Power System Sensitivity Analysis for Probabilistic Small Signal Stability Assessment in a Deregulated Environment

Zhao Yang Dong, Chee Khiang Pang, and Pei Zhang

Abstract: Deregulations and market practices in power industry have brought great challenges to the system planning area. In particular, they introduce a variety of uncertainties to system planning. New techniques are required to cope with such uncertainties. As a promising approach, probabilistic methods are attracting more and more attentions by system planners. In small signal stability analysis, generation control parameters play an important role in determining the stability margin. The objective of this paper is to investigate power system state matrix sensitivity characteristics with respect to system parameter uncertainties with analytical and numerical approaches and to identify those parameters have great impact on system eigenvalues, therefore, the system stability properties. Those identified parameter variations need to be investigated with priority. The results can be used to help Regional Transmission Organizations (RTOs) and Independent System Operators (ISOs) perform planning studies under the open access environment.

Keywords: Power system stability, sensitivity analysis, probabilistic small signal stability, power system modeling, eigenvalue analysis, open access and electricity market.

1. INTRODUCTION

Power systems are experiencing more and more uncertainties especially under a deregulated environment. The system uncertainty may come from various sources but the main contribution is from uncertainties in system parameters and forecasted loads. Because of deregulation, in many cases the ISO or ITO planners have no access to Independent Power Producer (IPP) facilities, and therefore can not perform filed test in order to measure the real system parameters. Consequently uncertainties are inevitably introduced into the ISO or ITO's planning process. This has resulted in challenges for system planners in an open assess electricity market. In order to have a comprehensive picture of the system stability in planning, probabilistic stability assessment is attracting more and more attention over the traditional deterministic approach. Sensitivity analysis is the first step for probabilistic small signal stability studies. Sensitivity analysis has been investigated in various aspects in [1-7] as will be detailed in Section II. However, these previous work did not investigate the computational efficiency of analytical and numerical approaches in sensitivity computation as will be discussed in this paper.

After comparing the results of both approaches, the paper proposes guidelines of selecting the parameter perturbation sizes in sensitivity analysis. The paper also identifies the parameters that have great impact on system stability. It implies that, according to sensitivity analysis results, the planners need to model those parameters as random variables when performing small signal stability analysis. Given the fact that a power system is a nonlinear, complex and interactive large scale system, the parametric sensitivity to the system state matrix is very complex. Sensitivity of some of the parameters that have direct entry to the system state matrix can be computed analytically by studying their contribution in the state matrix; however, for those parameters which do not have direct entry to the state matrix, e.g. active power components of load, sensitivity computation can be very complex.

This paper presents the techniques of computing the sensitivity matrix of the critical system eigenvalues to non-deterministic random variables of the system. Both analytical and numerical methods are investigated and compared. The paper is organized as

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follows: we first introduce the eigenvalue sensitivity to non-deterministic parameters; then present the techniques of calculating such sensitivities for parameters such as exciter gains; Case studies based on New England system is presented to further justify the techniques.

2. SYSTEM STABILITY AND SENSITIVITY ANALYSIS

Small signal stability is the ability of the power system to maintain synchronism under disturbances. To investigate the small signal stability of a power system, we need to model the dynamic components (e.g. generators) and their control systems (such as excitation control system, and speed governor systems) in detail. The accuracy of power system stability analysis depends on the accuracy of the models used. Using of more accurate models could result in increases of overall power system transfer capability and associated economic benefits. Under open access environment, the planners may not be able to obtain this information accurately as they used to be. It is therefore important to attempt mathematically modeling and analyzing these parameters probabilistic-cally; therefore, the planner can gain a better understanding of the system stability margin.

A complex pattern of oscillations can result proceeding system disturbances; and linear, time-invariant state-space models are widely accepted ([1-9]) as a useful means of studying perturbations of the system state variable from the nominal values at a specific operating point [2]. Sensitivity analysis is then typically undertaken by examining the change in the system state matrix, or the eigenvalue sensitivity, for a variation in the system parameter in question [3]. With the sensitivity analysis results, further probabilistic stability properties of the power system can be obtained to assist in system planning.

Probabilistic eigenvalue analysis of power system dynamics is often applied with the advantage of determining the probabilistic distributions of critical eigenvalues, and hence providing an overall probability of system dynamic instability [4,5]. The probabilistic approach to dynamic power system analysis first occurred in 1978. A 2-machine test system at a particular load level was used to determine the eigenvalue probabilities stemming from the known statistical attributes of variations of system parameters [5]. Since then, several papers have taken a probabilistic approach to power system stability analysis, using larger, multimachine systems [5]. [4] proposes a hybrid utilization of central moments and cumulants, in order to ensure the consideration of both the dependence among the input random variables and the correction for probabilistic densities of eigenvalues.

Probabilistic methods of dynamic power system analysis have also been extended to the application of PSS design [5], though this is not the intention of this paper. In [6], the uncertainty of load level is stressed as a major area of concern. As such, the system instability probability calculations are based on the probabilistic nature of load demand and circuit breaker operating time. However, this method is only applied to a one-machine infinite-bus system.

We use the following process for sensitivity and stability analysis. The first step is to model the power system properly, [8]. A power system can be modeled by differential and algebraic equations (DAEs) as following,

$$\begin{aligned} \dot{X} &= F(X,Y,u), \\ 0 &= G(X,Y,u), \end{aligned} \qquad (1)$$

where **X** is a vector of the state variables, **Y** is the vector of algebraic variables and system parameters, and **u** is the control input. The DAEs can be linearized and rearranged at operating points as shown below,

$$\Delta\dot{X} = A\Delta X \qquad (2)$$

The dynamics of the system can be described by the linearized differential equation. The stability of the system is therefore determined by the eigenvalues of the state matrix A. Based on the small signal stability theorem and system dynamics, critical eigenvalues of the system can be identified based on their mode of oscillations. With these critical eigenvalues of a power system, the small signal stability properties can be obtained at the particular equilibrium only. In order to assess the system small signal stability over a range of operating points, repeated computation is required so that the system state matrix and corresponding critical eigenvalues can be computed at each operating point to obtain an overall picture of the system small signal stability property. Given the complexity of a power system, the total number of possible parameter variations can be huge, and makes this approach of computing critical eigenvalues computationally inefficient and even impractical in some cases. This leads to the investigation of probabilistic small signal stability study.

If the system parameter variations can be described by probabilistic density functions, the probabilistic approach can bet used to find the probability of the system eigenvalues remain in the left half complex plane. In order to do so, it is necessary to compute the sensitivity of the eigenvalue to the system parameters which are subject to variation following certain probabilistic density functions [1,7].

When a power system is subject to small signal disturbances and perturbations, the state matrix A contains functions of non-deterministic variables. As such, these random variables will cause eigenvalues of A to be non-deterministic. After identifying the critical eigenvalues the sensitivities of the eigenvalues to system parameters forms the sensitivity matrix, [1-7].

3. ANALYTICAL METHOD

In this section, the eigenvalues' sensitivities to non-deterministic system parameters will be derived from the left eigenvector $\mathbf{w}_i$ and right eigenvector $\mathbf{v}_i$ of the same eigenvalue λ_i of the state matrix **A**. We have

$$\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i, \tag{3}$$

$$\mathbf{w}_i^T\mathbf{A} = \lambda_i \mathbf{w}_i^T. \tag{4}$$

The right eigenvectors $\mathbf{v}_i$ are also known as the mode shapes of the system and the left eigenvectors are actually the right eigenvectors of $\mathbf{A}^T$ [9]. For large scale power systems, the eigenvalues λ_i are distinct. Taking partial derivative of equation (3) with respect to non-deterministic system parameter $\mathbf{K}_j$:

$$\frac{\partial \mathbf{A}}{\partial \mathbf{K}_j}\mathbf{v}_i + \mathbf{A}\frac{\partial \mathbf{v}_i}{\partial \mathbf{K}_j} = \lambda_i \frac{\partial \mathbf{v}_i}{\partial \mathbf{K}_j} + \frac{\partial \lambda_i}{\partial \mathbf{K}_j}\mathbf{v}_i. \tag{5}$$

Now taking dot product of each term with the left eigenvector $\mathbf{w}_i^T$:

$$w_i^T \frac{\partial \mathbf{A}}{\partial \mathbf{K}_j}\mathbf{v}_i + w_i^T \mathbf{A}\frac{\partial \mathbf{v}_i}{\partial \mathbf{K}_j} = w_i^T \lambda_i \frac{\partial \mathbf{v}_i}{\partial \mathbf{K}_j} + w_i^T \frac{\partial \lambda_i}{\partial \mathbf{K}_j}\mathbf{v}_i. \tag{6}$$

Since $w_i^T \mathbf{A}\frac{\partial \mathbf{v}_i}{\partial \mathbf{K}_j} = \frac{\partial \mathbf{v}_i}{\partial \mathbf{K}_j}\mathbf{w}_i^T \lambda_i$ we have:

$$\frac{\partial \lambda_i}{\partial \mathbf{K}_j} = \frac{\mathbf{w}_i^T \frac{\partial \mathbf{A}}{\partial \mathbf{K}_j}\mathbf{v}_i}{\mathbf{w}_i^T \bullet \mathbf{v}_i} \tag{7}$$

or in matrix form

$$\Delta\Lambda = \mathbf{S}\cdot\Delta\Gamma, \tag{8}$$

where $\Delta\Gamma = [\Delta K_j]$, $\Delta\Lambda = [\Delta\lambda_i]$ and **S** is the eigenvalue sensitivity matrix.

In order to evaluate the sensitivity matrix of the system with respect to its non-deterministic parameters, the partial derivative of the matrix $\partial \mathbf{A}/\partial \mathbf{K}_j$ will have to be calculated first. The base values of $\mathbf{K}_j$ are obtained from conventional Newton-Raphson load flow solutions.

If the non-deterministic system parameters are states of the state matrix **A**, then the matrix $\partial \mathbf{A}/\partial \mathbf{K}_j$ can be obtained from an analytical or a direct approximation method as shown in the sequel. This approach is made possible as the matrix **A** can be expressed explicitly in terms of the state variables, hence the required system parameters.

However, if the non-deterministic parameters are not states of the system, an analytical solution though possible is proven to be computational intensive. As such, a partial finite difference approach is recommended and used.

3.1. Analytical method for control parameter K_A

If the required perturbed parameters appear explicitly in state matrix **A** (*e.g.* IEEE type 1 exciter – see Fig. 1 – voltage regulator gain K_A), $\partial \mathbf{A}/\partial K_A$ can be obtained by direct differentiation of elements in **A**, *i.e.*:

$$\frac{\partial \mathbf{A}}{\partial K_A} = \frac{\partial}{\partial K_A}\{a_{ij}\}, \tag{9}$$

where a_{ij} are entries in **A**. The sensitivity of the state matrix to the exciter voltage regulator gain K_A is obtained by (10) – (11).

$$\begin{bmatrix} \dot{\Delta V}_{ex1}^{(i)} \\ \dot{\Delta V}_{ex2}^{(i)} \\ \dot{\Delta V}_{ex3}^{(i)} \\ \dot{\Delta E}_{fd}^{(i)} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_R^{(i)}} & 0 & 0 & 0 \\ -\frac{K_A^{(i)}}{T_A^{(i)}} & -\frac{1}{T_A^{(i)}} & \frac{K_A^{(i)}T_A^{(i)}}{K_F^{(i)}T_F^{(i)}} & -\frac{K_A^{(i)}T_A^{(i)}}{K_F^{(i)}T_F^{(i)}} \\ 0 & 0 & -\frac{1}{T_F^{(i)}} & \frac{1}{T_F^{(i)}} \\ 0 & \frac{1}{T_E^{(i)}} & 0 & -\frac{K_E^{(i)}+S_E^{(i)}}{T_E^{(i)}} \end{bmatrix} \begin{bmatrix} \Delta V_{ex1}^{(i)} \\ \Delta V_{ex2}^{(i)} \\ \Delta V_{ex3}^{(i)} \\ \Delta E_{fd}^{(i)} \end{bmatrix} + \begin{bmatrix} \frac{V_{gx}^{(i)}}{T_R^{(i)}V_g^{(i)}} & \frac{V_{gy}^{(i)}}{T_R^{(i)}V_g^{(i)}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta V_{gx}^{(i)} \\ \Delta V_{gy}^{(i)} \end{bmatrix} \tag{10}$$

$$\frac{\partial A}{\partial K_A^{(i)}} = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & -\frac{1}{T_A^{(i)}} & 0 & \frac{T_A^{(i)}}{K_F^{(i)}T_F^{(i)}} & -\frac{T_A^{(i)}}{K_F^{(i)}T_F^{(i)}} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{11}$$

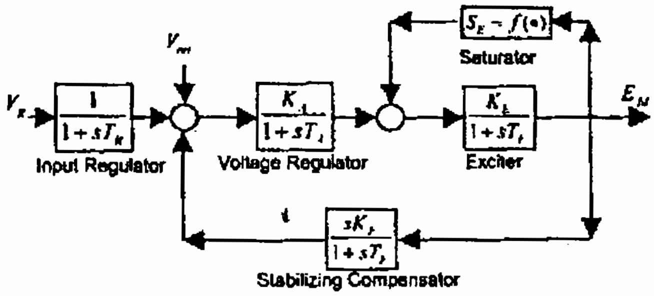

Fig. 1. Block diagram of the IEEE type 1 exciter.

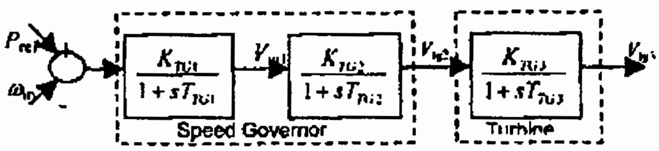

Fig. 2. Block diagram of speed governor system.

3.2. Analytical method for speed governor system gain K_{TG}

Normally speed governor systems have slow responses and do not significantly affect local modes with frequencies greater than 1 Hz. However, they may have negative damping effects on interarea mode oscillations in an interconnected power system. A simple governor model is used in this paper for sensitivity analysis [8,10] – see Fig. 2. The dynamic equations of a typical speed governor system installed at the i-th generator are given in (12).

$$\Delta\dot{X}_T = \begin{bmatrix} \frac{-1}{T_{TG4}} & 0 & 0 \\ \frac{K_{TG2}}{T_{TG5}} & \frac{-1}{T_{TG5}} & 0 \\ 0 & \frac{K_{TG3}}{T_{TG6}} & \frac{-1}{T_{TG6}} \end{bmatrix} \Delta X_T + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \Delta V_G + \begin{bmatrix} -\frac{K_{TG1}}{T_{TG4}} \\ 0 \\ 0 \end{bmatrix} \Delta\omega, \quad (12)$$

where let $\Delta X_T=[\Delta V_{tg1} \quad \Delta V_{tg2} \quad \Delta V_{tg3}]^T$ and $\Delta V_G = [\Delta V_x \ \Delta V_y]^T$. The interactions between the synchronous machines and the governor system have to be considered as in (13).

$$\Delta\dot{X}_{sg}(i) = \begin{bmatrix} & & \cdots & \\ 0 & 0 & \frac{(1-K_{TG2(i)}}{2H_{(i)}} & \frac{(1-K_{TG3(i)}}{2H_{(i)}} \end{bmatrix} \Delta X_{sg}(i) \quad (13)$$

Similar to that of the excitation system parameters, the speed governor system gains K_{TG1}, K_{TG2} and K_{TG3}'s individual contribution to the state matrix sensitivity are given in (14)–(18).

$$\frac{\partial A}{\partial K_{TG2}^{(i)}} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{T_{TG3}} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (14)$$

$$\frac{\partial A}{\partial K_{TG3}^{(i)}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{T_{TG6}} & 0 \end{bmatrix} \quad (15)$$

$$\frac{\partial A}{\partial K_{TG1}^{(i)}} = \begin{bmatrix} \frac{-1}{T_{TG4}} \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

$$\frac{\partial A}{\partial K_{TG2(i)}} = \begin{bmatrix} & & \cdots & \\ 0 & 0 & \frac{-1}{2H_{(i)}} & 0 \end{bmatrix} \quad (17)$$

$$\frac{\partial A}{\partial K_{TG3(i)}} = \begin{bmatrix} & & & \cdots \\ 0 & 0 & 0 & \frac{-1}{2H_{(i)}} \end{bmatrix} \quad (18)$$

For a governor system, the time constants T_{TGi} may have some deviation from their rated values over the time as well. Because these time constants are often denominators in the state matrix, they may have more impact than the system gains K_{TGi}. Their contribution to the partial derivative of the state matrix can be obtained similar to that of K_{TGi}.

So far the analytical approach only applies to parameters which have direct entry into the system state matrix such that an analytical form of the sensitivity can be accurately computed. However, there are parameters such as the rotor angles and load powers which do not have a direct entry into the state matrix but do have impact on the system state matrix. To find out the analytical form of sensitivity with respect to such parameters – e.g. $\partial A/\partial P_i$ is a very complex task. For a large scale system it may even becomes impractical to find out an analytical form. A numerical approach is more appropriate in view of computational efficiency and practicality. We next introduce the numerical approach to computing sensitivity.

4. NUMERICAL METHOD

The relationship of the system parameters to the system state matrix is very complex. Even though the impact from system parameters such as load does not directly appear in the system state matrix, however, the impact does exist indirectly. It is well known that when the load changes, the system voltages and voltage angles will also change to meet the power flow conditions.

A numerical approach is generally more desirable due to its simplicity in implementation and ability to meet the requirement of complex large scale systems. Based on our studies, we find that 1% perturbation is a good choice in most cases as parameter changes that are too large violate local linearity assumptions and parameter changes that are too small cause high round-off errors after division.

From the definition of partial derivative:

$$\frac{\partial \Lambda}{\partial K_j} = \lim_{\Delta K_j \to 0} \frac{\Lambda(K_j + \Delta K_j) - \Lambda(K_j)}{\Delta K_j}, \quad (19)$$

where $\Lambda(K_j+\Delta K_j)$ and $\Lambda(K_j)$ are the eigenvalue of the system matrix **A** after and before the parameter perturbation. Normally, only the critical eigenvalues need to be evaluated subject to small perturbations. We have

$$\frac{\partial \Lambda}{\partial K_j} = \frac{\Lambda(K_j + \Delta K_j) - \Lambda(K_j)}{\Delta K_j}, \quad (20)$$

if ΔK_j is small when compared to the entries in Λ.

The numerical approach applies to computing the sensitivity of eigenvalues with respect to the excitation system gains, governor system gains and time constants, which have been evaluated analytically in previous sections.

It is evident that analytical approach may become to complex for some parameters such as system loads. However, the numerical approach provides a convenient way to approximate the analytical sensitivity results. The amount of perturbation to perform the numerical approach, however, needs to be chosen carefully in order to ensure the robustness of the sensitivity computation. If it is too small, the system may not be able to respond to such perturbation numerically; if it is too large the system may not be able to maintain smooth change of states. In either case the final results will not be true approximation to the actual sensitivities. Subsequent case study of the New England test system will indicate a suitable choice of perturbations as acceptable level.

5. CASE STUDY

The New England system – see Fig. 3 - is used to test the derived algorithms in computing the eigenvalue sensitivities. Both the numerical and analytical approaches are applied to compute the eigenvalue sensitivity factor with respect to the selected power system parameters.

5.1. Numerical approach v.s. analytical approach

First the analytical approach of eigenvalue sensitivity computation with respect to K_A variations is performed as shown in Table 1. Both analytical and numerical approaches have been applied to study the eigenvalue sensitivity based on the New England system for different perturbation levels as given in Tables 1-5, which show that a 1% perturbation in numerical method can produce reasonably good results compared with the analytical methods. This is very important for large scale system analysis and for sensitivity computations with respect to other parameters not directly appear in the state matrix.

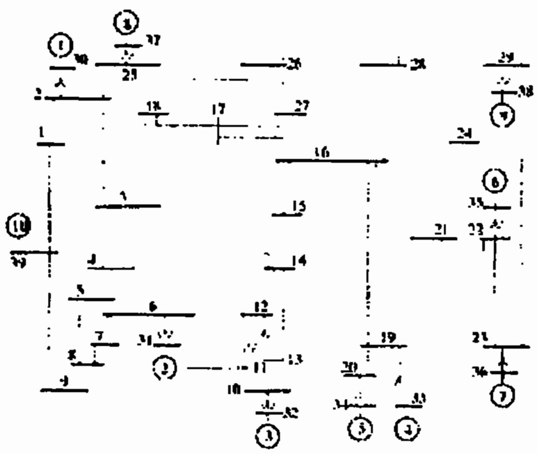

Fig. 3. The new england test system.

Table 1. Sensitivity factor of critical eigenvalue to exciter gain KA of generator at bus 30 using the analytical approach.

No	Oscillation Mode	Sensitivity Factor ($\times 10^{-3}$)
1	-0.3678 ± j8.7547	0.00078162184764
2	-0.4031 ± j8.6747	0.01106642351696
3	-0.3139 ± j8.4773	-0.00063404657501
4	-0.2745 ± j7.4595	0.00022037834264
5	0.0013 ± j6.9647	-0.00957289052384
6	-0.2493 ± j6.9965	-0.00513654645813
7	-0.2507 ± j6.3571	-0.00071777018732
8	-0.2600 ± j5.9958	-0.00214383955583
9	-0.2798 ± j3.8493	-0.06791209198673

Table 2. Sensitivity factor of critical eigenvalue to exciter gain K_A of generator at bus 30 using the numerical approach.

No	Oscillation Mode	Sensitivity Factor ($\times 10^{-3}$)
1	-0.3678 ± j8.7547	0.00078163731043
2	-0.4031 ± j8.6747	0.01106188979083
3	-0.3139 ± j8.4773	-0.00063335882650
4	-0.2745 ± j7.4595	0.00022046186654
5	0.0013 ± j6.9647	-0.00959517106325
6	-0.2493 ± j6.9965	-0.00513931605273
7	-0.2507 ± j6.3571	-0.00071194674489
8	-0.2600 ± j5.9958	-0.00208513232680
9	-0.2798 ± j3.8493	-0.06636688800743

Table 3. Sensitivity analysis errors for different exciter gain K_A perturbation levels.

No	Error (%) K_A from 5.0 to 6.0	Error (%) K_A from 5.0 to 5.1	Error (%) K_A from 5.0 to 5.01
1	0.008	0.003	0.0004
2	-0.98	-0.09	-0.009
3	-2.10	-0.2	-0.020
4	0.46	0.04	0.004
5	15.41	1.52	0.152
6	0.80	0.08	0.008
7	19.74	1.95	0.194
8	-14.88	-1.45	-0.145
9	-121.60	-11.92	-1.190

Table 4. Comparison of analytical to numerical approaches in computing the eigenvalue sensitivities (speed governor gain K_{TG1} at bus 30, sensitivity values $\times 10^{-3}$).

Mod. #	Numerical	Analytical	Error
1	-0.000272155	-0.000272677	0.191684%
2	0.016839917	0.016877105	0.220346%
3	-0.00044598	-0.000441561	1.000776%
4	0.000301912	0.000303428	0.499824%
5	0.498817537	0.498859015	0.008315%
6	-0.01390707	-0.013915327	0.059333%
7	0.00113737	0.001136542	0.072868%
8	0.070758053	0.070710436	0.067340%
9	0.043422405	0.043419504	0.006681%

Table 5. Percentage errors for eigenvalue sensitivity computation between numerical vs analytical methods at bus 30 (numerical method uses 1% perturbation).

No	K_A	K_{TG1}	K_{TG2}	K_{TG3}	T_{TG4}	T_{TG5}	T_{TG6}
1	0.002	0.1917	1.0752	0.6984	0.5102	1.6292	0.9819
2	0.041	0.2203	1.356	0.007	1.0787	0.997	1.4562
3	0.1085	1.0008	1.205	0.0087	1.4968	0.9993	1.0484
4	0.0379	0.4998	2.3475	0.0166	0.8038	1.0065	1.4636
5	0.2327	0.0083	0.0371	0.0042	0.7114	0.9841	1.082
6	0.0539	0.0593	1.0034	0.0174	0.2743	0.9695	1.1047
7	0.8113	0.0729	0.5967	0.1796	0.1166	0.7947	0.9689
8	2.7384	0.0673	0.5954	0.1815	0.0649	0.7923	0.9671
9	2.2753	0.0067	0.1604	0.0441	0.3598	0.9286	0.9419

Tables 6-7 show the complete sensitivity values with respect to governor time constant T_{TG6} and gain K_{TG2} for all generators and all critical modes.

It can be seen that even though the analytical approach does provide accurate sensitivity factor, but it carries with very complex numerical analysis and different analysis has to be done for different system parameters or even for different systems. Numerical approach is computationally simple and has been proven to be accurate enough as comparing with the analytical approach. Generally a 1% perturbation is able to generate acceptable results comparable to that of the analytical analysis results but saves significantly analytical time and possibility of errors. Because of the simplicity of the numerical approach, it will be used in sensitivity analysis for other parameters as well with a 1% perturbation.

5.2. The most sensitive parameters and machines

Based on the analytical and more importantly numerical methods discussed in previous sections, the eigenvalue sensitivity analysis is performed to all the system excitation system gains K_{Ah}, governor gains K_{TGi} and governor time constant T_{TGi} to find out the parameters which have the most impact on system eigenvalue variations, i.e. these parameters with which the system eigenvalue sensitivities are the highest. Tables 5-6 are selected complete eigenvalue sensitivities for all 9 synchronous generators. Tables 7-14 summarize the findings and identify the most sensitive parameters, critical eigenvalues and corresponding generators. By identifying such parameters and associated machines, the system operator is able to pay more attention to these identified sensitive parameters and machines to manage the system stability more efficiently and effectively.

Table. 6. Eigenvalue sensitivity analysis of the governor time constant T_{TG6}.

#	T_{TG6} at Machine 30	T_{TG6} at Machine 31	T_{TG6} at Machine 32	T_{TG6} at Machine 33	T_{TG6} at Machine 34	T_{TG6} at Machine 35	T_{TG6} at Machine 36	T_{TG6} at Machine 37	T_{TG6} at Machine 38
1	-1.11551E-06	-3.558E-11	-4.788E-11	6.6291E-08	3.80224E-08	-1.93559E-07	-0.000139367	5.07058E-08	-2.84968E-08
2	6.46559E-05	-4.0387E-09	-1.18767E-09	-3.71654E-06	-3.09263E-07	4.77072E-09	9.64761E-07	-3.36758E-07	-1.10567E-05
3	-1.95761E-06	-2.57155E-09	3.168E-11	-0.00016377	-3.88693E-05	1.12878E-07	-1.45484E-05	1.47502E-07	-1.57724E-06
4	1.76353E-06	-2.59354E-06	-5.21521E-08	-1.55722E-08	-2.07263E-08	-1.72079E-08	-2.90804E-06	-1.818E-11	2.34882E-08
5	0.003414985	-1.18294E-07	4.4259E-09	-3.76821E-08	1.09418E-05	-1.76852E-07	-4.33723E-06	6.69378E-07	-0.000150108
6	-8.95896E-05	-8.1947E-07	1.08412E-08	-2.99485E-07	-0.000156474	4.64143E-07	-0.000130285	-2.65358E-08	-5.22308E-05
7	8.58656E-06	-3.89898E-06	2.06921E-08	-1.86785E-06	-5.44971E-05	3.64671E-09	5.08404E-08	1.65754E-09	-0.000316083
8	0.000572916	-1.26273E-07	3.49368E-08	-2.40172E-05	-0.000387368	-7.10243E-08	-6.30757E-06	-1.02286E-07	-0.000482093
9	0.000838442	-1.08889E-06	-3.6804E-08	-9.65502E-05	-0.000242267	-3.66778E-07	-0.000118408	-5.26866E-07	-0.000349147

Table 7. Eigenvalue sensitivity of the governor gain K_{TG2}.

#	K_{TG2} at Machine 30	K_{TG2} at Machine 31	K_{TG2} at Machine 32	K_{TG2} at Machine 33	K_{TG2} at Machine 34	K_{TG2} at Machine 35	K_{TG2} at Machine 36	K_{TG2} at Machine 37	K_{TG2} at Machine 38
1	-0.000219504	2.06935E-06	4.53992E-07	0.001545566	-3.82499E-05	0.002097239	0.170009071	-0.000185777	5.3425E-06
2	0.006788523	0.000101141	1.16527E-05	0.000753583	0.000157565	-4.18803E-05	-0.001559526	0.001113672	0.00355742
3	-0.001191229	2.25112E-05	-1.59018E-07	0.181296538	0.069240347	-0.001011621	0.022957771	-0.000533284	0.000290418
4	0.000103514	0.026501181	0.000309328	2.01391E-05	5.19781E-05	0.000117185	0.003536287	1.79402E-07	4.75415E-06
5	0.336134994	-0.001874333	-4.24984E-05	3.27926E-05	-0.010366876	0.001057575	0.00768202	-0.001788257	0.03501701
6	-0.009386866	0.013020627	-6.905E-05	3.94168E-05	0.208190867	-0.003035191	0.114276293	6.4296E-05	0.017539022
7	0.000899972	0.050480814	-0.000180781	0.001762589	0.066927192	-1.87307E-05	-9.46254E-05	-9.72278E-06	0.034078409
8	0.055932815	-0.001087897	-0.00017022	0.017075462	0.404146387	0.000330395	0.005577833	0.00010949	0.067435278
9	0.03323575	0.007897174	5.57848E-05	0.037064593	0.138868907	0.000628983	0.043878354	0.000250131	0.041316363

It is shown in Tables 6 and 7 that T_{TG6} at bus 30 is the most sensitive one among all T_{TG6}s at critical mode 5 (0.003414985); and K_{TG1} at bus 34 is the most sensitive one at critical mode 8 (0.404146387). Similar analysis is performed to all other parameters at all machines. The results are given in Tables 8-12.

Table 8. Identify the most sensitive eigenvalue to exciter gain K_A at each generator.

Machine No	K_A	
	$d\lambda/dK_A(\times10^{-1})$	Critical Mode No
30	-0.06791209198673	9
31	0.34809738436733	7
32	0.32487814171438	9
33	-0.14168737887021	3
34	0.43863295050830	8
35	0.26266275761013	1
36	0.29353585500314	9
37	0.25569157094522	2
38	0.34868931208412	9
Maximum	0.43863295050830	(34, 8)

Table 9. Identify the most sensitive eigenvalue to speed governor gain K_{TG1} at each generator.

Machine No	K_{TG1}	
	$d\lambda/dK_{TG1}(\times10^{-1})$	Critical Mode No
30	0.49885901529951	5
31	0.09469911688998	4
32	-0.00000534910129	7
33	0.21556520763527	3
34	-0.19964981762726	9
35	-0.00002071690747	6
36	0.49600058177804	1
37	-0.00002595289485	5
38	0.09035773666933	7
Maximum	0.49885901529951	(30, 5)

Table 10. Identify the most sensitive eigenvalue to speed governor gain K_{TG3} at each generator.

Machine No	K_{TG3}	
	$d\lambda/dK_{TG3}(\times1)$	Critical Mode No
30	-0.00098710467616	5
31	-0.04169354401310	4
32	-0.00021299073944	4
33	-0.11734670906906	3
34	0.07513066243194	9
35	-0.00048199751110	6
36	-0.11193931004559	1
37	-0.00144899144764	2
38	-0.04415122152392	7
Maximum	-0.11734670906906	(33, 3)

Table 11. Identify the most sensitive eigenvalue to speed governor time constant T_{TG4} at each generator.

Machine No	T_{TG4}	
	$d\lambda/dT_{TG4}(\times1)$	Critical Mode No
30	-0.58393197169450	5
31	-0.00992954680167	4
32	-0.00000012095101	4
33	-0.02916175607370	3
34	0.79719080800131	8
35	-0.00000084238796	9
36	0.07860269814019	9
37	0.00000087982427	5
38	0.03049729669530	9
Maximum	0.79719080800131	(34, 8)

Table 12. Identify the most sensitive eigenvalue to speed governor time constant T_{TG5} at each generator.

Machine No	T_{TG5}	
	$d\lambda/dT_{TG5}(\times1)$	Critical Mode No
30	-0.00029901705056	5
31	0.09996306255147	7
32	-0.00000005215206	4
33	0.02636963298285	9
34	0.65079925596713	8
35	0.00000046414332	6
36	0.08358576995736	9
37	0.00000087982427	5
38	0.15184475179912	8
Maximum	0.65079925596713	(34, 8)

From Tables 6-12, we can conclude that (i) Machine 34 is most sensitive to parameter uncertainties subject to small signal stability; (ii) the governor system parameters are most sensitive for machine 34 to the 8th critical mode of oscillation; (iii) the governor time constants have the most impact on eigenvalue sensitivity as compared with other control system gains in a scale of 10^3; therefore more attention should be paid to these time constants than gains; (iv) excitation system gain K_A has about 10^{-3} times less impact on eigenvalue sensitivity to all buses as compared with governor time constants; (v) machines 30 and 33 are also sensitive to parameter uncertainties, however less than that of machine 34 ; (vi) the governor system gains have much more impact on eigenvalue sensitivity than excitation system gains in an approximately 10^3 times more. These observations indicate that the governor gains and time constants have more impact on eigenvalue sensitivity than excitation system gains, and therefore needs more attention to prevent small signal instability. It also indicates that machine 34 may need more attention in dispatch and maintenance to avoid possible small signal instability.

6. CONCLUSIONS

Sensitivity of the system eigenvalues to system parameters indicates the impact of such parameter to the system stability. It is preferable to have analytical approach to compute the eigenvalue sensitivity; however, the analytical form of sensitivity may be too complex for realistic computation for some system parameters and for large scale systems. With case study and mathematical analysis, the paper concludes that by proper perturbation level, the numerical approach, which is simple in algorithm, is able to produce accurate results as compared with the accurate analytical approach when computing the sensitivity factors. In addition, our research also finds out that the numerical approach can compute all of the sensitivity factors within one-time run, while the analytical method has to compute the sensitivity factor by each parameter. These findings can reduce the computational cost significantly for large scale systems and parameters whose relationship with the state matrix is complex.

In this paper, the critical eigenvalues' sensitivity matrix for non-deterministic power system parameters including excitation system gains is derived using analytical and numerical methods. Based on the case study with the New England system, we find out that less than 1% perturbation is reasonable for numerical approach to compute sensitivity factors. Larger perturbation value may cause significant errors.

The paper also indicates that the eigenvalues are most sensitive to the changes of governor gains and time constants. It again proves that the small signal stability analysis should model governor system. Since those parameters have significant impact on the stability margin, the variation of governor parameters needs to be modeled as random variables in probabilistic small signal stability analysis.

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