# Magic-angle effects in the interlayer magnetoresistance of quasi-one-dimensional metals due to interchain incoherence 

Urban Lundin* and Ross H. McKenzie<br>Department of Physics, University of Queensland, Brisbane Qld 4072, Australia

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#### Abstract

The dependence of the magnetoresistance of quasi-one-dimensional metals on the direction of the magnetic field show dips when the field is tilted at the so-called magic angles determined by the structural dimensions of the materials. There is currently no accepted explanation for these magic-angle effects. We present a possible explanation. Our model is based on the assumption that, the intralayer transport in the second most conducting direction has a small contribution from incoherent electrons. This incoherence is modeled by a small uncertainty in momentum perpendicular to the most conducting (chain) direction. Our model predicts the magic angles seen in interlayer transport measurements for different orientations of the field. We compare our results to predictions by other models and to experiment.


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## I. INTRODUCTION

There is a fundamental relation between quantum coherence of excitations properties and transport properties in strongly correlated metals. ${ }^{1-5}$ Scattering of electrons affects the transport, but also blurs information about the momentum of the electron, and therefore changes the coherence of the electrons, or quasiparticle excitations. Generally, the effect of strong electronic correlations and incoherent excitations are enhanced in systems of reduced dimensionality. A striking example of this are Luttinger liquids in one dimension. The quasi-one-dimensional Bechgaard salts $(\text { TMTST })_{2} \mathrm{X}\left(\mathrm{X}=\mathrm{PF}_{6}, \mathrm{ClO}_{4}, \mathrm{NO}_{3}, \ldots,\right)$ show a rich phase diagram ranging from field induced spin density waves to insulators and superconductors, depending on pressure and anion. ${ }^{6,7}$ The structures are highly anisotropic, and show interesting features as a magnetic field is applied. Recently, in some quasi-two-dimensional organic metals definitive signatures $^{8}$ of coherent interlayer transport have been seen. ${ }^{9,10}$

Lebed ${ }^{11}$ predicted that resistance maxima would occur when orbits along directions in the crystal are commensurate with the applied field at the so-called "magic angles" (MA) where $\tan \theta=l b / c$, where $\theta$ is the angle between the magnetic field, tilted in the $(y, z)$ plane, and the least conducting direction, $z, b$, and $c$ are lattice constants, and $l$ is an integer. The MA were later discovered, ${ }^{12}$ but not as maxima but as dips in the angular dependence of the magnetoresistance. MA effects are also seen ${ }^{13}$ in the (DMET-TSeF) ${ }_{2} \mathrm{X}$ family where $\mathrm{X}=\mathrm{AuCl}_{2}, \mathrm{AuI}_{2}$. The theory was later modified to explain why dips should be found. ${ }^{14}$ The idea presented is that periodic motion is induced at the MA and, provided there is even a small overlap in the direction of the applied field, the electron-electron interaction becomes more two dimensional which would produce a dip in the magnetoresistance (MR) at the MA. Alternative ideas and explanations have since then appeared in the literature. ${ }^{15-21}$ Most of them are based on a small overlap of electronic wave functions in the direction of the magnetic field, but suggestions based on a Luttinger liquid approach have also been made. ${ }^{22,23}$ The
theory developed by Osada, Kagoshima, and Miura ${ }^{15}$ captures many details of the experimental data. ${ }^{24}$ However, that theory requires the existence of very long-range hopping (for example, the second nearest-neighbor hopping integral in a tight-binding model is of the same order of magnitude as the next-nearest neighbor integral). Further, if $B_{x}=0, x$ being the most conducting direction, i.e., along the one-dimensional chain of molecules, the theory predicts that there would be no MA seen in the interlayer conductivity $\sigma_{z z}$. Further, it does not explain the dip in the MR when the magnetic field lies in the plane $\left(\theta=90^{\circ}\right)$. The data for in-plane magnetic field is affected by the fact that the sample is superconducting and the upper critical field $\mathrm{H}_{c 2}$ for an in-plane magnetic field is quite large. ${ }^{25}$ The consensus is that there is no accepted theory behind the appearance of the MA. The experimental situation is also unclear at the moment. Some groups report that the MR has the same behavior in all directions of the current. ${ }^{24,26,27}$ Whereas other experiments disagree, ${ }^{28}$ and claim that, due to defects in the crystals, the electrons are forced to travel in one direction past the defects, so that there can be a contribution from, e.g., the resistivity in the $x$ direction to the resistivity in the $z$ direction, producing an apparently similar angular behaviour of $\rho_{x x}$ and $\rho_{z z}$.

The MA effects are also seen in torque measurements, as measured by Naughton et al. ${ }^{12}$ This has been discussed theoretically by Yakovenko. ${ }^{29}$ Since the torque can be related to the free energy, it is likely that MA effects reflects the ground state electronic properties of the material. Further, a big Nernst signal has been detected at the MA, ${ }^{30}$ and has been discussed theoretically. ${ }^{31}$

The crystal is oriented so that $x$ is the most conducting direction, followed by the $y$ direction. The $(x, y)$ plane defines the layered structure. Typically, the hopping in the three different directions are estimated to be of the order ${ }^{7,32}$ $t_{x}: t_{y}: t_{z} \sim 2000 \mathrm{~K}: 200 \mathrm{~K}: 10 \mathrm{~K}$.

Here we present an alternative explanation of the occurrence of the MA. Our physical picture is the following. The strongly anisotropic structure of the material affects the coherence of the particles in the crystal, as well as increasing the effect of the electron correlations ${ }^{33,34}$ In a previous paper,
we discussed a model for transport in layered materials based on a coherence-incoherence crossover as a function of temperature. ${ }^{4}$ Along the $x$ direction the motion is assumed to be coherent. The least conducting $z$ direction is assumed to be incoherent, and in the $y$ direction, the motion is predominantly coherent, but with a small incoherent contribution. We will show that the loss of coherence in the $y$ direction is directly responsible for the MA in the conductivity measured along the $z$ direction. Even a small amount of incoherence gives rise to a sizable effect seen at the MA.

## II. MODEL

We model the system as a quasi-one-dimensional metal. We introduce coordinates such that $a$ is the lattice spacing in the $\hat{x}$ direction, $b$ in the $\hat{y}$, and $c$ in the $\hat{z}$, the layers lie in the $(x, y)$ plane. Due to the layered structure, the Hamiltonian is divided into intralayer and interlayer contributions

$$
\begin{equation*}
\mathcal{H}=\mathcal{H}_{\|}+\mathcal{H}_{\perp} \tag{1}
\end{equation*}
$$

where $\mathcal{H}_{\|}$describes the two-dimensional (2D) $(x, y)$ layer and includes all many-body interactions within each layer, and $\mathcal{H}_{\perp}=t_{\perp} \Sigma_{\langle i, j\rangle}\left(c_{i}^{\dagger} c_{j}+\right.$ h.c. $)$ describes the tunneling between nearest neighbors in the $z$ direction. Because of the layered crystal structure, we assume that Coulomb correlations between the layers are small, and the separation is valid. Later, we will further specify $\mathcal{H}_{\|}$for quasi-one-dimensional systems. If we have a magnetic field in the $(y, z)$ plane the vector potential $\vec{A}$ for the magnetic field $\vec{B}=\left(0, B_{y}, B_{z}\right)$ $=(0, B \sin \theta, B \cos \theta)$. In the Landau gauge $\vec{A}$ is

$$
\vec{A}=\left(z B_{y}, x B_{z}, 0\right)
$$

We are going to study transport in the $z$ direction, i.e., transport between the anisotropic two-dimensional layers. Let us consider two adjacent layers. The vector potential in the two layers are not equal but differ by a gauge transformation $\vec{A}_{2}=\vec{A}_{1}+\vec{\nabla} \Lambda, 1$ and 2 indicate the layer, and $\Lambda=c B_{y} x$. At small bias we can use linear response theory to calculate the current between the layers. At low temperatures, only electrons at the Fermienergy contribute to the conductivity in the least conducting direction, and it can be written as a function of only the in-plane Green's function ${ }^{8}$ due to the separation of intralayer and interlayer contributions in the Hamiltonian. Separating the current-current correlation function we get that the conductivity is given by ${ }^{8}$

$$
\begin{align*}
\sigma_{z z}= & \frac{e^{2} t_{\perp}^{2} c}{\hbar \pi L_{x} L_{y}} \int \mathrm{~d} \mathbf{r} \int \mathrm{~d} \mathbf{r}^{\prime}\left[G^{1+}\left(\mathbf{r}, \mathbf{r}^{\prime}, E_{F}\right) G^{2-}\left(\mathbf{r}^{\prime}, \mathbf{r}, E_{F}\right)\right. \\
& \left.+G^{1-}\left(\mathbf{r}^{\prime}, \mathbf{r}, E_{F}\right) G^{2+}\left(\mathbf{r}, \mathbf{r}^{\prime}, E_{F}\right)\right] \tag{2}
\end{align*}
$$

where $G^{1+}\left(\mathbf{r}, \mathbf{r}^{\prime}, E_{F}\right)$ denotes the electronic Green's function (GF) within a single layer. Here, $L_{x} L_{y}$ are the dimensions of the sample in the $x$, and $y$ direction respectively. There is an indirect dependence on the distance between the layers in $t_{\perp}$. We stress that this is a very general expression and contains all the many-body effects within each layer.

## A. Noninteracting Green's function for quasi-one-dimensional materials in a magnetic field

Let us now look at the Hamiltonian in the absence of electron-electron interactions. We assume that the spectra in the most conducting direction can be linearized. Then, the Hamiltonian for a layer in a tilted magnetic field is (see Ref. 8)

$$
\begin{align*}
\mathcal{H}_{\|}^{0}= & \alpha v_{F}\left(-i \hbar \partial_{x}+e z B \sin \theta\right) \\
& -2 t_{y} \cos \left[b\left(-i \hbar \partial_{y}-e x B \cos \theta\right)\right] \tag{3}
\end{align*}
$$

where $\alpha= \pm 1$ denotes which sheet of the Fermi surface the electron is on. $v_{F}$ is the Fermi velocity, and $t_{y}$ is the interchain hopping-integral. The wave function is written as

$$
\begin{equation*}
\psi(x, y, t)=\exp \left\{i\left[\frac{-\epsilon t}{\hbar}+k_{x} x+k_{y} y-\alpha \lambda \sin \left(k_{y} b-q x\right)\right]\right\}, \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
q=\frac{e b B \cos \theta}{\hbar}=\frac{\omega_{B}}{v_{F}}=\frac{\omega_{0} \cos \theta}{v_{F}} . \tag{5}
\end{equation*}
$$

$\omega_{B}$ is the frequency at which electrons traverse the quasi-one-dimensional sheets of the Fermi surface ${ }^{7}$ and

$$
\begin{equation*}
\lambda=\frac{2 t_{y}}{e b v_{F} B \cos \theta} \tag{6}
\end{equation*}
$$

is the wavelength of the real space oscillations of the electron trajectories on the Fermi surface. ${ }^{7}$ In a magnetic field the electron dispersion relation is independent of $t_{y}$,

$$
\begin{equation*}
\epsilon_{\alpha}\left(k_{x}, k_{y}\right)=\alpha \hbar k_{x} v_{F} \tag{7}
\end{equation*}
$$

All energies are relative to the Fermi energy. The GF can be calculated in a way similar to the one in Ref. 8, to give

$$
\begin{equation*}
G_{0}^{1+}\left(\mathbf{r}, \mathbf{r}^{\prime}, E\right)=-\frac{i L_{x}}{\hbar v_{F}} \sum_{k_{y}, \alpha} \alpha e^{i\left[k_{y}\left(y-y^{\prime}\right)+\alpha \lambda L\right]} e^{i\left|x-x^{\prime}\right| \hbar v_{F}(E+i \Gamma)} \tag{8}
\end{equation*}
$$

where

$$
L=\sin \left(k_{y} b-q x^{\prime}\right)-\sin \left(k_{y} b-q x\right),
$$

and $\Gamma$ is the electron scattering rate. The GF for the second layer differs by a gauge factor, $e^{i e / \hbar \Lambda\left(\mathrm{r}-\mathbf{r}^{\prime}\right)}$ and is

$$
\begin{align*}
G_{0}^{2+}\left(\mathbf{r}, \mathbf{r}^{\prime}, E\right)= & -\frac{i L_{x}}{\hbar v_{F}} \sum_{k_{y}, \alpha} \alpha e^{i\left[k_{y}\left(y-y^{\prime}\right)+\alpha \lambda L\right]} \\
& \times e^{i\left|x-x^{\prime}\right| \hbar v_{F}\left(E+e v_{F} C B \sin \theta+i \Gamma\right)} . \tag{9}
\end{align*}
$$

## B. Green's function containing incoherence

We now allow for the possibility that the motion in the interchain direction can be incoherent. The incoherence might come from polaron formation, strong electron-electron correlation, or any other many-body effect. In a formulation in terms of GFs a possible ansatz for the effect of incoher-
ence is that the noninteracting GF is multiplied by a $y$-dependent factor

$$
\begin{equation*}
G\left(\mathbf{r}, \mathbf{r}^{\prime}, \tau\right)=G_{0}\left(\mathbf{r}, \mathbf{r}^{\prime}, \tau\right) \sigma\left(y-y^{\prime}\right) \tag{10}
\end{equation*}
$$

where $\sigma\left(y-y^{\prime}\right)$ depends on the process by which coherence is lost. The validity of this special form of GF can be seen for polarons in, e.g., Refs. 4, 35, and 36, and for electronelectron interaction, in, e.g., Ref. 1 In a 2D strongly correlated model using the slave-boson approach ${ }^{1}$ the electronic GF factorizes into $G=G_{B} G_{F}$, where $G_{F}$ is the free Fermion GF, and $G_{B}\left(\mathbf{r}, \mathbf{r}^{\prime}\right)=\exp \left(-\left|\mathbf{r}-\mathbf{r}^{\prime}\right|^{2} / m_{B} T\right)$ is the bosonic GF containing the correlations, $m_{B}$ is the mass of the accompanying boson, and $T$ the temperature. If the 2D lattice is anisotropic (i.e., weakly coupled chains) the effect from the bosonic part will be even more pronounced. In a previous paper we studied transport in layered materials of polarons. ${ }^{4}$ For this case the GF contains two parts, one coherent, describing band motion of electrons weakly scattered by the phonons, and one incoherent, where localized polarons hop between sites. For the case of polarons Eq. (10) is valid. ${ }^{4,36}$ Here, we do not specify the process responsible for the loss of coherence, but will just assume the general form given in Eq. (10). The process involved in Eq. (10) is the following. When the electron moves in the $(x, y)$ layer the $k_{x}$ momentum is conserved. Hence, there is no $x$ dependence in the term describing the incoherent contribution, $\sigma\left(y-y^{\prime}\right)$. Instead it describes the change in momentum in the $y$ direction as the particle jumps between $y$ and $y^{\prime}$. The change in momentum is $\delta k_{y}$, which will be centered around zero so that for most of the time $k_{y}$ is unchanged. If the proposed form for the GF is correct, it could be visible in angle resolved photoemission spectra, which measures the spectral density. ${ }^{3,37,38}$ Later we will demonstrate that even a very small incoherent term gives rise to observable MA effects.

## C. Interlayer conductivity

Using the GFs, Eqs. (8) and (9) in Eq. (2), and the incoherence factor $\sigma\left(y-y^{\prime}\right)$ we get a general expression for the conductivity

$$
\begin{align*}
\sigma_{z z}= & \frac{e^{2} t_{\perp}^{2} c}{\hbar \pi L_{x} L_{y}} \int \mathrm{~d} \mathbf{r} \int \mathrm{~d} \mathbf{r}^{\prime} \sum_{\alpha, k_{y}, k_{y^{\prime}}} e^{i\left[\left(k_{y}-k_{y^{\prime}}\right)\left(y-y^{\prime}\right)+\alpha \lambda\left(L+L^{\prime}\right)\right]} \\
& \times\left|\sigma\left(y-y^{\prime}\right)\right|^{2} e^{-2\left|x-x^{\prime}\right| / \hbar v_{F} \Gamma}\left(e^{i S}+e^{-i S}\right) \tag{11}
\end{align*}
$$

where $S=e c B \sin \theta / \hbar\left|x-x^{\prime}\right|$ is the change in gauge potential associated with interlayer transport. The summation over $\alpha$, the two Fermi sheets the electrons moves on, can be done and simplified. This can be simplified to

$$
\begin{aligned}
\frac{1}{2} \sum_{\alpha} e^{i \alpha \lambda\left(L+L^{\prime}\right)}= & 2 \cos \left\{4 \lambda \sin \left[\left(k_{y}-k_{y^{\prime}}\right) b / 2\right]\right. \\
& \left.\times \cos \left[\left(k_{y}+k_{y^{\prime}}\right) b / 2\right]\right\} \\
& \times \cos \left\{4 \lambda \sin \left[\left(k_{y}-k_{y^{\prime}}\right) b / 2\right]\right. \\
& \left.\times \cos \left[\left(k_{y}+k_{y^{\prime}}\right) b / 2-q / 2\left(x-x^{\prime}\right)\right]\right\} \\
& +2 \sin \left\{4 \lambda \sin \left[\left(k_{y}-k_{y^{\prime}}\right) b / 2\right]\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.\times \cos \left[\left(k_{y}+k_{y^{\prime}}\right) b / 2\right]\right\} \\
& \times \sin \left\{4 \lambda \sin \left[\left(k_{y}-k_{y^{\prime}}\right) b / 2\right]\right. \\
& \left.\times \cos \left[\left(k_{y}+k_{y^{\prime}}\right) b / 2-q / 2\left(x-x^{\prime}\right)\right]\right\}
\end{aligned}
$$

Here, we introduce new variables, $k_{y}-k_{y^{\prime}}=k_{-}, k_{y}+k_{y^{\prime}}=k_{+}$, $x-x^{\prime}=x_{-}, x+x^{\prime}=x_{+}, y-y^{\prime}=y_{-}$, and $y+y^{\prime}=y_{+}$. We can then perform the integral over $x_{+}$to give $L_{x}$, and the integral over $y_{+}$to give $L_{y}$. We now use the representation of the trigonometric functions in terms of Bessel functions

$$
\begin{gathered}
\cos \left[A \cos \left(k_{+} b / 2-\Delta\right)\right] \\
=J_{0}[A]+2 \sum_{k=1}^{\infty}(-1)^{k} J_{2 k}[A] \cos \left[2 k\left(k_{+} b / 2-\Delta\right)\right], \\
\begin{aligned}
\sin \left[A \cos \left(k_{+} b / 2-\Delta\right)\right]= & 2 \sum_{k=0}^{\infty}(-1)^{k} J_{2 k+1}[A] \cos [(2 k+1) \\
& \left.\times\left(k_{+} b / 2-\Delta\right)\right],
\end{aligned}
\end{gathered}
$$

where $J_{l}$ is a Bessel function of order $l$. The summation over $k_{+}$can now be done by transforming it into an integral and we get

$$
\begin{align*}
\sigma_{z z}= & \frac{4 e^{2} t_{\perp}^{2} c}{\hbar b} \int d k_{-} \int d x_{-} e^{-\left|x_{-}\right| / \hbar v_{F} \Gamma} \cos \left(\frac{e c B \sin \theta}{\hbar}\left|x_{-}\right|\right) \\
& \times \sum_{l=0}^{\infty} J_{l}\left[4 \lambda \sin \left(\frac{k_{-} b}{2}\right)\right]^{2} \cos \left(l q x_{-}\right) f\left(k_{-}\right), \tag{12}
\end{align*}
$$

where we introduced the distribution function

$$
\begin{equation*}
f\left(k_{-}\right)=\int d y_{-} e^{i y_{-} k_{-}}\left|\sigma\left(y_{-}\right)\right|^{2}, \tag{13}
\end{equation*}
$$

describing the spread (incoherence) in the (interchain) $y$ direction. The final step is the integration in $x_{-}$, which gives us the final expression

$$
\begin{align*}
\sigma_{z z}(\theta)= & \sigma_{0} \sum_{l=-\infty}^{\infty} \frac{\Gamma^{2}}{\Gamma^{2}+e^{2} v_{F}^{2} B^{2}(b l \cos \theta-c \sin \theta)^{2}} \\
& \times \int d k_{-} J_{l}\left[4 \lambda \sin \left(\frac{k_{-} b}{2}\right)\right]^{2} f\left(k_{-}\right), \tag{14}
\end{align*}
$$

where we defined the conductivity in zero field, $\sigma_{0}$ $\equiv 8 t_{\perp}^{2} e^{2} c / v_{F} b \Gamma$. Equation (14) is the main result of this paper. This expression can be directly compared with those derived by other authors for alternate theories. ${ }^{16,17}$ The MA appears as peaks in $\sigma_{z z}$ (dips in the MR), when the denominator has a minima. This will occur at angles when

$$
\begin{equation*}
\tan \theta=\frac{b}{c} l, \tag{15}
\end{equation*}
$$

i.e., at the MA.

Recall that the function $f\left(k_{-}\right)$indicates the amount of incoherence in the $y$ direction. If we have coherent particles in the $y$ direction, then, $k_{y}$ is always conserved so that $k_{y^{\prime}}=k_{y}$, and the distribution will be a delta function $f\left(k_{-}\right)=\delta\left(k_{-}\right)$. The


FIG. 1. Interlayer magnetoresistance as a function of tilt angle $\theta$ in the $y-z$ plane. Even for a very small incoherent hopping between the chains of molecules the magic-angle effect is clearly seen. The parameter $k_{0} b$ is a dimensionless parameter describing spread in the distribution of momentum as the particle tunnels between the chains. $k_{0} b=0$ means full coherence, i.e., a delta-function distribution of $k_{y}$ values. The magnetic frequency $\omega_{0}=e b v_{F} B$ is the frequency at which the electrons traverse the open sheets of the Fermi surface. $\rho^{0}$ is the resistivity in zero field, we used $b=c$ and $\omega_{0} / \Gamma$ $=10$. We also included, as a comparison, the result when no incoherence is present $\left(k_{0} b=0\right)$, given by Eq. (16) in the text.
sum over the Bessel functions collapses to only the $l=0$ term, and the result is

$$
\begin{equation*}
\sigma_{z z}(B, \theta)=\sigma_{0} \frac{\Gamma^{2}}{\Gamma^{2}+\left(e c v_{F} B \sin \theta\right)^{2}} \tag{16}
\end{equation*}
$$

This agrees with the result from regular Boltzmann transport theory, ${ }^{39}$ and the MA effects are not seen.

If an incoherent term is present we will have some spread in $k_{-}$. To illustrate this we use $f\left(k_{-}\right)=1 / \sqrt{2 \pi} k_{0} e^{-\left(k_{-}\right) 2 / 2 k_{0}^{2}}$, meaning that the averaged momentum in the $y$ direction follow

$$
\begin{equation*}
\left\langle\left(k_{y}-k_{y^{\prime}}\right)^{2}\right\rangle=k_{0}^{2} \tag{17}
\end{equation*}
$$

$f\left(k_{-}\right)$has the property that it becomes a delta function if $k_{0}$ $\rightarrow 0$, i.e., when the quasi-particles in the $y$ direction are coherent. The momentum in the $x$ direction is conserved, $k_{x^{\prime}}$ $=k_{x}$. We stress that the effects we are discussing are not sensitive to the particular form of $f\left(k_{-}\right)$used, since it is an integrated quantity. $k_{0} b$ is a measure of how poorly the quasiparticle wave vector is defined in the interchain direction. The electrons are coherent in the $y$ direction of the order of $k_{0}^{-1}$, meaning that if, say, $k_{0} b=0.01$, then the electrons are coherent on the order of 100 lattice constants in the $y$ direction. Thus, a value used below $k_{0} b=0.01$ still represents very well-defined quasiparticles. A typical curve for the angular dependence of the interalyer magnetoresistance is shown in Fig. 1. The value of the other parameters $\omega_{0} / t_{y}$ and $\omega_{0} / \Gamma$ are taken from typical experimental values. The decay, $\Gamma=\hbar / \tau$,
comes from two experiments where the scattering time has been measured by magnetoresistance measurements and is $\tau=4.3 \mathrm{ps}$ in Ref. $40\left[(\mathrm{TMTSF})_{2} \mathrm{ClO}_{4}\right.$ at $T=0.5 \mathrm{~K}$ and ambient pressure] giving $\Gamma=0.15 \mathrm{meV}$, and $\tau=6.3 \mathrm{ps}$ in Ref. 41 [(TMTSF) ${ }_{2} \mathrm{PF}_{6}$ at $T=0.32 \mathrm{~K}$ and 8.2 kbar$]$ giving $\Gamma$ $=0.10 \mathrm{meV}$. The magnetic frequency $\omega_{0}=e b v_{F} B$ is given by the Fermi velocity $v_{F}=0.2 \mathrm{Mm} / \mathrm{s}$ in Ref. 15 , and is equal to 1.08 meV when the magnetic field 7 T and $b=7.711 \AA .{ }^{40}$ The hopping parameter in the $y$ direction, $t_{y}$ is given as 31 meV in Ref. 41 and Ref. 15, but 12 meV in Ref. 40. In our numerical examples we use $\omega_{0} / \Gamma=10, \omega_{0} / t_{y}=0.1$. The results are not that strongly dependent on the choice of these values, only the amplitude of the MA dips change. Here we have to point out that according to the experiments ${ }^{40}$ there should be a dip when $\theta=90^{\circ}$, which is absent in our theory (see Fig. 1). This dip occurs when $B$ is parallel to the layers, and is therefore not a MA, and cannot be described by our theory. As described in the introduction, it may be connected with the proximity to the superconducting state for the inplane magnetic field. ${ }^{25}$

Note that by comparing our theory to the one by Osada ${ }^{17}$ (which assumes noninteracting electrons) the incoherent term in the $y$ direction has a similar effect as a magnetic field in the $x$ direction. In particular we have

$$
\begin{equation*}
B_{x} \leftrightarrow \frac{\hbar k_{0}}{e c}, \tag{18}
\end{equation*}
$$

giving $B_{x} \sim 6.3 \mathrm{~T}$ if we use $k_{0} b=0.01$. We see that even a very small incoherent part, $k_{0} b=0.01$, corresponds to a relatively large fluctuating field in the $x$-direction $B_{x} \sim 6.3 \mathrm{~T}$. Thus, the larger the incoherence is (larger $k_{0} b$ ) the larger the corresponding effective field in the $x$ direction, and the larger the MA dips in the MR. This is consistent with the experimental result by Lee and Naughton, ${ }^{41}$ where an increasing $x$ component of the magnetic field increased the size of the MR oscillations at the MA.

## III. $\overrightarrow{\boldsymbol{B}}$ IN THE $(x, z)$ PLANE

If we instead apply the magnetic field in the $(x, z)$ plane the vector potential will be

$$
\vec{A}=\left(0, x B_{z}-z B_{x}, 0\right)
$$

The derivation is very similar to the one presented above with the only difference that the gauge potential does not have any component depending on $\left|x-x^{\prime}\right|$, but now depends on $y-y^{\prime}$ instead. The result is that the integral over $x_{-}$is simpler, but the integral over $y_{-}$has an additional factor. This factor can be absorbed in the $y_{-}$integral, the final result is

$$
\begin{align*}
\sigma_{z z}(B, \theta)= & \sigma_{0} \sum_{l=-\infty}^{\infty} \int d k_{-} J_{l}\left[4 \lambda \sin \left(\frac{k_{-} b}{2}\right)\right]^{2} \\
& \times \frac{\Gamma^{2}}{\Gamma^{2}+\left(e v_{F} b l B_{z}\right)^{2}} g\left(k_{-}\right), \tag{19}
\end{align*}
$$

where


FIG. 2. Interlayer magnetoresistance as a function of the magnetic field direction in the $x-z$ plane. $\theta$ is the angle between the field and the $z$ axis. The upper panel shows a numerical calculation of the so-called Danner-Kang-Chaikin oscillations, ${ }^{40}$ from Eq. (19) in the text. The theoretical curve can be compared with Fig. 1 from Ref. 40 shown in the lower panel, with experiments done on (TMTSF) ${ }_{2} \mathrm{ClO}_{4}$ at ambient pressure and $T=0.5 \mathrm{~K}$. The dip around zero degree below 3 T is due to the sample becoming superconducting. Note that $\theta$ denotes the angle between the magnetic field and the $x$ axis. We used $b=c, k_{0} b=0.001$, with $\omega_{0} / \Gamma=10$ and $\omega_{0} / t_{y}=0.1$ at $B=7 \mathrm{~T}$.

$$
\begin{equation*}
g\left(k_{-}\right)=\int d y_{-} e^{i y_{-}\left(k_{-}-e c B_{x} / \hbar\right)}\left|\sigma\left(y_{-}\right)\right|^{2}=f\left(k_{-}-\frac{e c B_{x}}{\hbar}\right) . \tag{20}
\end{equation*}
$$

The parameter $\lambda$ is $2 t_{y} / e b v_{F} B_{z}$. The so-called Danner-KangChaikin oscillations ${ }^{40}$ are observed provided that

$$
\begin{equation*}
\frac{e c B_{x}}{\hbar} \gtrdot k_{0} \tag{21}
\end{equation*}
$$

where $k_{0} b$ is the incoherence parameter. In Fig. 2 we compare the resulting resistivity $\left(1 / \sigma_{z z}\right)$ from Eq. (19) with an experimental curve. ${ }^{40} \mathrm{We}$ did not adapt the parameters to the experiment, but just want to illustrate that this type of oscillations do appear in the theory presented. Note that we have used a smaller value for the incoherence parameter $k_{0} b$ $=0.001$, compared to the value used in Fig. 1. This is justified by the fact that the experiment we compare with is performed for the $\mathrm{ClO}_{4}$ compound and the oscillations in the $y$ $-z$ plane are not as visible ${ }^{12}$ as for the $\mathrm{PF}_{6}$ compound indicating a smaller incoherence factor.


FIG. 3. Interlayer magnetoresistivity versus $y-z$ plane angle, $\alpha$, defined via $\tan \alpha=\sin \phi / \tan \theta$ (see top figure). The middle panel shows the result from our numerical calculation of conductivity using Eq. (22). Modulations appear at the magic angles as the angle $\alpha$ is increased. We used $b=7.581 \AA$ and $c=13.264 \AA$. The other parameters used are $k_{0} b=0.1, \omega_{0} / \Gamma=10$ and $\omega_{0} / t_{y}=0.1$. The theoretical curve can be compared with Fig. 4 from Ref. 41 shown in the lower panel. This is an experiment done at 0.32 K on (TMTSF) ${ }_{2} \mathrm{PF}_{6}$ with an applied hydrostatic pressure of 8.3 kbar to suppress the spin-density-wave state.

## IV. $\vec{B}$ IN THE $(x, y, z)$ PLANE

Combining the results from the calculations above we can get an expression for a field in a general direction, ( $B_{x}, B_{y}, B_{z}$ ). We get

$$
\begin{align*}
\sigma_{z z}(\theta, \phi)= & \sigma_{0} \sum_{l=-\infty}^{\infty} \frac{\Gamma}{\Gamma^{2}+e^{2} v_{F}^{2}\left(b l B_{z}-c B_{y}\right)^{2}} \\
& \times \int d k_{-} J_{l}\left[4 \lambda \sin \left(\frac{k_{-} b}{2}\right)\right]^{2} f\left(k_{-}-\frac{e c B_{x}}{\hbar}\right), \tag{22}
\end{align*}
$$

note that $\lambda$ is a function of $B_{z}$. In Fig. 3 we compare results from this expression with the experimental results of Lee and Naughton, ${ }^{41}$ by identifying the angles defined in Fig. 3, as follows:

$$
\left\{\begin{array}{c}
B_{x}=B \cos \theta \cos \phi  \tag{23}\\
B_{y}=B \cos \theta \sin \phi \\
B_{z}=B \sin \theta
\end{array},\right.
$$

where the definition of $\theta$ and $\phi$ follows Ref. 41, (see the upper panel in Fig. 3) As the angle $\theta$ between the $(x, y)$ plane and the direction of the field is increased, the oscillations start to appear. The similarities to Fig. 4 in Ref. 41 are striking. Changing the parameters in the model does not change the general features of this plot.

## V. DISCUSSION

In summary we have presented an explanation in terms of many-body effects of the appearance of magic angle effects in the interlayer magnetoresistance. The MA appears naturally from, even a small, incoherent contribution to the interchain hopping. The hopping in the most conducting direction is assumed to be coherent, and in the least conducting direction incoherent. Momentum can change in the direction between the one-dimensional chain of molecules. This is described by a distribution function which is centered around zero, letting most quasiparticles retain their momentum when hopping. We used an explicit form of the interlayer Green's function, which can be directly observed in a angle resolved photoemission spectra. Unlike previous explanations, ${ }^{15,24,39}$ the theory does not assume any long distance hopping between nonadjacent quasi-one-dimensional molecules in different layers, where the overlap is quite small, only a nearestneighbor interlayer overlap. The shape of the Fermi surface is not affected by the incoherence. Numerical calculations produce results similar to experimental results.

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*Electronic address: lundin@ physics.uq.edu.au
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