 top). The downstream flow velocity U_w is smaller than the theoretical velocity V_{max} because of energy losses. Results are summarized in Fig. 5 in terms of U_w/V_{max} as a function of H_{max}/d_c where d_c is the critical depth. Developing flow and uniform equilibrium flow calculations are shown for smooth chutes, uncontrolled stepped chutes and stepped chutes with pressurized intake for $\theta = 50°$. (The latter case was analysed using the results presented above.) Prototype smooth chute data (Aviemore dam) and laboratory stepped chute data are shown and compared with the simplified method. Despite some scatter, Fig. 5 provides a simple practical tool to estimate the residual flow velocity U_w as a function of the flow rate and upstream total head. The results may be easily extended for other slopes.

Figure 5 demonstrates consistently the greater residual velocity and kinetic energy at the end of a stepped chute with pressurized intake. For example, considering a stepped spillway with an upstream total head above spillway toe $H_{max} = 60$ m, flow rate $q_w = 20\,\mathrm{m^2/s}$, step height $h = 0.6$ m, and slope $\theta = 50°$. Figure 5 predicts that the residual velocity U_w equals 24 and 31 m/s for a stepped chute with uncontrolled crest and pressurized intake flow conditions ($\mathrm{Fr}_1 = 10$), respectively. That is, the residual kinetic energy is 60% greater with a gated intake operating for $\mathrm{Fr}_1 = 10$.

(a)

Uncontrolled broad-crest	Yasuda & Ohtsu, Chanson, Toombes & Gonzalez
Uncontrolled smooth ogee crest	Yasuda & Ohtsu, Chamani & Rajaratnam
Uncontrolled ogee crest, with small first steps in ogee development	BaCaRa, Matos, Shvainshtein
Pressurized intake	Andre, Boes



(b)

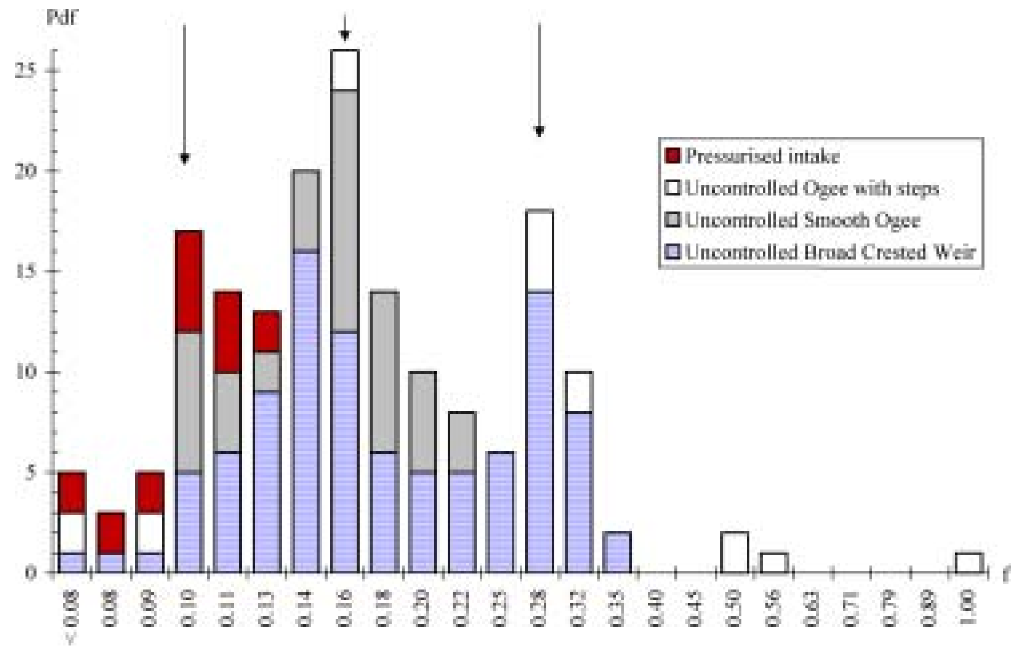


Figure 4 Darcy friction factor of skimming flows on stepped chute [179 data]. (a) Friction factor as function of the relative step roughness height. (b) Probability distribution function of stepped chute friction factor.

5 Conclusion

Skimming flow properties on stepped chutes are affected by the inflow conditions. Boundary layer calculations demonstrate that the location and flow depth at inception of free-surface aeration are substantially smaller than with an uncontrolled inflow chute, all other parameters being equal. The re-analysis of large-size experimental results shows that the equivalent Darcy friction factor is $f \sim 0.2$ in average on uncontrolled stepped chute and $f \sim 0.1$ on stepped chute with pressurized intake. A simple design chart (Fig. 5) is presented to estimate the residual flow energy, and the agreement of the calculations with experimental results is considered acceptable for preliminary design.

While this study demonstrate quantitative effects of inflow conditions on stepped chute flows, the basic mechanisms are not clearly understood. Overall the hydraulics of skimming

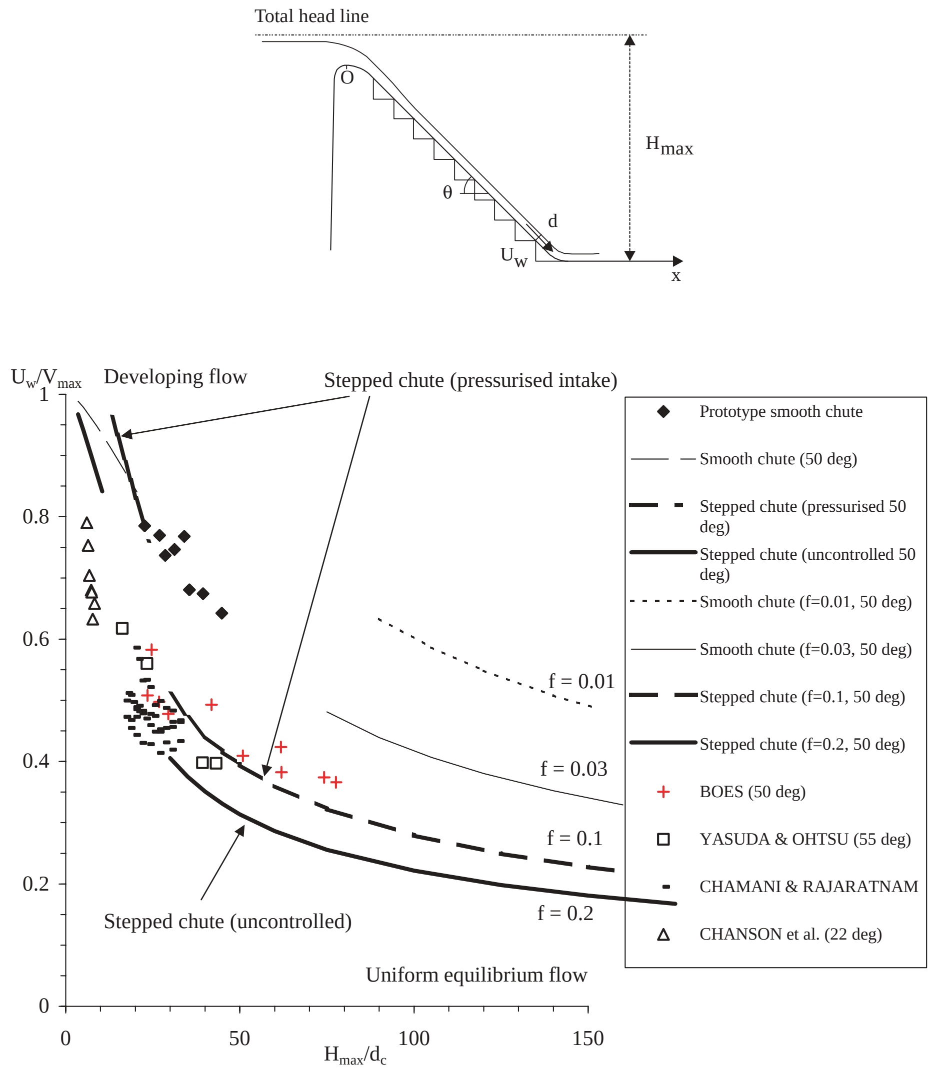


Figure 5 Residual flow velocity at the downstream end of the chute—comparison between smooth chute, stepped chute with pressurized intake and stepped chute with uncontrolled flow conditions

flows is significantly more complicated than traditional smooth chute hydraulic calculations. For example, some researchers (e.g. Matos, 2000; Chanson and Tombes, 2002a; Yasuda and Chanson, 2003; Gonzalez and Chanson, 2004) presented experimental results suggesting longitudinal oscillations of basic flow properties (velocity, depth, mean void fraction). Yasuda and Chanson (2003) proposed that these resulted from strong interference between vortex shedding in the shear layers behind each step edge and the free-surface.

Acknowledgments

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Appendix 1 Developing flow region calculations

At the upstream end of a stepped chute, a bottom turbulent boundary layer develops (Fig. 2). Its growth may be estimated as:

$$\frac{\delta}{x} = a\left(\frac{x}{k_s}\right)^{-b} \tag{A1}$$

where δ is the boundary layer thickness, x is the streamwise distance from the start of the growth of the boundary layer, k_s is the roughness height, and a and b are constants (e.g. Bauer, 1954; Cain and Wood, 1981). For a stepped profile, the roughness height is $k_s = h \cos\theta$. The velocity distribution is of the form:

$$\frac{V}{V_{max}} = \left(\frac{y}{\delta}\right)^{1/N} \tag{A2}$$

where V_{max} is the free-stream velocity in the ideal-fluid flow region (i.e. $\delta < y < d$) and y is the distance normal to the pseudo-bottom formed by the step edges. The Laser Doppler Anemometer (LDA) velocity data of Ohtsu and Yasuda (1997) showed that $N \sim 5$ in the developing boundary layer above a steep stepped chute. For an uncontrolled crest, the free-stream velocity is about:

$$V_{max} = \sqrt{2gx \sin\theta} \quad \text{(Uncontrolled crest)} \tag{A3}$$

For a pressurized intake, the free-stream velocity equals:

$$V_{\max} = \sqrt{2gx\ \sin\theta \left(1 + \frac{E_1}{x\sin\theta}\right)} \quad \text{(Pressurized intake)} \tag{A4}$$

where E_1 is the specific energy at the intake:

$$\frac{E_1}{d_c} = \mathrm{Fr}_1^{-2/3}\cos\theta + \frac{1}{2}\mathrm{Fr}_1^{4/3} \tag{A5}$$

where d_c is the critical depth and Fr_1 is the intake flow Froude number: $\mathrm{Fr}_1 = V_1/\sqrt{gd_1}$.

At the inception point ($x = x_I$), the combination of continuity and Bernoulli principles gives:

$$\frac{q_w}{V_{\max}\delta} = \frac{N}{N+1} \tag{A6}$$

where q_w is the flow rate per unit width. Combining Eq. (A6) with Eqs (A1)–(A4), it yields:

$$\left(\frac{x_I}{k_s}\right)^{3/2-b} = \frac{N+1}{Na\sqrt{2}}\frac{q_w}{\sqrt{g\sin\theta\, k_s^3}} \quad \text{(Uncontrolled crest)} \tag{A7}$$

$$\left(\frac{x_I}{k_s}\right)^{3/2-b} = \frac{N+1}{Na\sqrt{2}}\frac{q_w}{\sqrt{g\sin\theta\, k_s^3}} \times \frac{1}{\sqrt{1+(E_1/x_I\sin\theta)}} \quad \text{(Pressurized intake)} \tag{A8}$$

The boundary layer thickness equals the water depth d_I at inception. The continuity equation yields:

$$\left(\frac{d_I}{k_s}\right)^{(3-2b)/(2-2b)} = \frac{(N+1)a^{1/(2-2b)}}{N\sqrt{2}} \times \frac{q_w}{\sqrt{g\sin\theta\, k_s^3}} \quad \text{(Uncontrolled crest)} \tag{A9}$$

$$\left(\frac{d_I}{k_s}\right)^{(3-2b)/(2-2b)} = \frac{(N+1)a^{1/(2-2b)}}{N\sqrt{2}}\frac{q_w}{\sqrt{g\sin\theta\, k_s^3}} \times \frac{1}{\sqrt{1+(E_1/x_I\sin\theta)}} \times \text{(Pressurized intake)} \tag{A10}$$

Basically Eqs (A6)–(A9) which may be rewritten as:

$$\left(\frac{(x_I)_{pi}}{h\cos\theta}\right)^{3/2-b} = \left(\frac{(x_I)_{uc}}{h\cos\theta}\right)^{3/2-b} \frac{1}{\sqrt{1+\frac{F_*^{2/3}}{((x_I)_{pi}\ \sin\theta^{1/3}/h\ \cos\theta)}\left(\mathrm{Fr}_1^{-2/3}+\frac{1}{2}\mathrm{Fr}_1^{4/3}\right)}} \tag{A11}$$

$$\left(\frac{(d_I)_{pi}}{h\cos\theta}\right)^{(3-2b)/(2-2b)} = \left(\frac{(d_I)_{uc}}{h\cos\theta}\right)^{(3-2b)/(2-2b)} \times \frac{1}{\sqrt{1+\frac{F_*^{2/3}}{((x_I)_{pi}\ \sin\theta^{1/3}/h\ \cos\theta)}\left(\mathrm{Fr}_1^{-2/3}+\frac{1}{2}\mathrm{Fr}_1^{4/3}\right)}} \tag{A12}$$

where the subscripts "uc" and "pi" refer to uncontrolled inflow conditions and pressurized intake conditions, respectively, $F_* = q_w/\sqrt{g\sin\theta\,(h\cos\theta)^3}$ and Fr_1 is the intake flow Froude number.

For uncontrolled stepped chutes, Chanson (1995) compared successfully Eqs (A7) and (A9) with a large number of model and prototype data. If the data are compared with Eqs (A7) and (A9), the results of the statistical analysis yield $b = 0.1$.

Notation

a = Dimensionless constant
b = Dimensionless constant
D_H = Hydraulic diameter (m)
d = Flow depth (m) measured normal to the channel slope at the edge of a step
d_I = Flow depth (m) at the inception point of air entrainment
d_1 = Flow depth (m) immediately downstream of the channel intake
E = Specific energy (m)
$E1$ = Specific energy (m) at intake: $E_1 = d_1\cos\theta + V_1^2/2g$
Fr = Froude number defined as: $\mathrm{Fr} = V/\sqrt{gd}$
Fr_1 = Inflow Froude number: $\mathrm{Fr}_1 = V_1/\sqrt{gd_1}$
F_* = Dimensionless discharge: $F_* = q_w/\sqrt{g\sin\theta\,(h\cos\theta)^3}$
f = 1— Darcy–Weisbach friction factor; 2— equivalent Darcy friction factor estimate of stepped chute form drag
f_d = (equivalent) Darcy friction factor estimate of form drag
g = Acceleration due to gravity (m/s^2)
H = Total head (m)
$H_{\max}$ = Maximum total head (m) measured above the chute toe
h = Height of steps (m) (measured vertically)
K = Inverse of the spreading rate of a turbulent shear layer;
k_s = Step dimension (m) measured normal to the flow direction: $k_s = h\cos\theta$
N = Inverse of the velocity distribution exponent
q = Discharge per unit width (m^2/s)
Re = Reynolds number defined as: $\mathrm{Re} = VD_H/\nu_w$;
U_w = Residual flow velocity (m/s)
V = Velocity (m/s)
$V_{\max}$ = Free-stream velocity (m/s)
V_1 = Intake flow velocity (m/s)
W = Channel width (m)
x = Longitudinal distance (m) measured in the crest

x_I = Longitudinal distance (m) measured in the crest where free-surface aeration takes place

Greek symbols

δ = Boundary layer thickness (m)
ν = Kinematic viscosity (m^2/s)
θ = Channel slope

Subscripts

I = Inception point flow conditions
pi = Pressurized intake
uc = Uncontrolled inflow conditions
w = Water flow
1 = Inflow conditions

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