PHYSICAL REVIEW A, VOLUME 66, 033801

Violation of multiparticle Bell inequalities for low- and high-flux parametric amplification using both vacuum and entangled input states

M. D. Reid,[1] W. J. Munro,[2] and F. De Martini[3]
[1]Physics Department, University of Queensland, Brisbane, Australia
[2]Hewlett-Packard Laboratories, Filton Road, Stoke Gifford, Bristol, BS34 SQ2, United Kingdom
[3]Dipartimento di Fisica and INFM, Università di Roma "La Sapienza," Roma 00185, Italy
(Received 30 April 2001; published 4 September 2002)

We show how polarization measurements on the output fields generated by parametric down conversion will reveal a violation of multiparticle Bell inequalities, in the regime of both low- and high-output intensity. In this case, each spatially separated system, upon which a measurement is performed, is comprised of more than one particle. In view of the formal analogy with spin systems, the proposal provides an opportunity to test the predictions of quantum mechanics for spatially separated higher spin states. Here the quantum behavior possible even where measurements are performed on systems of large quantum (particle) number may be demonstrated. Our proposal applies to both vacuum-state signal and idler inputs, and also to the quantum-injected parametric amplifier as studied by De Martini *et al.* The effect of detector inefficiencies is included, and weaker Bell-Clauser-Horne inequalities are derived to enable realistic tests of local hidden variables with auxiliary assumptions for the multiparticle situation.

DOI: 10.1103/PhysRevA.66.033801 PACS number(s): 42.50.Dv, 03.65.Ud

I. INTRODUCTION

There is increasing evidence for the failure of "local realism" as defined originally by Einstein, Podolsky, and Rosen [1], Bohm [2] and Bell, Clauser and Shimony, and Greenberger [3–5]. For certain correlated quantum systems, Einstein, Podolsky, and Rosen (EPR) argued in their famous 1935 EPR paradox that "local realism" is sufficient to imply that the results of measurements are predetermined. These predetermined "hidden variables" exist to describe the value of a physical variable, whether or not the measurement is performed, and as such are not part of a quantum description. Bell later showed that the predictions of quantum mechanics for certain ideal quantum states could not be compatible with such local hidden variable theories. It is now widely accepted therefore, as a result of Bell's theorem and related experiments [6], that local realism must be rejected.

Recently three-photon states demonstrating a contradiction of quantum mechanics with local hidden variables have been generated [9]. A multiparticle entanglement involving four trapped ions has also been recently realized by Sackett *et al.* [7], and for atoms and photons in cavities by Rauschenbeutel *et al.* [8]. These experiments involve measurements performed on separated subsystems that are microscopic. Recently, the EPR paradox, itself a demonstration of entanglement, has been realized where each measurement is performed on a macroscopic system. Such experiments were performed initially by Ou *et al.* [10] using intracavity parametric oscillation below threshold, and have now been achieved for intense fields using parametric oscillation above threshold by Zhang *et al.* [11], and for pulsed fields by Silberhorn *et al.* [12]. There have been further theoretical proposals to demonstrate the macroscopic nature of EPR correlations [13,14]. However experimental efforts using clearly spatially separated systems, testing local realism directly through a violation of a Bell-type inequality, (or through the Greenberger-Horne-Zeilinger effect [5]), have so far primarily been confined to the most microscopic of systems, where each measurement is made on a system comprising only one particle. There has been very recent interest by Weinfurter and Zukowski [15] in devising and, by Lamas-Linares *et al.* [16], in realizing, strategies to test local realism for multiparticle situations.

A predicted incompatibility of quantum mechanics with local hidden variable theories for systems of potentially more than one particle per detector came with the work of Mermin [17], Garg and Mermin [17], and Mermin and Schwarz [18] who showed violations of Bell inequalities to be possible for a pair of spatially separated higher-spin j particles, where j can be arbitrarily large. The violation of a Bell inequality for multiphoton macroscopic systems was put forward by Drummond [19]. Such manifestations of irrefutably quantum behavior are contradictory to the notion that classical behavior is obtained in the limit where the quantum numbers, or particle numbers, become large. The work of Peres [20] has shown how the transition to classical behavior (local realism) is obtained through measurements that become increasingly fuzzy. To observe the failure of local realism it is generally necessary to perform measurements sufficiently accurate so as to resolve the $2j+1$ eigenvalues. The contradiction of quantum mechanics with local realism for multiparticle or higher-spin systems has since been explored theoretically in a number of works [21–24].

In this paper we present a proposal to test for multiphoton violations of local realism, by way of a violation of a Bell inequality, using parametric down conversion. Our proposal involves a four-mode parametric interaction, considered initially by Reid and Walls [25] and Horne *et al.* [25], as may be generated for example using two parametric amplifiers, or using two competing parametric processes. Such parametric interactions were used to demonstrate experimentally violations of a Bell-type inequality (for the single photon case) by

Rarity and Tapster [25], and there has been further experimental work [25,6]. While initially we consider vacuum inputs with two parametric amplifiers, our proposal is also formulated for the specific configuration of the quantum injected parametric amplifier [26]. Here "multiparticle Bell inequalities" refer to Bell-inequality tests applying to situations where each measurement is performed on a system of more than one particle. In our proposal the measurement is of the number of particles polarized "up" minus the number of particles polarized "down." Because of the formal analogy to a pair of spin j particles, our proposal allows a test of the predictions of quantum mechanics for the higher-spin states.

We will focus on two regimes of experimental operation. The first corresponds to relatively low interaction strength so that the mean signal/idler output is small and we have low incident photon numbers on polarizers which serve as the measurement apparatus. Here it is shown how certain measured probabilities of detection of precisely n photons transmitted through the polarizer can violate local realism, and represent a test of the established higher-spin results. Previous calculations [24] of this type were primarily confined to situations of extremely low-detection efficiency. Here the results are presented for higher efficiencies more compatible with current experimental proposals. The effect of detection efficiencies is calculated and (to also provide an experimental avenue where detection efficiencies are not sufficient to allow a test of the original stronger no "loophole" Bell inequality) we consider a weaker Bell-Clauser-Horne (Bell-CH) inequality as applied to the multiparticle situation.

Our second regime of interest is that of higher output signal/idler intensity, where many photons fall incident on the measurement apparatus. We present a proposal for a violation of a Bell inequality, where one measures the probability of a range of intensity output through the polarizer. The application of Bell inequality theorems, and the effect of detection inefficiencies on the violations predicted, to situations where many photons fall on a detector is relevant to the question of whether or not tests of local realism can be conducted in the experiments such as those performed by Smithey *et al.* [27]. In the Smithey *et al.* experiment, correlation of the photon number between two spatially separated but very intense fields is sufficient to give "squeezed" noise levels. Previous studies by Banaszek and Wodkiewicz [23] have demonstrated violations of Bell inequalities to be possible for certain measurements for the signal/idler outputs of the parametric amplifier. In these high-flux experiments, detection losses can be relatively small on a percentage basis, as compared to traditional Bell inequality experiments involving photon counting with low-incident photon numbers. The exact sensitivity of the violations to loss determines the feasibility of a multiparticle, no-loophole violation of a Bell inequality.

II. DERIVATION OF MULTIPARTICLE BELL INEQUALITIES

We consider a general situation as depicted in Fig. 1 of two pairs of spatially separated fields. The two modes at

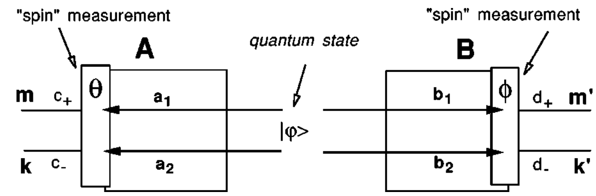

FIG. 1. Schematic diagram of the experimental arrangement to test the Bell inequality. Here m, k, and $m'k'$ are the results of measurement of $c_+^\dagger c_+$, $c_-^\dagger c_-$, $d_+^\dagger d_+$, and $d_-^\dagger d_-$, respectively. Binary outcomes $+1$ and -1 are defined and we measure joint and marginal probabilities $P_{++}^{AB}(\theta,\phi)$, $P_+^A(\theta)$, and $P_+^B(\phi)$ for obtaining $+1$.

location A are denoted by the boson operators a_1 and a_2, while the two modes at location B, spatially separated from A, are denoted by the boson operators b_1 and b_2. One can measure at A the photon numbers $c_+^\dagger c_+$ and $c_-^\dagger c_-$; and similarly at B one can measure, simultaneously, the photon numbers $d_+^\dagger d_+$ and $d_-^\dagger d_-$, where

$$c_+ = a_1 \cos\theta + a_2 \sin\theta,$$

$$c_- = -a_1 \sin\theta + a_2 \cos\theta,$$

$$d_+ = b_1 \cos\phi + b_2 \sin\phi,$$

$$d_- = -b_1 \sin\phi + b_2 \cos\phi. \tag{1}$$

These measurements may be made [6,25] with the use of two sets of polarizers, to produce the transformed fields c_+ and d_+, followed by photodetectors at A and B to determine the photon numbers $c_+^\dagger c_+$ and $d_+^\dagger d_+$, respectively. We note that each measurement at A corresponds to a certain choice of parameter θ. Similarly a measurement at B corresponds to a certain choice of ϕ. In our final proposal, the fields a_1 and b_1 will be the correlated signal/idler outputs of a single parametric amplifier with Hamiltonian $H = i\hbar g(a_1^\dagger b_1^\dagger - a_1 b_1)$, while a_2 and b_2 are the outputs of a second parametric amplifier with Hamiltonian $H = i\hbar g(a_2^\dagger b_2^\dagger - a_2 b_2)$.

Let us denote the outcome of the photon number measurements $c_+^\dagger c_+$, $c_-^\dagger c_-$, $d_+^\dagger d_+$, and $d_-^\dagger d_-$ as m, k, m', and k', respectively. We will classify the result of our measurements made at each of A and B as one of two possible outcomes. For certain outcomes m and k at A we will assign the value $+1$. (This choice of outcomes will be specified later.) Otherwise our result is -1. Similarly at B, certain values m' and k' are classified as result $+1$, while all other outcomes are designated -1. This binary classification of the results of the measurement is chosen to allow an easy application of Bell's theorem.

To establish Bell's result, one considers joint measurements where the photon numbers $c_+^\dagger c_+$, $c_-^\dagger c_-$, and $d_+^\dagger d_+$, $d_-^\dagger d_-$ are measured simultaneously at the spatially separated locations A and B, respectively. A joint measurement will give one of four outcomes, $+1$ or -1 for each particle. By performing many such measurements over an ensemble, one can experimentally determine the following: $P_{++}^{AB}(\theta,\phi)$ the probability of obtaining $+1$ for particle A

and $+1$ for particle B upon simultaneous measurement with θ at A and ϕ at B; $P_+^A(\theta)$ the marginal probability for obtaining the result $+1$ upon measurement with θ at A; and $P_+^B(\phi)$ the marginal probability of obtaining the result $+1$ upon measurement with ϕ at B.

Assuming a general local hidden variable theory then, we can write the measured probabilities as follows:

$$P_+^A(\theta)=\int \rho(\lambda)p_+^A(\theta,\lambda)d\lambda. \tag{2}$$

The probability of obtaining "+1" for B_ϕ^B is

$$P_+^B(\phi)=\int \rho(\lambda)p_+^B(\phi,\lambda)d\lambda. \tag{3}$$

The joint probability for obtaining "+1" for both of two simultaneous measurements with θ at A and ϕ at B is

$$P_{++}^{AB}(\theta,\phi)=\int \rho(\lambda)p_+^A(\theta,\lambda)p_+^B(\phi,\lambda)d\lambda. \tag{4}$$

Here, θ and ϕ denote the choice of measurement at the locations A and B, respectively. The independence of $p_+^A(\theta,\lambda)$ on ϕ, and $p_+^B(\phi,\lambda)$ on θ, follows from the locality assumption. The measurement made at B cannot instantaneously influence the system at A.

It is well known [3,4] that one can derive the following "strong" Bell-Clauser-Horne inequality from the assumptions of local realism made so far:

$$S=\frac{P_{++}^{AB}(\theta,\phi)-P_{++}^{AB}(\theta,\phi')+P_{++}^{AB}(\theta',\phi)+P_{++}^{AB}(\theta',\phi')}{P_+^A(\theta')+P_+^B(\phi)} \leqslant 1. \tag{5}$$

For situations that we consider in this paper of more than two outcomes, it is pointed out that other more general forms of Bell inequalities are also possible, and the study of the violation of these generalized inequalities [21,22] would be interesting. For our purposes, in this paper, the use of the traditional Bell-CH inequality, as presented originally in [19], is sufficient to demonstrate that violations are possible for multiparticle systems.

III. MULTIPARTICLE "SPIN" STATE VIOLATING BELL INEQUALITIES

Bell inequality violations have been proposed previously for macroscopic or multiparticle states [17–20,22,24]. Previous studies by Mermin, Peres, and others have considered violations by states of arbitrary spin j. There is a formal equivalence by way of the Schwinger representation to bosonic states of $N=2j$ photons [24]. For example, we consider the following N particle state:

$$|\varphi_N\rangle=\frac{1}{N!(N+1)^{1/2}}(a_1^\dagger b_1^\dagger+a_2^\dagger b_2^\dagger)^N|0\rangle|0\rangle, \tag{6}$$

where the boson operators a_1 and a_2 are as in Sec. II and Fig. 1. This state was presented, and shown to violate local realism where each measurement is performed on systems of N particles (where N can be macroscopic), by Drummond [19]. We introduce the Schwinger spin operators

$$\begin{aligned}
S_x^A&=(a_1a_2^\dagger+a_1^\dagger a_2)/2,\\
S_y^A&=(a_1a_2^\dagger-a_1^\dagger a_2)/2i,\\
S_z^A&=(a_2^\dagger a_2-a_1^\dagger a_1)/2,\\
S_x^B&=(b_1b_2^\dagger+b_1^\dagger b_2)/2,\\
S_y^B&=(b_1b_2^\dagger-b_1^\dagger b_2)/2i,\\
S_z^B&=(b_2^\dagger b_2-b_1^\dagger b_1)/2.
\end{aligned} \tag{7}$$

The photon number difference measurements at each detector corresponds in this formalism to a measurement of the "spin" component

$$\begin{aligned}
S_z^A(2\theta)&=(c_+^\dagger c_+-c_-^\dagger c_-)/2,\\
S_z^B(2\phi)&=(d_+^\dagger d_+-d_-^\dagger d_-)/2,
\end{aligned} \tag{8}$$

as determined by the polarizer angle θ or ϕ. Here, $S_z^A(2\theta)=S_z^A\cos 2\theta+S_x^A\sin 2\theta$ and $S_z^B(2\phi)=S_z^B\cos 2\phi+S_x^B\sin 2\phi$. The quantum state (6) can be written as

$$|\varphi_N\rangle=\frac{1}{(2j+1)^{1/2}}\sum_{m=-j}^{+j}|j,m\rangle_A|j,m\rangle_B, \tag{9}$$

where $|j,m\rangle_A$ and $|j,m\rangle_B$ are the eigenstates of S_A^2,S_z^A, and S_B^2,S_z^B, respectively, and $j=N/2$. The singlet state

$$|\varphi_N\rangle=\frac{1}{(2j+1)^{1/2}}\sum_{m=-j}^{+j}(-1)^{j-m}|j,m\rangle_A|j,-m\rangle_B \tag{10}$$

studied by previous authors is obtained upon substituting a_1 with $-a_1$, and interchanging b_1 and b_2 in the definitions of S_x^B, S_y^B, and S_z^B. The predictions as given in this paper of the quantum state (6) with measurements (7) and (8) using particular θ and ϕ will be identical to the predictions of the singlet state (10) above with measurements (7) and (8) but replacing ϕ and θ with ϕ_{spin} and θ_{spin} where $2\phi_{spin}=2\phi+\pi$ and $\theta_{spin}=-\theta$.

For the purpose of our particular experimental proposal we first demonstrate the failure of multiparticle local realism for the N states (6) as follows. We choose the following binary classification of outcomes. If the result m of the photon number measurement $c_+^\dagger c_+$ is greater than or equal to a certain fraction f of the total photon number $m+k$ detected at A, then we have the result $+1$. Otherwise our result is -1. The outcome of a measurement at the location B is classified as $+1$ or -1 in a similar manner. Violations of the Bell inequality (5) are found for a range of parameters as illustrated in Fig. 2. Here we have selected the following relation between the angles: $\phi-\theta=\theta'-\phi=\phi'-\theta'=\psi$ and $\phi'-\theta$

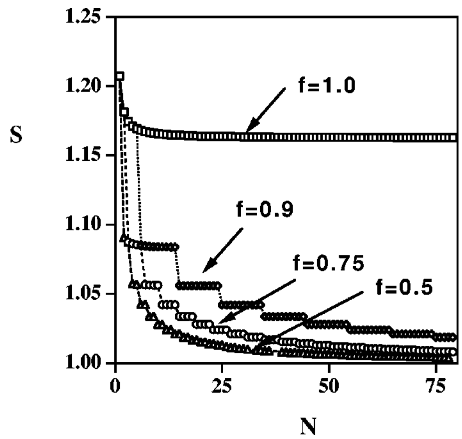

FIG. 2. Plot of S showing violation of the Bell inequality (5) [and Eq. (16)] versus N for the quantum state (6), using the arrangement depicted in Fig. 1. Our outcome at A is designated $+1$ if $m \geqslant fN$, and $+1$ for B if $m' \geqslant fN$, where f is a preselected fraction. The results are optimized with respect to the angle ψ as defined in the text. A violation is obtained when $S>1$. For $f=1$, the optimal angle ψ is 0.39,3.4,0.22,0.19, and 3.1 for $N=1,2,3,4$, and 80 respectively. Results for values of $f=0.5-x$ are identical to those for $f=0.5+x$.

$=3\psi$. This combination has been shown to be optimal for the cases $N=1$ [3,4] and for all N values with $f=1$ [19].

It is pointed out that other Bell-type inequality tests with multivalued outputs are possible [18,21,22]. Our particular classification in terms of binary events has been chosen initially since the $f=1$ case corresponds to the choice presented originally [19] which is known to give a strong violation even for high-particle numbers N, and which would seem feasible for moderate N values. The violation of the Bell inequality (5) is in fact greatest for $f=1$, where our result $+1$ at A, for example, corresponds to detecting all N photons in the c_+ mode. While this value of f gives the strongest violation, the actual probability of the $+1$ event in this case becomes increasingly small as N increases especially if detection inefficiencies are to be included as in later calculations. From this point of view, to look for the most feasible macroscopic experiment, the violations with reduced f become important.

We see that the magnitude of violation decreases with increasing f, so that the asymptotic value at $f=0.5$ is 1, meaning that the violation is lost. This case is interesting since the outcomes here are binned to give two binary outcomes that are, in the limit of N large, effectively macroscopically distinct. This is so because the probability of achieving a result of approximately equal photon numbers ($m \approx k$, $m' \approx k'$) becomes negligible. In this limit of a truly macroscopic experiment with macroscopically distinct outcomes, the violation of the Bell inequality is lost.

IV. EFFECT OF DETECTION INEFFICIENCIES: DERIVATION OF A WEAKER BELL INEQUALITY

The effect of loss through detection inefficiency is important, since this limits the experimental feasibility of a test of the Bell inequality. To date to our knowledge the "strong" inequality of the type (5) has not yet been violated [4] in any experiment involving photodetection, because of the detection inefficiencies which occur in photon counting experiments, although recent experiments by Rowe *et al.* [6] violate a true Bell inequality for trapped ions, with limited spatial separation.

It is well documented [3,4] that it is possible to derive, with the assumption of additional premises, a weaker form of the Bell-Clauser-Horne inequalities which have been violated in single photon counting experiments. Before proceeding to derive a "weak" Bell inequality for multiparticle detection, we outline the effect of detection inefficiencies on the violation, as shown in Fig. 2, of the strong Bell inequality (5).

We introduce a transmission parameter T, defining T as the probability that a single incoming photon will be detected, the intensity of the incoming field being reduced by the factor T. T is directly related to the detector efficiency η according to $T=\eta^2$. We model loss in the standard way by considering the measured field to be the transmitted output of an imaginary beam splitter with the input being the actual quantum field incident on the detector. The second input to the imaginary beam splitter is a vacuum field. Calculating the probabilities of this measured field is equivalent to using standard photocounting formulas which incorporate detection inefficiencies.

The following expression gives the final measured probability $P(m,k,m',k')$ for obtaining results m,k,m',k' upon measurement of $c_+^\dagger c_+$, $c_-^\dagger c_-$, and $d_+^\dagger d_+$, $d_-^\dagger d_-$, respectively. Here $P_Q(m_0,k_0,m_0',k_0')$ is the quantum probability for obtaining m_0,k_0,m_0',k_0' photons, upon measurement of $c_+^\dagger c_+$, $c_-^\dagger c_-$, and $d_+^\dagger d_+$, $d_-^\dagger d_-$, in the absence of detection losses. This quantum probability is derivable from Eq. (6):

$$\begin{aligned} P(m,k,m',k') = {} & T^{m+k+m'+k'} \sum_{r,q,s,t=0}^{\infty} (1-T)^{r+q+s+t} \\ & \times C_r^{m+r} C_q^{k+q} C_s^{m'+s} C_t^{k'+t} \\ & \times P_Q(m+r,k+q,m'+s,k'+t). \end{aligned} \tag{11}$$

Here, $C_r^{m+r}=(m+r)!/r!m!$, and r,q,s,t represent the number of photons lost. We also consider the measured marginal probability,

$$\begin{aligned} P^A(m,k) = {} & T^{m+k} \sum_{r,q=0}^{\infty} (1-T)^{r+q} C_r^{m+r} C_q^{k+q} \\ & \times P_Q^A(m+r,k+q), \end{aligned} \tag{12}$$

where $P_Q^A(m+r,k+q)$ represents the quantum probability for obtaining m_0,k_0 photons upon measurement of $c_+^\dagger c_+$ and $c_-^\dagger c_-$ in the absence of detection losses. This marginal quantum probability is derivable from Eq. (6).

With loss present there is a distinction between our actual quantum photon number m_0 present on the detectors, and the final readout photon number m, which is taken to be the result of the photon number measurement. (We must have

$m \leqslant m_0$). Therefore a number of quantum probabilities will contribute in the calculation for the final measured probability. This complicating effect may be avoided in the following manner. The outcome at A is labeled $+1$ only if $m \geqslant fN$ and $m+k=N$; and at B if $m' \geqslant fN$ and $m'+k'=N$. For N photons detected at each location A or B, we are restricted to the outcomes satisfying $m+k=m'+k'=N$ where loss has not occurred, for the given initial quantum state $|\varphi_N\rangle$. In this situation we get for the measured probabilities (11)

$$P(m,N-m,m',N-m') = T^{2N} P_Q(m,N-m,m',N-m') \tag{13}$$

and for the marginal

$$P^A(m,N-m) = T^N P_Q^A(m,N-m). \tag{14}$$

Here, $P_Q(m, \quad N-m, \quad m', \quad N-m')$ is the quantum probability (in the absence of loss) that measurement of $c_+^\dagger c_+$ and $d_+^\dagger d_+$, for the state $|\varphi_N\rangle$ of Eq. (6), will give results m and m', respectively. This quantum probability is calculated from the quantum amplitudes $C_{m,m'}^{(N)} = \langle \varphi_N | m \rangle_\theta | m' \rangle_\phi$, where $|m\rangle_\theta$, $|m'\rangle_\phi$ are eigenstates of $c_+^\dagger c_+$ and $d_+^\dagger d_+$, respectively, and is given by $P_Q(m, \quad N-m, \quad m', \quad N-m') = |C_{m,m'}^{(N)}|^2$. The quantum marginal for $|\varphi_N\rangle$ is $P_Q^A(m,N-m) = \Sigma_{m'=0}^N |C_{m,m'}^{(N)}|^2$.

The crucial effect of detection losses is that each measured joint probability contains the factor T^{2N} where $2N$ is the total number of photons $m+k+m'+k'$ detected. This implies immediately extreme sensitivity of the multiparticle strong Bell inequality (5) to loss, since this inequality involves the marginal which scales as T^N. In the presence of loss T, the predicted value for S [required to test the strong Bell inequality (5)] is $T^N S_0$ where S_0 is the value "S" for φ_N in the absence of loss as given graphically in Fig. 2. It is seen then that we require T to be $\sim (1/S_0)^{1/N}$ or larger in order to obtain the violations of the no loophole inequalities (5). For $N=2 S_0 = 1.18$, and this requires at least $T > \sqrt{1/1.18} = 0.92$. This figure is at the limits of current technology, and compares with the requirement $T > 0.83$ for $N=1$.

We now derive a multiparticle form of the weaker inequality so that we can also examine situations of significant detection loss. The result at A is $+1$ if the number of photons m detected at c_+ is fN or more, and if the total number of photons $m+k$ detected at A satisfies $m+k=N$; $p_+^A(\theta,\lambda)$ is the probability of this event given the hidden variable description λ. We define a probability, $p_+^A(-,\lambda)$, that the total photon number $m+k$ (at location A) is N, given that the system is described by the hidden variables λ. This total probability is then assumed to be independent of the choice of polarizer angle θ at A. Similarly we define a $p_+^B(-,\lambda)$, the probability that the total number of photons $m'+k'$ at B is N. This total probability is then assumed to be independent of the polarizer angle ϕ at B. We postulate as an additional premise that the hidden variable theories will satisfy

$$p_+^A(\theta,\lambda) \leqslant p_+^A(-,\lambda),$$

$$p_+^B(\phi,\lambda) \leqslant p_+^B(-,\lambda). \tag{15}$$

Using the procedure and theorems of the previous works of Clauser and Horne [4] one may derive from the postulate of local hidden variables and assumption (15) the following "weak" Clauser-Horne-Bell inequality, where the marginals are replaced by "one-sided" joint probabilities. Violation of this "weaker" Bell-CH inequality will only eliminate local hidden variable theories satisfying the auxiliary ("no enhancement") assumption (15).

$$S_W = \frac{P_{++}^{AB}(\theta,\phi) - P_{++}^{AB}(\theta,\phi') + P_{++}^{AB}(\theta',\phi) + P_{++}^{AB}(\theta',\phi')}{P_{++}^{AB}(\theta',-) + P_{++}^{AB}(-,\phi)} \leqslant 1. \tag{16}$$

Here we have defined "one-sided" experimental joint probabilities as follows: $P_{++}^{AB}(\theta',-)$ is the joint probability of obtaining $+1$ at A, with the polarizer at A set at θ', and of obtaining a total of $m'+k'=N$ photons at B. The joint probability $P_{++}^{AB}(-,\phi)$ is the probability of obtaining a total of $m+k=N$ photons at A, and of obtaining $+1$ at B, with the polarizer at B set at ϕ.

For the situation where the detected probabilities are taken to be the quantum probabilities calculated directly from Eq. (6), so that we are ignoring additional losses and noise which may come from the detection and measurement process, we have the same result for the weak and strong inequalities (5) and (16).

Now to consider detection losses, we notice that the detrimental effect of the T-scaling apparent in Eq. (11) is removed by considering the weaker inequality, in which the marginal is replaced by the one-sided joint probability. The quantum predictions for the one-sided probabilities are for example

$$\begin{aligned} P_{++}^{AB}(\theta',-) &= \sum_{m \geqslant fN}^{N} \sum_{m'=0}^{N} P(m,N-m,m',N-m') \\ &= T^{2N} P_Q^A(m,N-m), \end{aligned} \tag{17}$$

which we see from Eq. (13) is proportional to T^{2N}. Noting that $P_Q(m,N-m)$ is precisely the quantum marginal probability used in the strong inequality, we see that our predictions then for the violation of the weak inequality for the state (6) are as shown for the strong inequality in Fig. 2 [meaning that the value for S_W of Eq. (16) being given by the value of S as shown in Fig. 2].

To summarize then, to perform the Bell test in a practical situation where detection situations are present, but where we use as the input the quantum state (6), our apparatus is as depicted in Fig. 1. We classify our outcome to be $+1$ at A if $m \geqslant fN$ and also $m+k=N$; and $+1$ at B if $m' \geqslant fN$ and also $m'+k'=N$. A violation of the no-loophole Bell inequality (5) is possible only for high-detector efficiencies $T=\eta^2$. Violations of the weak inequality (16) (which involves an additional auxiliary assumption and therefore admits a loophole)

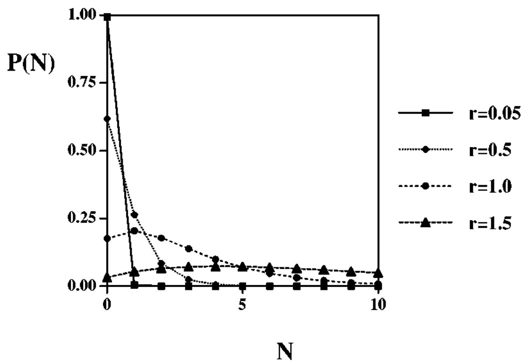

FIG. 3. Plot of $P(N)=|c_N|^2$ the probability that a total of N photons will be detected at each polarizer location.

however are still predicted, even with significant detector loss, the predictions being as given by Fig. 2, but replacing S with S_W.

V. PROPOSED EXPERIMENT TO DETECT VIOLATION OF MULTIPARTICLE BELL INEQUALITY USING PARAMETRIC DOWN-CONVERSION WITH AND WITHOUT ENTANGLED INPUTS

The prediction by quantum mechanics of the violation of a Bell inequality for the larger N states (6) has not been tested experimentally. For this reason we investigate how one may achieve related violations of Bell inequalities using parametric down-conversion. Previous work [24] has shown how such violations are possible in the regime of low amplification, but this work was limited to situations of very low-detection efficiencies.

We model the parametric down conversion by the Hamiltonian

$$H=i\hbar g(a_1^\dagger b_1^\dagger+a_2^\dagger b_2^\dagger)-i\hbar g(a_1 b_1+a_2 b_2). \tag{18}$$

Here, we consider two parametric processes to make a four-mode interaction [25], as may be achieved using two parametric amplifiers with Hamiltonians $H=i\hbar g(a_1^\dagger b_1^\dagger - a_1 b_1)$ and $H=i\hbar g(a_2^\dagger b_2^\dagger - a_2 b_2)$. The two outputs a_1, a_2 are input to the polarizer θ at A, while the two outputs b_1, b_2 are input to the polarizer ϕ at B. The time-dependent solution for the parametric process with vacuum inputs is

$$|\varphi\rangle=\sum_{N=0}^{\infty} c_N|\varphi_N\rangle, \tag{19}$$

where $c_N=\sqrt{(N+1)}\Gamma^N/\tilde{C}^2$ where $\tilde{C}\equiv\cosh r, \tilde{S}\equiv\sinh r$, $\Gamma\equiv\tilde{S}/\tilde{C}$, and "gain": $r=gt$. The probability that a total of N photons are detected at each location A and B is then $P(n)=|c_N|^2$ as plotted in Fig. 3.

The validity of the state (19), on which the predictions are based, depends on how well the Hamiltonian (18) describes the real parametric amplifier. While the model has been successful in predicting violations of weak Bell-CH inequalities for $N=1$, a chief limitation is the omission of absorption or loss which occurs in addition to the detector inefficiencies.

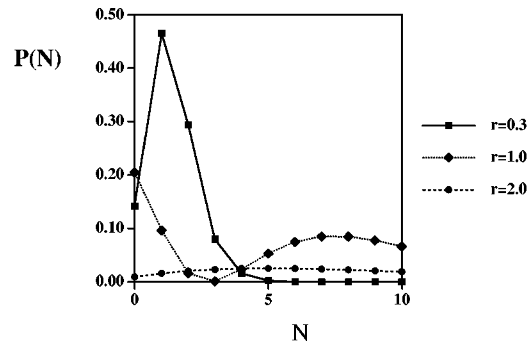

FIG. 4. Plot of $P(N)=|c_N|^2$ the probability that a total of N photons will be detected at each polarizer location, for the entangled state input.

The effect of asymmetric absorption on each mode will be to degrade the violation of the strong Bell inequalities, though we would expect the violation of the weaker Bell-CH inequalities to be less affected.

Of interest to us is the parametric output with the following polarization-entangled state as input:

$$|\varphi\rangle_{in}=\frac{1}{\sqrt{2}}(|1\rangle_{a_1}|1\rangle_{b_1}|0\rangle_{a_2}|0\rangle_{b_2}+|0\rangle_{a_1}|0\rangle_{b_1}|1\rangle_{a_2}|1\rangle_{b_2}). \tag{20}$$

This represents an example of the quantum-injected optical parametric amplifier realized experimentally by De Martini *et al.* [26]. The active nonlinear medium realizing the interaction (18) was a 2 mm BBO (beta-barium-borate) nonlinear crystal slab excited by a pulsed optical UV beam with wavelength $\lambda_p=345$ nm. The duration of each UV excitation pulses was 150 f sec and the average UV power was 0.3 W. The UV beam was second-harmonic generation generated by a mode-locked femtosecond Ti:Sa laser (Coherent MIRA) optionally amplified by a high-power Ti:Sa regenerative amplifier (Coherent REGA9000). The pulse repetition rate was 76.10^6 and 3.10^5 Hz, respectively, in absence and in presence of the regenerative amplification. The maximum OPA "gain" obtained by the apparatus was: $r\approx 0.3$ and $r\approx 5.1$, respectively, in absence and in presence of the laser amplification. These figures lead, respectively, to the following values of the parameters: $\tilde{C}=1.04$, $\Gamma=0.29$, and $\tilde{C}=82$, $\Gamma\approx 1$. The typical quantum efficiency of the detectors was in the range: $\eta^2\approx 0.4-0.6$. The final output state generated by this apparatus is expressed by the multiparticle entangled state (19) but where $c_N=[\sqrt{(N+1)}\Gamma^N/\tilde{C}^2]\times[(N-2\tilde{S}^2)/(\sqrt{2}\Gamma\tilde{C}^2)]$. The probability of an n photon output at each location A and B is then given by $P(n)=|c_n|^2$ as is plotted in Fig. 4, for various r.

There are a number of approaches one can use to detect the quantum violation of the Bell inequalities. The particular method preferred will depend on the interaction strength r and the degree of detection efficiency η.

We propose here first the following experiment making use of the double-channeled polarizers to detect the photon numbers of both orthogonal polarizations. This will allow the "selection" of a specified spin state $|\varphi_N\rangle$ and the observation

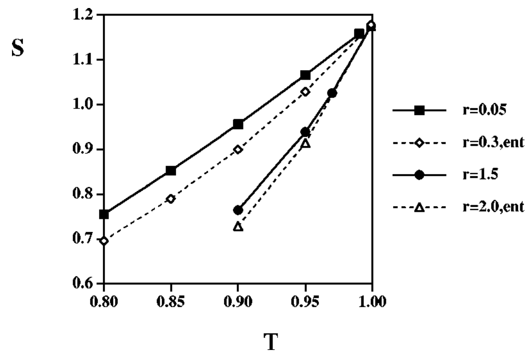

FIG. 5. Effect, for various parametric coupling r, of detection inefficiencies on the violation of the strong Bell inequality (5), for the scheme depicted in Fig. 1 with $N=2$. Here T models detector losses, T being the relative fraction of photons incident on each detector that are actually detected. The optimal angle ψ for $N=2$ is ~ 3.41. The curves labeled "ent" represent predictions for the entangled input state.

of the violation predicted in Fig. 2 for the strong or weak Bell inequalities. The experimental arrangement is as depicted in Fig. 1 but where the quantum source is that given by Eq. (19). Specifically we detect at locations A and B the photon numbers $c_+^\dagger c_+$, $c_-^\dagger c_-$, $d_+^\dagger d_+$, and $d_-^\dagger d_-$, where c_+, c_-, d_+, d_- are given by Eq. (1), and label the results m, k, m', and k', respectively. Our outcome is $+1$ at A if $m=N$ and $m+k=N$; and $+1$ at B if $m'=N$ and $m'+k'=N$. We measure $P_{++}^{AB}(\theta,\phi)$ and, if testing the no-loophole Bell inequality (5), the marginal probabilities $P_+^A(\theta)$ and $P_+^B(\phi)$. If testing the weaker Bell inequality (16), measurement is made of one-sided joint probabilities $P_+^A(\theta,-)$ and $P_+^B(-,\phi)$. We will show that a violation of the strong (no auxiliary-assumptions) Bell inequality is possible only for high $T=\eta^2$ (Fig. 5). The predicted violations of a weak inequality (16) will also be calculated and results are shown in Fig. 6.

The calculation of S as defined in Eq. (5) for the parametric amplifier state proceeds in a straightforward manner. We define in general $P_Q(m,k,m',k')$ as the probability of detecting m,k,m',k' photons upon measurements of $c_+^\dagger c_+$, $c_-^\dagger c_-$, $d_+^\dagger d_+$, and $d_-^\dagger d_-$, respectively, in the absence of loss. For $m+k=m'+k'=N$, we have

$$P_Q(m,N-m,m',N-m')=|c_N|^2|C_{m,m'}^{(N)}|^2. \qquad (21)$$

$|c_N|^2$ is defined in Eq. (19), and $|C_{m,m'}^{(N)}|^2$ is the probability that measurement of $c_+^\dagger c_+$ and $d_+^\dagger d_+$ for the state $|\varphi_N\rangle$ gives m and m', respectively. Our required probabilities are then given as follows $P_{++}^{AB}(\theta,\phi)=P_Q(N,0,N,0)=|c_N|^2|C_{N,N}^{(N)}|^2$ and $P_+^A(\theta)=\sum_{m'=0}^{\infty}P_Q(N,0,m',N-m')=\sum_{m'=0}^{\infty}|c_N|^2|C_{N,m'}^{(N)}|^2$. The detection of $m+k=N$ at A is correlated with $m'+k'=N$ at B. Immediately then it is apparent that the factors $|c_N|^2$ in the joint and marginal probabilities in the final form of the Bell parameter S for the strong inequality (5) will cancel. The predictions for the violation of Eq. (5), in the absence of loss, are as for the ideal spin state $|\varphi_N\rangle$. It is important to realize however that the actual probability of obtaining the event $+1$ is different in the parametric case, this probability being weighted by $|c_N|^2$, the probability of detecting $m+k=N$, that N photons are incident on each polarizer. While the joint probabilities are small, so is the true marginal, and we have a predicted violation of the strong Bell-Clauser-Horne inequality (5), without auxiliary assumptions.

The probabilities $P_Q(m,k,m',k')$ of Eq. (19) depend only on the angle difference $\phi-\theta$. We select the angle choice $\phi-\theta=\theta'-\phi=\phi'-\theta'=\psi$ and $\phi'-\theta=3\psi$ in line with previous work [19,24] with the states $|\varphi_N\rangle$.

Our first objective would be to detect violations of the inequality for relatively low N, $N=2$ say. The choice of $r\sim 1$ gives the maximum probability of obtaining an event where $m+k=2$, although $r\sim 0.5$ would give a reasonable probability. For the optimal choice of angle ψ (Fig. 2) the probability of an actual event $+1$ for $N=2$ and $r\sim 0.5$ is ~ 0.01. For perfect detection efficiency the level of violation is given by $S=1.181$ as indicated in Fig. 2.

We now need to consider the effect of detection inefficiencies. Our measured probabilities for obtaining m,k,m',k' at each detector are given by Eq. (11) where now the quantum probabilities are calculated from Eq. (19). We note that with the restriction $m+k=m'+k'=N$, and $m=N$ we get

$$P(N,0,m',N-m')$$
$$=T^{2N}\sum_{r,q,s,t=0}^{\infty}(1-T)^{r+q+s+t}C_r^{N+r}C_s^{m'+s}$$
$$\times C_t^{N-m'+t}P_Q(N+r,q,m'+s,N-m'+t), \qquad (22)$$

where from Eq. (21) we have

$$P_Q(N+r,q,m'+s,N-m'+t)$$
$$=\delta[r+q-(s+t)]|c_{N_0}|^2|C_{N+r,m'+s}^{(N_0?)}|^2, \qquad (23)$$

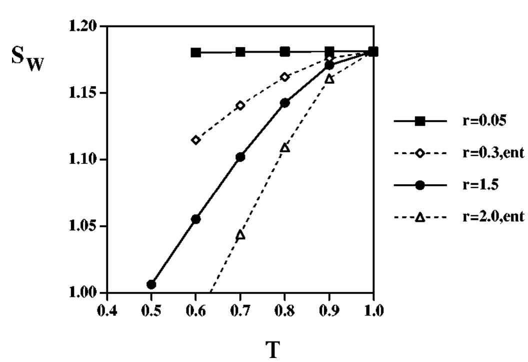

FIG. 6. Effect of detection inefficiencies on the violation of the weak Bell inequality (16), for the scheme depicted in Fig. 1 where $N=2$. Here, T represents detector losses, T being the relative fraction of incident photons actually detected. The optimal angle ψ for $N=2$ is ~ 3.41. The curves labeled "ent" are predictions for the entangled input state.

where $N_0 = N + r + q = N + s + t$. We note that for the quantum state (21) we require for nonzero probabilities $r + q = s + t$. The required joint probability $P^{AB}_{++}(\theta,\phi)$ becomes $P^{AB}_{++}(\theta,\phi) = P(N,0,N,0)$. The marginal probabilities needed for the strong Bell inequality (5) become for example $P^{A}_{+}(\theta) = P^{A}(N,0)$ where

$$P^{A}(N,0) = T^{N} \sum_{r,q=0}^{\infty} (1-T)^{r+q} C_r^{N+r} P_Q^{A}(N+r,q), \quad (24)$$

and

$$P_Q^{A}(N+r,q) = \sum_{m'=0}^{\infty} |c_{N_0}|^2 |C^{(N_0?)}_{N+r,m'}|^2, \quad (25)$$

where $N_0 = N + r + q$.

Figure 5 reveals the effect on the violation of the strong Bell inequality, for various r, and for $N = 2$. For the reasons discussed in the previous section, because the marginal probability scales as T^N while the joint probabilities scale as T^{2N}, the violation is lost for small detection loss.

To propose an experiment achievable with current detector efficiencies, we consider an appropriate weak Bell inequality. We define the joint probability $P^{AB}_{++}(\theta,\phi)$ of obtaining $m = N$ and $m + k = N$ at A, and $m' = N$ and $m' + k' = N$ at B. We define the joint one-sided probability $P^{AB}_{++}(\theta,-)$ of obtaining $m = N$ and $m + k = N$, and a total of $m' + k' = N$ photons at B. The one-sided probability $P^{AB}_{++}(-,\phi)$ is defined similarly. The auxiliary assumptions are made that for a hidden variable description λ, the probability $p^{A}_{+}(\theta,\lambda)$ of obtaining $m = N$ and $m + k = N$, and the probability $p^{A}_{+}(-,\lambda)$ of obtaining $m + k = N$ alone, satisfy

$$p^{A}_{+}(\theta,\lambda) \leqslant p^{A}_{+}(-,\lambda). \quad (26)$$

Also we assume $p^{A}_{+}(-,\lambda)$ is independent of θ. Similar assumptions are made for $p^{B}_{+}(\phi,\lambda)$ and $p^{B}_{+}(-,\lambda)$. With these assumptions the weaker inequality (16) is derivable. The one-sided probability used in the test of the weak inequality (16) is given by $P^{AB}_{++}(\theta,-) = P^{AB}(N,0;-)$ where

$$P^{AB}(N,0;-) = \sum_{m'=0}^{N} P(N,0,m',N-m'). \quad (27)$$

With a total of N photons detected at both locations A and B, we ensure all probabilities scale as T^{2N}.

The existence of the higher spin states $|\varphi_M\rangle$, where $M > N$, in the parametric output means that detector inefficiencies alter the violation of even the weak Bell inequality. Figure 6 illustrates the effect of detection inefficiencies on the violation of the weak inequalities (16), the effect being more significant for higher r values where the states $|\varphi_M\rangle$, where $M > N$, contribute more significantly. Smaller r values suffer the disadvantage however that the probability of an actual event $+1$ becomes small due to the small probability of $N = 2$ photons actually being incident on the polarizer. The sensitivity of the violations to loss is not so great that the experiment would be impossible for $r \sim 0.5$.

A point to be made concerns the alternative situation of a one-channeled polarizer where only the photon number m and m' can be detected. Here, the prediction is different due to the contribution of the $N+1$ spin state which can contribute an $m = N$ event (with $k = 1$) potentially decreasing the violation of the inequality.

VI. PROPOSED EXPERIMENT TO DETECT VIOLATION OF BELL INEQUALITY USING HIGH-FLUX PARAMETRIC DOWN -CONVERSION

As one increases the output intensities of the parametric device, the actual probability of detecting N photons transmitted through our polarizer decreases. In other words the probability of detecting the event $+1$, described in the last section, becomes smaller. To combat this we propose in this section that our outcome be a range of photon number values. Here we are interested in the regime of high amplification [27] where the output fluxes of signal and idler are high, and where one can use highly efficient photodiode detectors.

We now propose the following experiment. We detect at locations A and B the photon numbers $c_+^\dagger c_+$, $c_-^\dagger c_-$, $d_+^\dagger d_+$, and $d_-^\dagger d_-$, where c_+, c_-, d_+, d_- are given by Eq. (1). The mean photon number incident on each polarizer is $xm = \langle c_+^\dagger c_+\rangle + \langle c_-^\dagger c_-\rangle = \langle d_+^\dagger d_+\rangle + \langle d_-^\dagger d_-\rangle$ where $xm = 2\sinh^2(r)$. We denote the result for $c_+^\dagger c_+$ and $c_-^\dagger c_-$ at A by m and k, respectively, and the results of $d_+^\dagger d_+$ and $d_-^\dagger d_-$ at B by m' and k', respectively (Fig. 1). We define XM to be the integer nearest in value to the mean xm. We designate the result of the measurement at A to be $+1$ if our measured results m and k satisfy $m \geqslant XM$ and also $XM \leqslant m + k \leqslant XM + \Delta$. Otherwise our result is -1. Similarly, we define the result at B to be $+1$ if $m' \geqslant XM$ and $XM \leqslant m' + k' \leqslant XM + \Delta$.

By performing many such measurements over an ensemble, one can experimentally determine the following: $P^{AB}_{++}(\theta,\phi)$ the probability of obtaining $+1$ at A and $+1$ at B upon simultaneous measurement with θ at A and ϕ at B; $P^{A}_{+}(\theta)$ the marginal probability for obtaining the result $+1$ upon measurement with θ at A; and $P^{B}_{+}(\phi)$ the marginal probability of obtaining the result $+1$ upon measurement with ϕ at B.

Local hidden variables will predict, as discussed in Sec. II, the strong Bell inequality (5). We define $P(m,k,m',k')$ as the probability of detecting m, k, m', and k' photons for measurements of $c_+^\dagger c_+$, $c_-^\dagger c_-$, $d_+^\dagger d_+$, and $d_-^\dagger d_-$, respectively. The probability of results m and k upon measurement of $c_+^\dagger c_+$ and $c_-^\dagger c_-$ is defined as $P^{A}(m,k)$. We have in the absence of loss, where $m + k = m' + k' = N$ is ensured,

$$P(m,N-m,m',N-m') = |c_N|^2 |C^{(N)}_{m,m'}|^2,$$

$$P^{A}(m,N-m) = \sum_{m'=0}^{N} |c_N|^2 |C^{(N)}_{m,m'}|^2, \quad (28)$$

where all other probabilities are zero. Here, $|c_N|^2$ is defined in Eq. (19), and $|C^{(N)}_{m,m'}|^2$ is the probability that measurement of $c_+^\dagger c_+$ and $d_+^\dagger d_+$ for the state $|\varphi_N\rangle$ gives m and m', re-

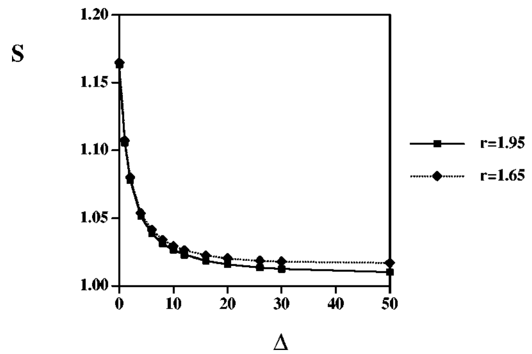

FIG. 7. Plot of violation of the strong Bell inequality where we designate the result of the measurement at A to be $+1$ if our measured results m and k satisfy $m \geqslant XM$ and also $XM-\Delta \leqslant m+k \leqslant XM+\Delta$. Otherwise our result is -1. Similarly we define the result at B to be $+1$ if $m' \geqslant XM$ and $XM \leqslant m'+k' \leqslant XM+\Delta$. For $r=1.65$ we have $XM=13$ and for $r=1.95$, $XM=24$.

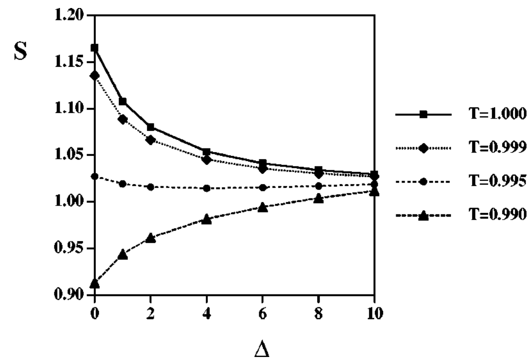

FIG. 8. Plot of the effect of detection losses on the violation of the no-loophole Bell inequality test (5) as explained in Fig. 7 above. Here, $r=1.65$ we have $XM=13$.

spectively, with no loss. The probability $P(m,m')$ of getting m and m' for $c_+^\dagger c_+$ and $d_+^\dagger d_+$, respectively, while the total $m+k$ is restricted to $XM \leqslant m+k \leqslant XM+\Delta$, and $m+k=m'+k'$ is restricted to $XM \leqslant m'+k' \leqslant XM+\Delta$, is given generally as

$$P^{AB}(m,m')=\sum_{k=XM-m}^{XM+\Delta-m}\ \sum_{k'=XM-m'}^{XM+\Delta-m'} P(m,k,m',k'). \tag{29}$$

The corresponding marginal probability is

$$P^{A}(m)=\sum_{k=XM-m}^{XM+\Delta-m} P^{A}(m,k). \tag{30}$$

Our required probabilities are then given as follows:

$$P_{++}^{AB}(\theta,\phi)=\sum_{m,m'=XM}^{XM+\Delta} P^{AB}(m,m') \tag{31}$$

and for the marginal

$$P_{+}^{A}(\theta)=\sum_{m=XM}^{XM+\Delta} P^{A}(m). \tag{32}$$

For the purpose of a weaker Bell inequality we also define a one-sided probability

$$P_{++}^{AB}(\theta,-)=\sum_{m=XM}^{XM+\Delta}\ \sum_{m'=0}^{XM+\Delta} P^{AB}(m,m'). \tag{33}$$

The probabilities $P(m,k,m',k')$ depend only on the angle difference $\phi-\theta$. We select the angle choice $\phi-\theta=\theta'-\phi=\phi'-\theta'=\psi$ and $\phi'-\theta=3\psi$ in line with previous work [3,19] with the states $|\varphi_N\rangle$.

Results for S, optimizing ψ to give maximum S, are presented in the Fig. 7. With the choice $\Delta=0$, we will get only one of the $|\varphi_N\rangle$ contributing. The results for S will be identical [19] to that obtained for the $|\varphi_{XM}\rangle$ state, where a clear violation of the Bell inequality (5) is obtained even for very large $N=XM$. The difficulty with such a situation however is that in the regime of higher r (where greater signal intensities are generated), the probability that the total number $m+k$ of photons $c_+^\dagger c_+ + c_-^\dagger c_-$ is precisely this fixed number is very small, making the probability of our $+1$ outcome tiny. We are more interested in situations where the intensity on the detectors is large but also where the probability that $XM \leqslant m+k \leqslant XM+\Delta$ is significant. This is achieved by increasing the range Δ. Violations of the Bell inequality are still possible ($S \geqslant 1$) but the degree of violation is reduced, the limiting value for large Δ approaching 1 as XM increases.

The sensitivity to loss can be evaluated by calculating in Eq. (29) [and in the equations for the marginal probabilities such as $P^A(m)$] the measured probabilities $P(m,k,m',k')$ and $P^A(m,k)$ as given by Eqs. (11) and (12). The effect on the violation of the no-loophole Bell inequality (5) is given in Figs. 8 and 9. Sensitivity is strong for low Δ but decreases as the range Δ increases. This provides a potential opportunity to test a strong no-auxiliary multiparticle Bell inequality for lower detector inefficiencies than indicated by the $\Delta=0$ regime discussed in the previous section.

VII. CONCLUSIONS

We have presented a proposal to test the predictions of quantum mechanics against those of local hidden variable

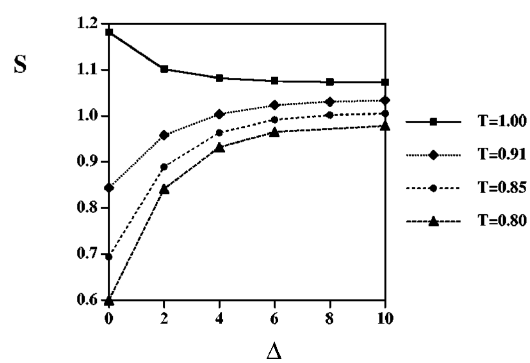

FIG. 9. Plot of the effect of detection losses on the violation of the no-loophole Bell inequality test (5) as explained in Fig. 7 above. Here, $r=0.9$ we have $XM=2$.

theories for multiparticle entangled states generated using-parametric down conversion, where measurement is made on systems of more than one particle. A calculation is given of the detector efficiencies required to test directly the ''no-loophole''multiparticle Bell inequality. In view of the limitation of current detector efficiencies, it is necessary to consider initially tests of a ''weaker'' Bell inequality derived with additional auxiliary assumptions, and to therefore extend previous such derivations to the multiparticle situation we consider here.

ACKNOWLEDGMENTS

F.D.M. acknowledges the Italian Ministero dell'Università e della Ricerca Scientifica e Tecnologica (MURST) and the FET European Network IST-2000-29681(ATESIT) on ''Quantum Information,'' for funding.

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, NJ, 1951).
[3] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1965); *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1988); Physics (Long Island City, N.Y.) **1**, 195 (1964).
[4] J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978), and references therein; J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969); J. F. Clauser and M. A. Horne, Phys. Rev. D **10**, 526 (1974).
[5] D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, Am. J. Phys. **58**, 1131 (1990).
[6] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982); A. Aspect, J. Dalibard, and G. Roger, *ibid.* **49**, 1804 (1982); Y. H. Shih and C. O. Alley, *ibid.* **61**, 2921 (1988); Z. Y. Ou and L. Mandel, *ibid.* **61**, 50 (1988); J. G. Rarity and P. R. Tapster, *ibid.* **64**, 2495 (1990); J. Brendel, E. Mohler, and W. Martienssen, Europhys. Lett. **20**, 575 (1992); P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, Phys. Rev. A **47**, 2472 (1993); T. E. Kiess, Y. H. Shih, A. V. Sergienko, and C. O. Alley, Phys. Rev. Lett. **71**, 3893 (1993); P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, *ibid.* **75**, 4337 (1995); D. V. Strekalov, T. B. Pittman, A. V. Sergienko, Y. H. Shih, and P. G. Kwiat, Phys. Rev. A **54**, 1 (1996); G. Weihs, T. Jennewein, C. Simon, H. Weinfurther, and A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1998); A. Zeilinger, Rev. Mod. Phys. **71**, 5288 (1998); M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, Nature (London) **409**, 791 (2001).
[7] C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland, and C. Monroe, Nature (London) **404**, 256 (2000).
[8] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. Raimond, and S. Haroche, Science **288**, 2024 (2000).
[9] J. W. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, Nature (London) **403**, 515 (2000).
[10] Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng, Phys. Rev. Lett. **68**, 3663 (1992).
[11] Yun Zhang, Hai Wang, Xiaoying Li, Jietai Jing, Changde Xie, and Kunchi Peng, Phys. Rev. A **62**, 023813 (2000).
[12] Ch. Silberhorn, P. K. Lam, O. Weiss, F. Koenig, N. Korolkova, and G. Leuchs, Phys. Rev. Lett. **86**, 4267 (2001).
[13] M. D. Reid, Europhys. Lett. **36**, 1 (1996); Quantum Semiclassic. Opt. **9**, 489 (1997); M. D. Reid and P. Deuar, Ann. Phys. (Leipzig) **265**, 52 (1998); M. D. Reid, Phys. Rev. A **62**, 022110 (2000); e-print quant-ph/0101050.
[14] V. Giovannetti, S. Mancini, and P. Tombesi, e-print quant-ph/0005066.
[15] H. Weinfurter and M. Zukowski, Phys. Rev. A **64**, 010102 (2001).
[16] A. Lamas-Linares, JC. Howell, and D. Bouwmeester, Nature (London) **412**, 6850 (20001).
[17] N. D. Mermin, Phys. Rev. D **22**, 356 (1980); A. Garg and N. D. Mermin, Phys. Rev. Lett. **49**, 901 (1982).
[18] N. D. Mermin and G. M. Schwarz, Found. Phys. **12**, 101 (1982).
[19] P. D. Drummond, Phys. Rev. Lett. **50**, 1407 (1983).
[20] A. Peres, Found. Phys. **22**, 1819 (1992); A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic, Dordrecht, 1993).
[21] S. L. Braunstein and C. M. Caves, Phys. Rev. Lett. **61**, 662 (1988).
[22] A. Garg and N. D. Mermin, Phys. Rev. Lett. **49**, 901 (1982); Phys. Rev. D **27**, 339 (1983); N. D. Mermin, *ibid.* **65**, 1838 (1990); S. M. Roy and V. Singh, *ibid.* **67**, 2761 (1991); A. Peres, Phys. Rev. A **46**, 4413 (1992); Found. Phys. **22**, 1819 (1992); M. D. Reid and W. J. Munro, Phys. Rev. Lett. **69**, 997 (1992); B. C. Sanders, Phys. Rev. A **45**, 6811 (1992); G. S. Agarwal, *ibid.* **47**, 4608 (1993); D. Home and A. S. Majumdar, *ibid.* **52**, 4959 (1995); C. Gerry, *ibid.* **54**, 2529 (1996); A. Beige, W. J. Munro, and P. L. Knight, *ibid.* **62**, 052102 (2000).
[23] K. Banaszek and K. Wodkiewicz, Phys. Rev. A **58**, 4345 (1998); Phys. Rev. Lett. **82**, 2009 (1999).
[24] W. J. Munro and M. D. Reid, Phys. Rev. A **47**, 4412 (1993).
[25] J. G. Rarity and P. R. Tapster, Phys. Rev. Lett. **64**, 2495 (1990); M. A. Horne, A. Shimony, and A. Zeilinger, *ibid.* **62**, 2209 (1989); M. D. Reid and D. F. Walls, Phys. Rev. A **34**, 1260 (1985); P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, Phys. Rev. Lett. **75**, 4337 (1995); M. Marte, *ibid.* **74**, 4815 (1995).
[26] F. De Martini, Phys. Rev. Lett. **81**, 2842 (1998); F. De Martini, V. Mussi, and F. Bovino, Opt. Commun. **179**, 581 (2000); F. De Martini and G. Di Giuseppe, Phys. Rev. Lett. (to be published).
[27] D. T. Smithey, M. Beck, M. Belsley, and M. G. Raymer, Phys. Rev. Lett. **69**, 2650 (1992).